This one ended up being really hard for them and way too long. I guess they hadn't all heard of perturbation theory etc. Shorten this one up.

Phys 245 Quantum Computation Homework 6

- 1. [31] Neutral atom tweezers as a QHO. Neutral atom tweezers function by shining a very red detuned laser onto an atom. In class, we derived the Stark shift due to such a laser as $E=-\frac{\hbar |\Omega|^2}{4\delta}$, where δ is the detuning from resonance. For a laser focused on an atom the Rabi frequency can be written as $\Omega=dE/\hbar$, where d is the atomic dipole moment (just a constant and a property of the atom) and E is the electric field of the laser. The intensity of the laser can be found from the electric field as $I=\frac{1}{2}c\epsilon_0|E|^2$. Assuming at the focus of the laser beam the electric field of the laser is described by a Gaussian beam of the form $E=E_0e^{-\frac{x^2+y^2}{w_0}}e^{i(kz-\omega t+\phi)}\hat{x}$. Do the following:
 - a. [10] Find the Stark shift on the atom
 - b. [10] Approximate the result from part (a) to describe the confining potential as a harmonic oscillator and identify the trap frequency ω .
 - c. [10] Assume that d=e a_o (e is the electron charge and a_o the Bohr radius), the laser power is 100 mW, the spot size is $w_o=10$ μ m, and the detuning is $\delta=10$ THz, what is ω ?
 - d. [1] What is $\hbar\omega$ in temperature units?
- 2. [40] An anharmonic oscillator. Suppose a harmonic oscillator with frequency ω is modified such that its potential goes from $V=\frac{1}{2}m\omega^2x^2 \to V=\frac{1}{2}m\omega^2x^2+\lambda x^4$. For this problem let $\omega=1$ and $\lambda=0.01$.
 - a. [10] Use perturbation theory to calculate the first-order correction to the states $|0\rangle$ and $|1\rangle$.
 - b. [15] Use perturbation theory to calculate the second-order correction to the energy. For this problem you'll have to deal with the problem of having an infinite basis. You'll need to choose a maximum n at which to truncate the basis. I recommend doing this step numerically and then exploring the effect of changing n_{max} on your final energy.
 - c. [15] Instead of using perturbation theory, write the Hamiltonian as an $n_{max} \times n_{max}$ matrix and diagonalize it to find the eigenenergies tip: QuTip has a built-in function for finding eigenvalues. For the same n_{max} do the methods of part (b) and part (c) agree or disagree?
- 3. [80] Composite quantum systems. In this exercise, we'll try out our new tools for dealing with composite quantum systems.

- a. [20] For a system composed of two qubits with qubit one in the state $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and qubit two in the state $|\psi_2\rangle = |0\rangle$, do the following:
 - i. [5] Write down the total state vector.
 - ii. [5] From the composite state vector, find the expectation value of the $\hat{\sigma}_z^{(1)}$, i.e. the expectation value of $\hat{\sigma}_z$ for qubit 1.
 - iii. [5] From the composite state vector, find the expectation value of the $\hat{\sigma}_z^{(2)}$, i.e. the expectation value of $\hat{\sigma}_z$ for qubit 2.
 - iv. [5] From the composite state vector, find the expectation value of $\hat{\sigma}_z^{(1)} \otimes \hat{\sigma}_z^{(2)}$.
- b. [15] For a system composed of two qubits with qubit one in the state $|\psi_1\rangle=\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ and qubit two in the state $|\psi_2\rangle=\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, do the following:
 - i. [5] Write down the total state vector.
 - ii. [5] From the composite state vector, find the expectation value of $\hat{\sigma}_z^{(1)} \otimes \hat{\sigma}_z^{(2)}$.
 - iii. [5] From the composite state vector, find the expectation value of $\hat{\sigma}_{\chi}^{(1)} \otimes \hat{\sigma}_{\chi}^{(2)}$.
- c. [10] Assume that the total state vector is $|\psi_{Tot}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and do the following:
 - i. [5] From the composite state vector, find the expectation value of $\hat{\sigma}_z^{(1)} \otimes \hat{\sigma}_z^{(2)}$.
 - ii. [5] From the composite state vector, find the expectation value of $\hat{\sigma}_{\chi}^{(1)} \otimes \hat{\sigma}_{\chi}^{(2)}$.
- d. [15] Write down the total statevector for a general three qubit system. That is, starting with $|\psi_1\rangle=(a_1|0\rangle+b_1|1\rangle)$, $|\psi_2\rangle=(a_2|0\rangle+b_2|1\rangle)$, and $|\psi_3\rangle=(a_3|0\rangle+b_3|1\rangle)$ find the total statevector in the composite Hilbert space.
- e. [20] For a system composed of a qubit in the state $|\psi\rangle=(a_1|0\rangle+b_1|1\rangle)$ and a quantum harmonic oscillator in a superposition of Fock states $|\phi\rangle=\frac{1}{\sqrt{3}}(|0\rangle+|1\rangle+|3\rangle)$, do the following:
 - i. [5] Write down the total state vector.
 - ii. [5] From the composite statevector, find the expectation value of $\hat{\sigma}_z$ for the qubit.
 - iii. [5] From the composite statevector, find the expectation value of \widehat{N} for the harmonic oscillator.
 - iv. [5] From the composite statevector, what is the probability of finding the system with qubit in state $|1\rangle$ and the harmonic oscillator in state $|3\rangle$?
- 4. [70] Jayne says! The Jaynes-Cummings Hamiltonian is:

$$H/\hbar = \frac{\omega_o}{2}\hat{\sigma}_z + \omega\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right) + \frac{g}{2}(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^{\dagger})$$

Implement the Jaynes-Cummings Hamiltonian in QuTip and use it to find the time dynamics of the following situations. For simplicity assume $\omega_o=2\pi$, and $g=\omega_o/100$. Since you cannot use an infinite dimensioned Hilbert space in QuTip you'll need to truncate the basis at some maximum $|n\rangle$. Make sure and choose that maximum n large enough to not affect your answer.

- a. [50] For $\omega = \omega_o$ find the time evolution of the population in the initial state for the following initial states over one period of oscillation:
 - i. [10] $|\psi(t=0)\rangle = |00\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its ground state
 - ii. [10] $|\psi(t=0)\rangle = |10\rangle$ -- that is the qubit in the ground state and the harmonic oscillator in its ground state
 - iii. [10] $|\psi(t=0)\rangle = |01\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its first excited state
 - iv. $[10] |\psi(t=0)\rangle = |05\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its fifth excited state
 - v. $[10] |\psi(t=0)\rangle = |16\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its fifth excited state
- b. [20] For $\omega=1.03~\omega_o$ find the time evolution of the population in the initial state for the following initial states over one period of oscillation:
 - i. [10] $|\psi(t=0)\rangle = |05\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its fifth excited state
 - ii. [10] $|\psi(t=0)\rangle = |16\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its fifth excited state