EC ENGR 279AS Homework 1

Spin echo, quantum measurement, and quantum logic spectroscopy
Due: January 22 – Submit online by noon

Spin echo and quantum measurement

Consider a spin-echo measurement using a single electronic spin S as in the paper by Maze, *et al.*, Nature **455**, 644 (2008). In an external dc magnetic field, B_{dc}, it is possible to address only two of the spin states (say, $|m_s = 0\rangle$ and $|m_s = 1\rangle$) and treat the electronic spin as a two level system.

a) In the absence of ^{13}C nuclear spins, determine the expectation value of the spin echo signal $P = |\langle\varphi_i|\varphi_f\rangle|^2$ as a function of the free evolution time τ in the presence of an applied ac field given by $\vec{B}(t) = B\sin(2\pi\nu t + \phi_0)\hat{z}$, where ν is the frequency of the ac field. (The pulse sequence is given by $\pi/2$ x-rotation, free precession for time $\tau/2$, π rotation around x axis, free precession for time $\tau/2$ and $\pi/2$ x-rotation). Assume that the first $\pi/2$ pulse is applied at $t = 0$ and that the initial state is $|\varphi_i\rangle = |m_s = 0\rangle$. Under what conditions (τ, ϕ_0) is the electronic spin most sensitive to the ac field at frequency ν? Provide a physical interpretation for your results.

b) Repeat the above analysis assuming that the second $\pi/2$ pulse rotates the probe spin around the y axis. Set the initial phase of the ac field to zero ($\phi_0 = 0$) and determine the signal, P, as a function of the field amplitude, B. Furthermore, determine projection-noise limited sensitivity of this magnetometer, i.e., the minimal detectable field at a frequency ν within total averaging time T using this method (neglect initialization and readout times).

c) In the presence of a spin environment, the spin echo signal collapses and revives. This can be used to detect and analyze proximal nuclear spins for nanoscale NMR. Assume that the probe spin is located near a surface consisting of a layer of proton spins as reported in Staudacher, *et al.*, Science **339**, 6119 (2013). This nuclear spin bath produces an effective field of the form $\vec{B}_{nuc} = B_{nuc}\sin(\omega\tau + \phi_0)\hat{z}$, where ϕ_0 is the random phase uniformly distributed from $-\pi$ to π and ω is the Larmor frequency of ^{1}H ($\omega = \gamma_n B_{dc}$, where $\gamma_n = 2\pi \times 4.26$ kHz is the proton gyromagnetic ratio). Use your result from part (a) to determine the spin echo signal P. For which values of ω and τ can collapses/revivals of the signal be observed?

Quantum logic spectroscopy

Consider two qubits (labeled A and B) that interact via the Ising Hamiltonian $H = g\sigma_z^A\sigma_z^B$ as described in in the paper by Maurer, *et al.*, Science **336**, 1283-1286 (2012). Assume we want to use qubit A for repetitive readout of qubit B. To do this we first prepare qubit A in an equal superposition state, such that the state of the composite system is $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0_A\rangle + |1_A\rangle) \otimes |\psi_B\rangle$, where $|\psi_B\rangle$ is the state of qubit B we wish to measure. We then let the system evolve for a time T, after which we apply a $\frac{\pi}{2}$ pulse on qubit A about some axis in the x-y plane. Such a $\frac{\pi}{2}$ pulse rotates the state $|0_A\rangle$ to $\frac{1}{\sqrt{2}}(|0_A\rangle + e^{i\phi}|1_A\rangle)$. Finally we measure the state of qubit A.

a) Write down the state of the composite system after the $\frac{\pi}{2}$ pulse and just before the final measurement.

b) Determine the values of T and ϕ such that measurement of qubit A corresponds to projective measurement of qubit B along the z axis. Which choice of rotation axis does the optimal phase ϕ correspond to?

c) Consider the situation where the measurement of qubit A is not perfect. After this initial measurement, we can repetitively measure the resulting state of B by iterating the above procedure N times. Suppose that the state of qubit A is measured by detecting the number of photons scattered by the qubit. In particular, if qubit A is in $|0_A\rangle$, we detect on average $\langle n_0\rangle = 0$ photons, while we detect $\langle n_1\rangle = \eta < 1$ photons if it's in $|1_A\rangle$. After N repetitions of the measurement, we determine that qubit B is in $|0_B\rangle$ if we detect zero photons, and $|1_B\rangle$ if we detect one or more photons. Define the measurement fidelity as $F = 1 - \mathrm{Max}[p^0_{err}, p^1_{err}]$, where p^0_{err} and p^1_{err} are the probabilities of making the wrong decision when the state is $|0_B\rangle$ and $|1_B\rangle$, respectively. Find F as a function of N.

c) *Extra credit:* Assume that instead, $\langle n_0\rangle = \alpha_0$, $\langle n_1\rangle = \alpha_1$, and $1 < \alpha_0 < \alpha_1$. In addition, the number of photons detected for each state follows a Poisson distribution. After N measurements the probability to detect n photons is given by $p(n|0)$ if the qubit B is in $|0_B\rangle$ and $p(n|1)$ if it is in $|1_B\rangle$. In general these distributions will overlap. Define n^* such that $p(n^*|0) = p(n^*|1)$ (see figure). In any given experiment, we associate outcomes with $n < n^*$ ($n > n^*$) with detection of $|0_B\rangle$ ($|1_B\rangle$). Find an expression for p_{err}, the total probability of making the wrong determination and numerically determine its scaling with N by plotting the result.