Lab Session 3: Inference and Estimation in MLE

Recitation

The main takeaways

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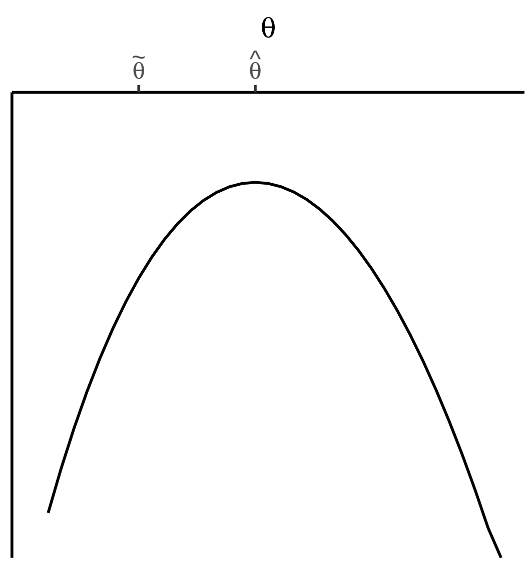
Uncertainty

Information matrix.

Hypothesis testing

Let's review the three hypothesis tests. Suppose we have a log-likelihood function that looks like the figure below. How can we visualize the three (asymptotically equivalent) tests in the figure?



Optional Exercise

Let's put what we learned this week into practice, using the following example.

Q1

The following data were generated by the Weibull distribution in the last question. The data are available in the shared dropbox folder.

$$f(x) = \alpha\beta x^{\beta-1}e^{-\alpha x^\beta}, \quad x \geq 0, \alpha, \beta > 0.$$

1.3043	0.49254	1.2742	1.4019	0.32556	0.29965	0.26423
1.0878	1.9461	0.47615	3.6454	0.15344	1.2357	0.96381
0.33453	1.1227	2.0296	1.2797	0.96080	2.0070	

a. Obtain the maximum likelihood estimates of α and β, and estimate the asymptotic covariance matrix for the estimates.
b. Carry out a Wald test of the hypothesis that $\beta = 1$.
c. Obtain the maximum likelihood estimate of α under the hypothesis that $\beta = 1$.
d. Using the results of Parts a and c, carry out a likelihood ratio test of the hypothesis that $\beta = 1$.
e. Carry out a Lagrange multiplier test of the hypothesis that $\beta = 1$.

For reference, the question is preceeded by the following question in Greene's textbook.

Q2

Suppose that x has the Weibull distribution

$$f(x) = \alpha\beta x^{\beta-1}e^{-\alpha x^\beta}, \quad x \geq 0, \alpha, \beta > 0.$$

a. Obtain the log-likelihood function for a random sample of n observations.
b. Obtain the likelihood equations for maximum likelihood estimation of α and β. Note that the first provides an explicit solution for α in terms of the data ant β. But, after inserting this in the second, we obtain only an implicit solution for β. How would you obtain the maximum likelihood estimators?

Excerpts from Greene. *Econometric Analysis*, 8th Edition. Exercise Q4 and Q5. Ch16.