Stevens Institute of Technology

Financial Engineering

Visualization of Volatility Surface and Swaption Market Monitor

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Abstract

There are two parts in this project. For the first part, local volatility surface is built using two different approaches, the first method is to calculate the local volatility by using option price, the second method is to calculate local volatility using implied volatility. Then the results from both methods and the comparison implied volatility surface are displayed in R shiny. In addition to the visualization, the offline study of two methods estimate the percentage of error of both methods. For the second part, a Swaption market monitor application is built based on SABR model and R Shiny dashboard. This monitor can offer both market quote volatility surface and SABR volatility surface. A backtesting strategy is exercised and the returns and profits are shown in UI. This information can offer traders some trading suggestions.
Keywords: SABR Model, Local volatility, Swaption Market Monitor

Contents

1 Introduction

The goal of this project is to compute and visualize the local volatility surfaces for European options, monitor the swaption market and build up an application that generates visualizations of the results. Generally, the volatility shows the dispersion of returns for a given security or market index, which represent the how volatile the price is. Therefore, the volatility is one of the most important parameter for pricing an option or swaption.

The implied volatility of an option contract is that value of the volatility of the underlying instrument which, when input in an option pricing model will return a theoretical value equal to the current market price of the option. The Black-sholes-merton (Black & Scholes, 1973) model assumes an implied volatility, however the local volatility is a function of both current asset level and time. The local volatility surface is a three-dimensional surface that the x axis is strike price, y axis is time and z axis is the local volatility. Considering there are close relationships between implied volatility, this project implemented two methods for local volatility estimation with implied volatility as data input. The first method is to use definition formula of local volatility, and the second method is to use the Dupire formula in the Piterbarg option pricing model.

Since the swaption market quote volatilities as prices, the SABR model can fit this market well. The SABR model (Stochastic alpha, beta and rho) is an extension of BlackScholess model and the CEV model (constant elasticity of variance model which is a stochastic volatility model). SABR model can be described in three equations. We will introduce the details of SABR model later in literature review. The prices of European call options in the SABR model are given by BlackScholess model. For a current forward rate f, strike K, and implied volatility the price of a European call option with maturity T is

$$C_B(f, K, \sigma_B, T) = e^{-rT}[fN(d1) - KN(d2)]$$

Where

$$d_{1,2} = \frac{lnf/K \pm \frac{1}{2}\sigma_B^2 T}{\sigma_B \sqrt{T}}$$

The volatility parameter σ_B is provided by the SABR model. With estimate of parameters of SABR model the implied volatility is directly calculated.

After calibration part, we obtain a series of model volatilities. Then, we calculate the confidence interval of these series of model volatilities and compare with the market quote volatilities. We exercise our trading strategy and visualize the results of comparison and return of our strategy by using R package shinny dashboard and plotly.

2 Background

2.1 Option

In finance, an option is a contract which gives the buyer (the owner or holder of the option) the right, but not the obligation, to buy or sell an underlying asset or instrument at a specific strike price on a specified date, depending on the form of the option. There are two types of option. A call option gives the holder the right to buy the underlying asset by a certain date for a certain price. A put option gives the holder the right to sell the underlying asset by a certain date for a certain price. The price in the contract is known as the exercise price or strike price; the date in the contract is known as the expiration date or maturity. American options can be exercised at any time up to the expiration date. European options can be exercised only on the expiration date itself. Hull (2006)

2.2 Swaption

Swaptions are essentially European calls and puts on the forward swap rate. It is the option to enter into a swap. A receiver swaption is the option to receive the fixed side of a swap and pay the floating leg starting at the swaption maturity. A payer swaption is the option to enter into a swap that pays the fixed side of a swap and receives the floating leg. For example, a "1 year into 5 year receiver swaption struck at 1.5%gives the holder the right to begin a 5 year maturity swap starting in 1 year where the holder receives 1.5% (semi-annual, 30/360, 5 year maturity) and pays 3M LIBOR (quarterly, act/360, 5 yr maturity). If the actual 5 year spot swap rate in 1 yr is less than the strike of 1.5%, the holder makes money.

The market quotes for swaptions are quoted as implied volatilities via Blacks formula. To use this formula, one first needs an expression for a forward starting swap. The market convention pricing of a receiver swaption uses Blacks formula for a put option on the swap rate, K is strike,t_s is expiry, T is tenor and $S(0, t_s, T)$ is forward swap rate.Chatterjee (2014)Clewlow and Strickland (1996)

$$
\begin{aligned}
Swaption_{receiver} &= DV01 * [KN(-d_2) - S(0, t_s, T)N(-d_1)] \\
d_1 &= \frac{ln(S(0, t_s, T)/K) + \sigma^2 t_s/2}{\sigma\sqrt{t_s}} \\
d_2 &= d_1 - \sigma\sqrt{t_s} \\
DV01 &= \sum_{i=1}^{n_{fixed}} \Delta_i df(0, t_i^{fixed})
\end{aligned}
$$

3 Literature review and Methodology

3.1 SABR Model

3.1.1 Description

In mathematical finance, the SABR model is a stochastic volatility model, which attempts to capture the volatility smile in derivatives markets. The name stands for "stochastic alpha, beta, rho", referring to the parameters of the model. The SABR model is widely used by practitioners in the financial industry, especially in the interest rate derivative markets. It was developed by Patrick S. Hagan, Deep Kumar, Andrew Lesniewski, and Diana Woodward. SABR model can be described in flowing three equations

$$\begin{aligned} df_t &= \alpha_t f_t^{\beta} dW_t^1 \\ d\alpha_t &= \nu \alpha_t dW_t^2 \\ E[dW_t^1 dW_t^2] &= \rho dt \end{aligned}$$

with initial values f_0 and $\alpha = \alpha_0$

In these equations, f_t is the forward rate of a specified instrument, α_t is the volatility of f_t, and W_t^1 and W_t^2 are correlated Brownian motions, with correlation ρ. β is the exponent for the forward rate and range in $[1, 0]$.

3.1.2 Solving the SABR Model

For our project the most important work is to calculate the volatilities of swaptions. By the paper *"Managing Smile Risk"* Hagan et al. (2002), the following approximation to the implied volatility resulting from the SABR model is presented:

$$\sigma_B(K,f) = \frac{\alpha_0 (1 + (\frac{(1-\beta)^2}{24} \frac{\alpha_0^2}{(fK)^{1-\beta}} + \frac{1}{4} \frac{\rho\beta\nu\alpha_0}{(fK)^{(1-\beta)/2}} + \frac{2-3\rho^2}{24}\nu^2)T}{(fk)^{(1-\beta)/2}(1 + \frac{(1-\beta)^2}{24} ln^2(\frac{f}{K}) + \frac{(1-\beta)^4}{1920} ln^4(\frac{f}{K}))} * \frac{z}{\chi(z)} \tag{1}$$

Where

$$\begin{aligned} z &= \frac{\nu}{\alpha_0}(fK)^{(1-\beta)/2} ln(\frac{f}{K}) \\ \chi(z) &= ln(\frac{\sqrt{1 - 2\rho z + z^2} + z - \rho}{1 - \rho}) \end{aligned}$$

In our project the swaptions are all ATM f=K, so the formula can be simplified to

$$\sigma_B(f,f) = \frac{\alpha_0}{f^{(1-\beta)}}(1 + (\frac{(1-\beta)^2}{24} \frac{\alpha_0^2}{f^{(2-2\beta)}} + \frac{1}{4} \frac{\rho\beta\nu\alpha_0}{f^{(1-\beta)}} + \frac{2-3\rho^2}{24}\nu^2)T) \tag{2}$$

In this formula the unknown parameters are α,β,ρ,and ν so next step is to estimate these parameters.

Estimating β
Taking logs on both sides of equation (2)to can obtain:

$$ln\sigma_B(f,f) = ln\alpha - (1-\beta)lnf + ln(1 + (\frac{(1-\beta)^2}{24}\frac{\alpha_0^2}{(fK)^{1-\beta}} + \frac{1}{4}\frac{\rho\beta\nu\alpha_0}{(fK)^{(1-\beta)/2}} + \frac{2-3\rho^2}{24}\nu^2)T) \tag{3}$$

Ignore the third term, the equation can be simplified to an approximation:

$$ln\sigma_{ATM} \approx ln\alpha - (1-\beta)lnf \tag{4}$$

By this formula, β can be estimated by a linear regression on two time series $ln\sigma_{ATM}$ and lnf_{ATM}. β can also be chosen from prior beliefs about which model is appropriate. Based on the analysis of Hansen, setting β to 0.5 can get better result in most cases. So in this project we fix β=0.5. Hansen (2011)

First Parameter Estimating α,ρ,ν
Once β has been estimated , the next step is to estimate the rest parametersα,ρ and ν. The first method is to estimate these three parameter in the same time. This goes to minimize the errors between the model results σ_{model} and market data σ_{market}. In this project SSE is used, thus this can be described as an equation:

$$(\widehat{\alpha},\widehat{\rho},\widehat{\nu}) = \arg\min_{\alpha,\rho,\nu}\sum_i [\alpha_i^{mkt} - \sigma_B(f_i,K_i;\alpha,\rho,\nu)]^2$$

Hansen (2011) To solve this,the optimize function nlmin in R is used in this project.

Second Parameter Estimating ρ,ν
The second method is to reduce the numbers of parameters which need to be estimated. Equation (2) can be transform into a polynomial of α

$$[\frac{(1-\beta)^2T}{24f^{2-2\beta}}]\alpha^3 + [\frac{\rho\beta\nu T}{4f^{1-beta}}]\alpha^2 + [1 + \frac{2-3\rho^2}{24}\nu^2T]\alpha - \sigma_{ATM}f^{1-\beta} = 0 \tag{5}$$

A single root can be find by solbing this equation, so α can be described by a $\alpha = \alpha(\rho,\nu)$. Hence ρ and ν are the parameters which need to be estimated, and then plug them into $\alpha(\rho,\nu)$ to obtain α. The same method is used as above, thus this can be described as

$$(\widehat{\alpha},\widehat{\rho},\widehat{\nu}) = \arg\min_{\alpha,\rho,\nu}\sum_i [\alpha_i^{mkt} - \sigma_B(f_i,K_i;\alpha(\rho,\nu),\rho,\nu)]^2$$

Rouah (Rouah)

3.2 Local volatilities

3.2.1 Definition of local volatility

The first method (The volatility surface) is to use the definition of local volatility

$$\sigma = \sqrt{\frac{\frac{\partial C}{\partial T}}{\frac{1}{2}K^2\frac{\partial^2 C}{\partial K^2}}}$$

Where C is the European call option price, K is strike price and T is expiry. First, Option price surface will be estimated using Black-Scholes formula, then, use the equation above to compute the local volatility. Gatheral (2011)

3.2.2 Local volatility in terms of implied volatility using Dupire formula in Piterbarg framework

Using Dupire formula in the Piterbarg model Labuschagne and von Boetticher (2016), there will three formulas for local volatility as function of option price surface for different scenarios of collateralization. Then, they can be transformed into one expression for local volatility in term of implied volatility.

$$\sigma^2(S,T) = \frac{\sigma_{imp}^2 + 2\sigma_{imp}T(\frac{\partial\sigma_{imp}}{\partial T} + (r_R(T) - q(T))K\frac{\partial\sigma_{imp}}{\partial K})}{(1 + Kd_1\sqrt{T}\frac{\partial\sigma_{imp}}{\partial K})^2 + K^2T\sigma_{imp}(\frac{\partial^2\sigma_{imp}}{\partial K^2} - d_1\sqrt{T}(\frac{\partial\sigma_{imp}}{\partial K})^2)}$$

Where:

$$d_1 = \frac{ln(\frac{S_0}{K}) + \frac{1}{2}\sigma_{imp}^2 T}{\sigma_{imp}\sqrt{T}}$$

3.3 Data collection

3.3.1 Option data

In this paper, the option implied volatilities data is downloaded from Bloomberg GV.

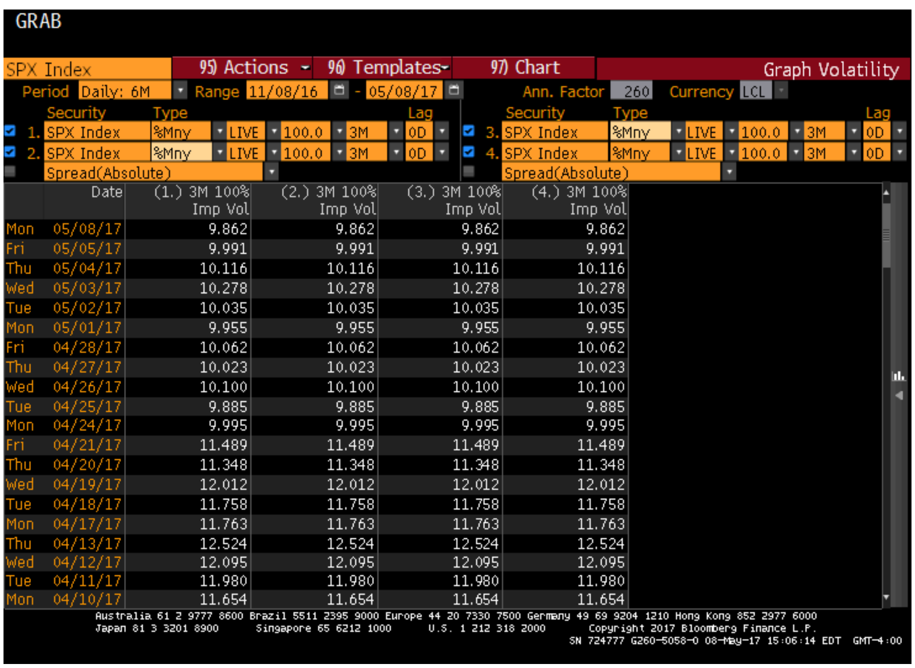


Figure 1: Screen shot of Bloomberg GV

In GV, user can input the name of security and choose Type as %Mny (moneyness). The 4th term is the value of moneyness, in the figure it shows 100, which means the

%Mny equal to 100%. %Mny ranges from 80% to 120%. The 5th term is the maturities of the option which range from 1M to 2Yr. After setting, user can click Actions and then Expert to excel to download these implied volatility data. For each maturity, there are two excel files which are strike price less or equal than 100% moneyness and strike price greater than 100% moneyness. In this case, there are 14 files downloaded from Bloomberg. To get data prepared to be used in the model, 14 files was combine into one dataset and convert the implied volatilities into percentage so that the implied volatility matrix (implied volatility respect to different time to maturity and percentages of moneyness) can be attained when selecting a specific date.

3.3.2 Swaption data

In this paper, ATM USD swapiton quote volatilities and forward swap rates are used to fit SABR model. Through the VCUB in Bloomberg we can have an overview of daily data of swaptions.

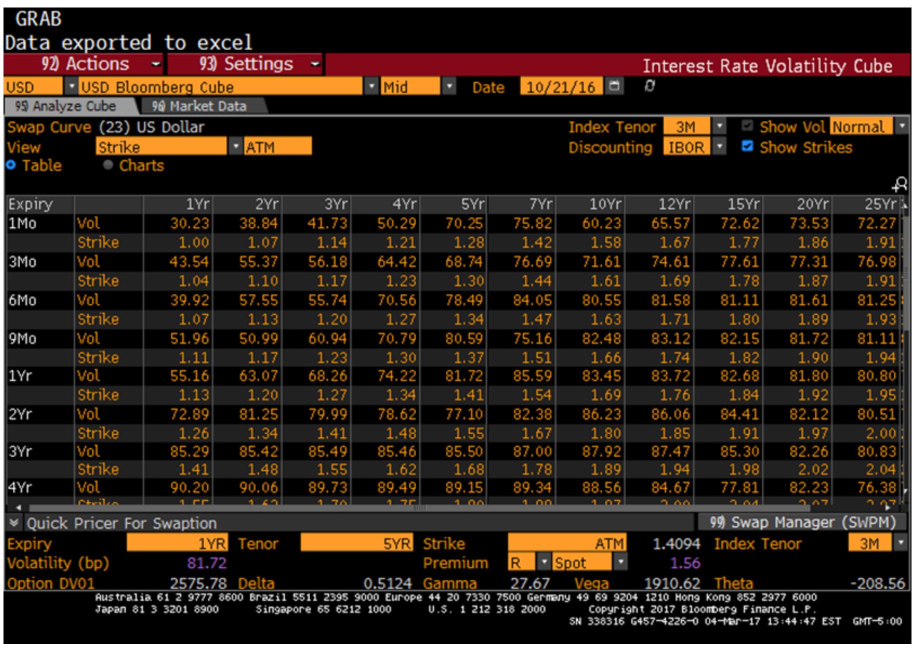


Figure 2: Daily data of swaptions

This table contains the quote volatilities and ATM strikes on a specified date. The stickers of all swaptions can be obtained from this table. The ticker contains instrument name, tenor and expiry (eg: USSV + Expiry + tenor).Where USSV means USD ATM swaption, Expiry range in 1M, 3M, 6M, 9M, 1Y, 2Y, 3Y, 4Y, 5Y, 6Y, 7Y, 8Y, 9Y, 10Y, 15Y, 20Y, 25Y, 30 Y (which are presented by A, C, F 01to 30). To download multiple daily data we need to use Bloomberg excel add in. By using the function BDH and tickers of swaptions and tickers of forward swap rate, we can download both swaptions quote volatilities and ATM strikes.

For example, *BDH(USSV0A1 BBIR Curncy, "PX_LAST","1/1/2016","1/1/2017", "Dir=V","Dts=H","Sort=A","Quote=C","QtTyp=Y","Days=T", "Per=cd",* *"DtFmt=D","UseDPDF=Y","cols=1;rows=261")* This formula will download the Last quote volatilities of the 1M into 1 Yr swaption from 1/1/2016 to 1/1/2017

Date	USSV0A1	USSV0A2	USSV0A3	USSV0A4	USSV0A5	USSV0A6	USSV0A7	USSV0A8	USSV0A9	USSV0A10	USSV0A12
2016/1/1	31.42	36.555	38.065	38.985	38.945	36.865	35.195	33.38	31.565	30.725	29.87
2016/1/4	34.525	37.57	38.96	40.6	39.485	37.215	35.535	33.715	32.23	30.97	30.1
2016/1/5	44.085	38.16	43.545	40.665	42.215	37.86	36.085	34.17	36.1	35.185	32.705
2016/1/6	45.135	39.735	41.405	42.485	41.995	39.43	37.475	35.42	33.75	32.325	31.33
2016/1/7	43.645	41.69	43.345	45.925	43.435	43.585	40.22	36.14	34.365	32.845	31.775
2016/1/8	48.99	49.99	48.98	45.835	50.975	41.965	45.525	37.35	35.485	39.685	36.745
2016/1/11	46.085	42.565	44.475	45.28	44.34	41.275	38.965	36.645	38.815	33.22	32.105
2016/1/12	43.42	43.505	45.76	49.835	45.87	42.72	42.115	37.9	40.305	34.355	33.2
2016/1/13	46.49	49.72	51.51	51.04	50.37	45.58	44.305	40.605	38.935	37.85	36.085
2016/1/14	47.49	48.765	50.655	50.62	47.08	43.665	41.055	38.495	36.44	34.74	34.615
2016/1/15	48.99	53.83	55.53	54.365	49.01	45.205	46.03	39.585	37.37	38.87	36.28
2016/1/18	51.17	47.32	49.8	50.33	48.545	45.125	42.15	39.54	37.325	35.545	34.295
2016/1/19	47.69	50.2	54.27	53.305	52.185	44.425	41.735	39.065	44.32	35.24	34.025
2016/1/20	49.395	55.04	54.52	55.73	54.415	50.995	43.165	40.35	47.505	36.32	35.05
2016/1/21	56.995	62.92	60.21	54.96	53.115	45.055	46.175	39.42	45.54	35.445	34.175
2016/1/22	53.06	52.025	52.935	48.9	47.44	43.93	41.345	38.76	36.715	35.01	33.795
2016/1/25	56.3	61.735	49.585	54.22	48.64	45.035	42.37	39.715	37.6	35.83	34.53
2016/1/26	56.9	62.365	61.145	54.77	48.805	48.775	48.945	39.87	44.315	35.97	34.66
2016/1/27	57.2	62.655	52.725	50.55	49.025	45.23	43.735	39.68	40.575	35.69	34.365

Figure 3: swaption quote volatilities data

Date	USSV0A1	USSV0A2	USSV0A3	USSV0A4	USSV0A5	USSV0A6	USSV0A7	USSV0A8	USSV0A9	USSV0A10	USSV0A12
2016/1/1	31.42	36.555	38.065	38.985	38.945	36.865	35.195	33.38	31.565	30.725	29.87
2016/1/4	34.525	37.57	38.96	40.6	39.485	37.215	35.535	33.715	32.23	30.97	30.1
2016/1/5	44.085	38.16	43.545	40.665	42.215	37.86	36.085	34.17	36.1	35.185	32.705
2016/1/6	45.135	39.735	41.405	42.485	41.995	39.43	37.475	35.42	33.75	32.325	31.33
2016/1/7	43.645	41.69	43.345	45.925	43.435	43.585	40.22	36.14	34.365	32.845	31.775
2016/1/8	48.99	49.99	48.98	45.835	50.975	41.965	45.525	37.35	35.485	39.685	36.745
2016/1/11	46.085	42.565	44.475	45.28	44.34	41.275	38.965	36.645	38.815	33.22	32.105
2016/1/12	43.42	43.505	45.76	49.835	45.87	42.72	42.115	37.9	40.305	34.355	33.2
2016/1/13	46.49	49.72	51.51	51.04	50.37	45.58	44.305	40.605	38.935	37.85	36.085
2016/1/14	47.49	48.765	50.655	50.62	47.08	43.665	41.055	38.495	36.44	34.74	34.615
2016/1/15	48.99	53.83	55.53	54.365	49.01	45.205	46.03	39.585	37.37	38.87	36.28
2016/1/18	51.17	47.32	49.8	50.33	48.545	45.125	42.15	39.54	37.325	35.545	34.295
2016/1/19	47.69	50.2	54.27	53.305	52.185	44.425	41.735	39.065	44.32	35.24	34.025
2016/1/20	49.395	55.04	54.52	55.73	54.415	50.995	43.165	40.35	47.505	36.32	35.05
2016/1/21	56.995	62.92	60.21	54.96	53.115	45.055	46.175	39.42	45.54	35.445	34.175
2016/1/22	53.06	52.025	52.935	48.9	47.44	43.93	41.345	38.76	36.715	35.01	33.795
2016/1/25	56.3	61.735	49.585	54.22	48.64	45.035	42.37	39.715	37.6	35.83	34.53
2016/1/26	56.9	62.365	61.145	54.77	48.805	48.775	48.945	39.87	44.315	35.97	34.66
2016/1/27	57.2	62.655	52.725	50.55	49.025	45.23	43.735	39.68	40.575	35.69	34.365

Figure 4: Forward swap rate data

For this project the daily data of swaption and forward swap rate ranges from 2016/1/1 to 2017/1/1.

3.3.3 Yield Curve

To calculate swaption price, a yield curve need to be built to calculate DV01, so downloading short term LIBOR rate and swap rate is necessary. This data can also be downloaded by using Bloomberg excel add-in and tickers.

	US0001M Index	US0003M Index	US0006M Index	US0009M Index	US0012M Index	USSWAP2 Curncy	USSWAP3 Curncy	USSWAP4 Curncy
Date	PX_LAST	PX_LAST	PX_LAST	PX_LAST	PX_LAST	PX_LAST	PX_LAST	PX_LAST
2016/1/4	0.4225	0.6117	0.84225	1.06025	1.16925	1.1529	1.385	1.5648
2016/1/5	0.422	0.6171	0.8512	1.0658	1.1731	1.1323	1.3568	1.5315
2016/1/6	0.4235	0.6201	0.8513	1.060666667	1.16535	1.0878	1.3023	1.4662
2016/1/7	0.4235	0.61685	0.8448	1.046333333	1.1471	1.043	1.2483	1.4135
2016/1/8	0.4238	0.6211	0.8508	1.053666667	1.1551	1.0195	1.2088	1.3653
2016/1/11	0.424	0.6221	0.8513	1.052833333	1.1536	1.024	1.2143	1.3772
2016/1/12	0.4245	0.6236	0.8548	1.055666667	1.1561	0.9959	1.1779	1.3323
2016/1/13	0.4255	0.622	0.8597	1.061966667	1.1631	0.9861	1.1648	1.3188
2016/1/14	0.4255	0.6211	0.8573	1.058166667	1.1586	0.9618	1.139	1.292
2016/1/15	0.4255	0.6196	0.84905	1.046416667	1.1451	0.91	1.0805	1.2345
2016/1/18	0.426	0.6238	0.85325	1.048616667	1.1463	0.8995	1.065	1.237
2016/1/19	0.425	0.6243	0.85775	1.052883333	1.15045	0.933	1.103	1.261
2016/1/20	0.4253	0.6213	0.85325	1.044716667	1.14045	0.8925	1.0555	1.2063

Figure 5: LIBOR rate and swap rate

For example US0001M index means 1M LIBOR rate, the rate on 2016/1/4 is 0.4225% and USSWAP2 Curncy means 2Year swap rate, the rate on 2016/1/4 is 1.16925%.

4 Model Implement

4.1 Local volatilities

4.1.1 Method 1

The core of the first method is to use the definition formula of local volatility $\sigma_{local} = \sqrt{\frac{\frac{\partial C}{\partial T}}{\frac{1}{2}K^2\frac{\partial^2 C}{\partial K^2}}}$ In order to find the local volatility using this equation, first, call option price was calculated using the Black-Scholes formula with the implied volatilities data. Then, the first order partial derivative term, such as$\frac{\partial C}{\partial T}$, it can be computed with simply using the difference between call option price divided by difference between respecting time for example $\frac{\partial y}{\partial x} = \frac{y_2 - y_1}{x_2 - x_1}$. For second order derivative term, such as$\frac{\partial^2 C}{\partial K^2}$, it can be estimated using finite central difference approximation method as $\frac{\partial^2 C_i}{\partial K^2} = \frac{C_{i+1} - 2C_i + C_{i-1}}{\Delta K^2}$. As all terms are known, the local volatility matrix can be calculated using the definition formula.Apostol (1974)

4.1.2 Method 2

Dupire formula is well-known to obtain local volatility in Black-Scholes-Merton model. When extending Dupire formula in the Piterbarg option pricing model, the local volatility can be express as option pricing in all three type of collateralization. In the further derivation of partially collateralized formula, the local volatility can be expressed in term of implied volatility $\sigma_{local} = \sqrt{\frac{\sigma_{imp}^2 + 2\sigma_{imp}T(\frac{\partial \sigma_{imp}}{\partial T}}{(1 + Kd_1\sqrt{T}\frac{\partial \sigma_{imp}}{\partial K})^2 + K^2 T\sigma_{imp}(\frac{\partial^2 \sigma_{imp}}{\partial K^2} - d_1\sqrt{T}(\frac{\partial \sigma_{imp}}{\partial K})^2)}}$ where $d_1 = \frac{ln(\frac{S_0}{K}) + \frac{1}{2}\sigma_{imp}^2 T}{\sigma_{imp}\sqrt{T}}$

4.1.3 Local volatility result and visualizations

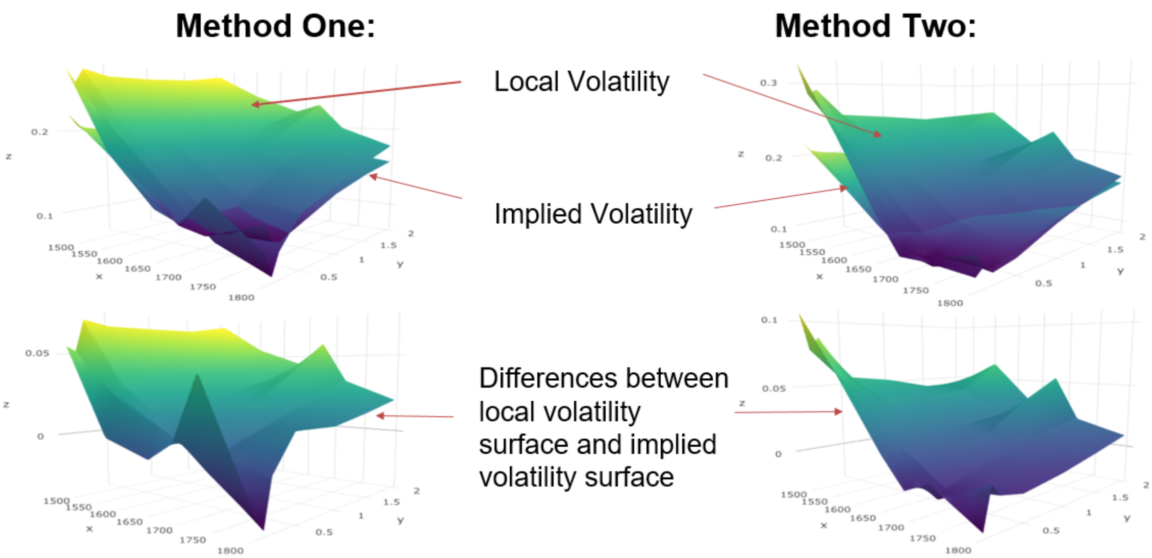


Figure 6: Local volatility, implied volatility and their difference comparison using method one and method two

This Figure above shows the comparison of local volatility over the implied volatility in various strike price and maturity for both method one and method two, in each plot, the x axis represents the strike price, y axis represent different expiry and z axis for upper plots are the implied and local volatility surfaces and x axis of lower plots are the difference from local volatility to the implied volatility. Therefore in both methods, local volatility is higher than the implied volatility at lower percentage of moneyness. In the shiny dashboard (http://fevl02.fsc.stevens.edu:3838/users/fe800/local%20vol/), there are two tabs in the left which are Local Volatility 1 and Local Volatility 1 which represents the results for two methodologies. In the right, users are able to select a specific date, the local volatility surface, implied volatility surface and the difference will be visualized. When the date is selected to be either Saturday or Sunday where is not data, it will display the results of previous Friday and the title will inform user of this date.

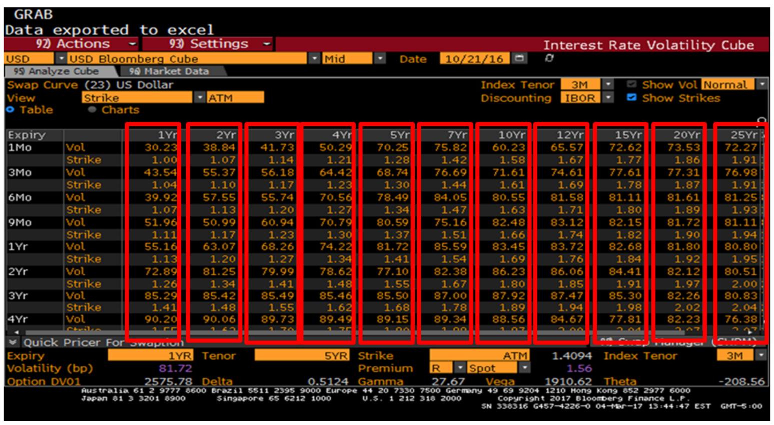
In addition, there is offline study performed to analysis the results from two different methods, which is to assess how much difference is the local volatility results give us with actual option price as bench mark. The method for error estimation is to first to select implied volatility data only for 95% to 105% moneyness, then take out three implied volatility data points and calculate local volatility using the remain data point in both methods, then find the local volatility surface and use Black-Scholes model to calculate option price using the three local volatilities where the data was not use, finally compare with pricing results with actual historical option prices downloaded from Bloomberg and find the average of the percentile of error. The error of first method is 5.66% and error for the second method is 3.70%. Herne, method two seems to give more accurate results in option pricing. The reason for first method lead greater error can be using pricing model before using definition formula of local volatility, because the pricing models cause error and it is just an estimation.

4.1.4 SABR model volatilities calibrations

The formula

$$\sigma_B(f,f) = \frac{\alpha_0}{f^{(1-\beta)}}(1 + (\frac{(1-\beta)^2}{24}\frac{\alpha_0^2}{f^{(2-2\beta)}} + \frac{1}{4}\frac{\rho\beta\nu\alpha_0}{f^{(1-\beta)}} + \frac{2-3\rho^2}{24}\nu^2)T)$$

is for European call option, this formula cannot be directly uesd for swaptions data. Since swaption is an option combine with a swap, we can use this formula to fit the option of the swaps which have the same maturity. Thus in this paper parameters are calibrated tenor by tenor.



Expiry		1Yr	2Yr	3Yr	4Yr	5Yr	7Yr	10Yr	12Yr	15Yr	20Yr	25Yr
1Mo	Vol	30.23	38.84	41.73	50.29	70.25	75.82	60.23	65.57	72.62	73.53	72.27
	Strike	1.00	1.07	1.14	1.21	1.28	1.42	1.58	1.67	1.77	1.86	1.91
3Mo	Vol	43.54	55.37	56.18	64.42	68.74	76.69	71.61	74.61	77.61	77.31	76.98
	Strike	1.04	1.10	1.17	1.23	1.30	1.44	1.61	1.69	1.78	1.87	1.91
6Mo	Vol	39.92	57.55	55.74	70.56	78.49	84.05	80.55	81.58	81.11	81.61	81.25
	Strike	1.07	1.13	1.20	1.27	1.34	1.47	1.63	1.71	1.80	1.89	1.93
9Mo	Vol	51.96	50.99	60.94	70.79	80.59	75.16	82.48	83.12	82.15	81.72	81.11
	Strike	1.11	1.17	1.23	1.30	1.37	1.51	1.66	1.74	1.82	1.90	1.94
1Yr	Vol	55.16	63.07	68.26	74.22	81.72	85.59	83.45	83.72	82.68	81.80	80.80
	Strike	1.13	1.20	1.27	1.34	1.41	1.54	1.69	1.76	1.84	1.92	1.95
2Yr	Vol	72.89	81.25	79.99	78.62	77.10	82.38	86.23	86.06	84.41	82.12	80.51
	Strike	1.26	1.34	1.41	1.48	1.55	1.67	1.80	1.85	1.91	1.97	2.00
3Yr	Vol	85.29	85.42	85.49	85.46	85.50	87.00	87.92	87.47	85.30	82.26	80.83
	Strike	1.41	1.48	1.55	1.62	1.68	1.78	1.89	1.94	1.98	2.02	2.04
4Yr	Vol	90.20	90.06	89.73	89.49	89.15	89.34	88.56	84.67	77.81	82.23	76.38

Figure 7: Swaption volatilities grouped by tenor

By using shinny dashboard and plotly, there are two pages: one shows the whole calibrations on a specified date and the other shows calibration and parameters of a specified tenor on a specified date.

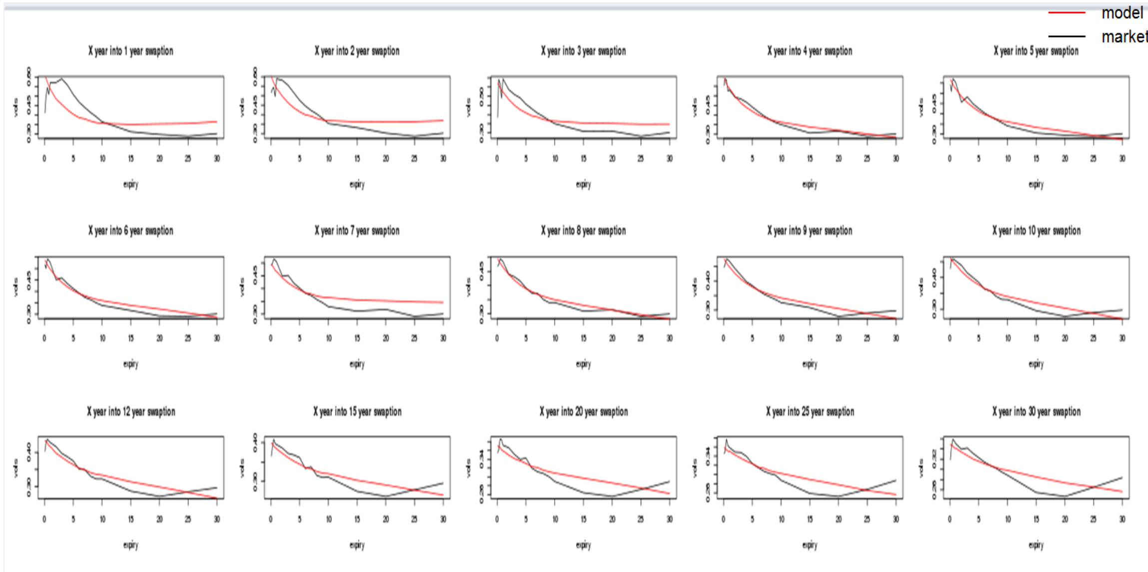


Figure 8: Whole calibration on a specified date

Figure 9: Calibration and parameters on a specified date

From these two figures, we find the swaptions which tenor is 1Y and 2Y cannot be fitted well. When the expiry moves from 1M to 1Y the market volatilities rise up from a low value, however the SABR model didnt present such tendency.

4.1.5 Visualization method

The visualization is built by shiny dashboard which is a advanced package based on shiny(a web application framework for R).

Shiny applications have two components: a user-interface definition(ui.r) and a server script(server.r). In user-interface part, it contains the framework of the web application. In server script, it defines all the inputs and outputs of the application. In input part, inputs can be defined in the code or initialize in the web application. In output part, outputs can be number, plot, text, html, etc.

Shiny dashboard is designed based on shiny, additionally added more default design and framework in user interface. The code of the shiny dashboard is similar with shiny. It mainly contains two parts too, app.r and ui.r. In ui.r part, it is only the overlook of the user interface. People usually put all the code in app.r include all the framework design input and output of the application. In this project, it have three part, app.r ui.r and global.r. The global part contains all the default function or data without any connect with the input from application. The reason why we add another part is that we upload our shiny application to the FE server. The application can be tested easily just but click the web link.

All the plots are built by plotly and ggplot2. These two R packages are graphic packages in R. Plotly is a package built based on ggplot2. There are mainly two advantages using plotly. The first one is the number in plot can be check easly. The second one is that it is much easier to use compared with ggplot2, because user dont have to define so many graphic pattern in plotly.

4.1.6 SABR model market monitor

The core of SABR model part is to build a market monitor for swaption market. The monitor will show user which swaption quote volatilities excess the model confidence interval. This project supposes that user will buy the swaption whose quote volatility excesses the lower limit of confidence interval and sell the swapiton whose quote volatility excesses the upper limit of confidence interval. The monitor also shows the total return of this strategy and the return of each swaption. In this part, firstly, for a specified date, the program will load previous 30 days data and use each day data to calibrate SABR model. Thus, 30 days SABR model volatility surfaces are obtained. Secondly, the program will calculate the confidence interval of these 30 days SABR volatilities by using formula

$$\alpha = 1 - C$$
$$Lowerlimit = \overline{\sigma_{model}} - Z_{\frac{\alpha}{2}} * std(\sigma_{model})$$
$$Upperlimit = \overline{\sigma_{model}} + Z_{\frac{\alpha}{2}} * std(\sigma_{model})$$

The swaptions whose quote volatilities excess the lower bound of confidence interval will be shown as a green marker on the SABR volatilities surface, and the swaptions whose quote volatilities excess the upper bound of confidence interval will be shown as a red marker. The strategy is to buy the swaption which is shown as green dot and sell the swaption which is shown as red markers. Then, the program will load the quote volatilities, forward swap rates and yield curve of next day, calculate the return of each swaption and sum them as a total return.

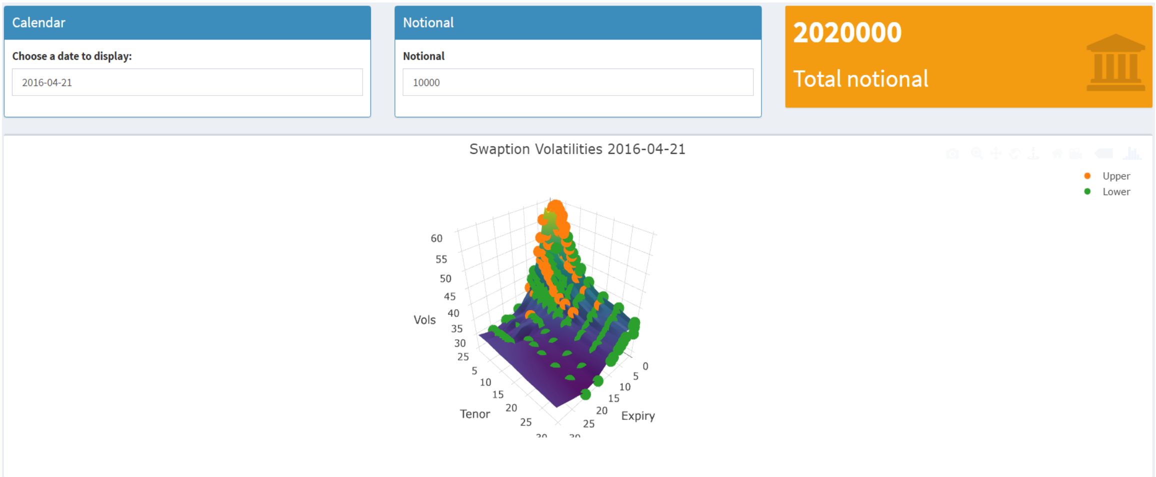


Figure 10: Shiny user interface

Where

$$
\begin{aligned}
return_{green} &= \frac{todayprice = tomorrowprice}{todayprice} \\
return_{red} &= \frac{tomorrowprice - todayprice}{todayprice} \\
return_{total} &= return_{green} + return_{red}
\end{aligned}
$$

By using shiny dashboard, we build an UI for visualization. First, user can choose a date from calendar and set a notional for one swaption. The application will take these two values as parameters for loading data and calculation. The results will be shown as 2 chart and 2 tables. The first chart contains the SABR volatilities surface and the swaption whose quote volatility excesses the SABR confidence interval as a marker, the orange markers means the volatility excesses the upper bound and the green markers means the volatility excesses the lower bound.

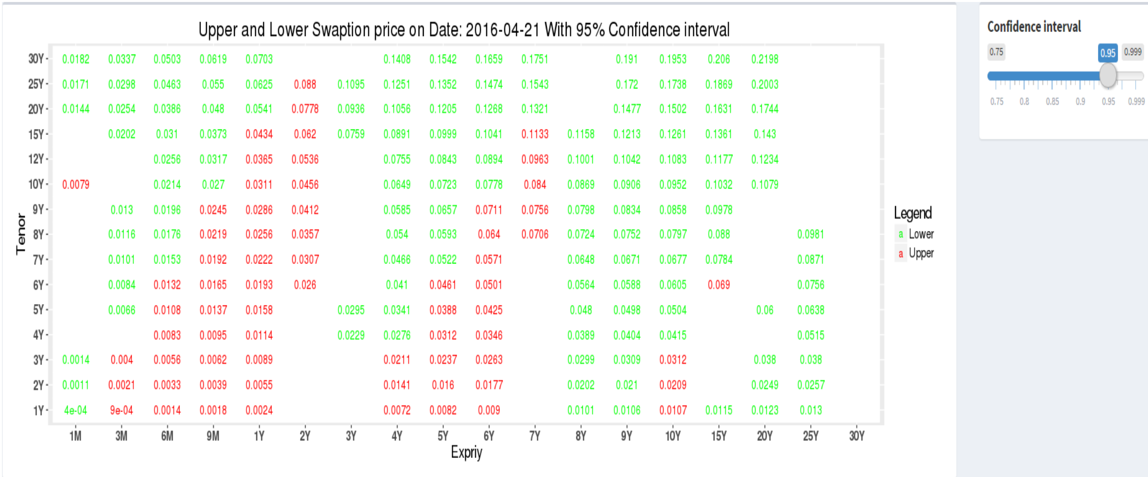


Figure 11: Prices of swaptions

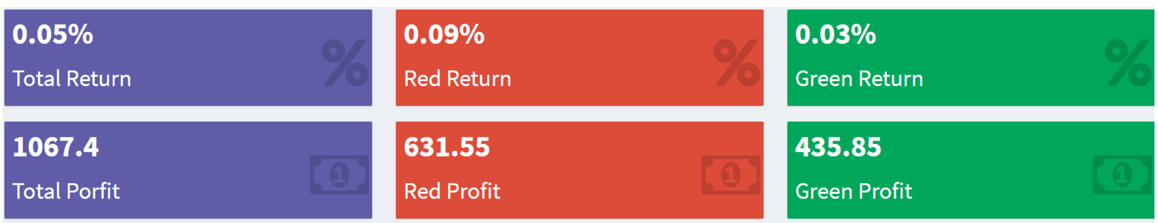


Figure 12: Return and profits of strategy

The first table, UI offers a value bar for user to set the confidence interval for the calculation. It also shows the total return, red return and green return of this strategy. The profits are calculated by the input notional.

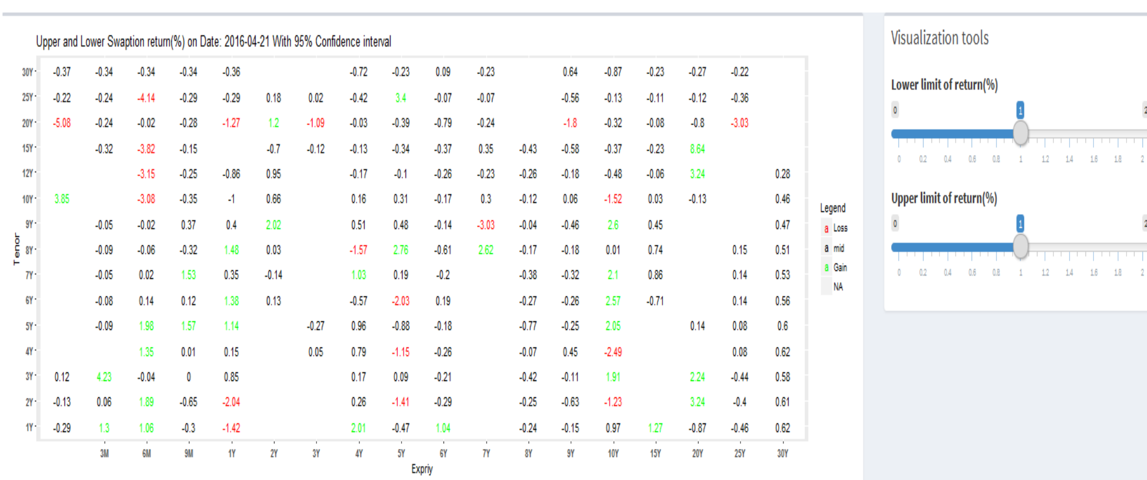


Figure 13: Shiny user interface

The second table shows the return of each swaption and two slide bars which control the lower limit of return and upper limit of return are offered. User can choose the values and the table will filter the swaption. The swaptions whose return is less than the lower limit are shown as a orange number and swaptions whose return is larger than the upper limit are shown as a blue number. The swaption whose return is between upper limit and lower limit is shown as black.

5 Conclusion

In the first part, the local volatility surface is estimated with two methods, first method is to use option price in the local volatility definition formula, and the second method is to use implied volatility in the extension of Piterbarg model. Local volatility, implied volatility surface and their differences can be visualized by simply just selecting a date on the calendar. The analyzation of the performance of these methods show the pricing error for the first method is 5.66% by using Black-Sholes option pricing model, and error for second method is 3.70%. The higher error first method might be cause by using Black-Sholes model to price the option before implement the local volatility definition formula.

Second part, in the SABR calibration part above, the results of the swaption whose expiry is small are not very good.The problem is that in the project the initial value is fixed when doing optimization of the objective function. The initial value will have a big effect on the calibration results, so a better methodology for setting initial value will make the results better. The value of beta will also have some effect on the results, in this project we fixed beta, so finding a good method to estimate beta improve the results for SABR model calibration. For the swaption monitor, the results of our strategy need to look into future this is impossible in the real world, since the only input of the SABR volatility formula that we need is forward swap rate, to improve this part we can use other model to predict the forward swap rate. For example, we can use Vasicek model. The SABR model swaption monitor we have already uploaded to Stevens server. For more information of our project, please check this website: http://fevl02.fsc.stevens.edu:3838/users/fe800/version%20final/

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