Practical Work 2 : US elections

Max Perraudin, Marie Dupont
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***Abstract*— The goal of this practical work is to represent the 2016 US elections trough a Linear Programming problem. The problem variables are the number of great electors in each state, which are optimized in function of the population. We applied different parameters in order to see the outcome of the election, but D. Trump won every time.**

I. Context

During the general election, Americans go to their polling place to cast their vote for president. But the tally of those votes —the popular vote— does not determine the winner. Instead, presidential elections use the Electoral College. To win the election, a candidate must receive a majority of electoral votes (270 out of 538). The entirety of the electoral college of a state votes for the candidate with the majority of popular votes. This system is called an indirect democracy, and can yield surprising results. In 2016, Donald Trump won despite not being the favorite and collecting less popular votes than Hillary Clinton.

In this exercise, we will modelize the distribution of great electors per state using a Linear Programming problem. In order to verify our model, we will compare it to reality, and then we will tune and change some parameters, which will allow us to possibly witness another outcome. We can, for example, give the right to vote to Puerto Rico or change the minimum number of great electors per state.

II. Presentation of the problem

Let be $x = (x_1, ..., x_p)$. Each coordinate of x represents the population of the state i and N represents the total number of electoral votes. In the general case, $N = 538$. We will minimize the representativity gap by solving the following Linear Programming problem :

$$min\{u - v \quad | \quad v \leq \frac{\alpha_i}{x_i} \leq u, \quad \sum_{1 \leq i \leq p} \alpha_i = N\}$$

where α_i represents the number of electoral voters in the state i.

Indeed, $\frac{\alpha_i}{x_i}$ represents the number of electoral voters per million of inhabitants per state. As shown on the map illustrating these numbers in every states, the states are not equally represented during the elections. That is why it could be interesting to minimize the gap between the states, and it can be done by minimizing the difference of this fraction in the different states.

III. Results

In our model we chose the following hypotheses. The population is composed of only people over 18 years old and in possession of the American citizenship. Puerto Rico can have the right of vote, and when it is the case we assume that they voted 100 percent in favor of Hillary Clinton. At the beginning of the simulation, we can set up if Puerto Rico votes or not and the minimum number of great electors per state. The District of Columbia deserves a special treatment. As for the 23rd amendment, the District of Columbia is granted electors in the Electoral College as though it were a state, though the district can never have more electors than the least-populous state. This means DC will always have the minimum number of electoral college members. This constraint is implemented in our simulation, but other constraints are impossible to take into account (see IV).

We tried different combinations of these parameters, which gave us the number of members of the electoral college per state, optimized for the previous LP problem. The map of the distribution of the electoral college

is showed in the annexes. The model is very accurate, if we compare it to the real map, also in annexes. Using the real parameters, the minimum number of members is 3, the total number of members is 538, and Puerto Rico does not vote. Our model gave 1 more vote to California, 4 more to Florida,

Puerto Rico	Min number	Winner	Nvotes
Yes	0	Trump	315
No	0	Trump	310
Yes	1	Trump	323
No	1	Trump	311
Yes	3	Trump	312
No	3	Trump	310

As shown in this table, D.Trump wins in all cases.Yet we can notice that Puerto Rico does not have so much influence on the results, especially in the case with three minimum great electors – which is the current minimum in the USA.

We also tried to vary the total number of Electoral College members, but every case yields the same results, with Donald Trump far ahead (over 300 votes)

One of the main problems is the over-representativity of low population states, such as Wyoming and Vermont (See VI). They have 3 electoral votes each, the minimum possible. According to our model when setting the minimum number to 0, Vermont and Wyoming should only have 1 electoral vote. This can affect other states that probably need more electoral votes due to their higher population.

IV. Limits of our model

Our model is not quite a perfect representation of reality. Usually, states apply the rule 'winner takes all', but Maine and Nebraska have a different approach. They allocate two electoral votes to the state popular vote winner, and then one electoral vote to the popular vote winner in each Congressional district. This can split the votes between the candidates. It happened in 2016 when Clinton won 3 votes and Trump won 1. We couldn't represent this case in our model.

Another limit of our model, is the honesty of the great electors. A faithless elector is a member of the Electoral College who does not vote for the presidential or vice presidential candidate for whom they had pledged to vote. That is, they break faith with the candidate they were pledged to, and vote for another candidate, or fail to vote. This happened during the elections of 2016 with two states : Washington and Texas. In Washington state, 4 voters defected out of the 12 supposed to vote for H. Clinton, and voted for other candidates. In Texas, out of the 38 members of the Electoral College, 2 vote for other candidates than D. Trump. This scenario is not supposed to happen. The members of the electoral college who defect run the risk of being subject to legal charges and political retaliation from their party.

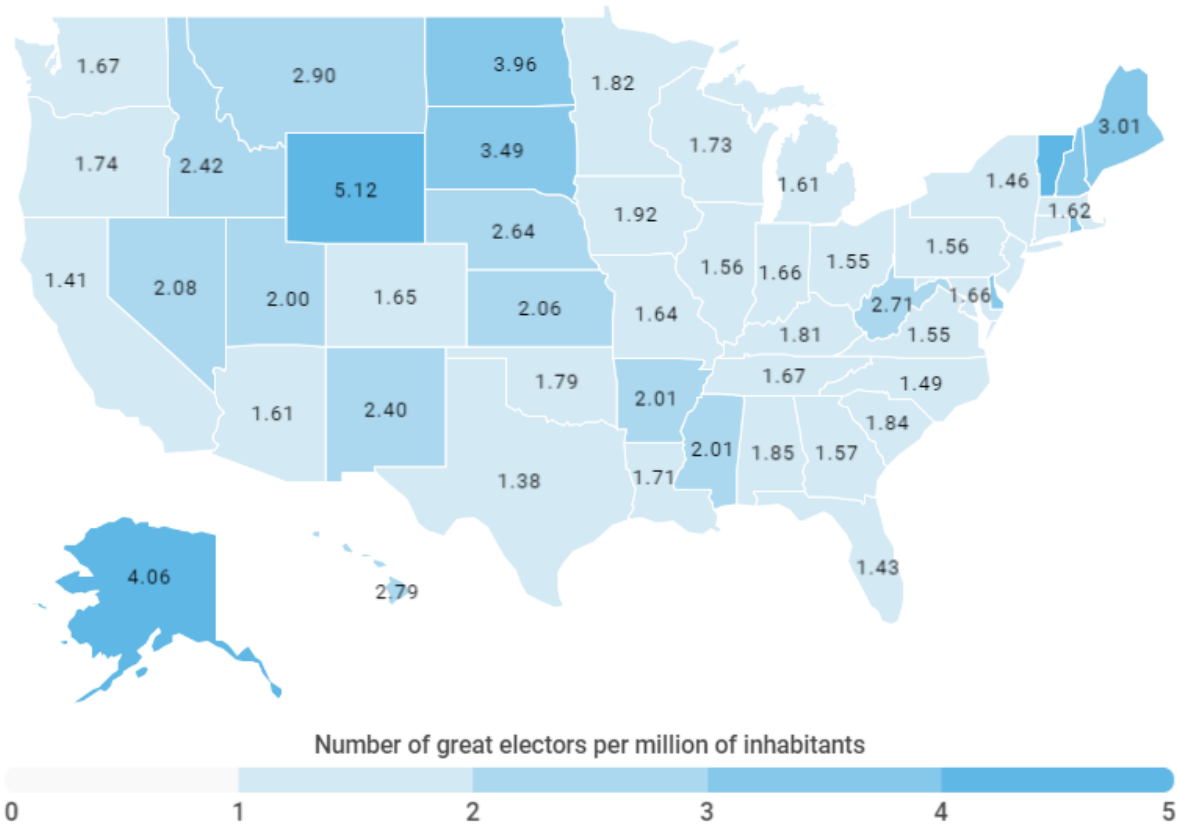
V. What would happen when using direct democracy ?

The current system is, obviously, based on the indirect democracy but we can wonder what would happen if we try to take into account the percentage of votes each candidate got in each state. We tried to create an hybrid between direct and indirect democracy, where the electoral college votes following the distribution of the popular votes. For example, in Connecticut 41% of the population voted for D. Trump, so 41% of the electoral college would vote for Trump. This system is closer to a direct democracy. The results of such a simulation were in favor of Hillary Clinton, as she obtained more popular votes than Donald Trump.

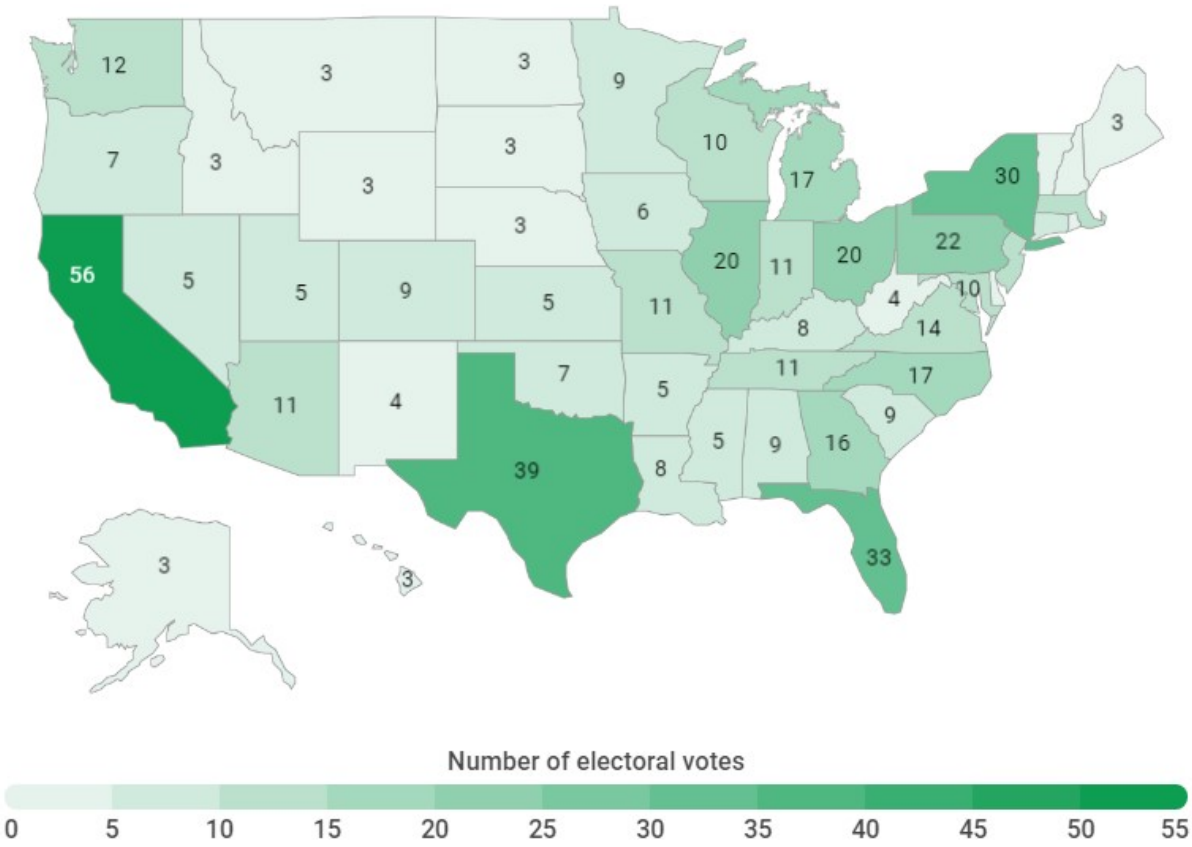
Puerto Rico	NMin	Clinton	Trump	Total
Yes	1	235	221	456
No	1	243	220	463
Yes	3	230	221	451
No	3	235	222	457

These results show that in all cases H.Clinton would win. It can also be noticed that the number of great electors who voted for Trump and Clinton (column Total) is not equal to the total number of great electors in USA. It means that other parties would (have a chance to) be represented. Moreover, the absolute majority is not reached, thus a second vote might be established.

USA map showing the number of great electors per million of inhabitants



Members of the electoral college per state according to our model



Difference in number of electoral votes per state between reality and model