Phys 140A – Fall 2024 – Homework 2

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Due Friday October 18 at 8pm via BruinLearn

General instructions, please read carefully. You are welcome to work collaboratively on these homework assignments and to ask/answer questions on how to approach the questions on BruinLearn Discussion Board. However, please do not share full solutions. Your final write-up must be completed independently.

Solid State Basics by Steven Simon = SBB

1. **Vibrations with parabolic dispersion.** In HW#1 you calculated the heat capacity for a 2D system with *N* atoms and volume *V* that had the typical linear dispersion, $\omega(\mathbf{k}) = v|\mathbf{k}|$. Now consider a 2D system with *N* atoms and volume *V* that has the following dispersion relation

$$\omega(\mathbf{k}) = \alpha|\mathbf{k}|^2, \qquad \text{Eq. 1}$$

where α is a constant with units of L^2T^{-1} (L = length; T = time).

   a. What is the density of states, $g(\omega)$? Recall, this is the number of vibrational modes, $d\nu$, per unit frequency, $d\omega$. In 2D, how many polarizations are possible for each mode?

   You can solve this following the similar logic as SSB section 2.2.2 or by first calculating the *total* number of vibrational modes for a given frequency, $\nu(\omega)$, then taking a derivative: $g(\omega) = d\nu/d\omega$.

   b. Calculate the Debye frequency, ω_D, for this 2D system. You can follow SSB 2.2.3 or you can first find the max wave vector, k_D, for a system of *N* atoms then use Eq. 1 to find the corresponding ω_D.

   c. Next let's calculate the average energy of this system by evaluating the following,

$$\langle E \rangle = p\sum_{\mathbf{k}}^{\mathbf{k}_D} \hbar\omega(\mathbf{k})\left[n(\hbar\omega/k_BT) + 1/2\right], \qquad \text{Eq. 2}$$

   where *p* is the number of polarizations for this dimension, $n(\hbar\omega/k_BT)$ is the Bose-Einstein distribution function, and $\omega(k)$ is the parabolic dispersion in Eq. 1.

   Hint: Convert the discrete sum to an integral then make a change-of-variables so that the integration is performed over a ***dimensionless*** *variable, x. Similar to HW1 you do not need to evaluate/compute this dimensionless integral right away. Your final answer for $\langle E \rangle$ should have units of energy!*

d. At **high temperature**, show that the average energy you calculated in (c) leads to a heat capacity, $C = \partial\langle E \rangle / \partial T$, that agrees with the result you found for the 2D system with linear dispersion in HW1, namely, $C_{2D} = 2Nk_B$.

e. At **low temperature**, show that the average energy you calculated in (c) leads to a heat capacity of the form $C = KT$, where K is a constant that depends on $\alpha, \hbar, k_B$. How does this differ from your low-temperature solution in HW1.

f. **Optional:** For students interested in a small challenge, show that for **any** integer spatial dimension *n* and **any** integer power-law dispersion, $\omega(k) = \alpha k^m$, that the high temperature limit yields a heat capacity that is **independent** of temperature and a low temperature heat capacity that goes as $C = KT^{n/m}$, where *K* is a constant.

2. SSB 3.1. Parts (b) and (c).

3. Consider a 3-dimensional material in the so-called “ultra-clean" limit with no scatterers, $\tau \to \infty$ (i.e. you can ignore the scattering/damping term). Start with the equation of motion for an electron in the presence of both an electric and magnetic field (top equation in SSB 3.1.2). The magnetic field is constant and points in the *z*-direction, $\mathbf{B} = B\hat{z}$ and the driving electric field along the x-direction is oscillating at frequency ω, $\mathbf{E}(t) = E_0 e^{i\omega t}\hat{x}$. If we look for oscillating solutions, $\mathbf{j} = \mathbf{j}_0 e^{i\omega t}, \mathbf{v} = \mathbf{v}_0 e^{i\omega t}, \mathbf{p} = \mathbf{p}_0 e^{i\omega t}$, find the explicit form the 3×3 resistivity matrix.

4. (Quick) Consider a system that has two species of charged carriers, electrons with charge $-e$, mass m_e, scattering time τ_e, and density n_e and holes (missing electrons) with charge $+e$, mass m_h, scattering time τ_h, and density n_h. In the presence of a constant electric field, the ***total*** current density is $\mathbf{j}_{tot} = \mathbf{j}_e + \mathbf{j}_h$. Using the common relations between current and electric field, and the form of the conductivity in the Drude model, find the ***total*** conductivity of the system.