Amazon Delivery Truck Simulation

Hanna Butt [*] Ashton Cole [†] Kelechi Emeruwa [‡]

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Abstract

Summary of whole paper. Note that this is not an introduction or context, but a summary.

1 Introduction

Imagine that a delivery company has a series of orders that it needs to fulfill. It has a truck that can stop at each address and make the delivery. The company naturally wants to save on time and fuel costs, so it tries to find the shortest path from its warehouse to cover all of the stops. This is the basic premise of the Traveling Salesman problem. Our team chose to solve it for our final project in COE 322: Scientific Computation at the University of Texas at Austin for the fall of 2022.

In Section 2 we discuss the algorithm that we used to solve the simple Traveling Salesman Problem, and expansions upon it to construct our final program. In Section 3, we display the outcomes of several test scenarios and discuss the results. Finally, in Section 4, we offer our final thoughts and reflect on ethical considerations. However, we will first further define the problem for our purposes and apply some limitations and assumptions in the following subsections.

1.1 Perfect is the Enemy of Good

One could simply test each of the $n!$ combinations of a list of n addresses to fund the one with the shortest total distance, but with 4 stops this becomes quite tedious, and after that nearly untenable. A computer could solve this faster, but the problem still becomes very computationally expensive at a factorial rate. The preferable alternative is to use algorithms to find good and better paths, significantly cutting down on calculations. Perhaps it will neglect the perfect solution, but in a world with finite resources, we have to make the trade-off to settle for less. The algorithms which we employ, detailed in Section 2 reflect this.

[*] HFB352, hannaf2020@gmail.com
[†] AVC687, ashtonc24@utexas.edu
[‡] KEE688, kelechi@utexas.edu

1.2 Earth is Flat

We also have to restrict the definitions of an address and the distance between addresses. Since this project is intended to focus on algorithms, not a turn-key solution for use in the real world, addresses are represented with Cartesian coordinates on a two-dimensional Euclidian plane. Distances are assumed to be Euclidian, i.e. “as the bird flies,” with travel times proportional thereto. The distance could alternatively be defined as the “Manhattan Distance,” i.e. the distance along a perfect rectilinear street grid. Although using real-world geographic data considering road layouts, speed limits, real-time traffic data, and Earth’s curvature would be more realistic, it would add significant complexity to our implementation without contributing much to the core principles of the solution.

1.3 TODO

MORE DEFINITIONS HERE: deliver by date, multiple days, multiple trucks

2 Methodology

To approach this problem, we wrote a library and scripts in C++. These were compiled with GNU’s `g++` on our local machines and Intel’s `icpc` on the Texas Advanced Computing Center’s ISP supercomputer. These scripts are outlined below.

- `traveling_salesman.h`: a header file for our TravelingSalesman library, defining all of the objects and algorithms used in the project
- `traveling_salesman.cpp`: an implementation file
- `tester.cpp`: a script which tests the functionality of our TravelingSalesman library and generates TikZ code for figures displayed in this report; this script is compiled as `tester.exe`
- `deliveries_generator.cpp`: a script which generates `.dat` files containing lists of orders to be processed by `delivery_truck_simulation.exe`; this script is compiled as `deliveries_generator.exe`
- `delivery_truck_simulation.cpp`: a script which represents a hypothetical final product for use in industry, as described in Section 2.4; this script is compiled as `delivery_truck_simulation.exe`

We began our project by writing the header and implementation files, coupled with tests in our tester file. After we confirmed that all of our objects and algorithms functioned as expected, we designed a main program to best parallel a real world application of solving the Traveling Salesman Problem. The structures and algorithms that we developed are further detailed in this section.

In Section 2.1, we outline the structure of the classes that we used to represent and solve the problem. In Section 2.2, we describe the algorithms we used to solve the simple Traveling Salesman Problem. In Section 2.3, we describe the expansion of the problem to account for optimizing multiple delivery routes. Finally, in Section 2.4, we describe how we combined our algorithms into a final product for a hypothetical user.

2.1 Object-Oriented Structure

Our scripts took advantage of C++'s object-oriented capabilities to organize the problem. This section provides a brief overview; the contents of these classes are not described exhaustively.

Each delivery stop is represented by an `Address` object, which has two-dimensional integer Cartesian coordinates `i` and `j` representing the location of the address, an an integer `deliver_by` which describes the day by which the order is supposed to be delivered, or the stop passed-by. The class can also calculate the distance to other `Address`es, using either the Euclidian distance $\sqrt{i^2 + j^2}$ or Manhattan distance $|i| + |j|$. In our implementation, we use the Euclidian distance, but it could easily be replaced with another formula.

A list of `Address`es is represented by an `AddressList` object, which holds the objects in a `std::vector<Address>` instance variable called `address_list`. This class can add, remove, and rearrange `Address`es. It does not accept duplicate `Address`es, i.e. those with the same coordinates. If the user attempts to add an order to the same `Address` with different `deliver_by` due dates, then the lesser value is accepted. This parallels orders being combined in real life. Note that the preference for the earlier date is based on the assumption that at the time that the `Address`es are added to the `AddressList`, they are available to be delivered.

The `Route` class extends the `AddressList` class by including a `hub` instance of type `Address`. This represents the starting and ending point of the `Route`. This class contains several functions to solve variants of the Traveling Salesman problem.

2.2 Traveling Salesman Problem

Having developed a strong Object-Oriented skeleton we can explore algorithms to address the Traveling Salesman Problem. An intuitive approach we could adopt is called the *greedy algorithm* also known as the *nearest neighbor algorithm*. [1, p. 458] The greedy algorithm works to develop an optimal route by traversing (from a starting point) to the next closest point in a list of points until all points in the list have been visited. To achieve this optimal route, the greedy algorithm must determine which point is closest to the current point at each iteration. This can be accomplished with the help of our *index_closest_to()* method. Now we can iterate through the list of addresses and at each iteration calculate the next closest address until we have visited all the addresses in our list. To prevent visiting the same address more than once, we can make another list, and pop elements from our current list into our new (optimized) list. Since we call our *index_closest_to()* method n times (where n is the length of our address list) and the method itself has a time complexity of O(n) we arrive at a Big-O time complexity of O(n^2) for this greedy algorithm . The figure below illustrates the effect our greedy algorithm has on developing a more optimized route. 8. The improvements from the greedy algorithm seem to suggest that

Figure 1: insert improvement from greedy alog

it will play an important role in developing our Amazon route scheduling algorithm. On the other hand, additional testing demonstrates how the greedy algorithm alone can fail to provide the most accurate solutions:

Figure 2: insert deficiency of greedy algorithm

Although this deficiency may seem minimal with a few routes, at scale , the costs incurred due to longer distances traveled and longer delivery times may prove to be a great burden for Amazon, therefore we'd like to explore better methods. One approach we can adopt is based on the opt-2 heuristic. The opt-2 heuristic suggests that optimal routes are generally not "entangled" (i.e no intersections). Therefore, if we can work to "detangle" a given list of points we can find its optimal path. Both pathes can be seen in the figure below.

Figure 3: ** insert entangled vs detangled route and its total distance.

Rather than aim to algorithmically identify the intersections in a given path, we can try to "detangle" a path by reversing segments of it and checking to see if such a modification generates a more optimal path. The figure below shows how this is done visually.

To implement this in code, we employ the strategy in Listing 1.

Listing 1: opt2 Algorithm

```
For all possible segments in our path:
    Make the new path:
        Original start + reversed segment + original end
        If new path is shorter keep it
```

In code this equates to:

```
Route Route::opt2(){
    AddressList address_list(address_vec);
    double current_length = address_list.length();
    for (int m=1 ; m<address_list.size(); m++){
        for (int n=0; n <= m; n++){
```

Figure 4: ** insert single pass of reversage of a path

```
            AddressList new_list(address_list.reverse(n, m+1));
            if ( new_list.length() < address_list.length() ){
                address_list = new_list;
                current_length = new_list.length();
            }
        }
    }
    Route new_route(address_list, hub);
    return new_route;
}
```

In this implementation, we utilized the *reverse()* method from the C++ standard library reverses a given segment of a vector in place. Below we compare the results from our greedy algorithm with that of our opt-2 algorithm:

Figure 5: compare greedy w/opt 2 (use previous greedy deficiency example too)

The opt-2 algorithm seems to be a reasonable replacement for the greedy algorithm. However, we should also consider this algorithm's efficiency. Calling the *reverse()* and *length()* methods of an `AddressList` of size n leads to an average time complexity of O(n) for each. Moreover, because this opt-2 algorithm evaluates a total of $\frac{n^2}{2}$ combinations.[1] the overall time complexity of this algorithm is $O(2n^3)$ which simplifies to $O(n^3)$. Fortunately the efficiency of this algorithm can be slightly improved. Instead of reversing segments of our route we can choose to swap pairs of Addresses instead Additionally, rather than call *length()* to compare the *total distances* between the modified and original path, we can focus our comparisons on just the *change in distance* caused by each swap. In code, this results in the following modified opt2 algorithm:

Listing 2: Opt-2 Algorithm (optimized)

```
Route Route::opt2(){
    // this code needs to be modified
    AddressList address_list(address_vec);
    double current_length = address_list.length();
    for (int m=1 ; m<address_list.size(); m++){
        for (int n=0; n <= m; n++){
            AddressList new_list(address_list.reverse(n, m+1));
```

[1]The estimation comes from the fact that the total number of combinations is equivalent to the sum of a triangular number sequence

```
            if ( new_list.length() < address_list.length() ){
                address_list = new_list;
                current_length = new_list.length();
            }
        }
    }
    Route new_route(address_list, hub);
    return new_route;
}
```

Consequently, the time complexity is now reduced to $O(n^2)$ (same as our greedy algorithm!) and yields the same results as before:

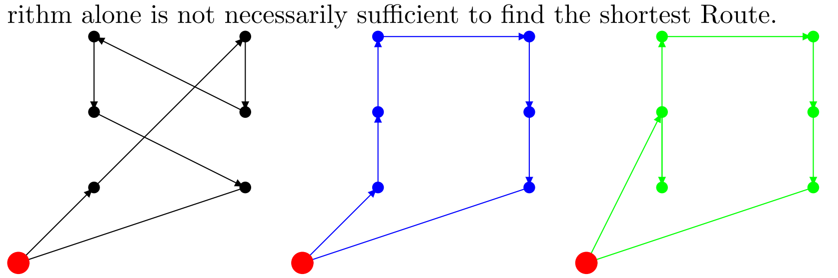
Figure 6: ** make sure it yields same results, then copy paste from previous figures **

Thus, our opt-2 algorithm seems to be a substantial improvement from the greedy algorithm. Yet, even this approach seems to not be heavily reliable. See below:

Figure 7: ** include optimized opt2 yielding poor solution results

Thus, to develop more optimal solutions we could choose to employ our greedy algorithm followed by opt-2. In addition, because both algorithms have a time complexity of $O(n^2)$ the overall time complexity of running both algorithms would be $O(2n^2)$ which simplifies back to $O(n^2)$. Although this approach is well-suited for single delivery routes, we can extend the opt-heuristic to optimize two routes simultaneously as well. In other words, we can further shorten the paths of two routes by swapping segments of one out for segments of the other. This may be more desirable, as it introduces more flexibility, and thus could lead to identifying potentially shorter paths.

Figure 8: An unsorted Route is optimized through both the greedy algorithm (blue) and the opt2 algorithm (green). This demonstrates how the opt2 algorithm alone is not necessarily sufficient to find the shortest Route.



2.3 Multiple Traveling Salesmen Problem

Optimizing multiple routes simultaneously extends our Traveling Salesman Problem to the Multiple Travelling Salesman Problem (MTSP). Seeing how computationally expensive our initial opt2 algorithm was in section 2.2 , one can imagine how much greater this costs would become for an algorithm that optimized multiple routes simeltaneously. For this reason, and others we will, aim to optimize only two routes at once. As mentioned in section 2.2 we can achieve this by extending the opt-2 heuristic to multiple routes, as seen in Listing 3

Listing 3: opt2 Multi Algorithm

```
For all possible segments for both routes
    swap both routes with the following variations:
        1. Reverse route 1 then swap
        2. Reverse route 2 then swap
        3. Just swap
        4. Reverse both routes then swap
    If the distance from any of the variations is
   improved
        keep the modification.
```

As mentioned previously in this section, we can expect the time complexity of this algorithm to be larger than that of the single route opt-2 algorithm. The multi-path opt-2 algorithm uses four for-loops (two for each route). Further, at each iteration of the innermost loop, we call our reverse and swap methods several times. Both methods have a time complexity of O(n) (where n is the number of addresses in each route). Since those methods are called in total nine times, the time complexity of the innermost loops is O(9n) which simplifies to O(n). Consequently, when we consider our four for-loops the overall time complexity becomes $O(n^5)$ [2]. It's important to note that the time complexity of this algorithm can also be improved, but at a cost. For example, two for-loops are created to account for total possible starting and ending of each segment per route. To improve runtime, we could simply give each segment a fixed length, thereby reducing the time complexity to $O(n^3)$. However, this comes at the cost of accuracy, as our algorithm may ignore more optimal solutions due to this constraint. Therefore, therein lies a tradeoff between accuracy and efficiency that we must consider when choosing which approach/algorithm to use. The figures below demonstrate this tradeoff visually:

Figure 9: **include graph. Caption 1: Multi-path opt-2 algorithm with permitting swaps of fixed segment length 3 ($O(n^3)$) Caption 2: Multi-path opt-2 algorithm with permitting swaps of varied segment lengths ($O(n^5)$). ****

In prioritizing accuracy, we will choose to pay the computational costs and

[2]Futhermore, the time complexity would be $O(m^5+n^5)$ If we consider routes of differing lengths m and n

use a multi-path-op2 algorithm that swaps segments of varied lengths efficient mulit-path opt-2 algorithm may require. Additionally, we can also choose to use this algorithm to optimize two routes belonging to a single driver (that is, spread over two different days). In this case, we may also want to consider optimizing for the total distance traveled by both routes, rather than the *individual* distance traveled by both routes. The figures below demonstrate how this subtle change in constraints produces dramatically different results:

Figure 10: *** include graphs use example test case from book: Caption 1: Multi-path opt-2 algorithm optimizing for the individual distance traveled by both routes. Caption 2: Multi-path opt-2 algorithm optimizing for the total distance traveled by both routes. ****INCLUDE INDIVIDUAL LENGTHS AND TOTAL DISTANCES FOR EACH CAPTION

Notice how in Figure XXX (caption 2) we achieve a smaller total distance at the expense of increasing the individual distance of route XXX. Whereas in Figure XXX (caption 1) we optimize for the individual distances of each route at the expense of having a larger total distance (from taking the sum of both routes). As a result, it seems we may want to optimize for total distance and individual distance on a case-by-case basis. For example, in the scenario of scheduling routes for multiple (in this case two) Amazon truck drivers it may seem more logical to optimize for individual distances. This is because having lopsided routes similar to that shown in Figure XXX (caption 1) can lead to undelivered packages (in the event that one route is so long all points cannot be visited in a single shift of work). Conversely, in the scenario that we were trying to optimize two routes belonging to the same truck driver, it seems more logical to optimize for total distance traveled across both routes (although this is assuming delivery timing is not a factor)[3]. For our final product, we will explore ways to utilize the multi-path opt2 algorithm (including the others previously discussed) in optimzing truck routes for drivers.

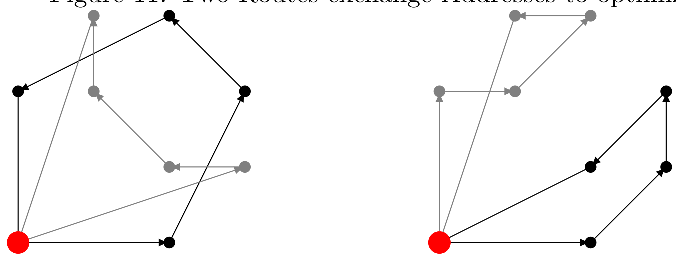
Talk about how swap algorithm works. Figure 11 provides a simple example of how this algorithm improves the total distance. (Now prove it with data! What are the distances before and after?) ALSO: Talk about how the gray and black Addresses are symmetrical, but the trucks take different Routes. Is this okay, or does one or both of them need to go through greedy/opt2???

2.4 Developing the Final Product

After our team implemented our solutions to the Single and Multiple Traveling Salesman Problems, we decided to explore dynamicism by constructing a simplified route allocator for a delivery company. Every morning, a regional fulfilment

[3]this is because swapping segments containing different delivery times would lead to some packages being delivered early while others being delivered too late. We don't mind early deliveries, but late deliveries are unacceptable

Figure 11: Two Routes exchange Addresses to optimize their distances.



center recives a list of orders to fulfill, corresponding to packages available onsite. It invokes our program, which combines these with unfulfilled orders from the previous day, and delegates them amongst a predetermined number of trucks. The `Routes` are then optimized individually and between one another. At this point, their distances are measured. If a `Route` exceeds a predetermined distance limit, `Addresses` are removed from the `Route` based on their `deliver_by` due date, until it falls within an acceptable length. The delivery routes are then exported to documents for the drivers, the unfulfilled orders are saved to a file which overwrites the old one, and performance statistics are compiled into a report for management.

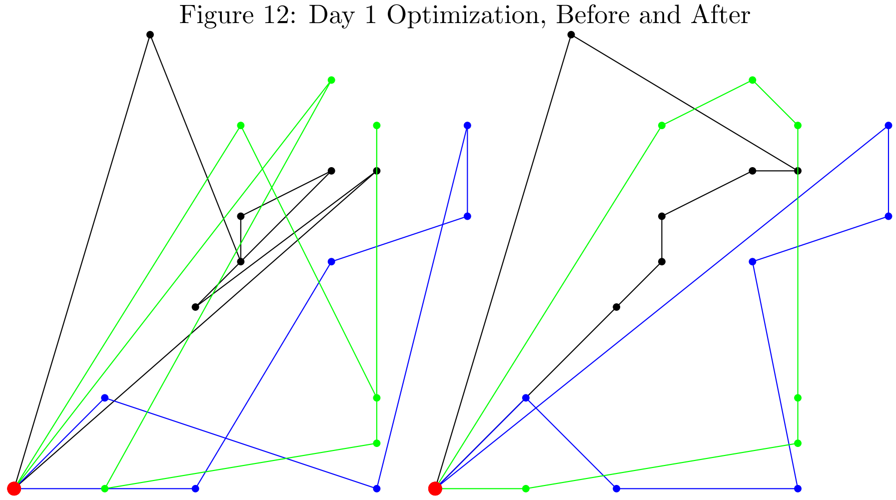
All of the tasks to be completed before the start of a business day are modularized in a single function. It requires parameters specifying input and output file locations, the number of trucks available, the maximum permissible route distance, the hub address, and a boolean which allows the user to specify if data should be output from intermediate optimization steps. Most of the harder tasks of the simulation have already been solved in our library. This even includes the repetitive tasks of reading or exporting a `Route` or `AddressList` from or to a file, based on a given file path string. To demonstrate this program, we constructed a simulation with pre-generated daily orders spanning several days. The results of this simulation are discussed in Section 3.

3 Results

For our main simulation, we generated randomized order data spanning 9 days. Each day has 18 orders with coordinates spanning from 0 to 10 excluding the hub, i.e. $\{(i,j) \in \mathbb{Z}^2 | 0 \leq i \leq 10 \cap 0 \leq j \leq 10\} - \{(0,0)\}$. Delivery due dates range from 1 to 7 days after the order was generated. For this test, 3 trucks were used, and the distance limit was set to 35.0 for each truck. Our program took 795 milliseconds to execute locally, and XXXXXX milliseconds on the ISP machine. The resulting outputs, spread across over 100 data files, would be quite tedious to present in full in this report, so the results will be condensed into tables and figures. To view samples of each form of output, please refer to Appendix A.

The first thing to consider from the results is whether the optimization performed as expected. As seen in Figure 12, stops are exchanged within and between truck routes until they all take efficient paths. Since no orders were left undelivered on Day 1, all of the Addresses are still present afterwards. This can be verified for the other days as well.

Figure 12: Day 1 Optimization, Before and After



Reports and data files can also be examined for their accuracy; refer to Listings 4 and 5 to see that they correspond to the black unsorted route. On the other hand, the job assignment in Listing 6 corresponds to the black sorted route, and the order numbers in the status report in Listing 7 match the number of stops and distances of the sorted routes.

From there, we can study the numbers provided in the status reports. The numbers are condensed into Table 1.

Table 1: Status Report Output Data

	Delivered			Undelivered		
	Early	On Time	Late	Not Due	Due Tomorrow	Overdue
Day 1	18	0	0	0	0	0
Day 2	16	0	0	0	0	0
Day 3						
Day 4						
Day 5						
Day 6						
Day 7						
Day 8						
Day 9						

3.1 Varying the Distance Limit

Note that if the distance limit dropped below $2 * 10\sqrt{2} \approx 28.284$, then some orders would be forever out of range.

3.2 Varying the Number of Trucks Deployed

One important question that a delivery company might want to consider is how many trucks to deploy on a given day. More trucks can cover a greater cumulative distance and deliver more orders, but

3.3 Performance Data

go over execution time and stuff

4 Conclusion

Talk about what we learned, how this all applies to industry, ideas to scale the problem up, ethics, &c.

Talk about how our simulation is flawed, e.g. address removal isn't intelligent (purely by due date); orders aren't held back to be grouped with later-arriving orders; after stops are removed, the routes arent re optimized; main simulation program isn't super flexible (e.g. to change document names, formattig, you have to go into the code; dat files have to be in a specific format, can't be a database); no UI for ease of use

A Sample Output Files

A variety of data files are used as inputs, intermediates, and outputs of our main program and scripts. They fall into 4 main categories. Data files like Listing 4 are used to input Addresses into the program and save output Routes from the program. The example is clearly a Route, since it starts and ends at the same location, in this case, the origin. TikZ files like Listing 5 are used to automate plotting figures with the TikZ package in LaTeX. Job assignments like Listing 6 format Routes such that human drivers can read them. They also offer some statistics and custom messages. In this case, an affirmation is distributed to drivers to increase morale. Finally, status reports like Listing 7 condense essential information like delivery numbers for managers to evaluate the hub's performance.

Listing 4: Sample Data File

```
0 0 0
8 7 16
4 4 6
7 7 2
```

```
5 6 6
5 5 2
3 10 8
0 0 0
```

Listing 5: Sample TikZ File

```
\draw [black] (0, 0) -- (8, 7);
\filldraw [black] (0, 0) circle (2pt);
\draw [black] (8, 7) --(4, 4);
\filldraw [black] (8, 7) circle (2pt);
\draw [black] (4, 4) --(7, 7);
\filldraw [black] (4, 4) circle (2pt);
\draw [black] (7, 7) --(5, 6);
\filldraw [black] (7, 7) circle (2pt);
\draw [black] (5, 6) --(5, 5);
\filldraw [black] (5, 6) circle (2pt);
\draw [black] (5, 5) --(3, 10);
\filldraw [black] (5, 5) circle (2pt);
\draw [black] (3, 10) --(0, 0);
\filldraw (3, 10) [black] circle (2pt);
\filldraw [red] (0, 0) circle (4pt);
```

Listing 6: Sample Job Assignment

```
..\Delivery Truck Simulation Data\Jobs\day1_truck1.txt

Today's Route

Start at hub: 0 0
4 4
5 5
5 6
7 7
8 7
3 10
Finish at hub: 0 0

Stats

Length: 27.578394
Stops: 6

Have a nice day! You are ~not~ a corporate wage slave :-)
```

Listing 7: Sample Status Report

```
Status Report for Day 1

Number of trucks: 3
Truck distance limit: 35

TRUCK DATA
Truck 1: 6 deliveries, distance 27.5784
Truck 2: 6 deliveries, distance 32.7244
Truck 3: 6 deliveries, distance 28.167

ORDER DATA
Delivered: 18
18 early, 0 on time, 0 late
Unfulfilled: 0
0 not due, 0 due tomorrow, 0 overdue
```

References

[1] Victor Eijkhout. *Introduction to Scientific Programming.* 2022.