Week 4: Recitation Session (pols602)

Proofs

Example from https://deopurkar.github.io/teaching/algebra1/cheng.pdf

Example 3. *Prove by induction that* $\forall n \in \mathbb{N}, 1 + \cdots + n = \dfrac{n(n+1)}{2}$

BEGINNING Principle of Induction

MIDDLE

$$\begin{aligned} \text{for } n = 1, \text{ LHS} &= 1 \\ \text{RHS} &= \frac{1(1+1)}{2} \\ &= 1 \end{aligned}$$

$\therefore$ result is true for $n = 1$

If result is true for $n = k$ then

$$\begin{aligned} 1 + \cdots + k + (k+1) &= \frac{k(k+1)}{2} + (k+1) \\ &= \frac{k(k+1) + 2(k+1)}{2} \\ &= \frac{(k+1)(k+2)}{2} \qquad \text{i.e. result true for } n = k+1 \end{aligned}$$

$\therefore$ result true for $k \implies$ result true for $k+1$

END $\therefore$ by the Principle of Induction, the result is true for all $n \in \mathbb{N}$ $\square$

3 sections (not necessary to write these section names down in an actual proof)

Beginning

- Define the main variables etc
    - e.g. L in A = LL’
- Note what is already given or is inferred (to the extent necessary)
    - e.g. “Both AB and BA exists.”

      e.g. If A is n x m matrix, B is a m x n matrix.
    - <u>No new assumptions.</u>

Middle

- The main part of the proof.
- As you transform equations, make sure to mention which properties or formula you relied on.
- Remember to define any variables you introduce here.

End

- Briefly conclude by referencing what you wanted to prove. Don’t just end with a long equation.

  e.g. Hence, $A^{-1} = (L^{-1})'L^{-1}$.

<u>A common mistake</u>

- Starting the proof from what you need to prove.

  You might have solved the problem that way. But when writing proofs, start from what is given and conclude with the proposition in question.
- Not referencing equation numbers. Which equations together prove your point?