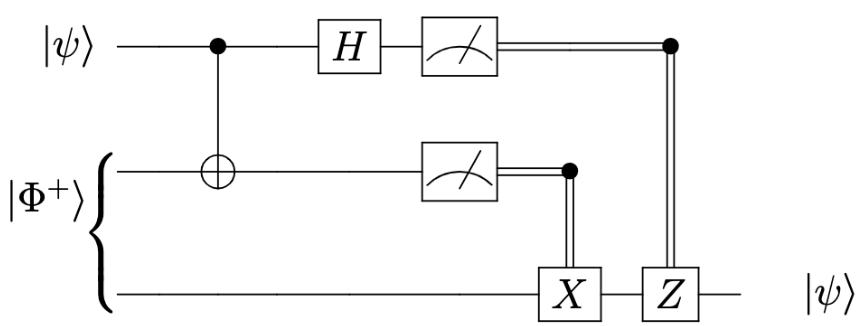
Phys 245 Quantum Computation
Homework 8

1. *[50] Universal Single Qubit Gates.* In this problem, we'll explore how the $\hat{T}$ and $\hat{H}$ gate can be used to effect an arbitrary single qubit gate.
    a. [5] Show that the up to a global phase, the $\hat{T}$ gate can be written as a rotation about the $\hat{z}$ axis and find the angle of rotation.
    b. [15] Show that the up to a global phase, the combination of $\hat{H}\hat{T}\hat{H}$ gates can be written as a rotation about the $\hat{x}$ axis and find the angle of rotation.
    c. [25] Find the evolution due to the combination of gates $\hat{T}\hat{H}\hat{T}\hat{H}$ and show that it can be written as $e^{-\iota\frac{\theta}{2}(\hat{n}\cdot\vec{\sigma})}$ and determine $\hat{n}$ – remember $\hat{n}$ needs to be normalized -- and θ.
    d. [5] Given that the θ you just found is irrational, how can this be used to produce an arbitrary single qubit rotation?

2. *[50] Beam me up, Scotty!* The usual set up for quantum teleportation is to imagine two folks, Alice and Bob. Alice and Bob share a pair of qubits, with one qubit each, that are in the $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(\,|00\rangle + |11\rangle)$ Bell state. The details of how they got in this state aren't important for us, but you could imagine they were entangled then Bob put his in his suitcase and flew off to the Moon or wherever. Alice also has an additional qubit in some state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. The goal of quantum teleportation is to put Bob's qubit into the state $|\psi\rangle$ using only classical communication. A quantum circuit that accomplishes this teleportation is:



    a.) [30] Work this out by hand and show that the teleportation is indeed accomplished by this circuit.
    b.) [20] Work this in QuTip using an instance of QubitCircuit() and show that the teleportation is indeed accomplished by this circuit.

3. [20] *1+1 = ?* A [two-bit half adder circuit]() is a Boolean logic circuit that adds two bits and outputs their sum. Design a quantum version of this circuit and implement it in QuTip to show it works.
4. [20] *Grover loves math.* Assume we are using Grover's algorithm to solve an unstructured search problem with only one solution $|q\rangle$.

a. [15] Show that when starting in an equal superposition state $|s\rangle = \cos\theta\,|s'\rangle + \sin\theta\,|q\rangle$, after applying the oracle $\hat{O}$ and diffuser $\widehat{D}$ operators the system is in the state: $\widehat{D}\hat{O}|s\rangle = \cos 3\theta\,|s'\rangle + \sin 3\theta\,|q\rangle$. Here $|s'\rangle$ is an equal superposition of all states except the solution $|q\rangle$.

b. [5] After one application of the Grover's step, i.e. $\hat{G} = \widehat{D}\hat{O}$, what is the probability of finding the system in the solution state if a measurement is performed for a total state space dimension of N = 4?