SLT coding exercise #1

Locally Linear Embedding

https://gitlab.vis.ethz.ch/vwegmayr/slt-coding-exercises

Due on Monday, March 6th, 2017

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Contents

The Model

The model section is intended to allow you to recapitulate the essential ingredients used in Locally Linear Embedding. Write down the *necessary* equations to specify Locally Linear Embedding and and shortly explain the variables that are involved. This section should only introduce the equations, their solution should be outlined in the implementation section.
Hard limit: One page

Suppose the data consists of N real valued vectors X_i, each of dimensionality D. For each data point we find its K nearest neighbors as measured by Euclidean distance. For each data point X_i we want to find weights W_{ij} for each of the K nearest neighbor such that we minimize the reconstruction error function

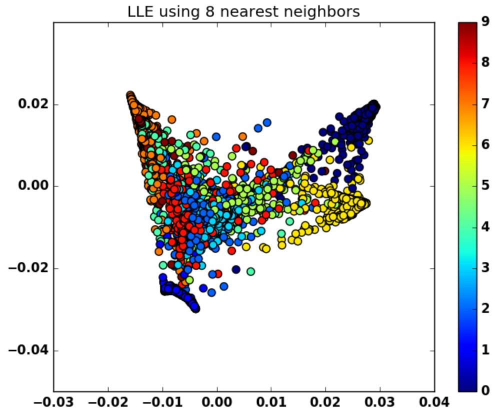
$$\mathcal{E}(W) = \sum_i \left| X_i - \sum_j W_{ij} X_j \right|^2$$

subject to two constraints: first, that each data point is reconstructed only from its neigbors, enforcing $W_{ij} = 0$ if X_j does not belong to this set; second, that the rows of the weight matrix sum to one.
We choose a d¡¡D and find d dimensional coordinates Y_i for each data point X_i. We do this by minimizing the embedding cost function

$$\Phi(Y) = \sum_i \left| Y_i - \sum_j W_{ij} Y_j \right|^2$$

The Questions

Answer b: Yes, some clusters appear. See examples in figure 1 below.



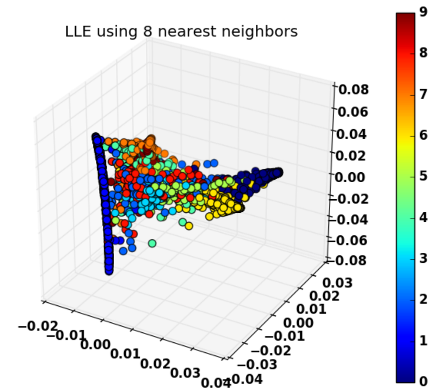


Figure 1: There are some visible clusters in the results.

Answer c: We can see that the diagonal elements are all non-zero.

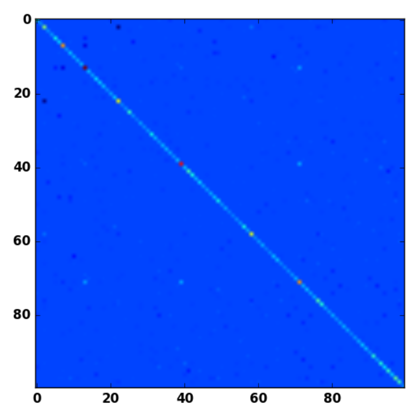


Figure 2: Plotting part of the matrix M

Answer d: When we change K, the number of nearest neighbors we can see some differences. If we only use 1 neighbor for each data point we can see that no visible clusters form (figure 3). Then as we increase the number of nearest neighbors we start seeing clusters even with K=3 (figure 4). The clusters seem to become more clear when we increase K.

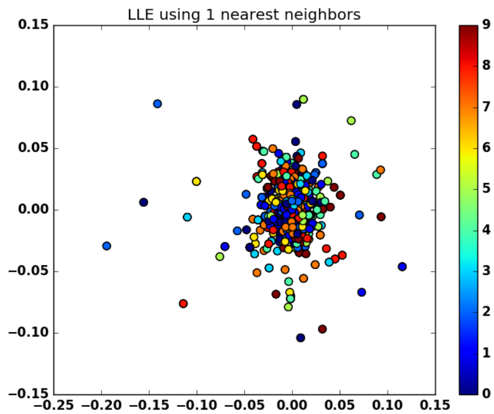


Figure 3: Using only 1 nearest neighbor

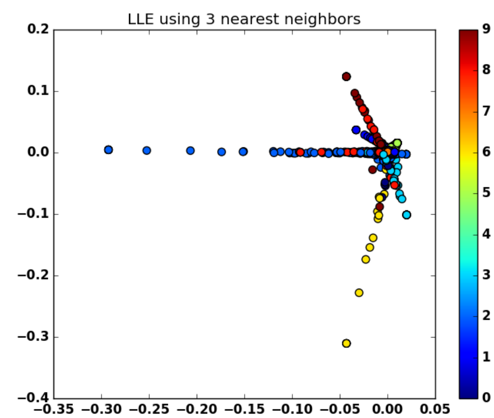


Figure 4: Using 3 nearest neighbor

Figures 5,6 and 7 show the use of 7 nearest neighbors with Euclidean, Hamming and Chebyshev distance metrics respectively.

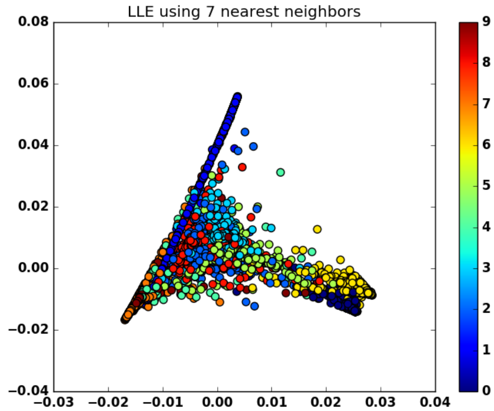


Figure 5: K=7, Euclidean distance

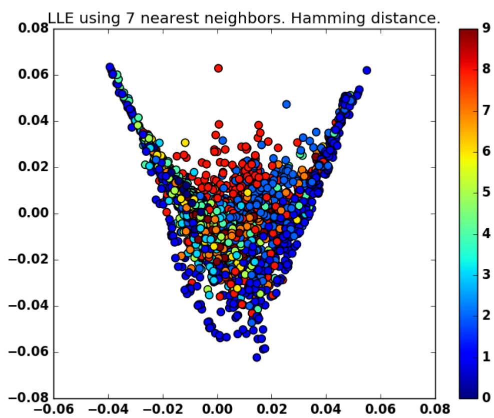


Figure 6: K=7, Hamming distance

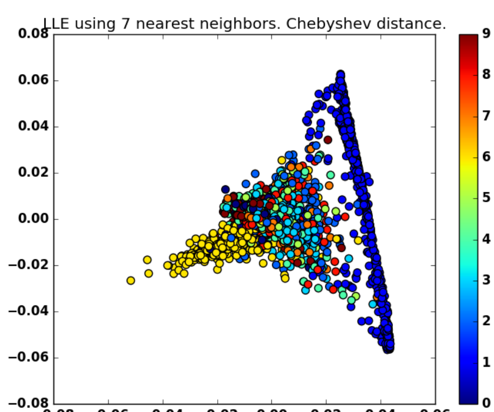


Figure 7: K=7, Chebyshev distance

Answer e: One method would be to find the point's K nearest neighbors in the embedding space and find the reconstruction weights as in the high dimensional space. Then use these K nearest neighbors and weights in the high dimensional space to reconstruct the data-point. This depends highly on the dimensionality of the embedding space, it works better for higher dimensions. See figure 8 where a data point is reconstructed using this method. The figures also show the original image for comparison.

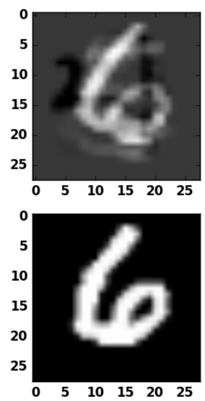


Figure 8: Reconstruction of a data point. Upper image shows the reconstruction, the lower image shows the original

The Implementation

The implementation involves finding eigenvectors of a very large matrix. For the MNIST dataset this could mean a 60000 by 60000 square matrix which my implementation would take a very long time to compute. When answering the questions in the previous questions I used only 5000 samples which my implementation of LLE took only about 10 seconds to solve. Link to my git branch:

Your Page

Your Answer YOUR GIT BRANCH