Lab2

November 26, 2023

Keith Farrugia 11104L

1 Imports

[ ]:
```
import os
from scipy import signal
import numpy as np
import math
from scipy.io import wavfile
from matplotlib import pyplot as plt
import IPython.display as ipd
from typing import List
```

2 Fourier Synthesis

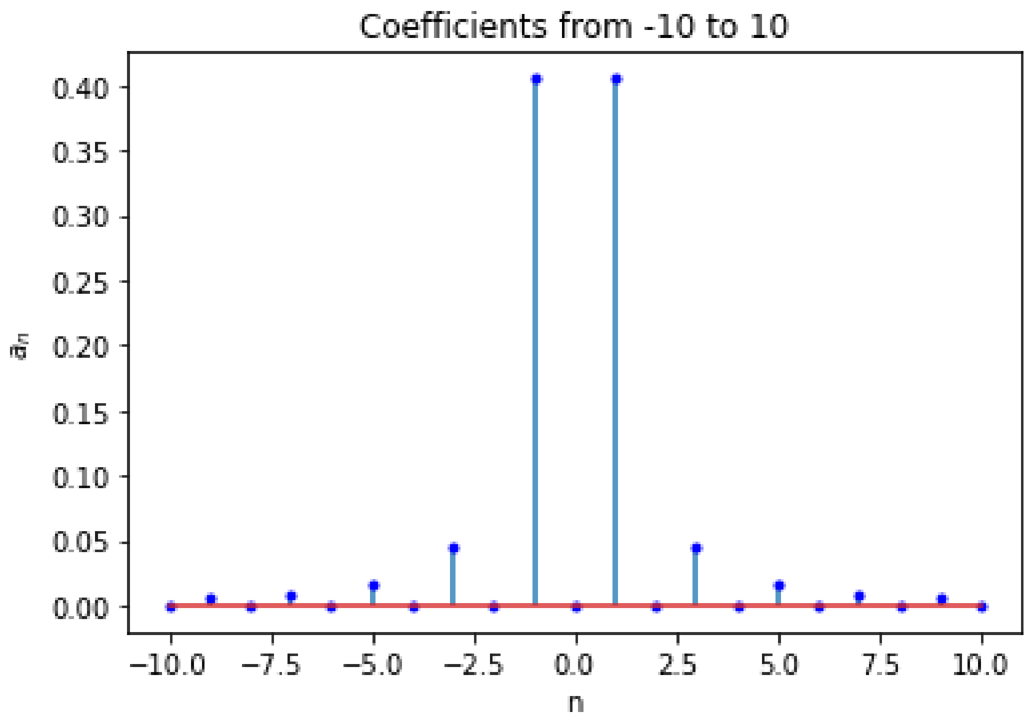
2.1 Function a_n

[ ]:
```
def a_n (n : int) :
    if (n % 2 == 0):
        return 0
    elif (n % 2 == 1) :
        return 4/(math.pow (math.pi,2) * math.pow(n, 2))
```

2.2 Coefficients s an for n $[-10, 10]$

[ ]:
```
def calcCoeff (range_start:int, range_end:int):
    array = [0] * (np.absolute(range_start) + np.absolute(range_end)+1)

    for i in range (0, len(array)) :
        array[i] = a_n(range_start + i)
    return array

y = calcCoeff(-10, 10)

x = np.arange(-10, 11)
```

```python
plt.figure()
plt.stem(x,y,markerfmt='b.')
plt.xlabel('n')
plt.ylabel('$a{_n}$')
plt.title('Coefficients from -10 to 10')
plt.show()
```



2.3 Simplify the truncated function

$$\hat{f}(t, \omega_0, nmax) = \sum_{k=-n_{max}}^{n_{max}} a_k e^{jkw_0 t}$$

$$\hat{f}(t, \omega_0, nmax) = \sum_{k=-n_{max}}^{n_{max}} a_k cos(kw_0 t) + a_k jsin(kw_0 t)$$

In order to plot the function, it first must be simplified.

Since the function An() is symetric about 0 the summation can be simplified in the following manner

$$\hat{f}(t, \omega_0, nmax) = \sum_{k=-n_{max}}^{n_{max}} a_k e^{jkw_0 t}$$

$$\hat{f}(t, \omega_0, nmax) = a_0 e^{j(0)w_0 t} + \sum_{k=1}^{n_{max}} a_k e^{jkw_0 t} + a_k e^{j(-k)w_0 t}$$

$$\hat{f}(t, \omega_0, nmax) = a_0 * 1 + \sum_{k=1}^{n_{max}} a_k e^{jkw_0 t} + a_k e^{-jkw_0 t}$$

But at 0:$a_n = 0$

$$\hat{f}(t, \omega_0, nmax) = \sum_{k=1}^{n_{max}} a_k e^{jkw_0 t} + a_k e^{-jkw_0 t}$$

$$\hat{f}(t, \omega_0, nmax) = \sum_{k=1}^{n_{max}} a_k cos(kw_0 t) + a_k jsin(kw_0 t) + a_k cos(-kw_0 t) + a_k jsin(-kw_0 t)$$

Since sin(-x) = -sin(x) and cos(-x) = cos(x)

$$\hat{f}(t, \omega_0, nmax) = \sum_{k=1}^{n_{max}} a_k cos(kw_0 t) + a_k jsin(kw_0 t) + a_k cos(kw_0 t) + -a_k jsin(kw_0 t)$$

hence the final statement

$$\hat{f}(t, \omega_0, nmax) = 2 \sum_{k=1}^{n_{max}} a_k cos(kw_0 t)$$

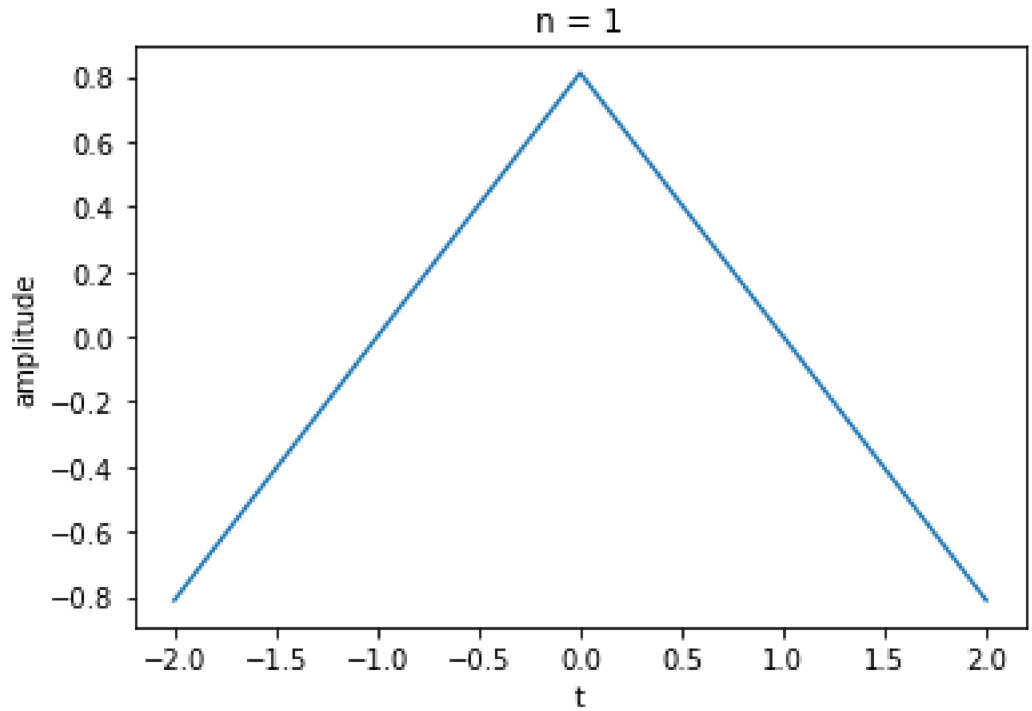
2.4 Implementing $\hat{f}$ (fhat)

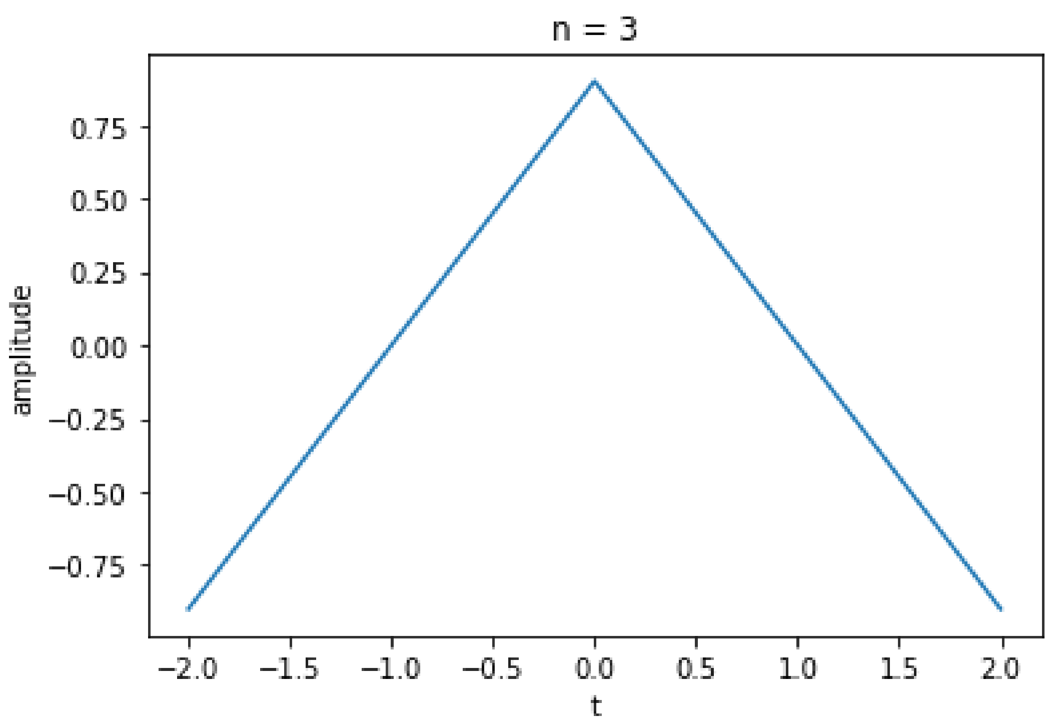
[ ]:
```
def fhat (t:int, omega0:int , n_max:int):
    total = 0
    for k in range(1, n_max+1):
        total += a_n(k)*np.cos(k*omega0*t)
    return 2*total
```

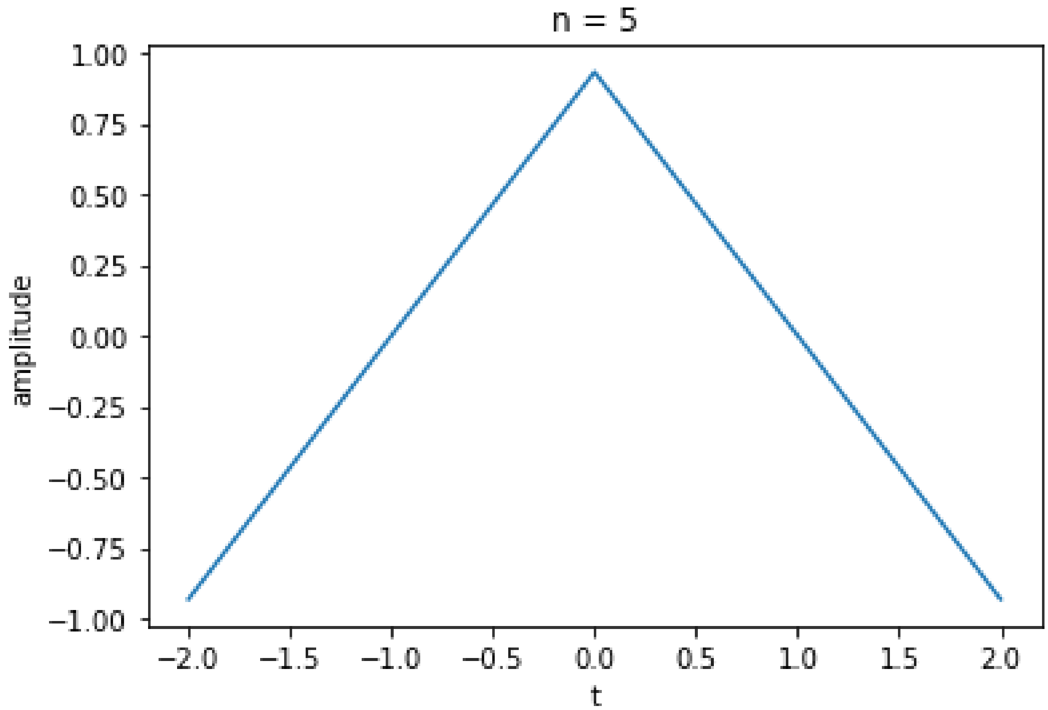
2.5 Plotting of truncated Fourier Synthesis

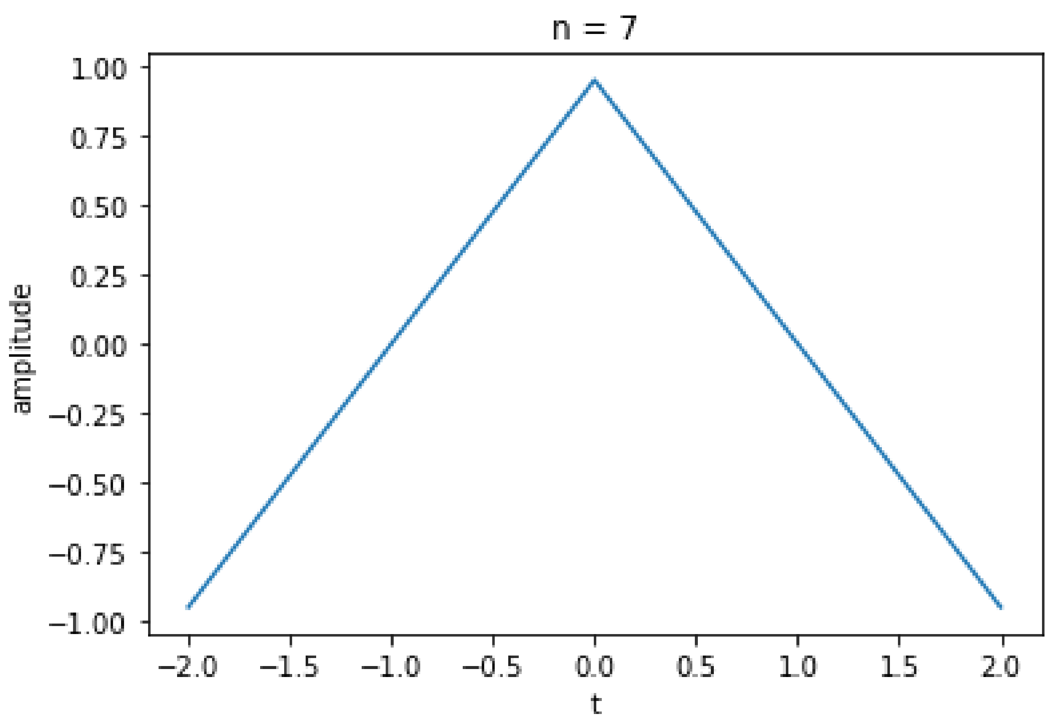
[ ]:
```
def fhat_array(t_start:int, t_end:int, n_max:int):
    period_T:int = 4
    omega0:int = (2*math.pi) / period_T
    array = [0] * (np.absolute(t_start) + np.absolute(t_end)+1)
    for t in range (0, len(array)) :
        array[t] = fhat(t_start+t, omega0, n_max)
    return array

def plot_Arrays(n_max):
    x = np.arange(-2, 3)
    for n in range(0, len(n_max)):
        y = fhat_array(-2, 2, n_max[n])
        plt.figure()
        plt.plot(x,y)
        plt.title('n = ' + str(n_max[n]))
        plt.xlabel('t')
        plt.ylabel('amplitude')
        plt.show()
```

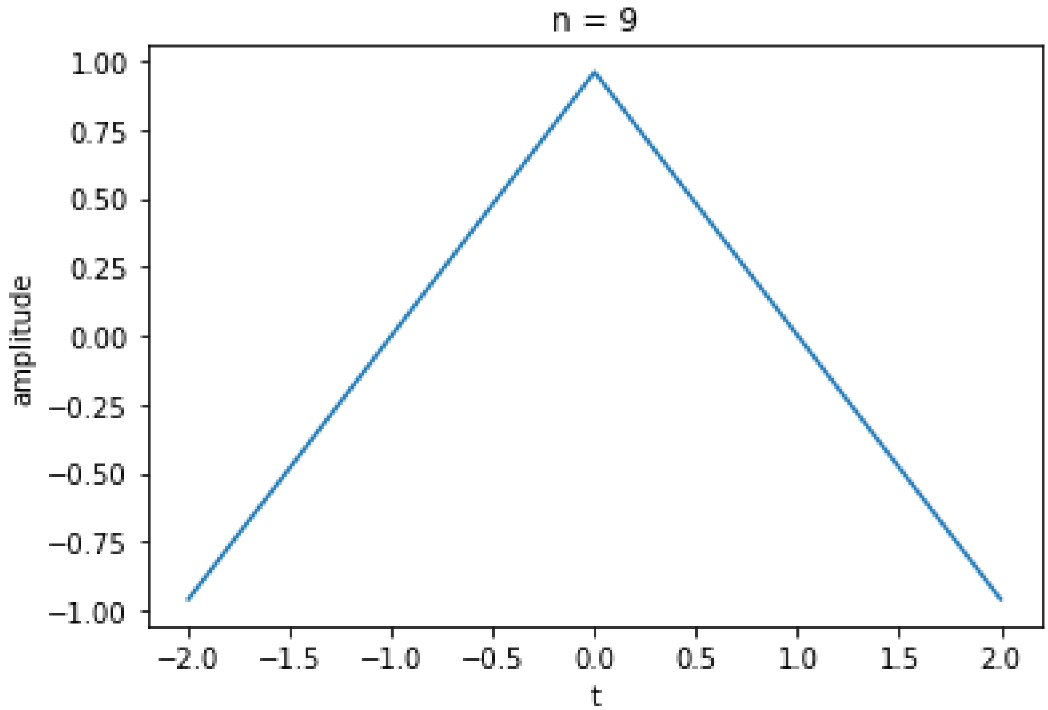
```
plot_Arrays([1,3,5,7,9])
```





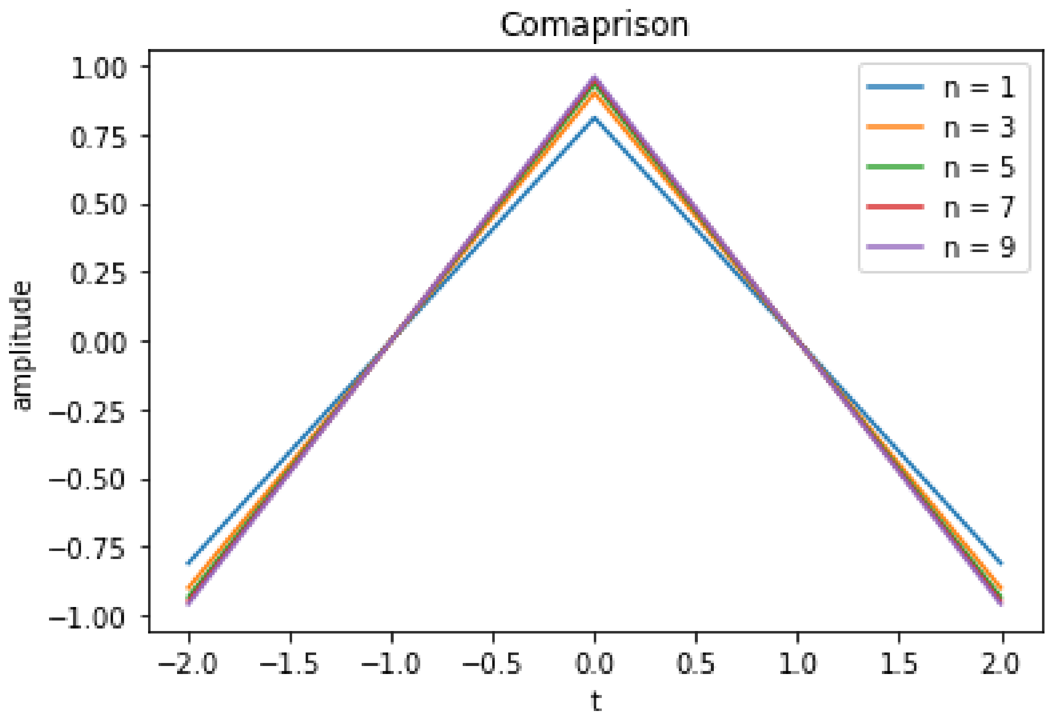

n = 5
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
amplitude
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
t


n = 7
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
amplitude
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
t



2.5.1 Comparison of truncated Fourier Synthesis

[ ]:
```
def plot_Arrays_Together(n_max):
    x = np.arange(-2, 3)
    plt.figure()
    plt.subplot()
    plt.title('Comaprison')
    for n in range(0, len(n_max)):
        y = fhat_array(-2, 2, n_max[n])
        plt.plot(x,y, label = 'n = ' + str(n_max[n]))
        plt.xlabel('t')
        plt.ylabel('amplitude')
    plt.legend()
    plt.show()

plot_Arrays_Together([1,3,5,7,9])
```



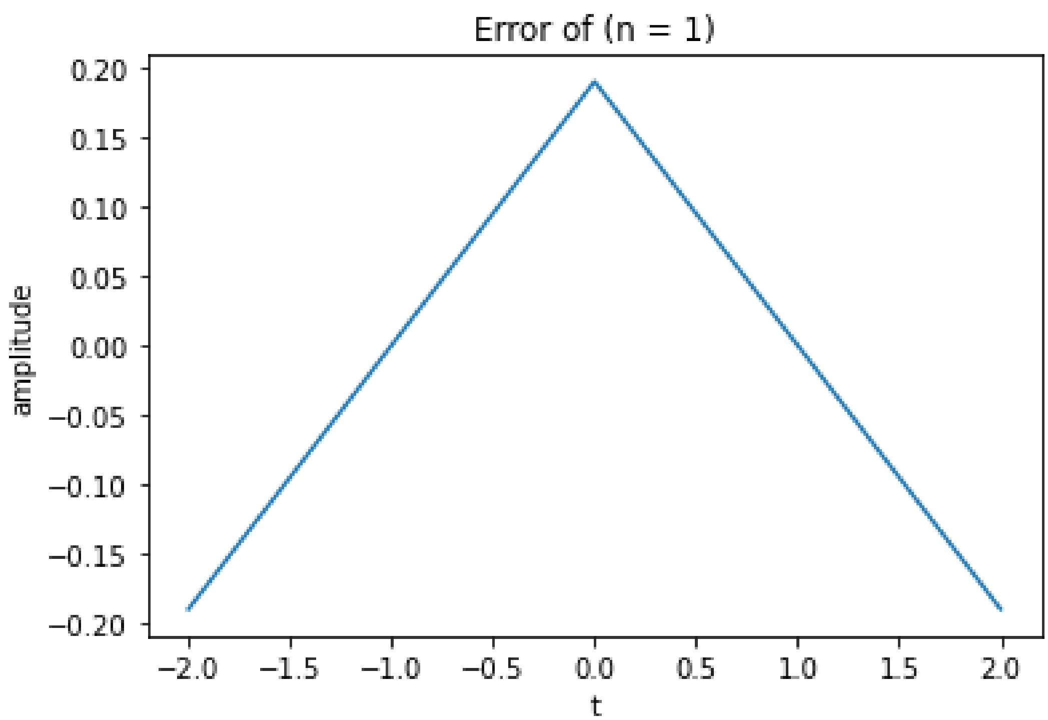
3 Error Analysis

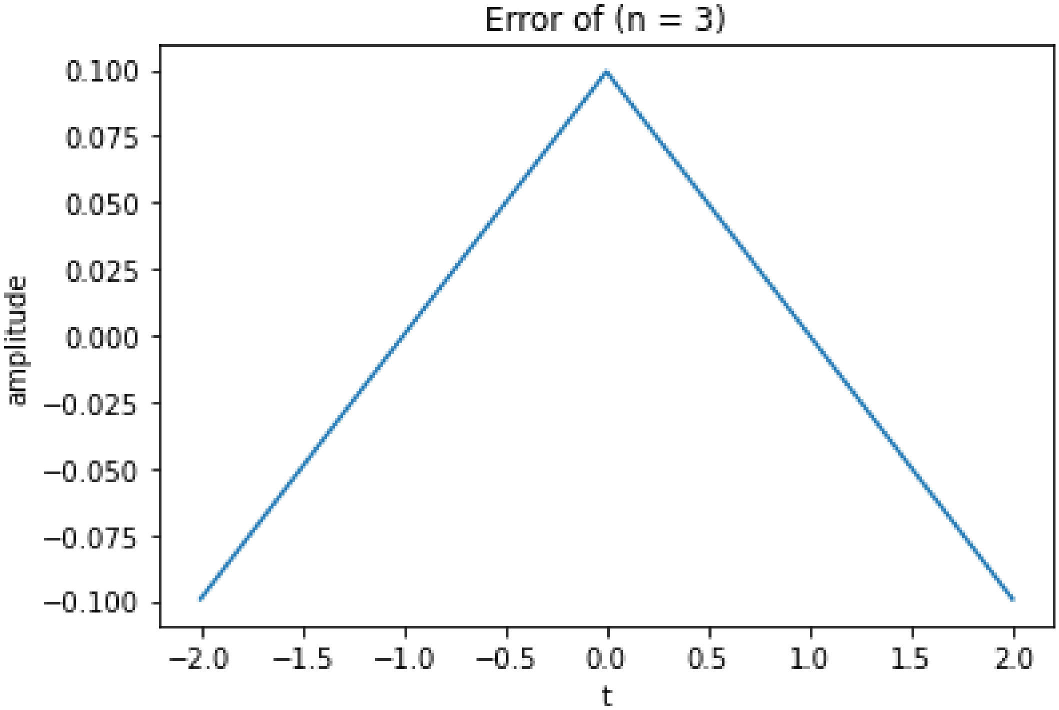
3.1 Defnining f(t)

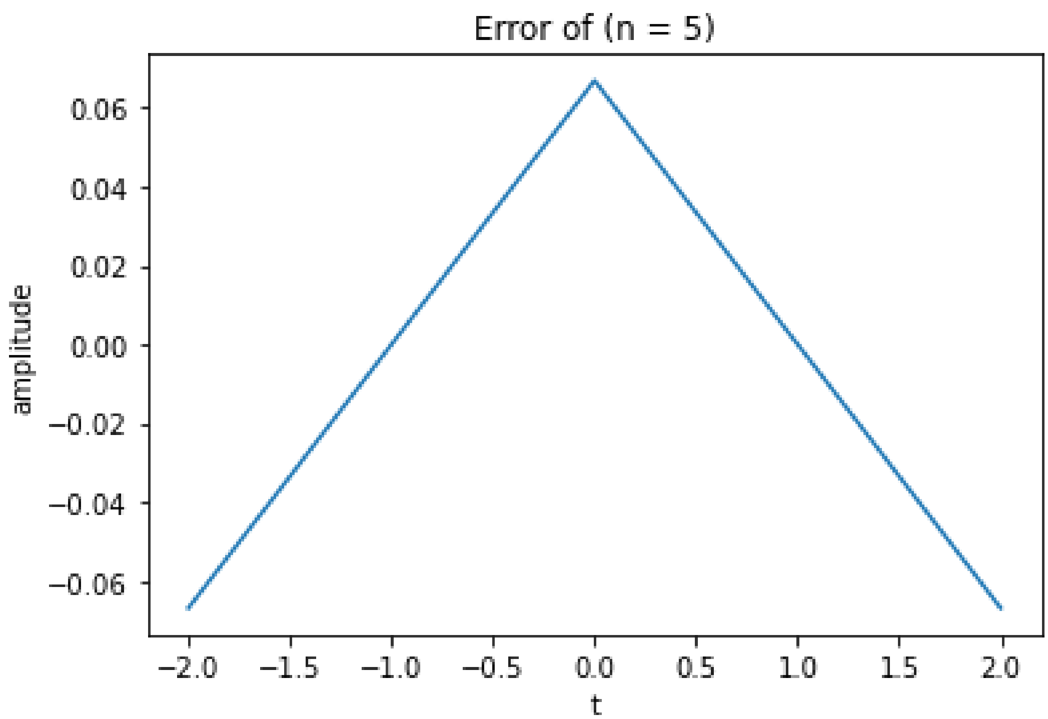
[ ]:
```
def f(t):
    if  (t >=  0 & t <= 2):
        return 1-t
    elif(t >= -2 & t <= 0):
        return 1+t
    else:
        return 0

def f_array (t_start:int, t_end:int):
    array = [0] * (np.absolute(t_start) + np.absolute(t_end)+1)
    for i in range(0, len(array)):
        array[i] = f(t_start + i)

    return array
```

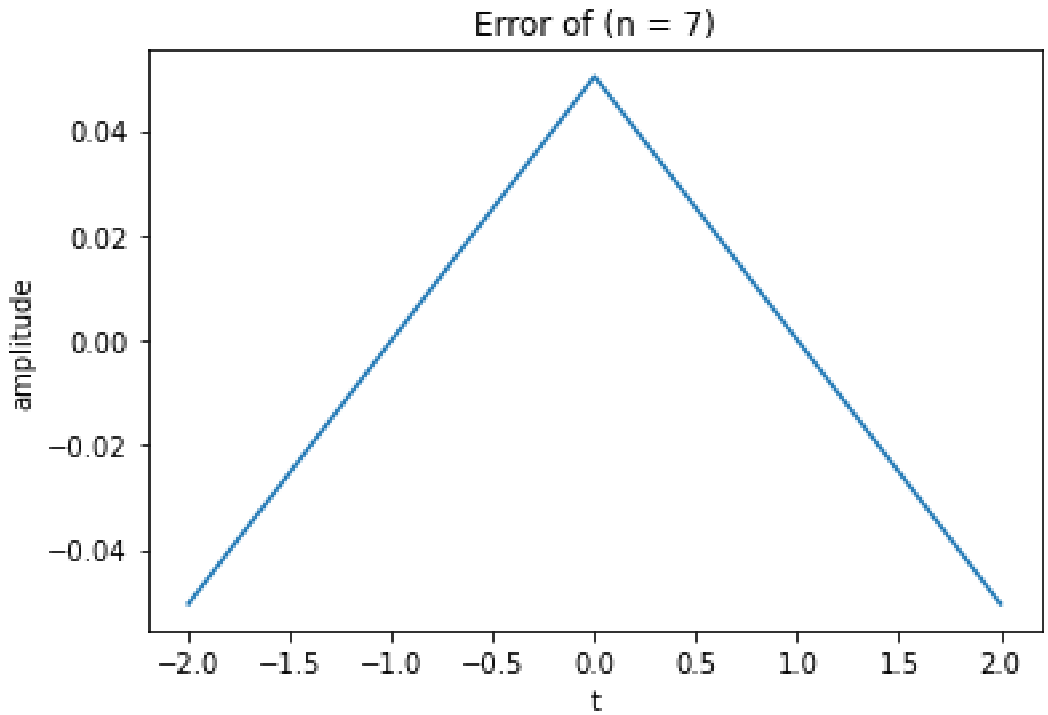
3.2 Plotting the errors

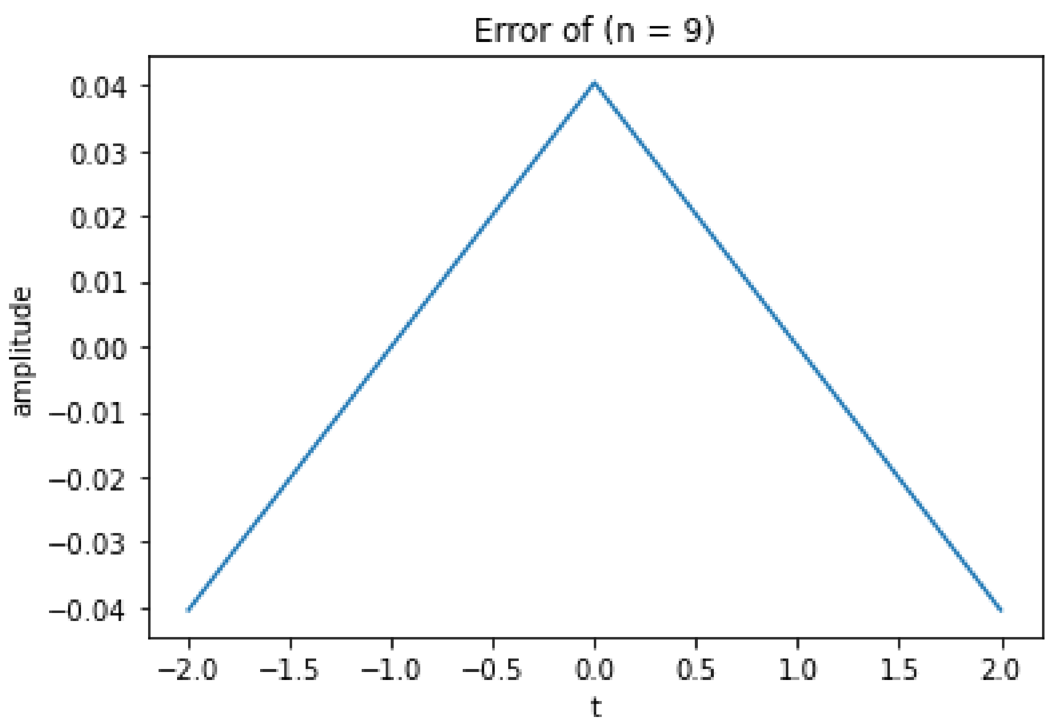
[ ]:
```
def calc_error(y_1, y_2):
    array = [0] * (min(len(y_1), len(y_2)))
    for i in range (0, min(len(y_1), len(y_2))):
        array[i] = y_1[i] - y_2[i]
    return array

y_f = f_array (-2, 2)

def plot_Error_Arrays(n_max):
    x = np.arange(-2, 3)
    for n in range(0, len(n_max)):

        y_fhat = fhat_array (-2, 2, n_max[n])
        y_error = calc_error(y_f,y_fhat)

        plt.figure()
        plt.plot(x,y_error)
        plt.title('Error of (n = ' + str(n_max[n])+')')
        plt.xlabel('t')
        plt.ylabel('amplitude')
        plt.show()
plot_Error_Arrays([1,3,5,7,9])
```




Error of (n = 3)
amplitude
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
t

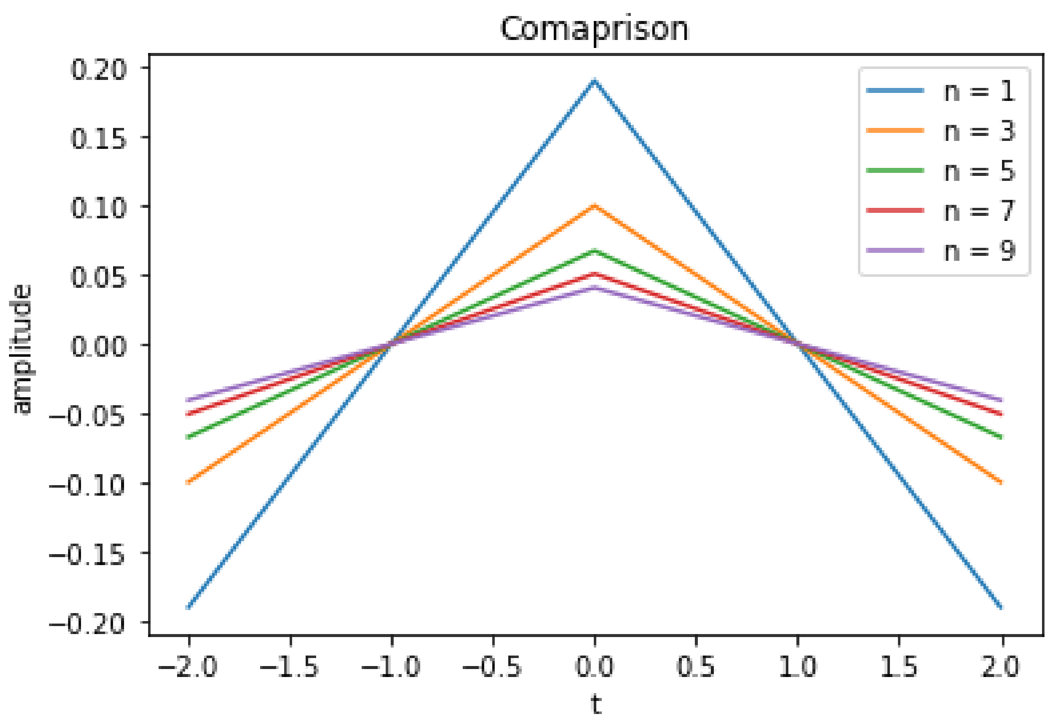

Error of (n = 5)
amplitude
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
t


Error of (n = 7)
0.04
0.02
0.00
−0.02
−0.04
amplitude
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
t


Error of (n = 9)
0.04
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
−0.04
amplitude
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
t

3.3 Comparing errors of truncated Fourier Syntesis

[ ]:
```
def plot_Arrays_Together_Error(n_max):
    x = np.arange(-2, 3)
    plt.figure()
    plt.subplot()
    plt.title('Comaprison')
    y_f = f_array (-2, 2)
    for n in range(0, len(n_max)):
        y_fhat = fhat_array (-2, 2, n_max[n])
        y_error = calc_error(y_f,y_fhat)
        plt.plot(x,y_error, label = 'n = ' + str(n_max[n]))
        plt.xlabel('t')
        plt.ylabel('amplitude')
    plt.legend()
    plt.show()

plot_Arrays_Together_Error([1,3,5,7,9])
```

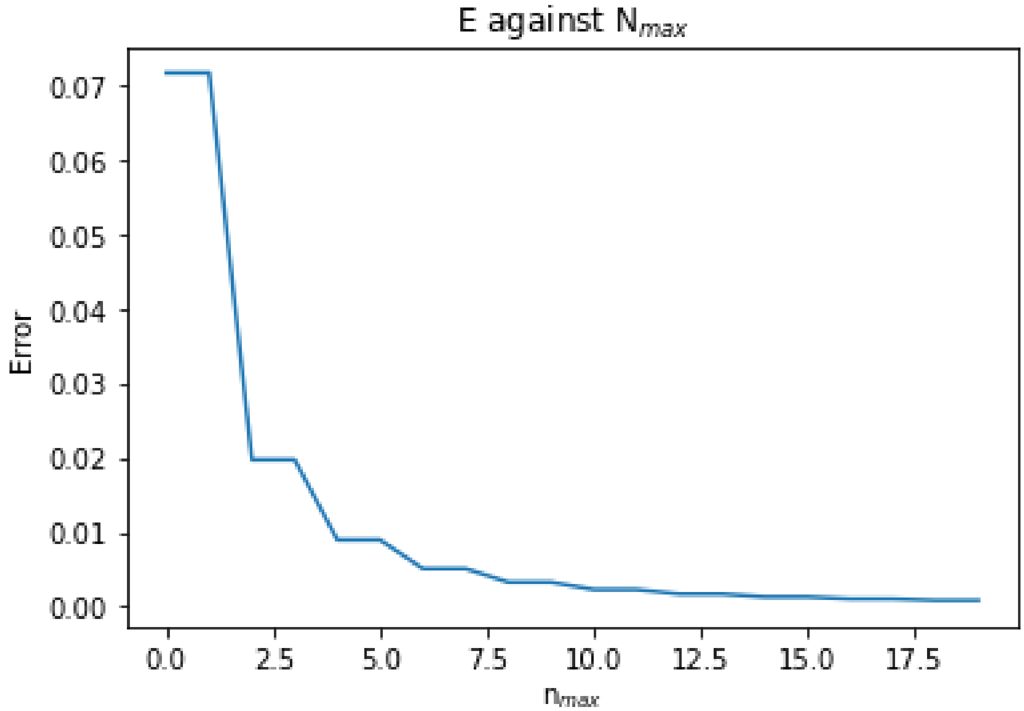


3.4 Function E

[ ]:
```
def E(y_1, y_2, x):
    array = [0] * (min(len(y_1), len(y_2)))
    for i in range (0, min(len(y_1), len(y_2))):
        array[i] = np.power(np.absolute(y_1[i] - y_2[i]),2)

    return np.trapz(x, array)
```

3.5 Plotting Of Values

[ ]:
```
def E_array(n_max):
    y_f = f_array (-2, 2)
    x = np.arange(-2, 3)
    e_array = [0]*(len(n_max))
    for n in range(0, len(n_max)):
        y_fhat = fhat_array(-2, 2, n_max[n])
        e_array[n] = E(y_f, y_fhat, x)
    return e_array

e_array = E_array(np.arange(1,21))
plt.figure()
plt.plot(e_array)
plt.title('E against N${_{max}}$')
plt.xlabel('n${_{max}}$')
plt.ylabel('Error')
plt.show()
```



3.6 Results

The results show that as the number of coefficients increases, the error values decrease. The graph becomes more accurate to the one plotted from F(). It can also be seen that although the error is decreasing, the more coefficiencts that are added the less of an effect it will have hence giving diminishing returns.