Writing Proofs in Problem Sets

Here are basic things to look out for when writing proofs.

Structuring Proofs

Beginning

Note what is already given (to the extent you need) e.g. "Both AB and BA exists."

Remember to define any new variables you introduce. e.g. L in $A = LL'$.

Middle

The main part of the proof.

As you transform equations, make sure to mention which properties or formula you are relying on.

Remember to define any variables you introduce here.

End

Briefly conclude by referencing what you wanted to prove. Don't just end your proof with a long equation with no final remark.

e.g. "Hence, A = B".

Common mistakes

Be careful not to add new assumptions in proofs.

Starting the proof from what you need to prove. You may have solved the problem that way. But when writing proofs, start from what is given and conclude with the proposition that you are asked to prove.

When you say an equation proves your point, refer to the equation number and explain how it supports your point.

Example

Taken from https://deopurkar.github.io/teaching/algebra1/cheng.pdf

Example 3. *Prove by induction that* $\forall n \in \mathbb{N}, 1 + \cdots + n = \dfrac{n(n+1)}{2}$

BEGINNING Principle of Induction

MIDDLE

$$\begin{aligned} \text{for } n = 1,\ \text{LHS} &= 1 \\ \text{RHS} &= \frac{1(1+1)}{2} \\ &= 1 \end{aligned}$$

$\therefore$ result is true for $n = 1$

If result is true for $n = k$ then

$$\begin{aligned} 1 + \cdots + k + (k+1) &= \frac{k(k+1)}{2} + (k+1) \\ &= \frac{k(k+1) + 2(k+1)}{2} \\ &= \frac{(k+1)(k+2)}{2} \qquad \text{i.e. result true for } n = k+1 \end{aligned}$$

$\therefore$ result true for $k \implies$ result true for $k+1$

END $\therefore$ by the Principle of Induction, the result is true for all $n \in \mathbb{N}$ $\square$