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78) LangProps

1) AB = A -> True

   Explanation: A = {a, aba, bba}; B = {λ} -> AB = {a, aba, bba}

2) AB = ∅ -> True

   Explanation: A = {∅}; B = {∅}

3) AB = BA -> True

   Explanation: A and B must be identical: A = {a,ab}; B = {a,ab} -> AB = {aa,aba} and BA = {aa,aba}

4) A* = A and A ≠ Σ* -> True

   Explanation: A is a proper subset of Σ*

5) AA = A -> True

   Explanation: A = {λ} so AA = {λλ} or AA = {λ}

6) AA ⊆ A but AA ≠ A -> False

   Explanation: AA cannot be a subset of A unless A = {λ} but in that case AA would equal A

7) AA ⊄ A -> True

   Explanation: A contains more than λ: A = {a,b} -> AA = {aa,ab,ba,bb}

8) A ⊆ AA but AA ≠ A -> True

   Explanation: A contains more than λ: A = {a,b} -> AA = {aa,ab,ba,bb}

9) A* ⊂ A, where ⊂ means proper subset -> False

   Explanation: By definition A is the set of all strings A*

79) LangSubset

1) forall A,B,C, A(B∪C)⊆ AB∪AC -> True

   Explanation: A = {a,aa},B={b,bb},C={c,cc} then B∪C = {b,bb,c,cc}

$A(B \cup C)$ = {ab,abb,ac,acc,aab,aabb,aac,aacc}

AB = {ab,abb,aab,aabb} AC = {ac,acc,aac,aacc}

$AB \cup AC$ = {ab,abb,aab,aab,ac,acc,aac,aacc}

Because concatenation is associative A(BC) is the same as (AB)C and Union is associative $AB \cup C$ is the same as $A \cup BC$ so the combination of the two associative operations allow this expression to be true.

2) forall A,B,C, $A(B \cup C) \supseteq AB \cup AC$ -> True

Explanation: s is an element in $AB \cup AC$ and we want to show that $s \in A(B \cup C)$. if $s2 \in AB$: s can be written as s1s2 if $s1 \in A$ and $s2 \in B$, then $s2 \in (B \cup C)$ -> $w \in A(B \cup C)$

if $s2 \in AC$: s = s1s2 with $s1 \in A$ and $s2 \in C$. $s2 \in (B \cup C)$ and then $s \in A(B \cup C)$

3) forall A,B,C, $A(B \cap C) \subseteq AB \cap AC$ -> True

Explanation: Intersection is associative and commutative and concatenation is associative.

Lets take an arbitrary string $s \in AB \cap AC$ then by definition of concatenation s = s1s2.

If $s1 \in A$ or ($s2 \in B$ and $s2 \in C$) then $s2 \in (B \cap C)$ and s is in $A(B \cap C)$

or

If $s2 \in A$ or ($s1 \in B$ and $s1 \in C$) then $s1 \in (B \cap C)$ and s is in $A(B \cap C)$

4) forall A,B,C, $A(B \cap C) \supseteq AB \cap AC$ -> False

Explanation: A = {a,b,c} B = {b,c} C = {λ}

AB = {ab,ac,bb,bc,cb,cc}

AC = {a,b,c}

$(B \cap C) = \emptyset$

$A(B \cap C)$ = {a,b,c}

$AB \cap AC = \emptyset$

95) Delta first

$\hat{\delta}$ (q,ax) = $\hat{\delta}$ (δ(q,a),x)

$q \xrightarrow{a} \delta(q,a) \xrightarrow{x} \hat{\delta}(\delta(q,a),x)$: Delta First

100) DFAUnion

1) M1 = (Σ,Q1,δ1,s1,F1); M2 = (Σ,Q2,δ2,s2,F2)

M1' = (Σ,Q1,δ1,s1,(Q1-F1))

M2' = (Σ,Q2,δ2,s2,(Q2-F2))

2) M' = L(M') = (Σ,(Q1 X Q2),δm,(s1,s2),((Q1-F1) X (Q2-F2)))

3) M'' = (Σ,(Q1 X Q2),δm,(s1,s2),(Q1 X Q2) - ((Q1-F1) X (Q2-F2)))

M'' and the DFA from the product construction for union as described in the text are compliments of each other.

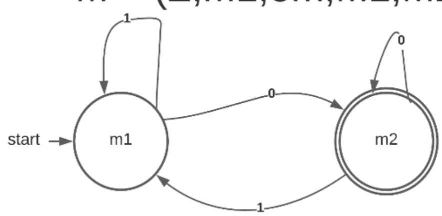
102) DFAPuzzle

See below for pictures.

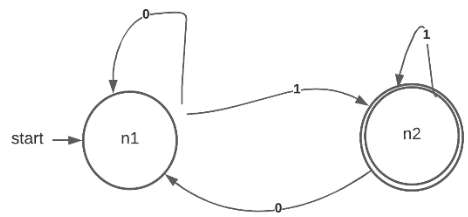
Because L(M)∩L(N) = Ø the accepted state is not reachable. The reason for this is because L(M) has only one accepting state that is reached if the string ends in 0 and L(N) has only one accepting state that is reached if the string ends in 1. A string cannot end in both 0 and 1 therefor there is no possible way to reach the accepting state for L(M)∩L(N).

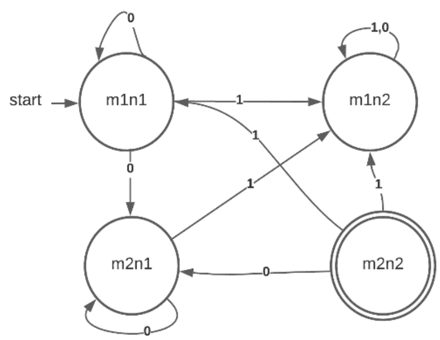
102) DFAPuzzle

$M = (\Sigma, M1, \delta m, M1, M2)$



$N = (\Sigma, N1, \delta n, N1, N2)$





$P = (\Sigma, (Qm \times Qn), \delta p, (m1, n1), (M2 \times N2))$