Phys 140A – Fall 2024 – Homework 1

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Due Friday October 11 at 8pm via BruinLearn

General instructions, please read carefully. You are welcome to work collaboratively on these homework assignments and to ask/answer questions on how to approach the questions on Campuswire. However, please do not share full solutions. Your final write-up must be completed independently.

Solid State Basics by Steven Simon = SBB

1. SSB Q2.1 (Einstein Solid) – you can skip the final part of the problem that asks you to sketch the heat capacity.

(a) Hint: You will need to express the vector squares p and x in terms of their components. For an ideal gas at equilibrium, we can safely assume that $\langle p_x^2 \rangle = \langle p_y^2 \rangle = \langle p_z^2 \rangle$ (and similarly for their squared displacements). Note that for some variable, x:

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2} dx = \sqrt{\pi / \alpha}$$

Recall the useful formula we derived in discussion that relates the average internal energy to the partition function.

(b) Hint: Calculate the average internal energy $\langle U \rangle$ before explaining the relationship with Bose statistics. You can perform this entire calculation for 1-dimension before generalizing to 3-dimensions.

2. SSB Q2.2 (Debye Theory I) – **only part a**.

   This question asks that you derive the Debye expression for the heat capacity in three-dimensions, i.e. the steps found in SSB pgs. 11-14. The point is to be able to justify each step to yourself conceptually (*i.e.* the significance of the cut-off frequency, what the density of states *means*).

3. SSB Q2.3 (Debye Theory II)

   This question has you re-trace the conceptual steps from 2.2 above to derive relations for a two-dimensional solid. How many modes are there? Where does dimensionality enter into the derivation in pgs. 11-14? Do not attempt to evaluate the integral for *K* in the low temperature limit.

4. Mini-Project #1. ***Plotting heat capacity.*** Note: The use of Python, NumPy, Matplotlib is strongly encouraged over software such as Excel.

On BruinLearn you will find a (real) heat capacity data set for a newly discovered material, La_2Cu_2In [data generously provided by Prof. Alannah Hallas of the University of British Columbia[1]]. The first column is temperature in units of [K] and the second column is raw (unnormalized) heat capacity in units of [μJ/K]. The sample's mass was measured to be 22.30 mg. Note that you do not need to worry about error analysis for this question.

(a) Produce a graph of heat capacity vs. temperature, where the heat capacity is normalized in units of [J/mol-K]. Hint: to normalize the data you will need to use the molar mass and the mass of the sample. You can calculate the molar mass using the Lenntech calculator (https://www.lenntech.com/calculators/molecular/molecular-weight-calculator.html).

(b) Include a dashed, labelled horizontal line in your graph at the Dulong-Petit limit. Hint: How many vibrational modes do you expect per formula unit of La_2Cu_2In? Count the number of atoms per formula unit!

(c) Using **only** the data below 10 K, estimate the Debye temperature using the relationship for the low temperature heat capacity $C = \gamma T + nR\frac{12\pi^4}{5\,T_{Debye}^3}T^3$ where *n* is the number of atoms per formula unit. In order to accurately estimate the Debye temperature, one should plot C/T vs T^2 and fit the data to a straight line. [Note: Fit to the part of the curve that is most linear!] Compare the slope of that line to the equation above. Add to your graph a dashed vertical line at the calculated Debye temperature T_{Debye}. (Note that for now we are not sure of the physical interpretation of the T-linear γ term — it's related to the electrons, but we'll get to that later!).

(d) Write ~3-4 sentences describing your plot in terms of the analysis performed in steps (a-c above). This should be similar to a descriptive figure caption.

For this question you don't not need to "show your work" – only turn in your graph and the brief description.

[1] https://hallas.phas.ubc.ca/home