CS3133 Foundations of Computer Science HW3

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Symbols for Convenience: ∅ ≠ Σ λ ⊆ ⊄ ϵ δ ≡ ≢ ∉ ∃

Collaboration with Samuel Parks and Kush Shah
Went to Noelle's Office Hours and Professor's Office Hours

129. EquivPractice)

1) A = {w | w contains abb }
   a. ab $\not\equiv_A$ ba

      z = b

      abb ϵ A

      bab ∉ A

   b. λ $\not\equiv_A$ abb

      z = λ

      λ λ ∉ A

      abb ϵ A

   c. λ $\not\equiv_A$ ba

      z = bb

      bb ∉ A

      babb ϵ A

   d. abb $\equiv_A$ babba

      For any given string z,

      abbz ϵ A

      and

babbaz ϵ A

This is because both strings already contain ‘abb’ and therefore will always be part of the language.

Therefore abb $\equiv_A$ babba

2) B = { w | |w| is even }

   a. aab $\not\equiv_B$ ab

      z = b

      aabb ϵ B

      abb $\notin$ B

   b. $\lambda \not\equiv_B$ a

      z = a

      a $\notin$ B

      aa ϵ B

3) C = { $a^i b^j$ | i < j }

   a. ab $\not\equiv_C$ ba

      z = λ

      ab ϵ C

      ba $\notin$ C

   b. $\lambda \not\equiv_C$ abb

      z = a

      a ϵ C

      abba $\notin$ C

   c. bba $\equiv_C$ ba

      For any string z,

bbaz $\notin$ C

and

baz $\notin$ C

This is because both bba and ba have an ‘a’ following a ‘b’ which means they are not in the language.

Therefore, bba $\equiv_C$ ba

145 NonRegPractice)

a) $A = \{ a^n b^{2n} \mid n >= 0 \}$

An infinite collection of strings that are all distinguishable from each other exists and is of the form:

λ , a, aa, aaa, aaaa, aaaaa, …

Or, textually, all strings in the language **a***.

Given two distinct strings from this collection: x and y.

Let i be the number of a’s in x.
Let j be the number of a’s in y.

$i \neq j$ because x and y are distinct.

Let $z = b^{2i}$

$xz \in A$ because $xz = a^i b^{2i}$

$yz \notin A$ because $yz = a^j b^{2i}$ and $i \neq j$

Therefore $x \equiv_A y$.

Therefore, A is nonregular.

b) $B = \{ a^n b^m c^n \mid n, m >= 0 \}$

An infinite collection of strings that are all distinguishable from each other exists and is of the form:

b, ab, aab, aaab, …

Or, textually, all strings in the language **a*b**

Given two distinct strings from this collection: x and y.

Let r = the number of a's in x
Let j = the number of a's in y

$i \neq j$ because x and y are distinct.

Let $z = c^i$

$xz \in B$ because $xz = a^i bc^i$

$yz \notin B$ because $yz = a^j bc^i$ and $i \neq j$

Therefore $x \equiv_B y$.

Therefore, B is nonregular.

c) $C = \{ a^n b^m \mid n <= m \}$

An infinite collection of strings that are all distinguishable from each other exists and is of the form:

λ , a, aa, aaa, aaaa, aaaaa, …

Or, textually, all strings in the language **a***.

Given two distinct strings from this collection: x and y.

Let $|x| > |y|$
Let i be the number of a's in x.
Let j be the number of a's in y.

$r > i$ because $|x| > |y|$.

Let $z = b^j$

$xz = a^i b^j \notin C$
$yz = a^j b^j \in C$

Therefore $x \equiv_C y$.

Therefore, C is nonregular.

d) $D = \{ a^i b^n c^n \mid i >= 0, n >= 0 \}$

An infinite collection of strings that are all distinguishable from each other exists and is of the form:

a, ab, abb, abbb, abbbb, …

Or, textually, all strings in the language **ab***.

Given two distinct strings from this collection: x and y.

Let r be the number of b's in x.
Let j be the number of b's in y.

$r \neq j$ because x and y are distinct.

Let $z = c^r$

$xz = ab^r c^r \in D$

$xy = ab^j c^r \notin D$

Therefore $x \equiv_D y$.

Therefore, D is nonregular.

e) $E = \{ w \in \{ a, b \}^* \mid \exists x \in \{ a, b \}^*, xx = w \}$

An infinite collection of strings that are all distinguishable from each other exists and is of the form:

λ, a, aa, aaa, aaaa, …

Or, textually, all strings in the language **a***.

Given two distinct strings from this collection: x and y.

Let i be the number of a's in x.
Let j be the number of a's in y.

$i \neq j$ because x and y are distinct.

Let $z = ba^ib$

$xz = a^iba^ib \in E$

$yz = a^jba^ib \notin E$

Therefore $x \equiv_E y$.

Therefore, E is nonregular.

f) $R = \{ w \in \{ a, b \}^* \mid \exists x \in \{ a, b \}^*, xx^R = w \}$ (^{R} means its reversed)

An infinite collection of strings that are all distinguishable from each other exists and is of the form:

λ, a, aa, aaa, aaaa, …

Or, textually, all strings in the language **a***.

Given two distinct strings from this collection: x and y.

Let i be the number of a's in x.
Let j be the number of a's in y.

$i \neq j$ because x and y are distinct.

Let $z = bba^i$

$xz = a^ibba^i \in E$

$yz = a^jbba^i \notin E$

Therefore $x \equiv_F y$.

Therefore, F is nonregular.

g) G = The set of strings of a's and b's whose length is a perfect square.

An infinite collection of strings that are all distinguishable from each other exists and is of the form:

λ, a, aaaa, aaaaaaaaa, …

Or textually, $a^{(n\wedge 2)}$ where n is a Natural Number.

Let $x = a^{(i\wedge 2)}$

Let $y = a^{(j^2)}$

Where $i < j$

Let $z = a^{(2i + 1)}$

$xz = a^{(i^2)}a^{(2i + 1)} = a^{i^2 + 2i + 1} = a^{(i + 1)^2} \in G$

$yz = a^{(j^2)}a^{(2i + 1)} = a^{(j^2 + 2i + 1)} \notin G$

$a^{(j^2 + 2i + 1)} \notin G$ because $j^2 + 2i + 1$ cannot be a perfect square.

This is because $j^2 < j^2 + 2i + 1 < (j + 1)^2$, since $(j + 1)^2 = j^2 + 2j + 1$ and $i < j$.

Therefore, $x \equiv_G y$.

Therefore, G is nonregular.

h) $K = \{ a^n \mid n \text{ is a perfect cube} \}$

An infinite collection of strings that are all distinguishable from each other exists and is of the form:

λ, a, aaaaaaaa, aaaaaaaaaaaaaaaaaaaaaaaaaaa, …

Or textually, $a^{(n^3)}$ where n is a Natural Number.

Let $x = a^{(i^3)}$
Let $y = a^{(j^3)}$

Where $i < j$

Let $z = a^{(3i^2 + 3i + 1)}$

$xz = a^{(i^3)}a^{(3i^2 + 3i + 1)} = a^{(i^3 + 3i^2 + 3i + 1)} = a^{(i^3)} \in H$

$yz = a^{(j^3)}a^{(3i^2 + 3i + 1)} = a^{(j^3 + 3i^2 + 3i + 1)} \notin H$

$a^{(j^3 + 3i^2 + 3i + 1)} \notin H$ because $j^3 + 3i^2 + 3i + 1$ cannot be a perfect square.

This is because $j^3 < j^3 + 3i^2 + 3i + 1 < (j + 1)^3$, since $(j + 1)^3 = j^3 + 3j^2 + 3j + 1$ and $i < j$.

Therefore, $x \equiv_H y$.

Therefore, H is nonregular.

i) $I = \{ a^n \mid n \text{ is a power of } 2 \}$

An infinite collection of strings that are all distinguishable from each other exists and is of the form:

a, aa, aaaa, aaaaaaaa, …

Or textually, $a^{(2^m)}$ where m is a Natural Number.

Let $x = a^{(2^i)}$
Let $y = a^{(2^j)}$

$i < j$

Let $z = a^{(2^j - 2^i)}$

$xz = a^{(2^i)}\, a^{(2^j - 2^i)} = a^{(2^j)} \in I$

$yz = a^{(2^j)}\, a^{(2^j - 2^i)} = a^{(2^{(j+1)} - 2^i)} \notin I$

$a^{(2^{(j+1)} - 2^i)} \notin I$ because $2^{(j+1)} - 2^i$ cannot be a power of 2, for simplicity I have an example given below, after that example I include a more detailed proof of this.

<u>Example:</u>

If j = 3 and i = 2:

$2^i = 2^2 = 4$

$2^j = 2^3 = 8$

$2^{(j+1)} = 2^4 = 16$

$2^{(j+1)} - 2^i = 16 - 4 = 12.$

12 is not a power of 2.

<u>Proof:</u>

Given that: $2^{(j+1)} = 2^j + 2^j$, $i < j$

Therefore $2^i < 2^j$.

Therefore $2^j + 2^i < 2^{(j+1)}$

Moving around terms we get: $2^j < 2^{(j+1)} - 2^i$

Therefore $2^j < 2^{(j+1)} - 2^i < 2^{(j+1)}$, and since there are no powers of 2 between. 2^j and $2^{(j+1)}$, $2^{(j+1)} - 2^i$ cannot be a power of 2.

Therefore, $x \equiv_I y$.

Therefore, I is nonregular.

147 Bounded Exponents)

a) $A = \{ a^i b^j \mid i >= j \text{ and } j <= 100 \}$

This language is regular.

It is regular by theorem because it is a union of regular expressions.

The language can be rewritten as the union of the distinct languages: $a^n a^* b^n \mid 0 <= n <= 100$.

All languages $a^n a^* b^n$ where $0 <= n <= 100$ are regular, because for any n the language can be written as a RegularExpression of the form: $\mathbf{a^n a^* b^n}$.

Therefore, since A is a union of regular languages, A is regular.

b) $B = \{ a^i b^j \mid i >= j \text{ and } j >= 100 \}$

An infinite collection of strings that are all distinguishable from each other exists and is of the form:

$a^{100}, a^{101}, a^{102}, \ldots$

Or, textually, any strings of 100 or more a's.

Given two distinct strings from this collection: x and y.

Let $|x| > |y|$
Let i be the number of a's in x.
Let j be the number of a's in y.

$i \neq j$ because x and y are distinct.
$i > j$ because $|x| > |y|$.

Let $z = b^i$

$xz = a^i b^i \epsilon B$

$yz = a^j b^i \notin B$

Therefore $x \equiv_D y$.

Therefore B is nonregular.

156 AcceptEveryEvenDFA)

Algorithm on DFA M to see if it accepts all even strings:

On input M;

CONSTRUCT K accepting precisely even-length strings;
CONSTRUCT DFA P as P = M intersect K;
call DFA equivalence on P and K;
return answer;

Reasoning for algorithm:

K = DFA accepting only even length strings

If M accepts all even length strings then:
L(K) n L(M) = K

159 DifferInfiniteDFA)

Algorithm on DFAs M and N to see if they differ on infinitely many inputs:

CONSTRUCT DFA M' = complement of M;
CONSTRUCT DFA N' = complenet of N';
CONSTRUCT DFA P = M' intersect N;
CONSTRUCT DFA K = N' intersect M;
CONSTRUCT DFA T = P union K;
call DFA infinite on T;
return answer;

Reasoning for algorithm:

L(P) = L(M') n L(N) is the set of all inputs that N accepts and M doesn't.
L(K) = L(N') n L(M) is the set of all inputs that M accepts and N doesn't.
L(T) = L(P) u L(K) is therefore all inputs that M and N disagree on.

If L(T) is infinite therefore, M and N differ on infinitely many inputs.