Phys 245 Quantum Computation
Homework 2

1. [25+5] *The Zeeman effect*. Suppose we have an electron in an uniform magnetic field described by $\vec{B} = B_x\hat{x} + B_z\,\hat{z}$. The electron has a magnetic moment $\vec{\mu} = -\frac{g\mu_B}{\hbar}(\hat{S}_x\hat{x} + \hat{S}_y\hat{y} + \hat{S}_z\hat{z})$ and it interacts with this magnetic field according to the Hamiltonian: $\widehat{H} = -\vec{\mu}\cdot\vec{B}$. Calculate the following:
   a.) [10] In the given coordinate system, calculating the eigenvalues and eigenvectors of the electron spin for arbitrary values of B_x and B_z.
   b.) [5] Find the eigenvectors and eigenvalues in the limit of $B_z \gg B_x$.
   c.) [5] Find the eigenvectors and eigenvalues in the limit of $B_x \gg B_z$.
   d.) [5] Show how the Hamiltonian can be written in a form $\widehat{H} = A\hat{\sigma}_z$, determine A, write down the eigenvectors and eigenvalues, and determine the direction of this new z-axis.
   e.) [5] Bonus: Suppose your system is in the positive energy eigenstate of the Hamiltonian in part d and you make a measurement of the spin along the original z-direction (i.e. the z-direction in parts (a)-(c). What is the expectation value of that measurement?

2. [20] The Zeeman Effect and the Bloch Sphere. Take your answer from 1(a) and plot on the Bloch Sphere, using e.g. QuTIP. The positive eigenvector for the several points ranging from B_x/B_z = 0 and B_x/B_z = 10.

3. [30] Calculate eigenvalues and eigenvectors of the three Pauli matrices:
   a. [10] $\hat{\sigma}_x$
   b. [10] $\hat{\sigma}_y$
   c. [10] $\hat{\sigma}_z$.

4. [30] Calculate the following (remember those are matrix exponentials):
   a. [5] $\hat{\sigma}_x^2$
   b. [5] $\hat{\sigma}_y^2$
   c. [5] $\hat{\sigma}_z^2$
   d. [5] $\exp(\imath\,\theta\,\hat{\sigma}_x)$
   e. [5] $\exp(\imath\,\theta\,\hat{\sigma}_y)$
   f. [5] $\exp(\imath\,\theta\,\hat{\sigma}_z)$

5. [30] Rabi flopping with detuning. In class, we found the Rabi flopping evolution for a qubit with resonant drive. Now, redo that derivation but with $\delta \neq 0$. You may assume $\phi = 0$. Find:
   a. [25] Show that if the system initially starts in $|\psi\rangle = |0\rangle$, the probability to find it in $|0\rangle$ at a later time is: $P_0 = 1 - \frac{\Omega^2}{\Omega'^2}\sin^2\left(\frac{\Omega' t}{2}\right)$, where the generalized Rabi frequency is $\Omega' = \sqrt{\Omega^2 + \delta^2}$.
   b. [5] Suppose $\Omega t = \pi$. Plot the probability of being in $|1\rangle$ as a function of δ.