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Department of Astronomy
800 Dongchuan Road
Shanghai, PRC, 200240
Phone:
Email: yingzu@sjtu.edu.cn

MNRAS Submission: ref. MN-21-4671-MJ
Title: The Sphere of Influence of the Bright Central Galaxies in the Diffuse Light of SDSS Clusters
Authors: Xiaokai Chen, Ying Zu, Zhiwei Shao, Huanyuan Shan

Dear Editors and the Referee,

We thank the referee for the valuable comments that have substantially improved our manuscript, especially on the potential impact of the PSF on our ICL measurement. We list our responses to the referee's questions/comments as follows, and provide a marked-up version of the pdf to highlight our modifications to the manuscript.

Comment [1]

> To be clear, the referee doesn't think the PSF issue will make this paper meaningless. The method is innovative regardless of this issue, but some conclusions have to be reviewed. Here is a straightforward way to check this: - On Figure 3, the r-band SB at $10\,kpc$ is $\sim 22\,mag\,/\,arcsec^2$, at $100\,kpc$ is $\sim 27\,mag\,/\,arcsec^2$, and at $500\,kpc$ is $\sim 30\,mag\,/\,arcsec^2$. - At z=0.2 (0.3), $1\,arcsec$ corresponds to 3.3 (4.5)kpc. - These correspond to angular ranges of $\sim 23\,arcsec$, $\sim 2030\,arcsec$, and ~ 100-$150\,arcsec$. - If the authors could look at Figure 8 of Infante-Sainz et al. (2020) and focus on the relative SB of the r-band PSF model, the difference between $23\,arcsec$ to $\sim 100\,arcsec$ is also around $8.0\,mag\,/\,arcsec^2$. - While this is just a naive estimate, it demonstrates that the range of SB explored here is similar to that of the PSF model itself. It doesn't mean the outer light profile has to be dominated by the PSF wide-angle structure, **but** it suggests that it is certainly a possibility.
> For these reasons, the authors should spend some time thinking about how to address this issue. The most reliable way, in the referee's opinion, would be: - Take the PSF models from the Infante-Sainz work. Scale the PSF based on the redshift distribution of the clusters and create an equivalent stacked PSF model. - Model the stacked images in 2-D after considering the PSF convolution. Should use a flexible model here, e.g., multiple-Gaussian or Sersic. - Estimate the 1-D profile on the model image without PSF effect, and proceed with the analysis.
> The referee is aware that this approach could be quite time-consuming. And it also creates new problems for the analysis, such as the difficulty of estimating the profiles' statistical uncertainties. So the authors are encouraged to explore other approaches as well **as long as** they can 1) demonstrate the impact of wide-angle PSF structure; 2) discuss how it does affect the conclusions.
> The PSF outer structure systematic is particularly important for the results related to the comparison between the low- and high-BCG mass samples. In Figure 9, the two profiles are very different

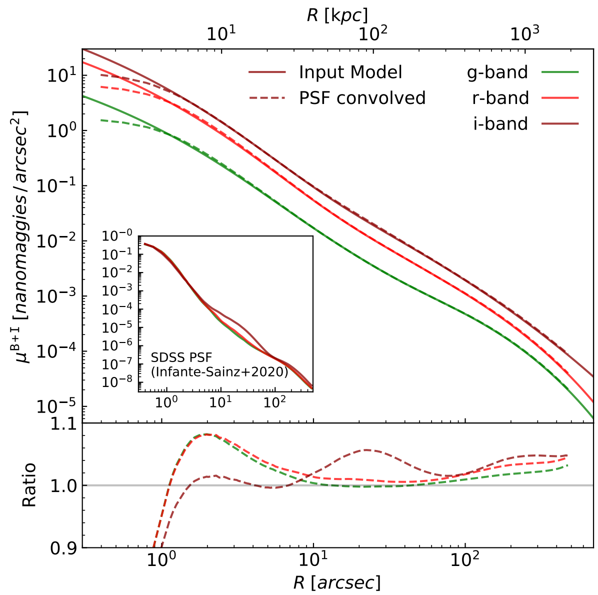

Figure 1: Impact of PSF on the surface brightness profile of BCG and ICL($\mu^{\mathrm{B+I}}$). *Top*: surface brightness profiles in the g (green curves), r (red curves), and i-band (maroon curves). Solid curves are the best-fitting three-Sérsic model used as the input profile before convolving with the PSF, and dashed curves are results after convolving with the PSF derived by Infante-Sainz et al. 2020 (the inset panel). The PSF in each band is normalized to have unit total flux. *Bottom*: The ratio between the dashed and solid curves in the top panel. The bottom and top x-axes are in units of arc sec and kpc (physical), respectively. This test shows that the impact of the SDSS PSF is negligible on scales above 100 kpc at z=0.25 compared to the statistical uncertainties.

> in the high-SB part but become increasingly similar at the low-SB end. The current conclusion from this comparison is interesting. **But** the referee needs to see evidence that these similar SB levels in the $r{\sim}29-32\, mag\,/\,arcsec^2$ regime are **, not** dominated by the PSF scattered light.

`authors' response:`

We thank the referee for pointing out the lack of discussion on the impact of the PSF in the previous version. We have performed a series of tests designed to thoroughly address the referee's concern, but before delving into the various tests we would like to clarify the reasoning behind our original lack of such discussion, as the referee seemed to be surprised by our omission and may have thought it was due to negligence — It was not due to the lack of awareness of the PSF effect or simple negligence, but because we expect a minimal impact (5%) of the PSF on the surface bright profiles, informed by a test we performed earlier (similar to the Figure 18 of Zhang et al. 2019 using DES images; an analogous version for our SDSS measurement will be shown later below). Therefore, we did not discuss the PSF in the previous version mainly because we *thought* it could be a distraction to the readers given its negligible impact, but now we agree with the referee that indeed we should design more tests to demonstrate that our results are insensitive to the PSF wide-angle effect and discuss the test results in the paper.

In the following, we will inspect the impacts of PSF on:

- the overall surface brightness profile in three bands,
- the amplitude of the large-scale ICL,
- the transitional stellar mass component,
- the comparison between high- and low-M_*^{BCG} subsamples,

using the SDSS PSFs derived by Infante-Sainz et al. (2020), as suggested by the referee.

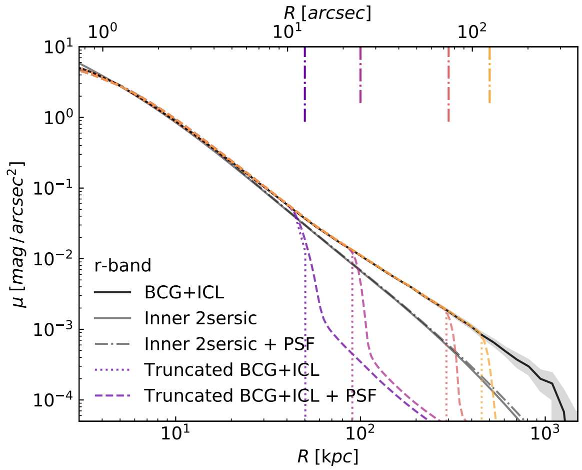

Figure 2: Contribution of the scattered light in the large-scale ICL. Black solid curve with shaded band is our observed surface brightness profile in the r-band. We truncated the surface brightness profile (dotted curves) at $50\,$kpc (average BCG radius), $100\,$kpc (the transition component), $300\,$kpc (BCG sphere of influence) and $500\,$kpc (corresponding to surface brightness around 30 mag/arc sec^2), respectively. Vertical dot-dashed lines on top indicate the truncation radii. Dashed curves of the same colors are the results after convolving with the r-band PSF. Gray solid curve is the sum of the two inner Sérsic components (solid) of our input surface brightness profile in r-band and gray dot-dashed its convolution with the PSF. This test shows that the wide-angle effect of the PSF cannot mimic the observed large-scale diffuse light.

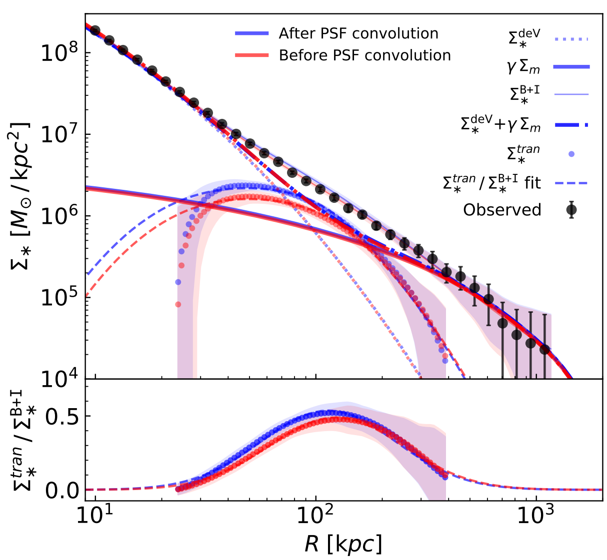

Figure 3: PSF impact on our decomposition of Σ_* of BCG+ICL. *Top*: Black solid circles with errorbars are the observed stellar surface density profile from the paper. Red and blue solid curves are the stellar surface density profiles inferred from the best–fitting three-Sérsic models in the gri bands, before and after PSF convolution, respectively. The rest of the red (blue) curves show the decomposition before (after) PSF convolution. *Bottom*: The ratio profiles between $\Sigma_*^{\rm tran}$ and $\Sigma_*^{\rm B+I}$. Clearly the transitional component stays roughly unchanged before and after the PSF convolution.

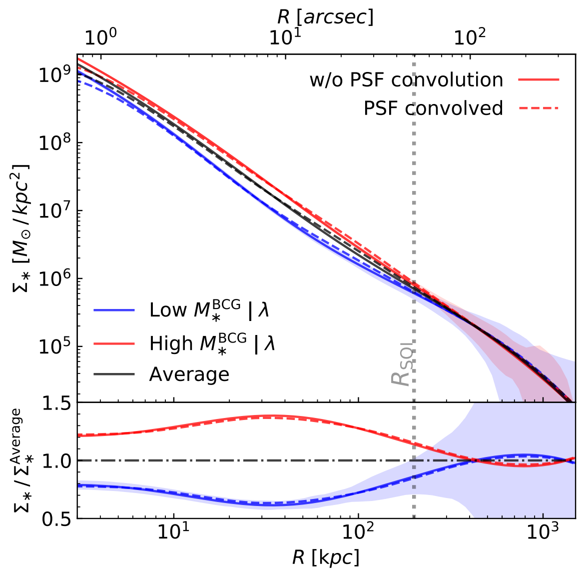

Figure 4: Σ_* of Low (blue curves) and High (red curves) $M_*^{\rm BCG}$ subsamples before (solid curves) and after (dashed curves) PSF convolution. The vertical gray dotted line indicates the R_{SOI} addressed in our manuscript

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We first access the impact of PSF on the overall amplitude of the surface brightness profiles in Figure 1. Following Zhang et al. 2019, we fit the surface brightness profile measured in each of the three bands (despite of the PSF effect) with three Sérsic components, and assume the best-fitting three-Sérsic profile as our input surface brightness profile *without the PSF effect*, shown as the maroon, red, and green solid curves for the i-, r-, and g-bands, respectively, in the upper sub-panel. We then adopt the PSF models inferred by the Infante-Sainz et al. 2020 (shown in the inset panel) to convolve with the solid curves in each band. The results *with the PSF effect included* are shown as the dashed curves. In the bottom sub-panel, we show the ratio profiles between the dashed (w/ PSF) and the solid (w/out PSF) curves. Similar to the finding by Zhang et al. 2019, we find that the impacts of PSF on the g- and r-bands are less than 2% on scales between 10 and 300 kpc, and less than 4% on scale beyond 300 kpc, both much smaller than the statistical uncertainties. The impact of PSF on the i-band profile is slightly larger, due to the small bump feature at 30 arcsec in the i-band PSF, but is still below 6% across all scales above 10 kpc. Therefore, consistent with the findings from Zhang et al. 2019, we expect the ICL amplitude (on scales above 300 kpc) to be enhanced by less than 5%, much smaller than the statistical uncertainties (above 15 %).

We next investigate the possibility of the large-scale diffuse ICL signal being caused by the wide-angle PSF effect in Figure 2. We use the sum of the two inner Sérsic components as the input profile (gray solid) and convolve it with the PSF, yielding the gray dot-dashed curve. Comparing the gray dot-dashed curve with the original three-Sérsic profile derived from the data, we can immediately see that the convolution with PSF cannot produce a significant diffuse ICL that may mimic the outermost Sérsic component on large scales. We also truncate the three-Sérsic model at four different radii (dotted) and convolve each truncated profile with the PSF, finding that the contribution of the scattered light to the large-scale diffuse ICL is negligible. Therefore, it is impossible for the scattered light to produce our measured ICL signal on large scales.

Thirdly, we examine the impact of PSF on the inference of the transitional component in Figure 3, plotted in the same format as the Figure 6 in the paper. Data points with errorbars are the original surface stellar mass profile measured in the paper without the PSF correction, which can be decomposed into a de Vaucouleurs' profile (red thin dotted), a scaled dark matter density profile (red thick solid), and a transitional component (red circles). Following the philosophy of the test above, we re-measure the surface stellar mass profile using the surface density profiles convolved with the SDSS PSFs in three bands. The results are shown as the corresponding blue curves and circles. Clearly, the fractional contribution of the transitional component, indicated by the ratio profiles in the bottom sub-panel, is insensitive to the impact of the PSF (compare the amplitude and centroid of the blue vs. red).

Finally, we check whether the main conclusion of the paper, the BCG sphere of influence, would be affected by the PSF. Again, we include the PSF effect in the stellar surface density profile of each subsample (dashed curves) and compare them with the original measurements (solid curves). The ratio profile between the high- and low-$M_*^{\rm BCG}$ subsamples stays almost unchanged before and after the PSF convolution. This is reassuring and as expected, because the impact of PSF on the two subsamples should be partially cancelled out in the ratio profile.

To summarise, the amplitude of the large-scale diffuse light profile can be enhanced by $1-5\%$ due to the SDSS PSF, but this enhancement is much smaller than the statistical uncertainties (above 15%) and therefore cannot mimic the ICL signal. More important, the inferred transitional component and the comparison between the high- and low-$M_*^{\rm BCG}$ surface stellar mass profiles are insensitive to the PSF effect — the main conclusions of our paper is unaffected by the PSF correction.

As the referee correctly pointed out, a rigorous, iterative correction of the PSF effect could be complicated and time-consuming; since we have demonstrated the impact of PSF on our conclusions is negligible, we decide to follow the practice of Zhang et al. 2019 and leave the surface brightness profile measurements uncorrected for PSF. We have nonetheless revised the manuscript by adding a paragraph where we describe the wide-angle effect of the SDSS PSF, cite the relevant papers that the referee suggested, and describing the tests that we performed to demonstrate the minimal impact of the PSF.

Comment [2]

> Generally speaking, even without the PSF outer structure issue, the Tal & van Dokkum, Wang et al., and de Souza & Kauffmann papers are still relevant and should be cited.

`authors' response:`

We have added citations to the relevant papers suggestd by the referee.

Comment [3]

> Regarding the rising stellar velocity dispersion profiles around the BCGs, several references are missing, e.g., Kelson et al. 2002; Bender et al. 2015; Veale et al. 2018.

`authors' response:`

We have added citations to the relevant papers suggestd by the referee.

Comment [4]

> Section 2.1: "cModel magnitudes provide a more robust estimate of the total flux": There are still systematic issues regarding the cModel flux: it still cannot recover the true "total" flux of a BCG. This is partially due to the limitation of the SDSS depth, but it is also related to the intrinsic limitation of the cModel assumptions. e.g., Bernardi et al. 2017.

`authors' response:`

We thank the referee for pointing out this particular problem with the cModel magnitudes, and have included a citation to the Bernardi et al. 2017 in the relevant text.

Comment [5]

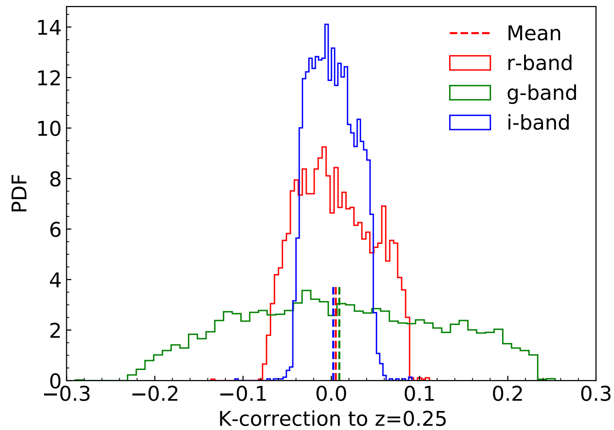

Figure 5: Histogram of the k-corrections of the BCGs in the g (green line), r (red line), and i-band (blue line) from the cluster redshifts to $z = 0.25$. Vertical dashed lines show the average k-corrections in each of the three bands.

> Section 3.2: Why only correct the Galactic extinction on the stacked images? And how did the authors achieve that, given that the clusters are all over the SDSS footprint?

`authors' response:`

We compute the average A_v of all the clusters in our sample and apply the extinction correction to the stacked profiles. Performing extinction correction over individual clusters is difficult because the fluxes of many pixels at large radii are negative after sky subtraction. We clarify this in the revised draft.

Comment [6]

> Section 3.2: It would be great to know the typical K-correction values for the chosen redshift range to evaluate whether it could be a source of uncertainty. I think it is ok to use the BCGs' color to estimate this value.

`authors' response:`

Using the `kcorrect_v4.3` (Blanton et al.2007), we measure the k-corrections of our BCGs in the three bands from their cluster redshifts to $z = 0.25$, as shown by Figure 5. As expected, the mean k-correction is close to zero in each band. In particular, the scatter in i-band is ~ 0.025 mag, translating into a $1 - 2\%$ uncertainty in stellar mass, hence an insignificant contribution to the uncertainty in the stellar surface density profiles.

We have revised the manuscript to mention the size of k-correction and its impact on uncertainties.

Comment [7]

> Section 3.2: What's the resampling method? Just a quick few words would do.

`authors' response:`

We have revised the text in Section 3.2 to clarify our jackknife resampling method.

Comment [8]

Section 3.3: "Contributions from the unmasked satellites....". This is not a **minor** issue: another source of the contribution could be the diffuse emission in the Milky Way (e.g., the Galactic cirrus). These emissions are almost everywhere once reaching the extremely low SB level probed by this stacking analysis. Its strength varies strongly with position (e.g., Bilek et al. 2020 and the references within). The authors used the statistical background subtraction method to deal with the excess of flux from whatever sources that are not the ICL + satellites. This method, in principle, should work for Galactic cirrus. Comparing the distributions of Av values for the cluster positions and random positions will help demonstrate that. But I believe the authors should be aware of this systematics and discuss its impact.

`authors' response:`

We thank the referee for bringing up the Galactic cirrus. We agreed with the referee that the effect of cirrus should be negligible due to our random background subtraction method. We have revised the relevant text to briefly describe its impact.

Comment [9]

Section 3.3: About the extraction of SB profile, the authors should mention how did they 1) align the BCGs (randomly? or rotate to align the major axis?) 2) extract the 1-D profile (circular aperture? or elliptical?). The BCG + ICL structures are known to have highly non-circular shapes in the outskirt (e.g., Montes et al. 2019, 2021). The ellipticity could reach 0.4 or even higher values. A circular aperture could potentially bias the profile significantly in the outskirt when taking the mean values within the annulus. Randomly aligning the BCGs could reduce this bias, but the authors should still mention this, in my opinion. Tal & van Dokkum align the galaxies before the stacking and demonstrate the rising ellipticity in the outskirt of LRGs. The authors could potentially do the same, which also helps provide a more robust 1-D profile. Maybe the authors could at least discuss this? Why not do this?

`authors' response:`

To answer the referee's questions: 1) we did not align the BCGs during the image stacking; and 2) we adopted the circular aperture for measuring the 1D profile. We agree with the referee that presumably we can align the BCGs by their major axes or use a non-circular aperture to stablize the 1D profiles, but the simple choices of non-alignment and circular aperture are adequate for the goal of the current paper. Furthermore, rotating the image frames or using non-spherical apertures before stacking adds quite a lot of complexities to the pixelation pipeline, so we decided not to implement this feature unless it is absolutely necessary for the sciences. We plan on investigating the effect of BCG alignments and non-circular apertures in our future work, and have clarified our choices in the current manuscript.

Comment [10]

Both Tal & van Dokkum and Zhang et al. 2019 (DES redMaPPer stacking work) suggest that there is still a significant fraction of luminosity outside, say 100 kpc. But the "true ICL fraction" derived from this work seems to be on the lower end than these works. What's the authors' take on this? Is this due to the different definitions of "ICL"? Like removing the unmasked flux from the satellites or not? A quantitative comparison would be appreciated too,

`authors' response:`

In Zhang et al. 2019, the luminosity fraction of BCG+ICL is $\sim 44 \pm 17\%$, but they did not estimate the fraction of undetected galaxies. If we add the undetected galaxies into the BCG and ICL component, our mass fraction is about 21%, comparable with the lower end of Zhang's estimation. The Tal&van Dokkum 2011 measurements are for lower-mass systems, so it is a bit difficult to do a direct comparison with our work.

Comment [11]

> Section 4.4: About the Yang et al. 2012 CSMF, are the stellar mass measurements consistent with the ones derived by the authors?

`authors' response:`

While they are consistent within 0.1 dex, the difference does not matter here because we are only making use of the *shape* of the CSMF from Yang et al. 2012. The amplitude of the CSMF is drived instead from our Σ_g measurement.

Sincerely,

Ying Zu