CS3133 Foundations of Computer Science HW1

Keith DeSantis

8/25/21

Collaboration with Samuel Parks and Kush Shah

Went to Office Hours

LangProps)

For each part decide if there can be languages A and B with given property. If so, give a specific example, if not, explain why not.

1) $AB = A$

   Can exist. If $B = \{\lambda\}$ then $AB = A$.

2) $AB = \emptyset$

   Can exist. If A and $B = \emptyset$.

3) $AB = BA$

   Can exist. If $A = B$.

4) $A^* = A$ and $A \neq \Sigma^*$

   Can exist. If $A = \emptyset$ or $A = \{\lambda\}$.

5) $AA = A$

   Can exist. If $A = \emptyset$ or $A = \{\lambda\}$.

6) $AA \subseteq A$ but $AA \neq A$

   Can exist.
   Example: A = { b^n | n >= 1 }
   AA = { b^n | n >= 2 },
   Therefore, $AA \subseteq A$ and $AA \neq A$

7) $AA \not\subset A$.

   Can exist. A = {“computer”}, AA = {“computercomputer”}.

8) $A \subseteq AA$ but $AA \neq A$.

   Can exist. $A = \{\lambda, a\}$, $AA = \{\lambda, a, aa\}$.

9) $A^* \subset A$

Can't exist. By definition, $A \subseteq A^*$, therefore A^* cannot be a proper subset of A.

LangSubset)

1) For all A ,B, C, $A(B \cup C) \subseteq AB \cup AC$
   Let $y \in A(B \cup C)$.
   Let $y = ax$ where: $a \in A$ and $x \in (B \cup C)$.
   Two cases for x:

   $x \in B$:
   In this case y is made of a and one element of B, therefore is an element of AB and an element of $AB \cup AC$.

   $x \in C$:
   In this case y is a concatenation of a and one element of C, therefore is an element of AC and an element of $AB \cup AC$.
   Therefore, $A(B \cup C) \subseteq AB \cup AC$.

2) For all A, B, C, $AB \cup AC \subseteq A(B \cup C)$

   Let $y \in AB \cup AC$.

   There are two cases:

   1) $y \in AB$:
   Then y can be written as ab where $a \in A$ and $b \in B$. ab is an element in $A(B \cup C)$, so $y \in A(B \cup C)$.
   2) $y \in AC$:
   Then y can be written as ac where $a \in A$ and $c \in C$. ac is an element in $A(B \cup C)$ so $y \in A(B \cup C)$.

   Therefore $AB \cup AC \subseteq A(B \cup C)$.

3) For all A, B, C $A(B \cap C) \subseteq AB \cap AC$

   Let $y \in A(B \cap C)$.
   y can be written as ax, where $x \in (B \cap C)$.
   Since $x \in B$, $ax \in AB$.
   Similarly, $x \in C$, $ax \in AC$.
   This leads to $x \in AB \cap AC$.
   Therefore, $A(B \cap C) \subseteq AB \cap AC$.

4) For all A, B, C $AB \cap AC \subseteq A(B \cap C)$

Not true.

Counter example:

A = { a, aa }
B = { c }
C = { ac }

Therefore,
AB = { ac, aac }
AC = { aac, aaac }

AB n AC = { aac }

However,

A(B n C) = A∅ = ∅

So, AB n AC ⊄ A(B n C)

DeltaFirst)

Definition:

$\delta^\wedge(q, \lambda) = q$

$\delta^\wedge(q, ax) = \delta^\wedge(\delta(q, a), x)$ for $x \in \Sigma^*$, $a \in \Sigma$

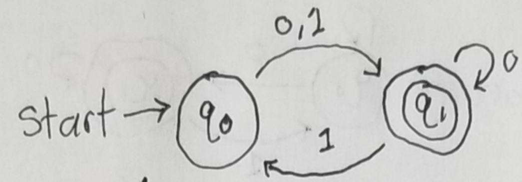
Picture:

$q \xrightarrow{a} \delta(q, a) \xrightarrow{x}\!\!\!\rightarrow \delta^\wedge(\delta(q, a), x)$

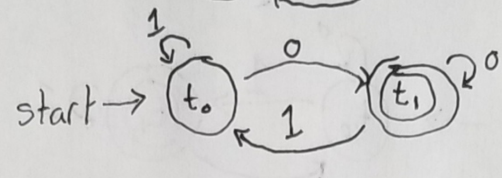
From q, reading the symbol a brings the DFA to state δ(q, a). From δ(q, a), reading in the string x brings the DFA to state δ^(δ(q, a), x).

DFAUnion)

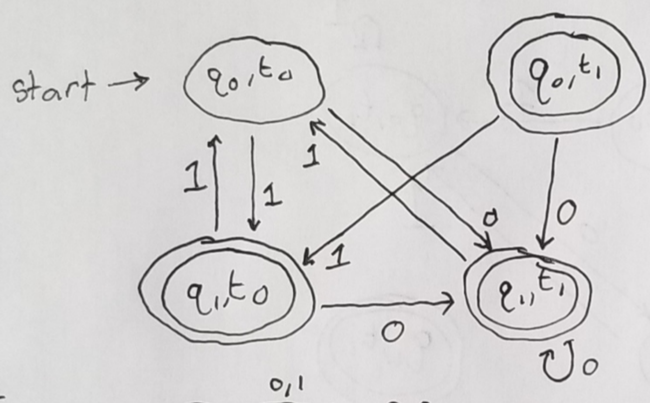
M_1:



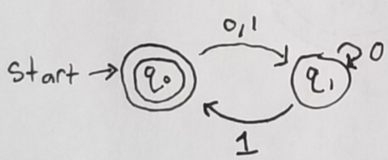
M_2:



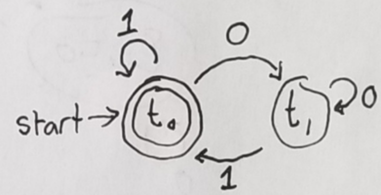
$M_1 \cup M_2$ with Union Construction:



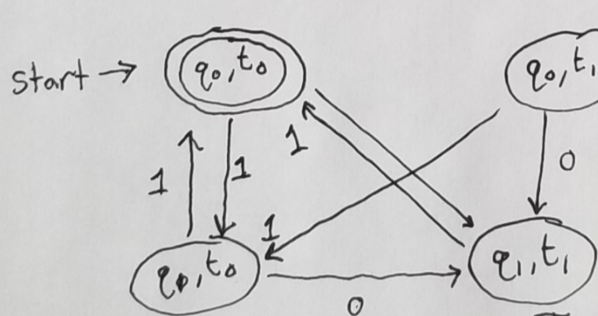
$\overline{M_1}$:



$\overline{M_2}$:



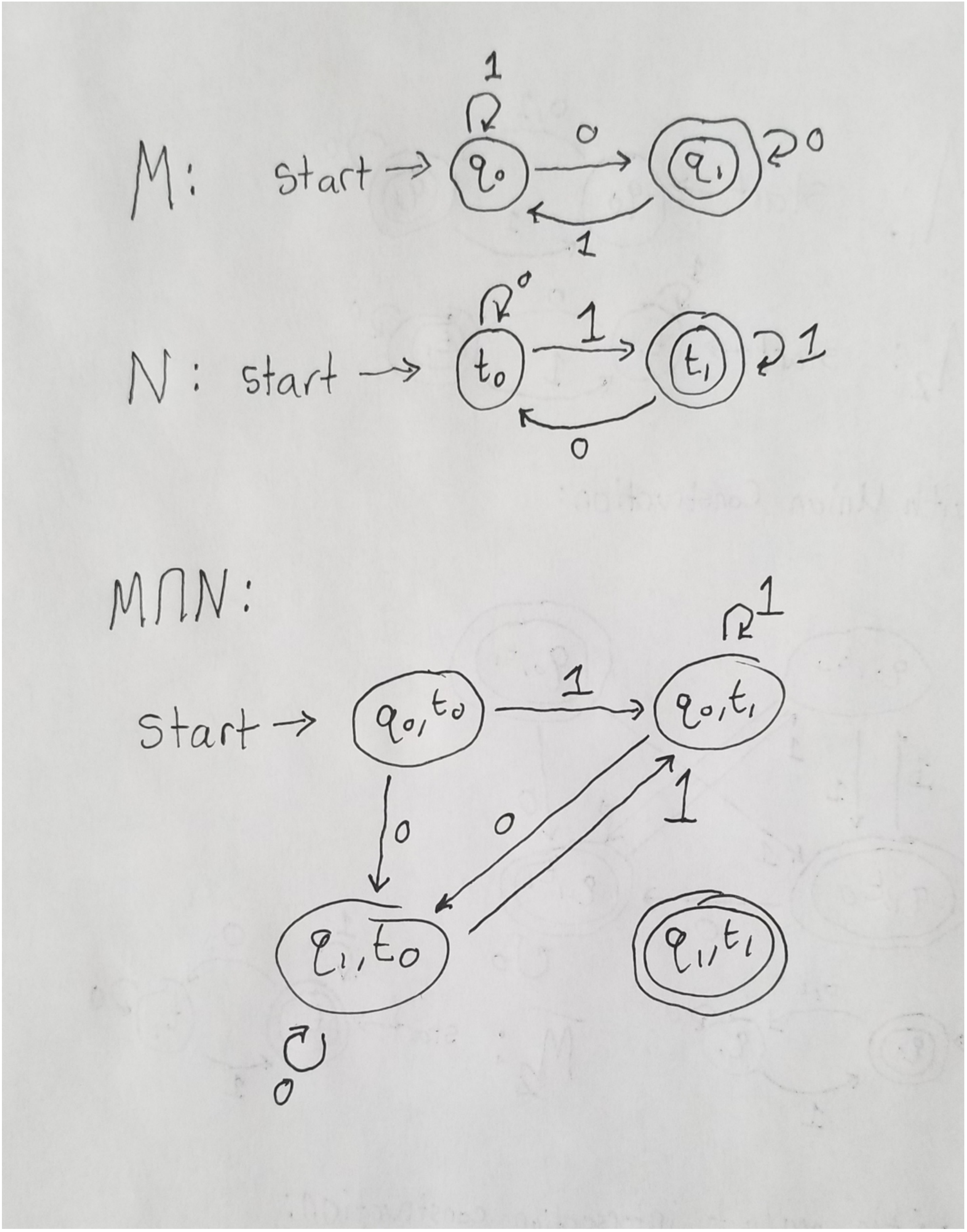
$\overline{M_1} \cap \overline{M_2}$ with product intersection construction:



Therefore, $\overline{\overline{M_1} \cap \overline{M_2}} = M_1 \cup M_2$.

After drawing out the example above, it was clear that the relationship between M''and M1 U M2 is that they are the same DFA, since M1' n M2' (aka M') is simply the complement of M1 U M2, as is visualized in the above drawings. It follows that L(M'') = L(M1 U M2).

DFAPuzzle)



M:
start
q_0
q_1
1
0
1
0
N:
start
t_0
t_1
0
1
0
1
$M \cap N$:
start
q_0, t_0
q_0, t_1
q_1, t_0
q_1, t_1
1
1
0
0
1
0

Since L(M) n L(N) = ∅, there are no possible strings that the DFA of M n N will accept. Accordingly, the diagram shows there is no way to reach the only accepting state of (q1,t1).