Physics Other: Homework 1

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Fall 2024

Problem 1.3.1

Equation 1.14 is

$$U(S,V) = C\left(\frac{e^{\frac{S}{nR}}}{V}\right)^{\frac{2}{3}} \tag{1}$$

Then, using the definition of temperature

$$\begin{aligned}
T &= \left(\frac{\partial S}{\partial U}\right)^{-1} \\
\frac{\partial S}{\partial U} &= \partial_U nR\ln\left(\frac{VU^{\frac{3}{2}}}{C^{\frac{3}{2}}}\right) \\
&= \frac{3nR}{2U} \\
\implies T &= \frac{2}{3nR}U
\end{aligned} \tag{2}$$

We now apply the Legendre transform to obtain the Helmholtz free energy potential

$$\begin{aligned}
F(T,V,N) = U - TS& \\
&= \frac{3nRT}{2} - T\left(nR\ln\left(\frac{VU^{\frac{3}{2}}}{C^{\frac{3}{2}}}\right)\right) \\
&= \frac{3nRT}{2}\left(1 - \frac{2}{3}\ln V - \ln U + \ln C\right) \\
&= \frac{3nRT}{2}\left((1+\ln C) - \ln\left(\frac{3nRT}{2}\right) - \frac{2}{3}\ln V\right)
\end{aligned} \tag{3}$$

We can now take the partial derivatives with respect to F

$$\frac{\partial F}{\partial T} = \frac{3nR}{2}() \tag{4}$$

Problem 1.6.1

We want to find the partition function for the classical harmonic oscillator, where

$$E(x,p) = \frac{p^2}{2m} + \frac{1}{2}m\omega_0^2x^2 \tag{5}$$

Then the partition function is, for a continuous E

$$Z = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left(-\beta\left(\frac{p^2}{2m} + \frac{1}{2}m\omega_0^2x^2\right)\right)dxdp \tag{6}$$

This is just two Gaussian integrals, which evaluate to

$$Z = \frac{2\pi}{\beta\sqrt{\omega_0}} \tag{7}$$

Problem 1.6.2

For the quantum harmonic oscillator the partition function becomes

$$\begin{aligned} Z &= \sum_i e^{-\beta E_i} \\ &= \sum_n e^{-\beta\hbar\omega_0\left(n+\frac{1}{2}\right)} \\ &= e^{-\beta\frac{\hbar\omega_0}{2}} \sum_n \exp\left(-\beta\hbar\omega_0\right)^n \\ &= e^{-\beta\frac{\hbar\omega_0}{2}} \left(1 - e^{-\beta\hbar\omega_0}\right)^{-1} \\ &= e^{-\beta\frac{\hbar\omega_0}{2}} n_{BE}\left(\hbar\omega_0\right) \end{aligned} \tag{8}$$

Problem 1.6.3

The classical average energy is

$$\langle E\rangle = \frac{1}{Z} \tag{9}$$

Problem 1.7.1

Problem 1.8.1