Homework 3

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1. Use the definition of the limit of a sequence to show that:

(i) $\{\frac{n^2+n+1}{3n^2+1}\} \to \frac{1}{3}$

$$\left|\frac{n^2+n+1}{3n^2+1} - \frac{1}{3}\right| < \epsilon$$

$$\left|\frac{n+\frac{2}{3}}{3n^2+1}\right| < \epsilon$$

$$\frac{n+\frac{2}{3}}{3n^2+1} < \epsilon$$

positive for $n \in \mathbb{N}$

$$\frac{n+\frac{2}{3}}{\epsilon} < 3n^2+1$$

$$\frac{1}{\epsilon'} < 3n^2+1$$

$$\frac{1-\epsilon'}{3\epsilon'} < n^2$$

$$n \geq \left\lceil \sqrt{\frac{1-\epsilon'}{3\epsilon'}} \right\rceil = N_\epsilon$$

$$\exists N_\epsilon : \forall n \geq N_\epsilon \Rightarrow \left|\frac{n^2+n+1}{3n^2+1} - \frac{1}{3}\right| < \epsilon$$

$$\therefore$$

$$\boxed{\{\frac{n^2+n+1}{3n^2+1}\} \to \frac{1}{3}}$$

(ii) $\{10 - \frac{1}{\sqrt{n+\sqrt{n+5}}}\} \to 10$

$$x$$