that all states are ergodic. We have

$$\rho_1 = \frac{\lambda}{\mu}\rho_0,$$

$$\rho_{n+1} = \frac{\lambda+\mu}{\mu}\rho_n - \frac{\lambda}{\mu}\rho_{n-1}, \qquad 1 \le n < K-1,$$

$$\rho_k = \frac{\lambda}{\mu}\rho_{k-1}.$$

Thus, for all possible values of n, and setting $\rho = \lambda/\mu$,

$$\rho_n = (\frac{\lambda}{\mu})^n \rho_0 = \rho^n \rho_0, \qquad 0 \le n \le K.$$

Notice,in particular, that this holds when $n = K$. We now need to compute ρ_0. We use

$$\sum_{n=0}^{K} \rho_n = 1$$

and thus

$$\rho_0 = \frac{1}{\sum_{n=0}^{K} \rho^n}.$$

The divisor is a finite geometric series and

$$\sum_{n=0}^{K} \rho^n = \frac{1-\rho^{K+1}}{1-\rho} \quad if\ \rho \ne 1$$

and is equal to $K+1$ if $\rho = 1$. Hence

$$\rho_0 = \begin{cases} (1-\rho)/(1-\rho^{K+1}) & if \rho \ne 1 \\ 1/(K+1) & if \rho = 1. \end{cases}$$

and is equal to $1/(K+1)$ if $\rho = 1$. Thus, the steady-state solution always exists, even for $\rho \ge 1$. The system is stable for all positive values of λ and μ . When $\lambda > \mu$, the number of customers in the system will increase, but it is bound from above by K. Also notice that ρ no longer represents the utilization. We shall derive an expression for this in just a moment. Finally, notice what happens as $K \to \infty$ and $\rho < 1$. We have

$$\lim_{K\to\infty} \frac{1-\rho}{1-\rho^{K+1}}\rho^n = (1-\rho)\rho^n,$$

which is the result previously obtained for the $M/M/1$ queue.

Example 11.12 Perhaps the simples example we can present is the $M/M/1/1$ queue. This gives rise to a two-state birth-death process, as illustrated in Figure 11.20

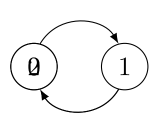

Figure 11.20. State transition diagram for the $M/M/1/1$ queue.