Amazon Delivery Truck Simulation

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Abstract

Summary of whole paper. Note that this is not an introduction or context, but a summary.

1 Introduction

Imagine that a delivery company has a series of orders that it needs to fulfill. It has a truck that can stop at each address and make the delivery. The company naturally wants to save on time and fuel costs, so it tries to find the shortest path from its warehouse to cover all of the stops. This is the basic premise of the Traveling Salesman problem. Our team chose to solve it for our final project in COE 322: Scientific Computation at the University of Texas at Austin for the fall of 2022.

In Section 2 we discuss the algorithm that we used to solve the simple Traveling Salesman Problem, and expansions upon it to construct our final program. In Section 3, we display the outcomes of several test scenarios and discuss the results. Finally, in Section 4, we offer our final thoughts and reflect on ethical considerations. However, we will first further define the problem for our purposes and apply some limitations and assumptions in the following subsections.

1.1 Perfect is the Enemy of Good

One could simply test each of the $n!$ combinations of a list of n addresses to fund the one with the shortest total distance, but with 4 stops this becomes quite tedious, and after that nearly untenable. A computer could solve this faster, but the problem still becomes very computationally expensive at a factorial rate. The preferable alternative is to use algorithms to find good and better paths, significantly cutting down on calculations. Perhaps it will neglect the perfect solution, but in a world with finite resources, we have to make the trade-off to settle for less. The algorithms which we employ, detailed in Section 2 reflect this.

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1.2 Earth is Flat

We also have to restrict the definitions of an address and the distance between addresses. Since this project is intended to focus on algorithms, not a turn-key solution for use in the real world, addresses are represented with Cartesian coordinates on a two-dimensional Euclidian plane. Distances are assumed to be Euclidian, i.e. "as the bird flies," with travel times proportional thereto. The distance could alternatively be defined as the "Manhattan Distance," i.e. the distance along a perfect rectilinear street grid. Although using real-world geographic data considering road layouts, speed limits, real-time traffic data, and Earth's curvature would be more realistic, it would add significant complexity to our implementation without contributing much to the core principles of the solution.

1.3 TODO

MORE DEFINITIONS HERE: deliver by date, multiple days, multiple trucks

2 Methodology

To approach this problem, we wrote a library and scripts in C++. These were compiled with GNU's `g++` on our local machines and Intel's `icpc` on the Texas Advanced Computing Center's ISP supercomputer. These scripts are outlined below.

- `traveling_salesman.h`: a header file for our TravelingSalesman library, defining all of the objects and algorithms used in the project
- `traveling_salesman.cpp`: an implementation file
- `tester.cpp`: a script which tests the functionality of our TravelingSalesman library and generates TikZ code for some figures displayed in this report
- `deliveries_generator.cpp`: a script which generates `.dat` files containing lists of orders to be processed by `delivery_truck_simulation.exe`
- `delivery_truck_simulation.cpp`: a script which represents a hypothetical final product for use in industry, as described in Section 2.4
- `experimental.cpp`: a script which runs multiple simulations like the one in `delivery_truck_simulation.cpp` to study the impact of simulation parameters on order backlogging
- `plotter.m`: an auxiliary MATLAB script to plot experimental data

We began our project by writing the header and implementation files, coupled with tests in our tester file. After we confirmed that all of our objects and algorithms functioned as expected, we designed a main program to best parallel a real world application of solving the Traveling Salesman Problem. The structures and algorithms that we developed are further detailed in this section.

In Section 2.1, we outline the structure of the classes that we used to represent and solve the problem. In Section 2.2, we describe the algorithms we used to solve the simple Traveling Salesman Problem. In Section 2.3, we describe the expansion of the problem to account for optimizing multiple delivery routes. Finally, in Section 2.4, we describe how we combined our algorithms into a final product for a hypothetical user.

2.1 Object-Oriented Structure

Our scripts took advantage of C++'s object-oriented capabilities to organize the problem. This section provides a brief overview; the contents of these classes are not described exhaustively.

Each delivery stop is represented by an `Address` object, which has two-dimensional integer Cartesian coordinates `i` and `j` representing the location of the address, an an integer `deliver_by` which describes the day by which the order is supposed to be delivered, or the stop passed-by. The class can also calculate the distance to other `Address`es, using either the Euclidian distance $\sqrt{i^2 + j^2}$ or Manhattan distance $|i| + |j|$. In our implementation, we use the Euclidian distance, but it could easily be replaced with another formula.

A list of `Address`es is represented by an `AddressList` object, which holds the objects in a `std::vector<Address>` instance variable called `address_list`. This class can add, remove, and rearrange `Address`es. It does not accept duplicate `Address`es, i.e. those with the same coordinates. If the user attempts to add an order to the same `Address` with different `deliver_by` due dates, then the lesser value is accepted. This parallels orders being combined in real life. Note that the preference for the earlier date is based on the assumption that at the time that the `Address`es are added to the `AddressList`, they are available to be delivered.

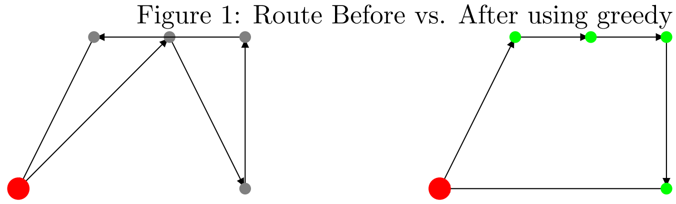
The `Route` class extends the `AddressList` class by including a `hub` instance of type `Address`. This represents the starting and ending point of the `Route`. This class contains several functions to solve variants of the Traveling Salesman problem.

2.2 Traveling Salesman Problem

Having developed a strong Object-Oriented skeleton we can explore algorithms to address the Traveling Salesman Problem. An intuitive approach we could adopt is called the *greedy algorithm* also known as the *nearest neighbor algorithm.* [1, p. 458] The greedy algorithm works to develop an optimal route by traversing (from a starting point) to the next closest point in a list of points until all points in the list have been visited. To achieve this optimal route, the greedy algorithm must determine which point is closest to the current point at each iteration. This can be accomplished with the help of our *index_closest_to()*

method. Now we can iterate through the list of addresses and at each iteration calculate the next closest address until we have visited all the addresses in our list. To prevent visiting the same address more than once, we can make another list, and pop elements from our current list into our new (optimized) list. Since we call our *index_closest_to()* method n times (where n is the length of our address list) and the method itself has a time complexity of O(n) we arrive at a Big-O time complexity of O(n^2) for this greedy algorithm . The figure below illustrates the effect our greedy algorithm has on developing a more optimized route.

Figure 1: Route Before vs. After using greedy algorithm



The improvements from the greedy algorithm seem to suggest that it will play an important role in developing our Amazon route scheduling algorithm. With this in mind, we attempted explore other local search methods that may be more effective than the greedy approach. One approach we can adopt is based on the opt-2 heuristic. The opt-2 heuristic suggests that optimal routes are generally not "entangled" (i.e no intersections). Therefore, if we can work to "detangle" a given list of points we can find its optimal path. Rather than aim to algorithmically identify the intersections in a given path, we can try to "detangle" a path by reversing segments of it and checking to see if such a modification generates a more optimal path. To implement this in code, we employ the follwing strategy:

Listing 1: opt2 Algorithm

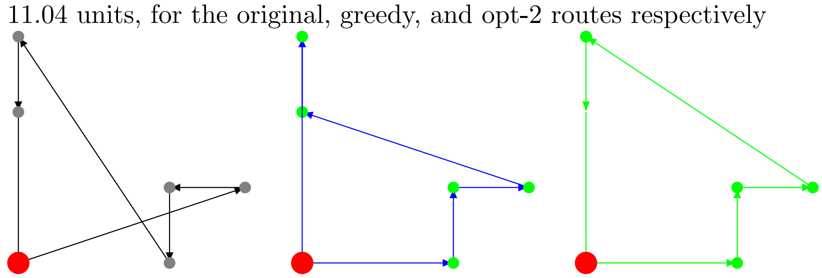
```
For all possible segments in our path:
    Make the new path:
        Original start + reversed segment + original
   end
        If new path is shorter keep it
```

In code this equates to:

```cpp
Route Route::opt2(){
    AddressList address_list(address_vec);
    double current_length = address_list.length();
    for (int m=1 ; m<address_list.size(); m++){
        for (int n=0; n <= m; n++){
            AddressList new_list(address_list.reverse(n, m+1));
            if ( new_list.length() < address_list.length() ){
                address_list = new_list;
                current_length = new_list.length();
            }
        }
    }
    Route new_route(address_list, hub);
    return new_route;
}
```

In this implementation, we utilized the *reverse()* method from the C++ standard library which reverses a given range of a vector in place. Below we compare the results from our greedy algorithm with that of our opt-2 algorithm:

Figure 2: Unoptimized Route (black) vs. greedy algorithm (blue) vs. opt-2 algorithm (green) Route is optimized through both the greedy algorithm and the opt2 algorithm. The total distances of each route were 11.76, 11.16, and 11.04 units, for the original, greedy, and opt-2 routes respectively

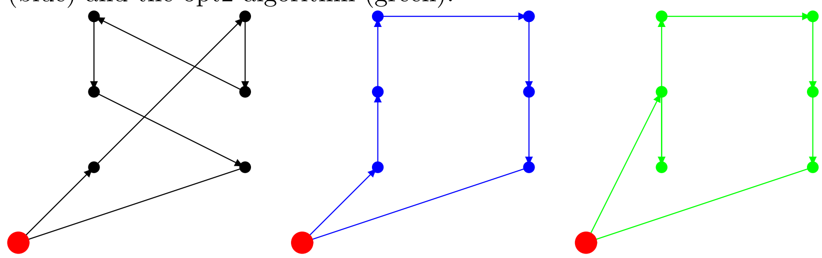


The opt-2 algorithm seems to produce slightly better solutions than our greedy algorithm. However, we should also consider this algorithm's efficiency. Calling the *reverse()* and *length()* methods of an `AddressList` of size n leads to an average time complexity of O(n) for each. Moreover, because this opt-2 algorithm evaluates a total of $\frac{n^2}{2}$ combinations.[1] the overall time complexity of this algorithm is O(2n^3) which simplifies to O(n^3). Fortunately the efficiency of this algorithm can be slightly improved. Instead of reversing segments of our route we can choose to swap pairs of `Addresses` instead. Additionally, rather than call *length()* to compare the *total distances* between the modified

[1] The estimation comes from the fact that the total number of combinations is equivalent to the sum of a triangular number sequence.

and original path, we can focus our comparisons on just the *change in distance* caused by each swap. Consequently, the time complexity can be reduced to $O(n^2)$ (same as our greedy algorithm!). Thus, our opt-2 algorithm seems to be a noticeable improvement from the greedy algorithm. Yet, even this approach may not always produce accurate results. Figure 3 demonstrates this below:

Figure 3: An unsorted Route is optimized through both the greedy algorithm (blue) and the opt2 algorithm (green).



Thus, to develop more optimal solutions we could choose to employ our greedy algorithm followed by opt-2. In addition, because both algorithms have a time complexity of $O(n^2)$ the overall time complexity of running both algorithms would be $O(2n^2)$ which simplifies back to $O(n^2)$. Although this approach is well-suited for single delivery routes, we can extend the opt-heuristic to optimize two routes simultaneously as well. In other words, we can further shorten the paths of two routes by swapping segments of one out for segments of the other. This may be more desirable, as it introduces more flexibility, and thus could lead to identifying potentially shorter paths.

2.3 Multiple Traveling Salesmen Problem

Optimizing multiple routes simultaneously extends our Traveling Salesman Problem to the Multiple Travelling Salesman Problem (MTSP). Seeing how computationally expensive our initial opt2 algorithm was in section 2.2 , one can imagine how much greater this costs would become for an algorithm that optimized multiple routes simeltaneously. For this reason, and others we will, aim to optimize only two routes at once. As mentioned in section 2.2 we can achieve this by extending the opt-2 heuristic to multiple routes, as seen in Listing 2

Listing 2: opt2 Multi Algorithm

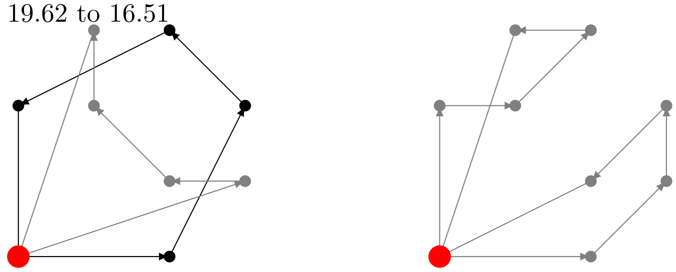
```
For all possible segments for both routes
    swap both routes with the following variations:
        1. Reverse route 1 then swap
        2. Reverse route 2 then swap
        3. Just swap
        4. Reverse both routes then swap
    If the distance from any of the variations is
   improved
```

```
keep the modification.
```

As mentioned previously in this section, we can expect the time complexity of this algorithm to be larger than that of the single route opt-2 algorithm. The multi-path opt-2 algorithm uses four for-loops (two for each route). Further, at each iteration of the innermost loop, we call our reverse and swap methods several times. Both methods have a time complexity of O(n) (where n is the number of addresses in each route). Since those methods are called in total nine times, the time complexity of the innermost loops is O(9n) which simplifies to O(n). Consequently, when we consider our four for-loops the overall time complexity becomes $O(n^5)$. [2] It's important to note that the time complexity of this algorithm can also be improved, but at a cost. For example, two for-loops are created to account for total possible starting and ending of each segment per route. To improve runtime, we could simply give each segment a fixed length, thereby reducing the time complexity to $O(n^3)$. However, this comes at the cost of accuracy, as our algorithm may ignore more optimal solutions due to this constraint. Therefore, therein lies a tradeoff between accuracy and efficiency that we must consider when choosing which approach/algorithm to use.

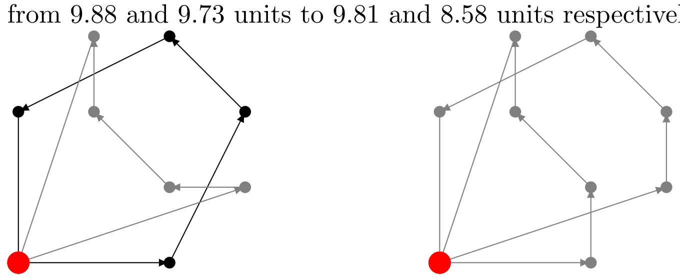
In prioritizing accuracy, we will choose to pay the computational costs and use a multi-path-op2 algorithm that swaps segments of varied lengths efficient mulit-path opt-2 algorithm may require. Additionally, we can also choose to use this algorithm to optimize two routes belonging to a single driver (that is, spread over two different days). In this case, we may also want to consider optimizing for the total distance traveled by both routes, rather than the *individual* distance traveled by both routes. The figures below demonstrate how this subtle change in constraints produces dramatically different results:

Figure 4: Two Routes exchange Addresses to optimize the total distance from 19.62 to 16.51



[2]Futhermore, the time complexity would be $O(m^5+n^5)$ If we consider routes of differing lengths m and n.

Figure 5: Two Routes exchange Addresses to optimize their individual distance from 9.88 and 9.73 units to 9.81 and 8.58 units respectively.



Notice how in Figure 4 we achieve a smaller total distance at the expense of increasing the individual distance of route 1 of the routes. Whereas in Figure 5 we optimize for the individual distances of each route at the expense of having a larger total distance (from taking the sum of both routes). As a result, it seems we may want to optimize for total distance and individual distance on a case-by-case basis. For example, in the scenario of scheduling routes for multiple (in this case two) Amazon truck drivers it may seem more logical to optimize for individual distances. This is because having lopsided routes similar to that shown in Figure 5 can lead to undelivered packages (in the event that one route is so long all points cannot be visited in a single shift of work). Conversely, in the scenario that we were trying to optimize two routes belonging to the same truck driver, it seems more logical to optimize for total distance traveled across both routes (although this is assuming delivery timing is not a factor)[3]. For our final product, we will strategically use the algoritms discussed to most effectivly optimize routes for truck drivers.

2.4 Developing the Final Product

After our team implemented our solutions to the Single and Multiple Traveling Salesman Problems, we decided to explore dynamicism by constructing a simplified route allocator for a delivery company. Every morning, a regional fulfilment center recives a list of orders to fulfill, corresponding to packages available onsite. It invokes our program, which combines these with unfulfilled orders from the previous day, and delegates them amongst a predetermined number of trucks. The `Route`s are then optimized individually and between one another. At this point, their distances are measured. If a `Route` exceeds a predetermined distance limit, `Address`es are removed from the `Route` based on their `deliver_by` due date, until it falls within an acceptable length. The delivery routes are then exported to documents for the drivers, the unfulfilled orders are saved to a file which overwrites the old one, and performance statistics are compiled into a report for management.

[3]this is because swapping segments containing different delivery times would lead to some packages being delivered early while others being delivered too late. We don't mind early deliveries, but late deliveries are unacceptable.

All of the tasks to be completed before the start of a business day are modularized in a single function. It requires parameters specifying input and output file locations, the number of trucks available, the maximum permissible route distance, the hub address, and a boolean which allows the user to specify if data should be output from intermediate optimization steps. Most of the harder tasks of the simulation have already been solved in our library. This even includes the repetitive tasks of reading or exporting a `Route` or `AddressList` from or to a file, based on a given file path string. To demonstrate this program, we constructed a simulation with 18 pre-generated daily orders spanning 9 days.

In addition, we used a pared-down version, with limited file output, to experiment with variations in simulation parameters. Each simulation is a sigle data point, drawing from 20 daily orders across 30 days. The same order data was used for each data point. The nature and results of these simulations are discussed in Section 3.

3 Results

For our main simulation, we generated randomized order data spanning 9 days. Each day has 18 orders with coordinates spanning from 0 to 10 excluding the hub, i.e. $\{(i,j) \in \mathbb{Z}^2 | 0 \leq i \leq 10 \cap 0 \leq j \leq 10\} - \{(0,0)\}$. Delivery due dates range from 1 to 7 days after the order was generated. For this test, 3 trucks were used, and the distance limit was set to 35.0 for each truck. The resulting outputs, spread across over 100 data files, would be quite tedious to present in full in this report, so the results will be condensed into tables and figures. To view samples of each form of output, please refer to Appendix A.

The first thing to consider from the results is whether the optimization performed as expected. As seen in Figure 6, stops are exchanged within and between truck routes until they all take efficient paths. Since no orders were left undelivered on Day 1, all of the Addresses are still present afterwards. This can be verified for the other days as well.

Reports and data files can also be examined for their accuracy; refer to Listings 3 and 4 to see that they correspond to the black unsorted route. On the other hand, the job assignment in Listing 5 corresponds to the black sorted route, and the order numbers in the status report in Listing 6 match the number of stops and distances of the sorted routes.

From there, we can study the numbers provided in the status reports. Delivery fulfillment statistics are condensed into Table 1. We can utilize this to analyze the hub's performance. In general, the hob is keeping up with the order inflow rate. Some days it isn't able to deliver everything, but it eventually catches up. This could be an important experimental metric for a delivery organization. Parameters like the number of trucks and the maximum delivery distance could be adjusted to find critical points where orders get added to the backlog faster than they are removed.

At this point, the main problem is solved. For a given list of orders, our program can find an optimal solution within a certain distance range and eliminate

Figure 6: Day 1 Optimization, Before and After

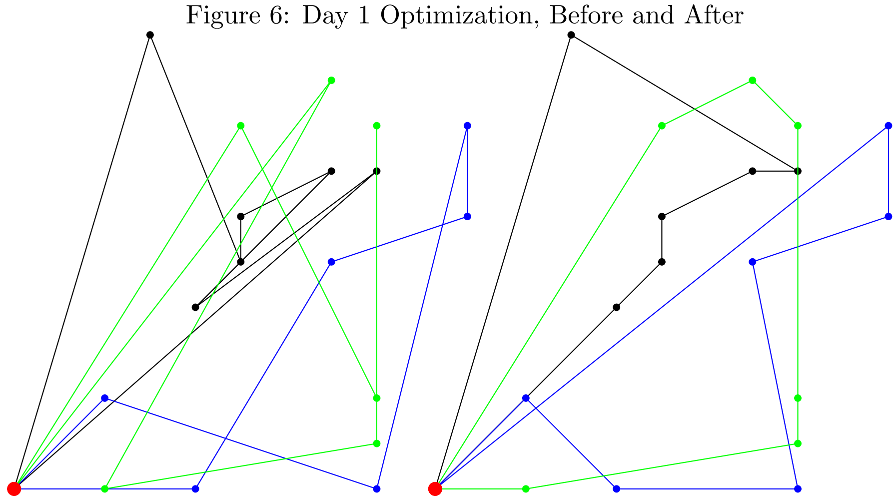


Table 1: Status Report Output Data

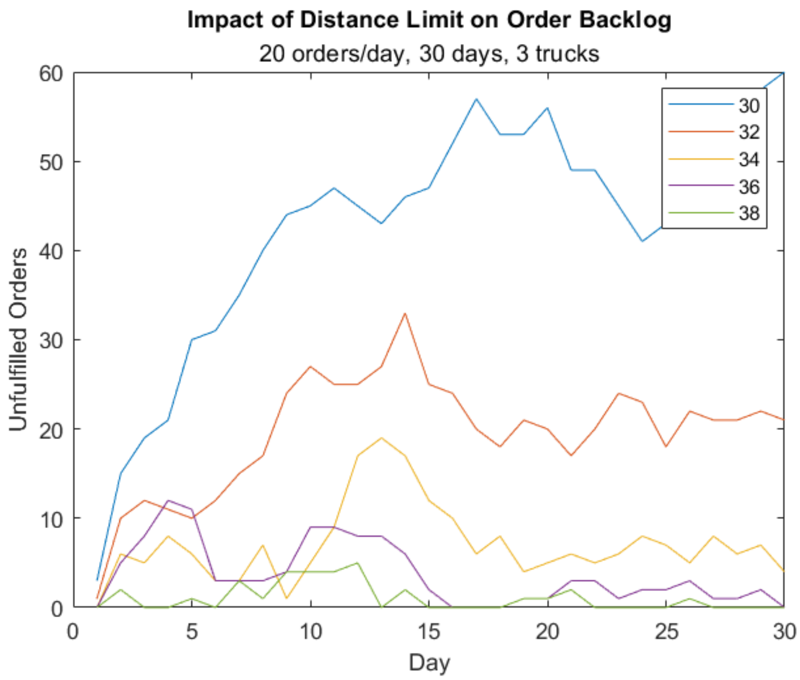
	Delivered			Undelivered		
	Early	On Time	Late	Not Due	Due Tomorrow	Overdue
Day 1	18	0	0	0	0	0
Day 2	16	0	0	0	0	0
Day 3	10	0	0	8	0	0
Day 4	16	0	0	7	0	0
Day 5	16	0	0	6	0	0
Day 6	23	0	0	0	0	0
Day 7	12	0	0	4	0	0
Day 8	19	0	0	1	0	0
Day 9	17	0	0	1	0	0

low-priority orders to keep delivery routes within a certain distance constraint. Now we will extend our investigation into modifying parameters of the simulation. As mentioned before, at a certain point, the simulation will likely start backlogging orders. If we find this point, managers could leverage it to maximize efficiency while maintaining customer satisfaction. In Section 3.1, we investigate how varying the maximum permissible route distance impacts backlogs. In section 3.2, we investigate how varying the number of trucks deployed impacts backlogs. Finally, in Section 3.3, we briefly reflect on the execution times of the programs.

3.1 Varying the Distance Limit

First, we consider varying the distance limit for each truck. Note that for a delivery zone ranging from coordinates 0 to d with a hub at $(0, 0)$, if the distance limit were dropped below $2 * d\sqrt{2}$, then some orders would be forever out of range. This sets a lower limit for our maximum permissible distance at $2 \cdot 10\sqrt{2} \approx 28.28$. However, because there are more orders for the hub to process than trucks, the limit will likely be higher. Figure 7 shows that for 3 trucks receiving 20 orders per day in a 10-by-10 grid, orders start to maintain a constant backlog below a minimum distance of 34. After that point, unfulfilled orders climb signigicantly. Interestingly, after an initial climb, backlogs tend to level off. The exact cause of this would require more investigation, but it's possible that once the backlog is high, it is counteracted by new orders occurring at the same locations as old ones, thus being merged.

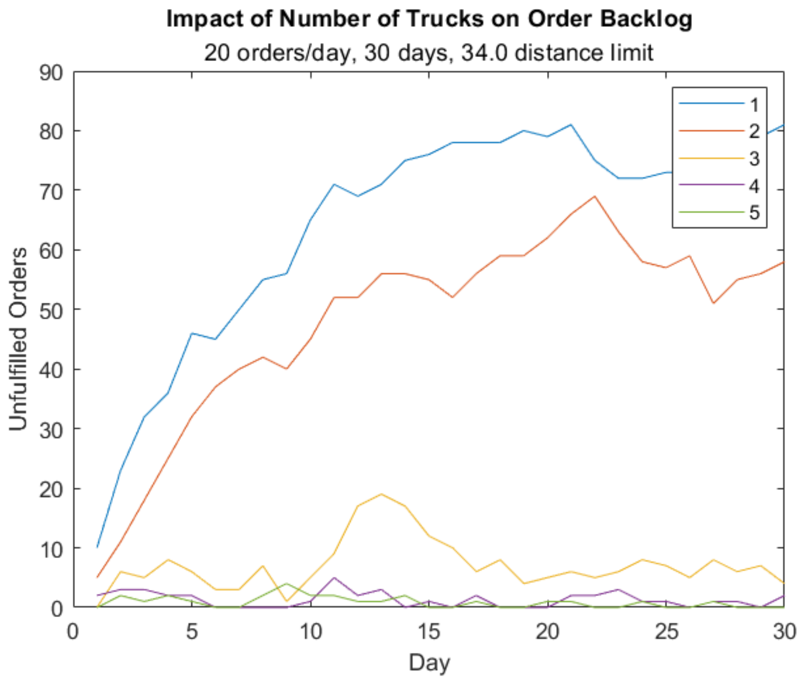
Figure 7: The order backlog builds up as the route distance limit drops below 34.



3.2 Varying the Number of Trucks Deployed

Now, we consider varying the number of trucks. Obviously, a delivery company would be interested in minimizing the number of trucks it has to deploy. A similar experiment was conducted on the same data as before. Here, we find a more striking threshold. Figure 8 shows that once the number of trucks drops below 3, the backlog explodes, albeit with a similar leveling-off as before. What this means is that the delivery company could not get away with short-staffing for very long before being overwhelmed.

Figure 8: The order backlog builds up as the number of trucks deployed drops below 3.



3.3 Performance Data

Finally, for each of the simulations, i.e. our main program and our experimental one, we recorded the total execution time both on a laptop computer and on the ISP machine. Our main program took 795 milliseconds on a laptop and 75 milliseconds on the supercomputer. The experimental series took 958,455 and 199,371 milliseconds, respectively. Clearly, the supercomputer offers a significant speed advantage in carrying out the programs.

4 Conclusion

Talk about what we learned, how this all applies to industry, ideas to scale the problem up, ethics, &c.

Talk about how our simulation is flawed, e.g. address removal isn't intelligent (purely by due date); orders aren't held back to be grouped with later-arriving orders; after stops are removed, the routes arent re optimized; main simulation program isn't super flexible (e.g. to change document names, formattig, you have to go into the code; dat files have to be in a specific format, can't be a database); no UI for ease of use

A Sample Output Files

A variety of data files are used as inputs, intermediates, and outputs of our main program and scripts. They fall into 4 main categories. Data files like Listing 3 are used to input Addresses into the program and save output Routes from the program. The example is clearly a Route, since it starts and ends at the same location, in this case, the origin. TikZ files like Listing 4 are used to automate plotting figures with the TikZ package in LaTeX. Job assignments like Listing 5 format Routes such that human drivers can read them. They also offer some statistics and custom messages. In this case, an affirmation is distributed to drivers to increase morale. Finally, status reports like Listing 6 condense essential information like delivery numbers for managers to evaluate the hub's performance.

Listing 3: Sample Data File

```
0 0 0
8 7 16
4 4 6
7 7 2
5 6 6
5 5 2
3 10 8
0 0 0
```

Listing 4: Sample TikZ File

```
\draw [black] (0, 0) -- (8, 7);
\filldraw [black] (0, 0) circle (2pt);
\draw [black] (8, 7) --(4, 4);
\filldraw [black] (8, 7) circle (2pt);
\draw [black] (4, 4) --(7, 7);
\filldraw [black] (4, 4) circle (2pt);
\draw [black] (7, 7) --(5, 6);
\filldraw [black] (7, 7) circle (2pt);
\draw [black] (5, 6) --(5, 5);
\filldraw [black] (5, 6) circle (2pt);
\draw [black] (5, 5) --(3, 10);
\filldraw [black] (5, 5) circle (2pt);
\draw [black] (3, 10) --(0, 0);
\filldraw (3, 10) [black] circle (2pt);
\filldraw [red] (0, 0) circle (4pt);
```

Listing 5: Sample Job Assignment

```
..\Delivery Truck Simulation Data\Jobs\day1_truck1.txt

Today's Route
```

```

Start at hub: 0 0
4 4
5 5
5 6
7 7
8 7
3 10
Finish at hub: 0 0

Stats

Length: 27.578394
Stops: 6

Have a nice day! You are ~not~ a corporate wage slave
    :-)
```

Listing 6: Sample Status Report

```
Status Report for Day 1

Number of trucks: 3
Truck distance limit: 35

TRUCK DATA
Truck 1: 6 deliveries, distance 27.5784
Truck 2: 6 deliveries, distance 32.7244
Truck 3: 6 deliveries, distance 28.167

ORDER DATA
Delivered: 18
18 early, 0 on time, 0 late
Unfulfilled: 0
0 not due, 0 due tomorrow, 0 overdue
```

References

[1] Victor Eijkhout. *Introduction to Scientific Programming*. 2022.