Weighting and Marginal Structural Models

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June 2020

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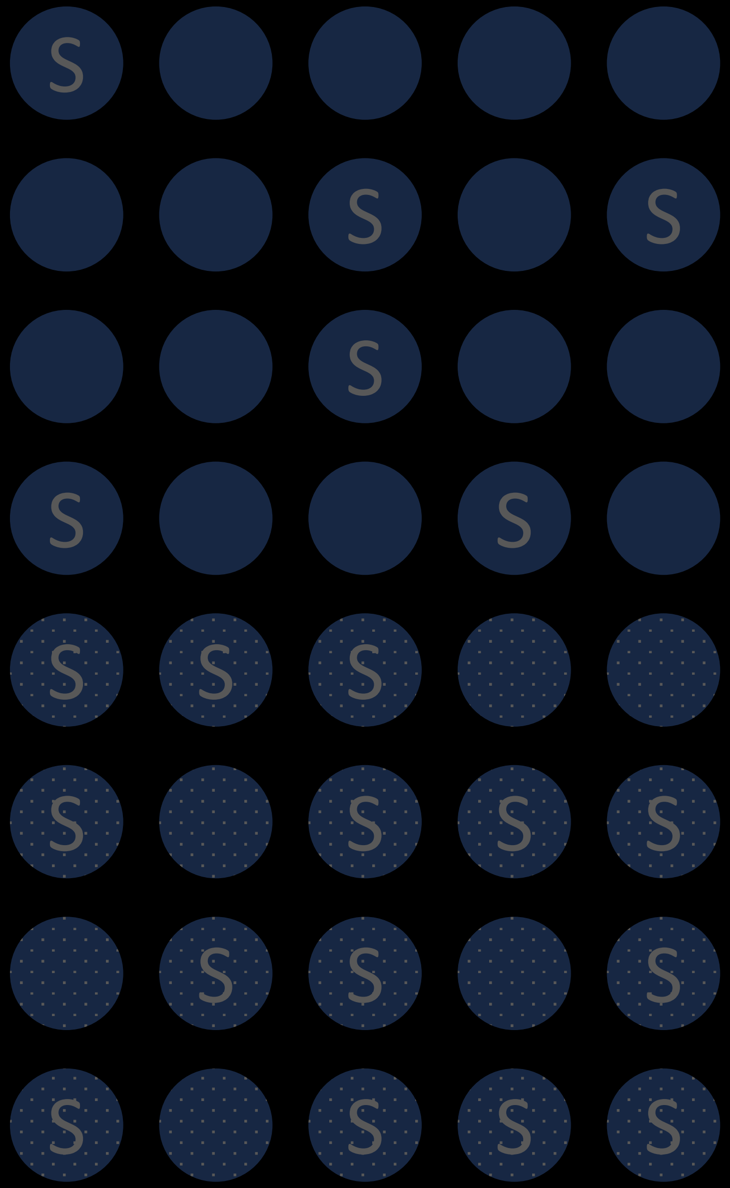
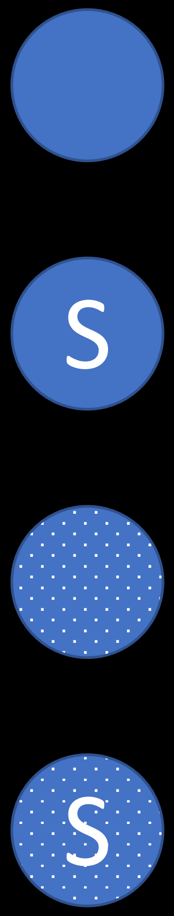




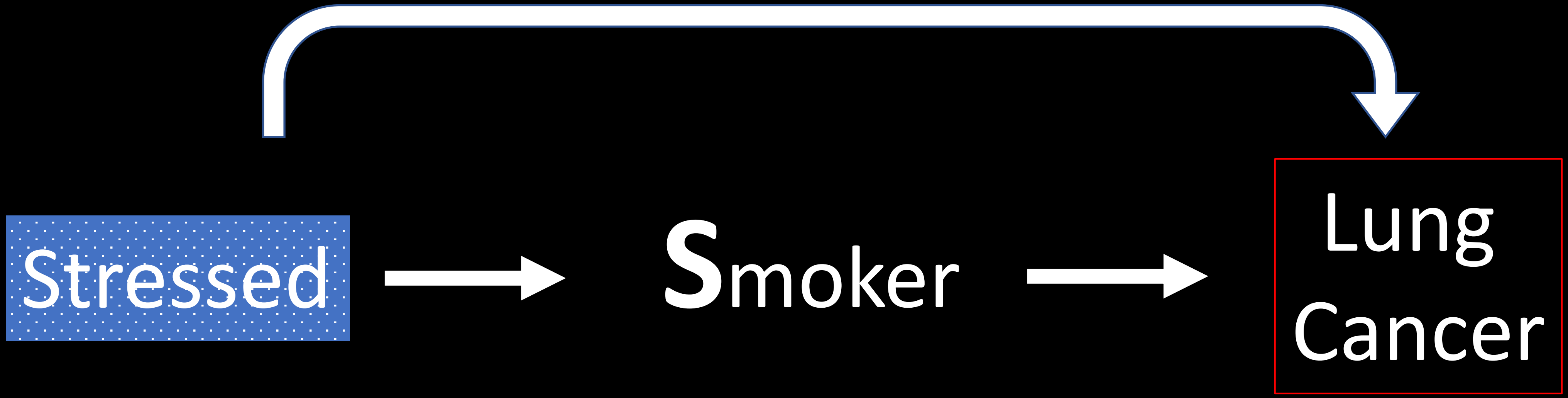
Unstressed Non-Smoker

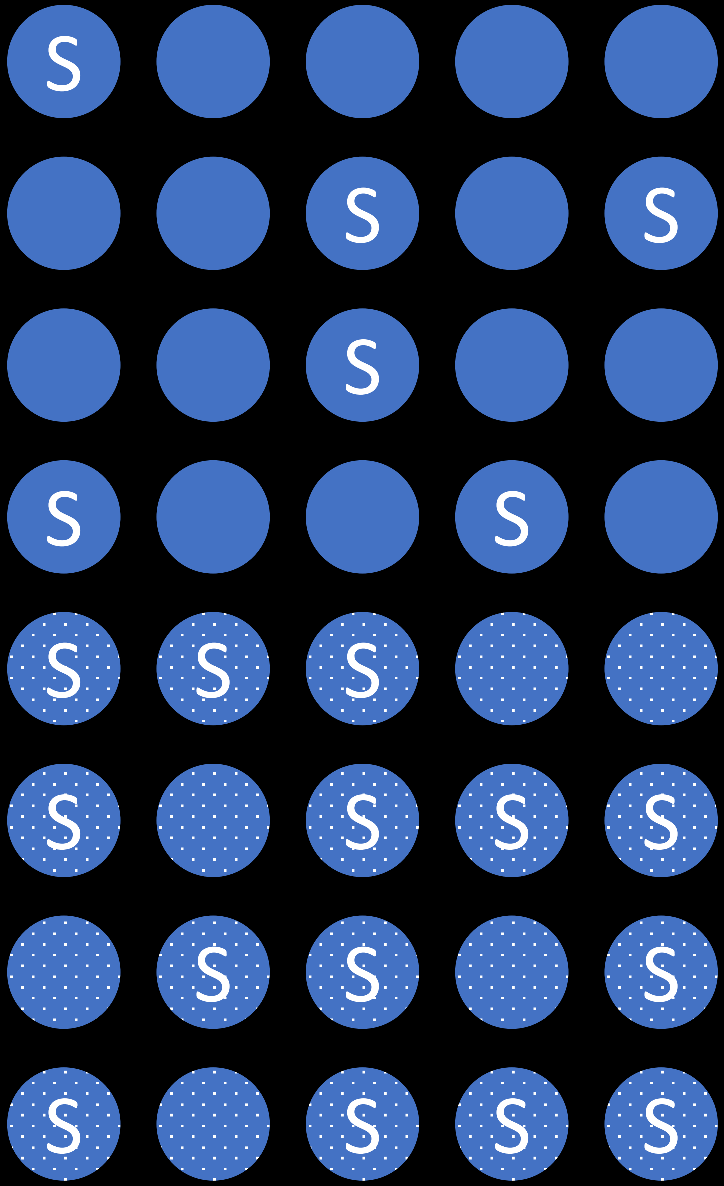
Unstressed Smoker

Stressed Non-Smoker

Stressed Smoker

We are interested in the effect of smoking on lung cancer

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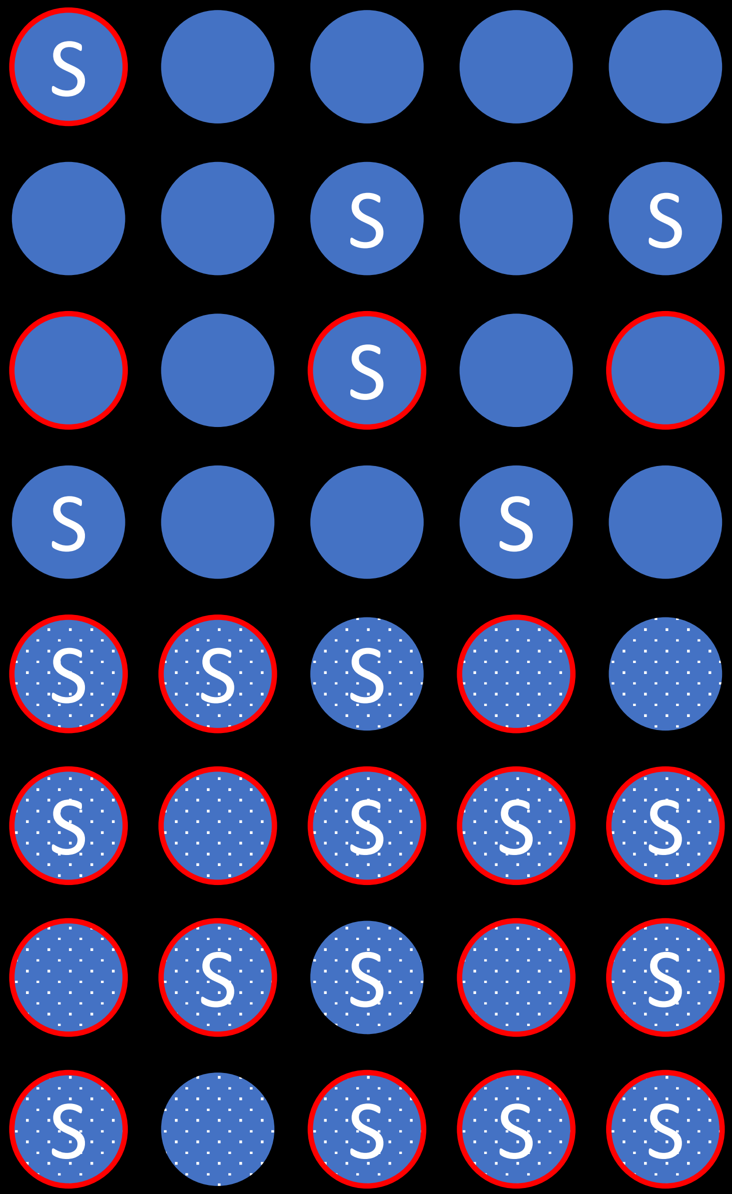

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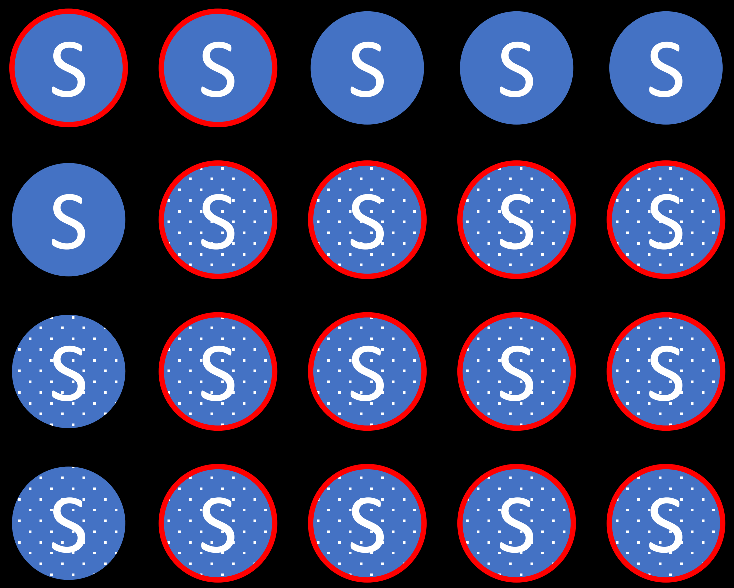
The effect of smoking on lung cancer

$$\frac{14}{20} - \frac{6}{20} = \frac{8}{20}\,?$$



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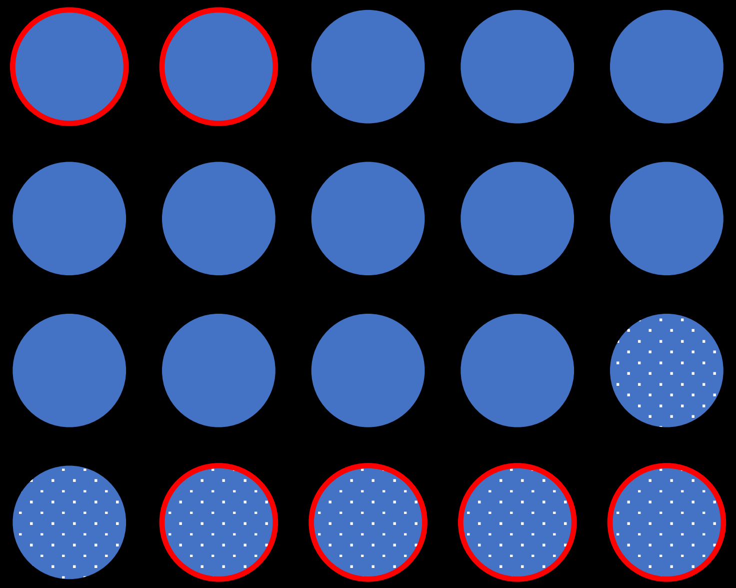
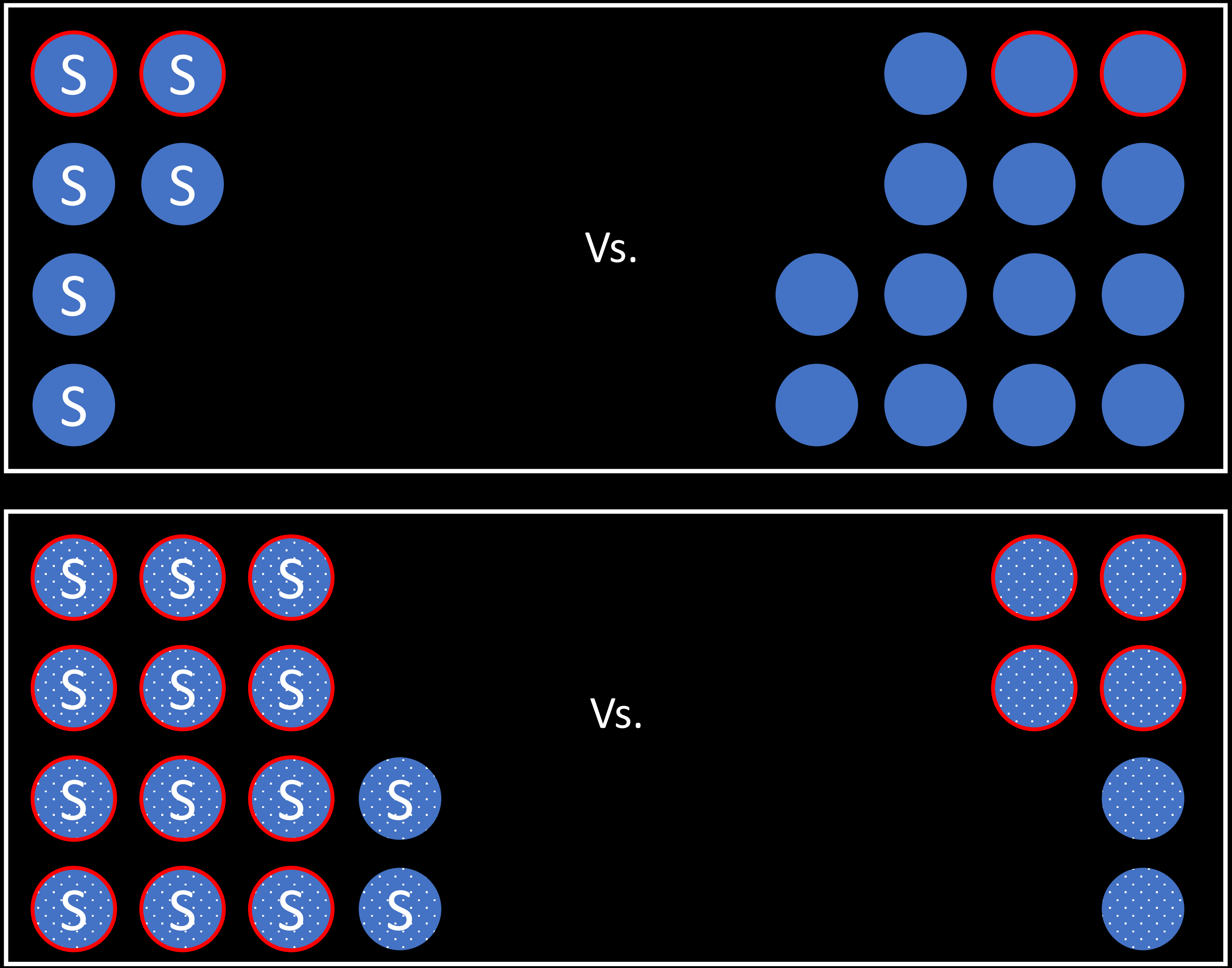


Linear Regression

Linear Regression Stratifies the population



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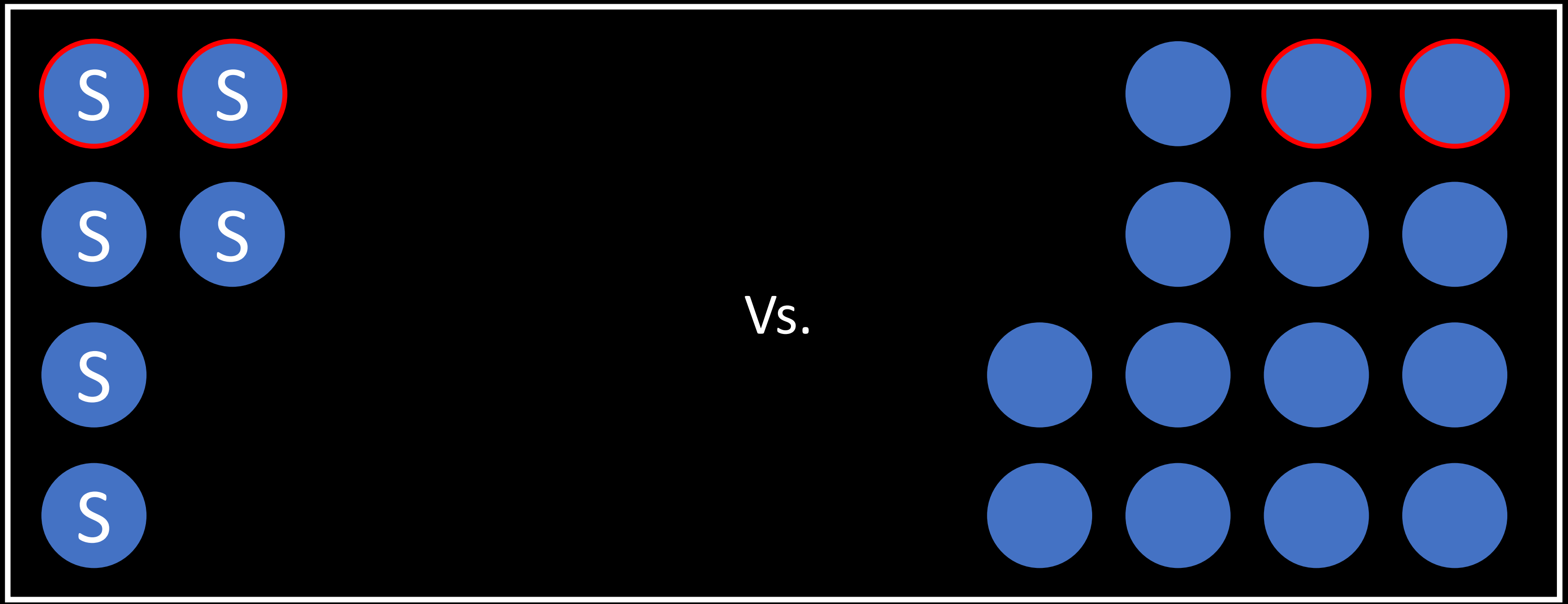
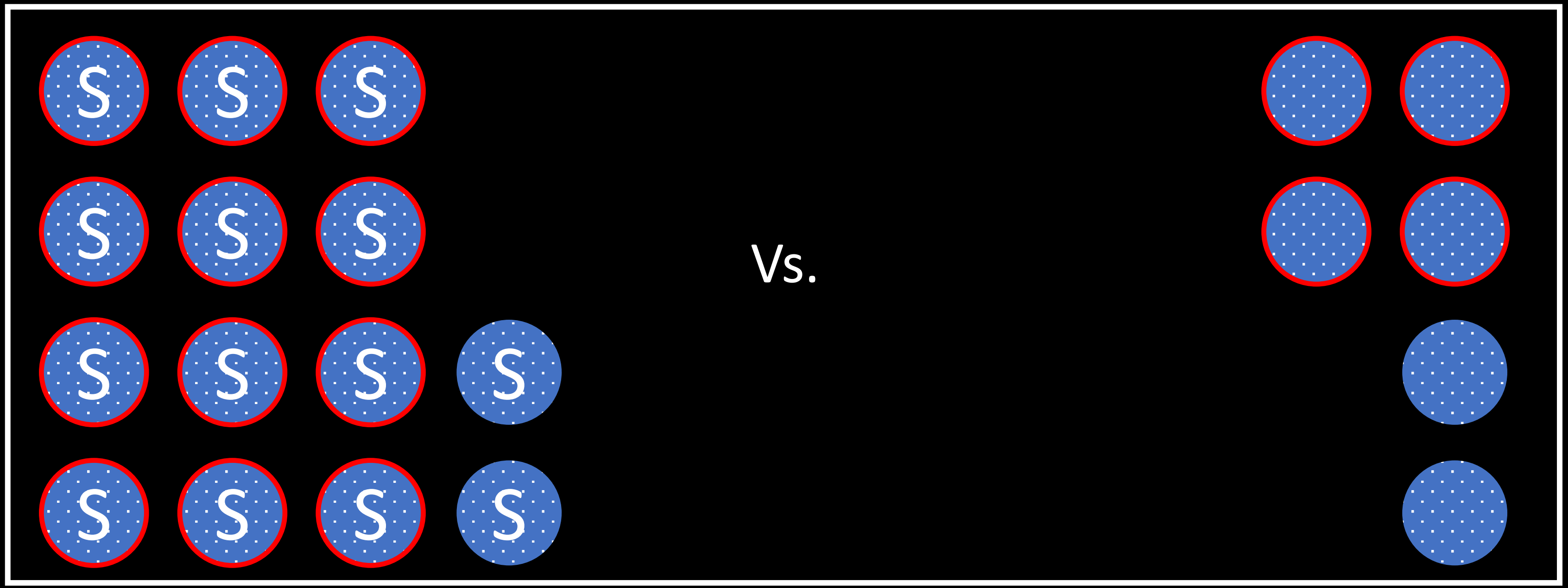
Effect of smoking among the unstressed

$$\frac{2}{6} - \frac{2}{14} = \frac{4}{21}$$



Effect of smoking among the stressed

$$\frac{12}{14} - \frac{4}{6} = \frac{4}{21}$$



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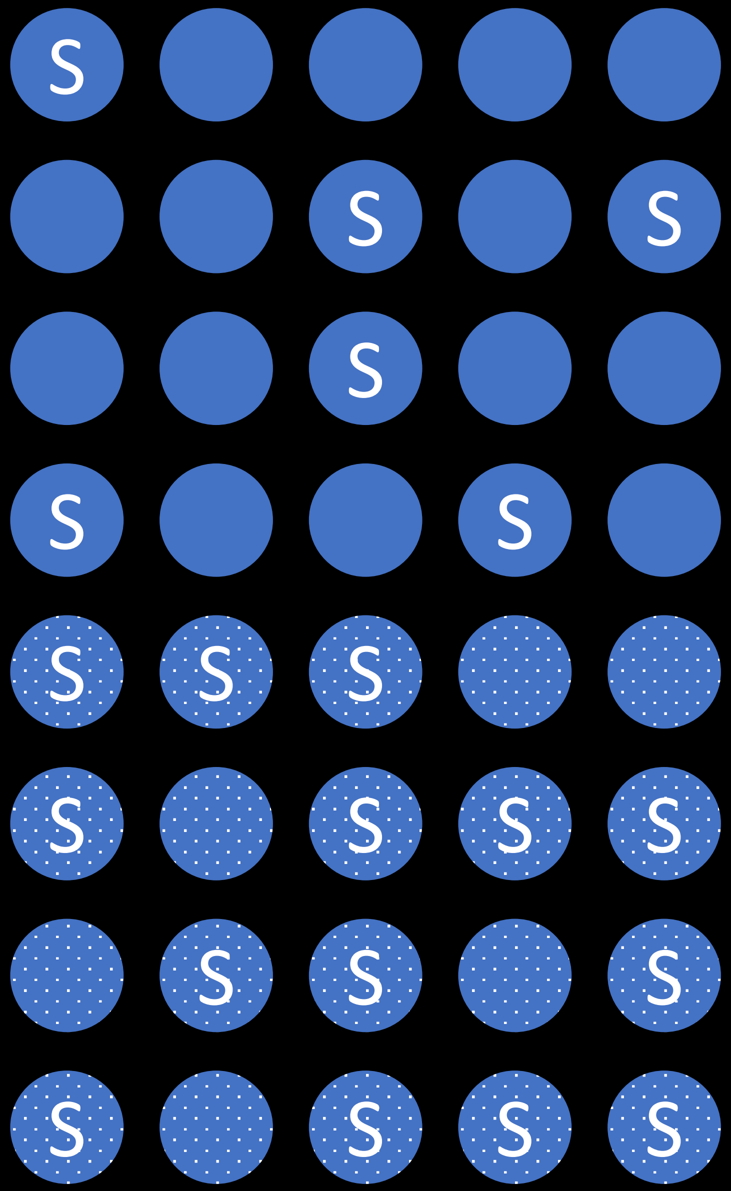
Regression is a model for the stratum-specific mean of the outcome

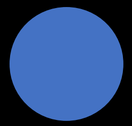
$$\hat{E}[Y|\, Smoking, Stress] = \frac{2}{14} + \frac{4}{21} Smoking + \frac{22}{42} Stress$$

Weighting for Single Treatment



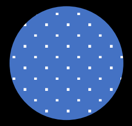
Imagine an RCT that assigns Smoking as follows:

$$P[Smoker = 1 \mid Stress = 0] = \frac{6}{20}$$

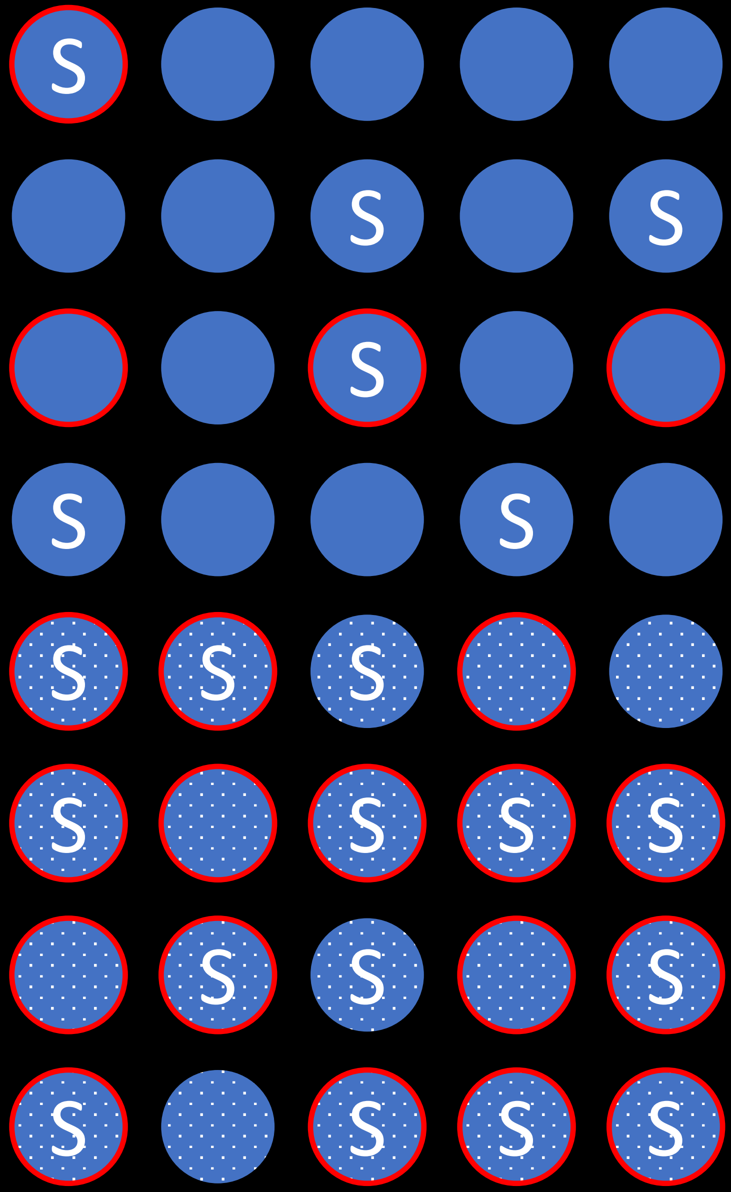


$$P[Smoker = 0 \mid Stress = 0] = \frac{14}{20}$$

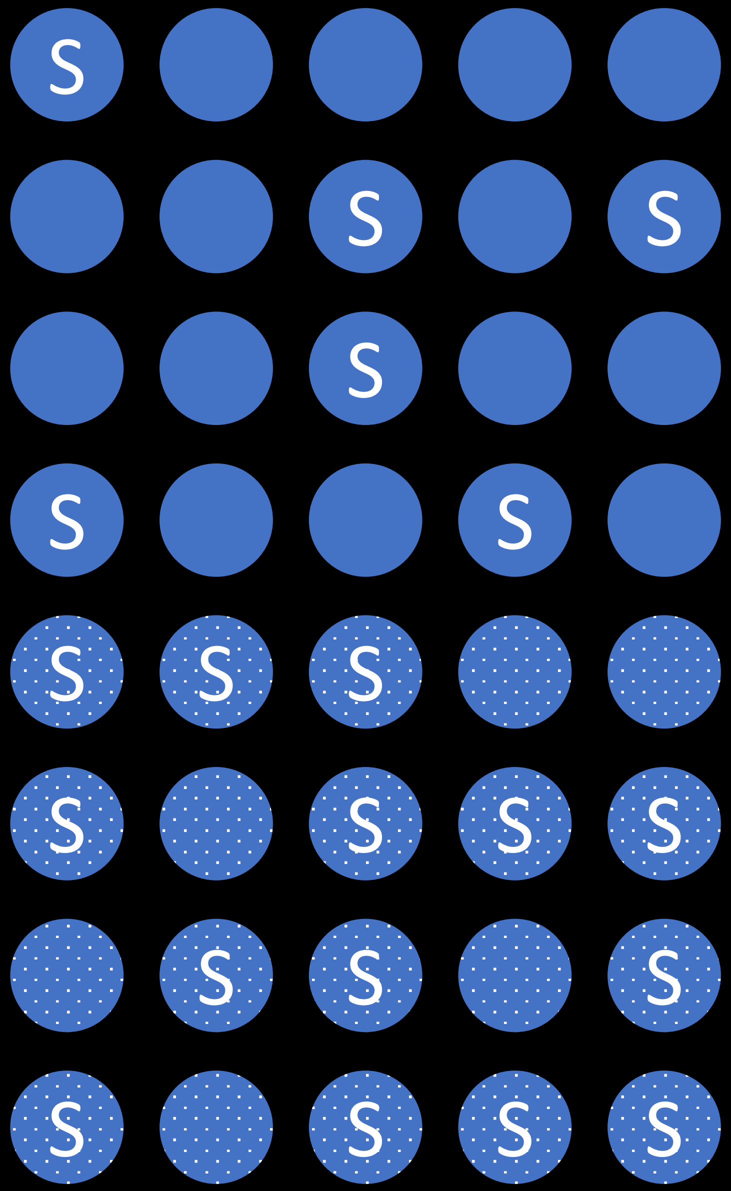
$$P[Smoker = 1 \mid Stress = 1] = \frac{14}{20}$$



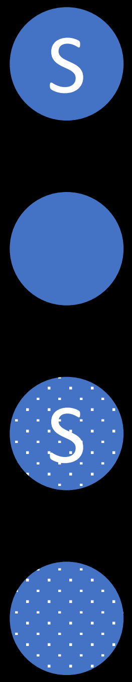
$$P[Smoker = 0 \mid Stress = 1] = \frac{6}{20}$$



The RCT follows the cohort and assesses lung cancer



We decide to undo the randomization scheme using weights

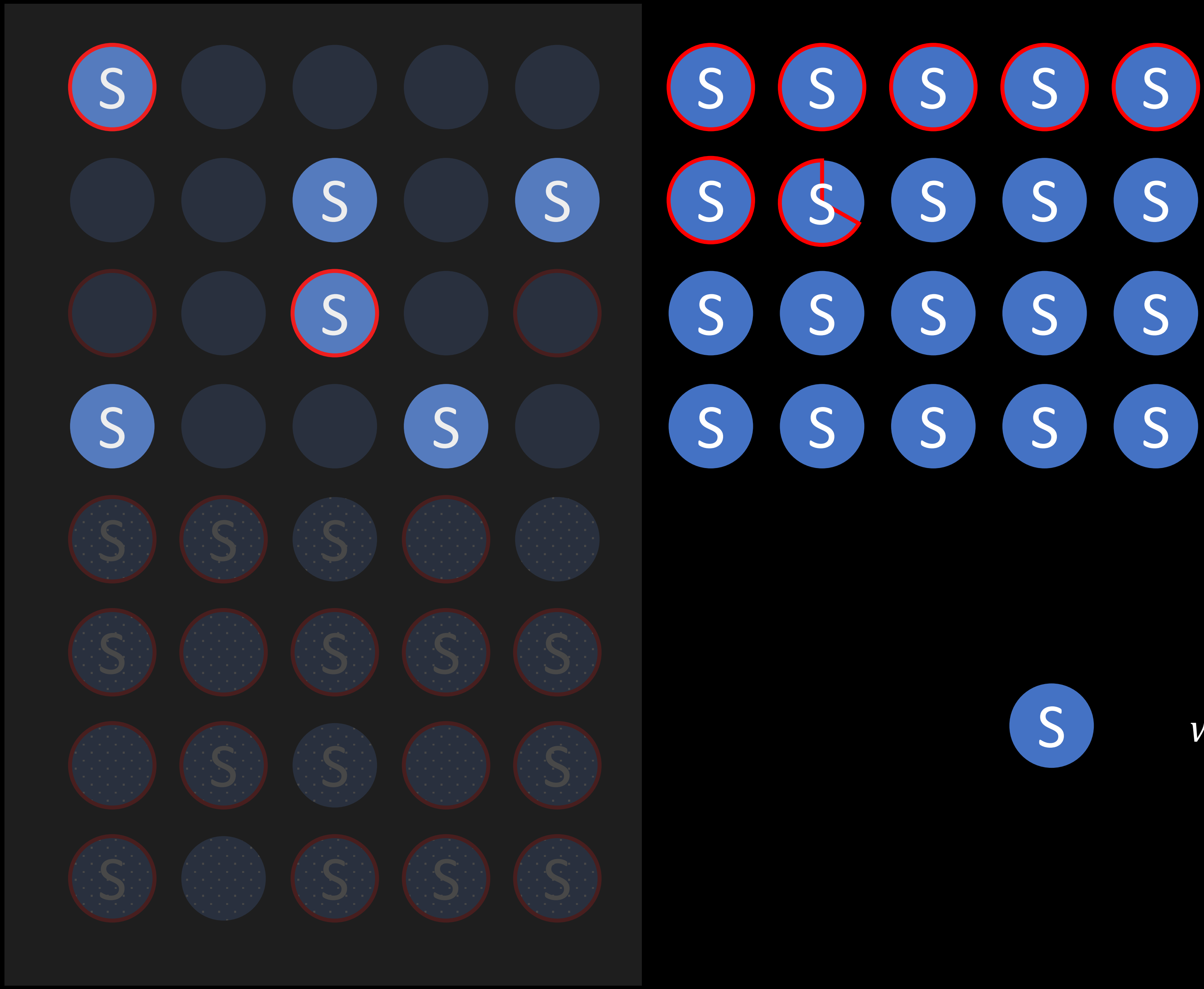


$$w = \frac{1}{P[Smoker = 1 \mid Stress = 0]} = \frac{20}{6}$$

$$w = \frac{1}{P[Smoker = 0 \mid Stress = 0]} = \frac{20}{14}$$

$$w = \frac{1}{P[Smoker = 1 \mid Stress = 1]} = \frac{20}{14}$$

$$w = \frac{1}{P[Smoker = 0 \mid Stress = 1]} = \frac{20}{6}$$



$$w = \frac{20}{6}$$

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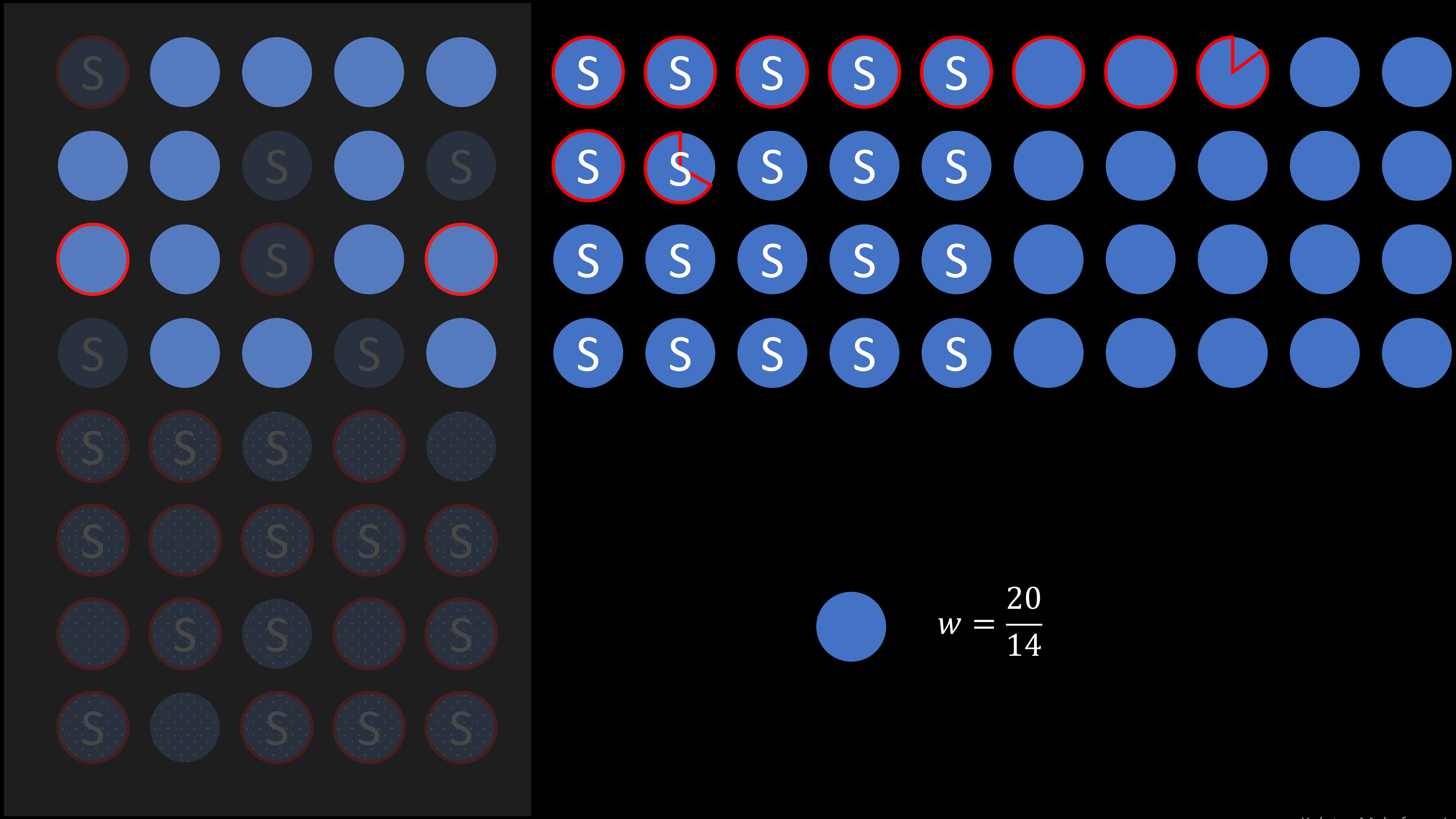
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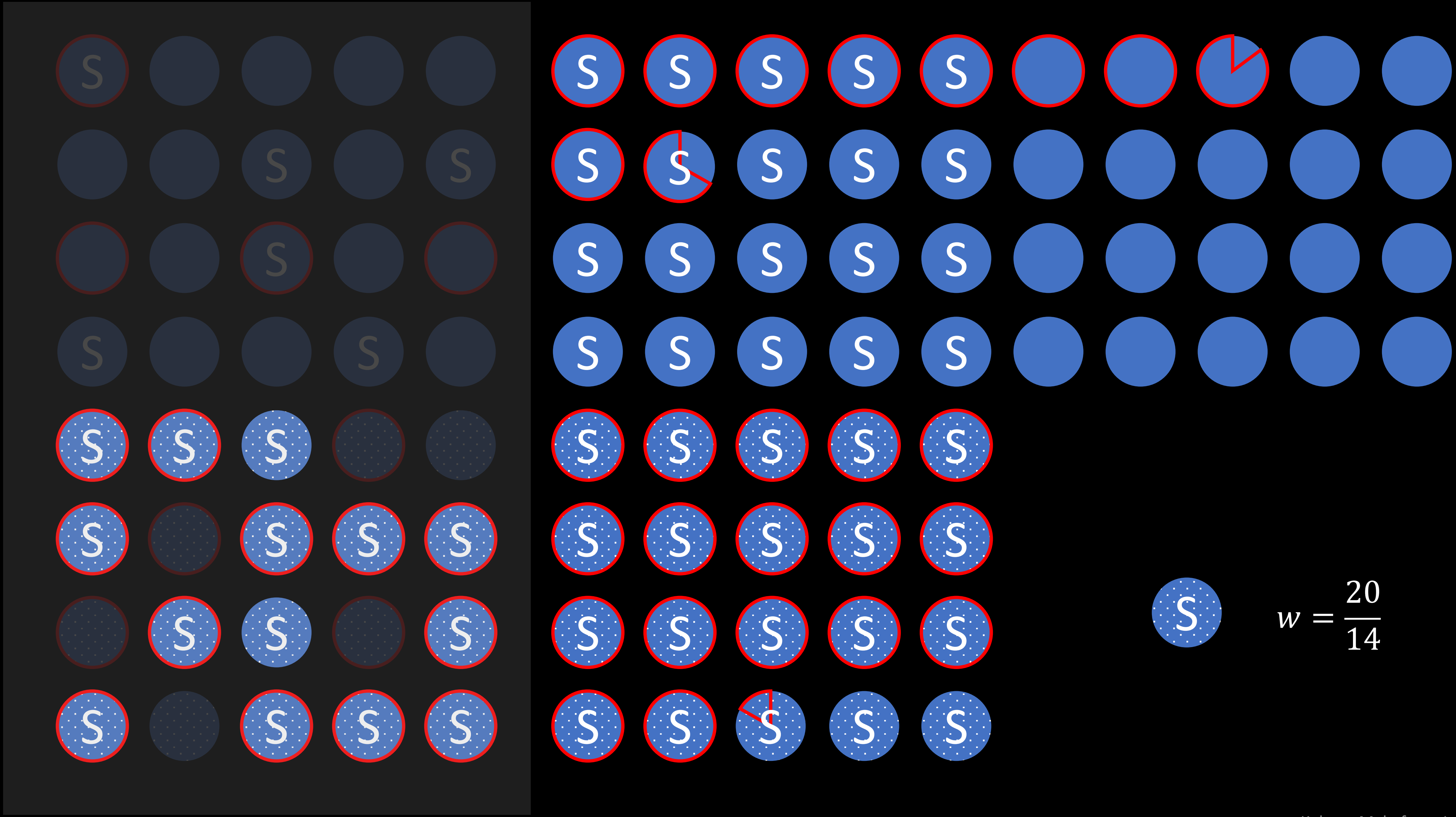
$w = \frac{20}{14}$



$$w = \frac{20}{14}$$

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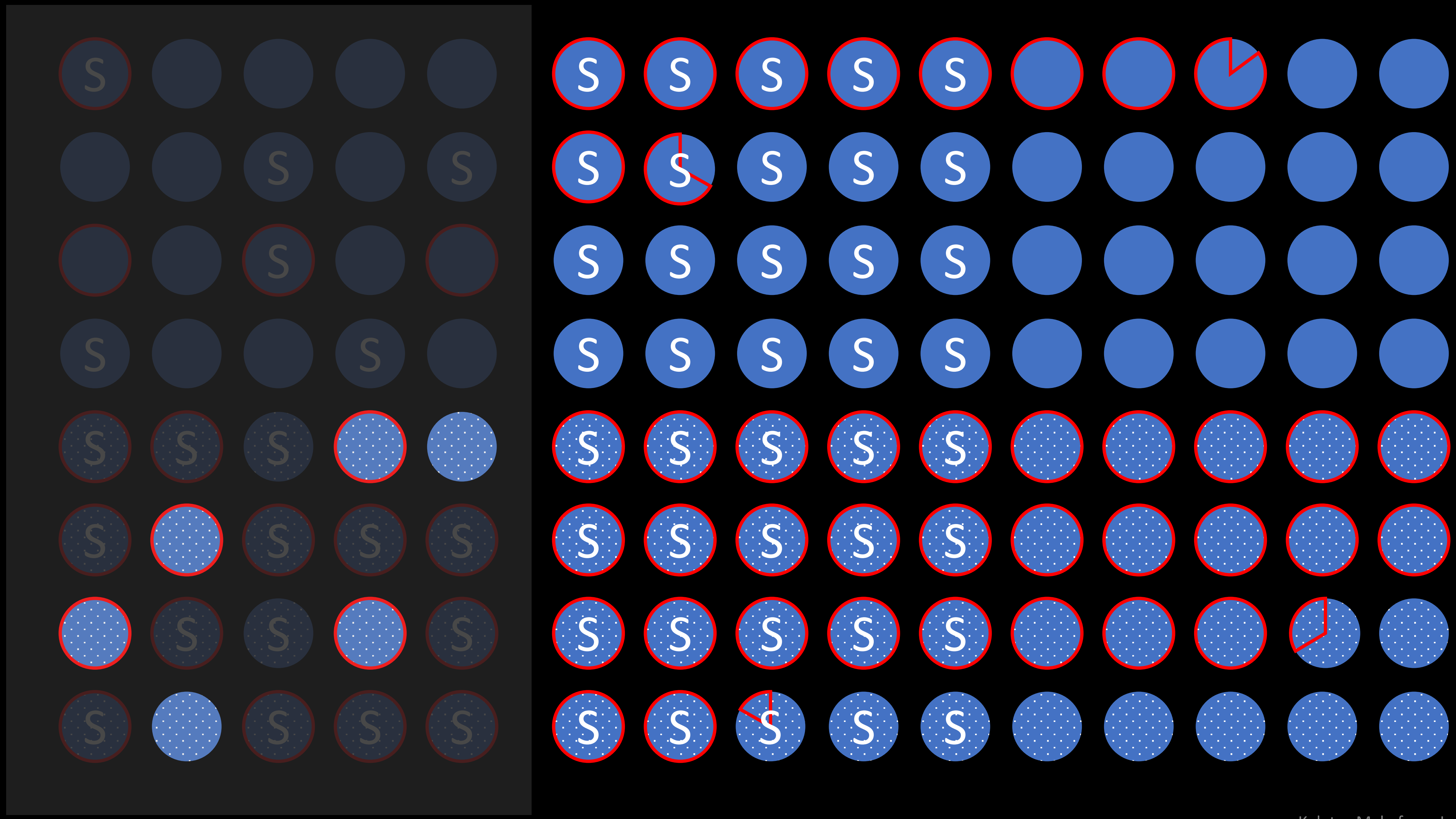

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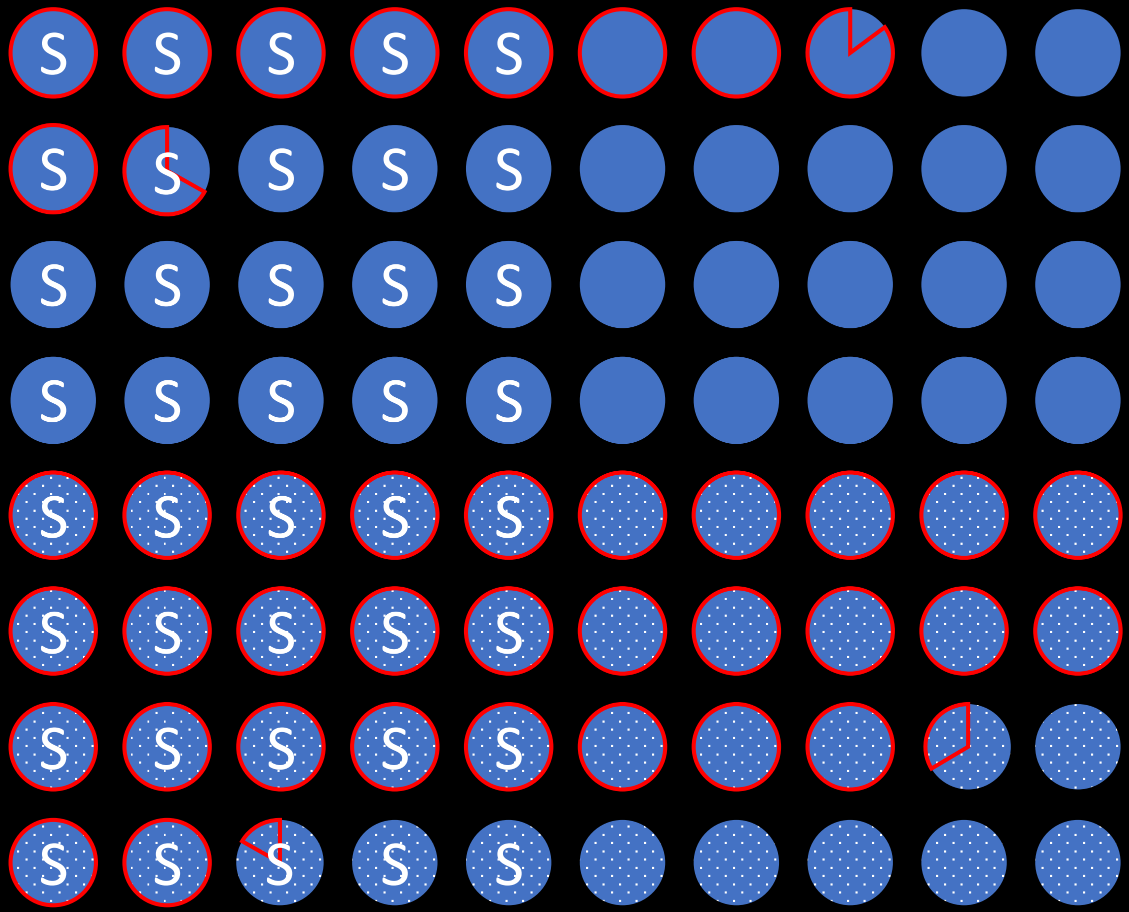
In our pseudo population, the distribution of stress is the same among smokers as it is among non-smokers



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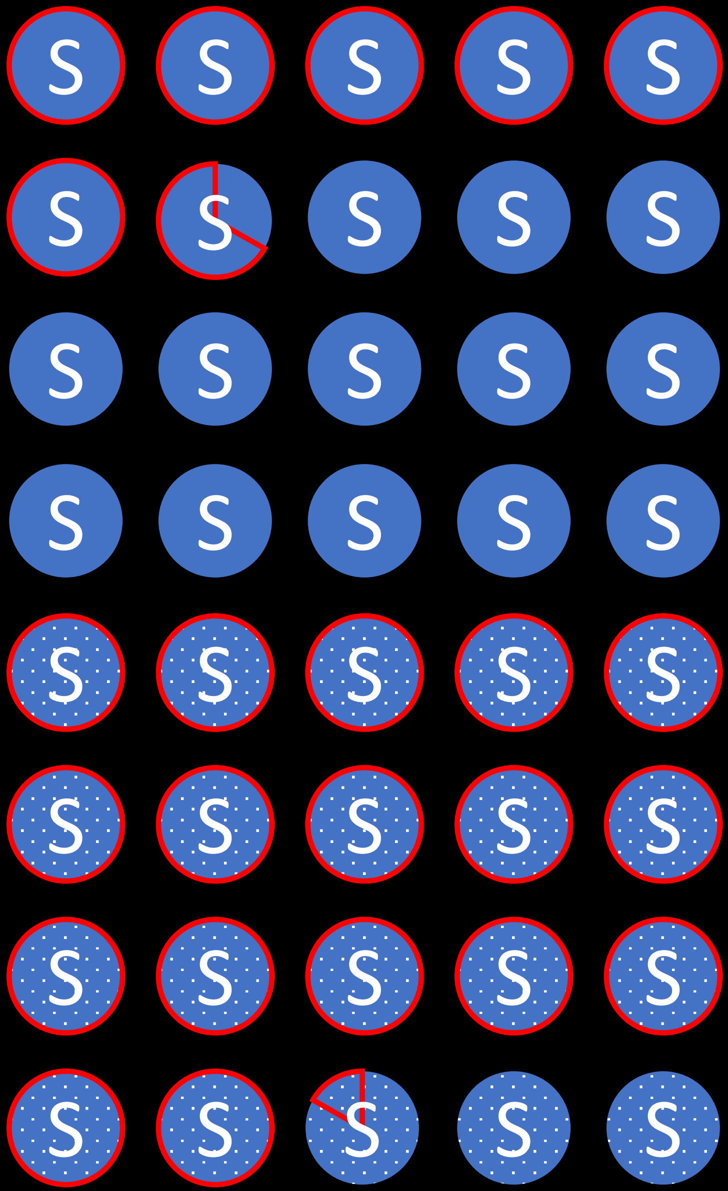

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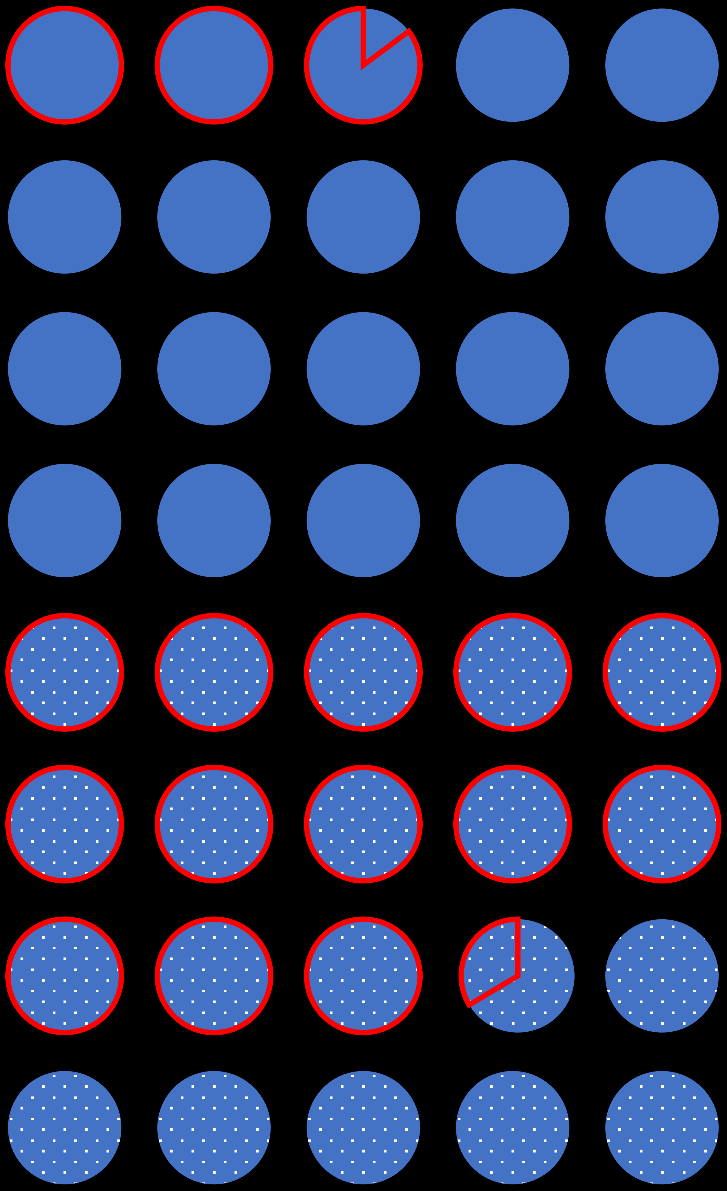

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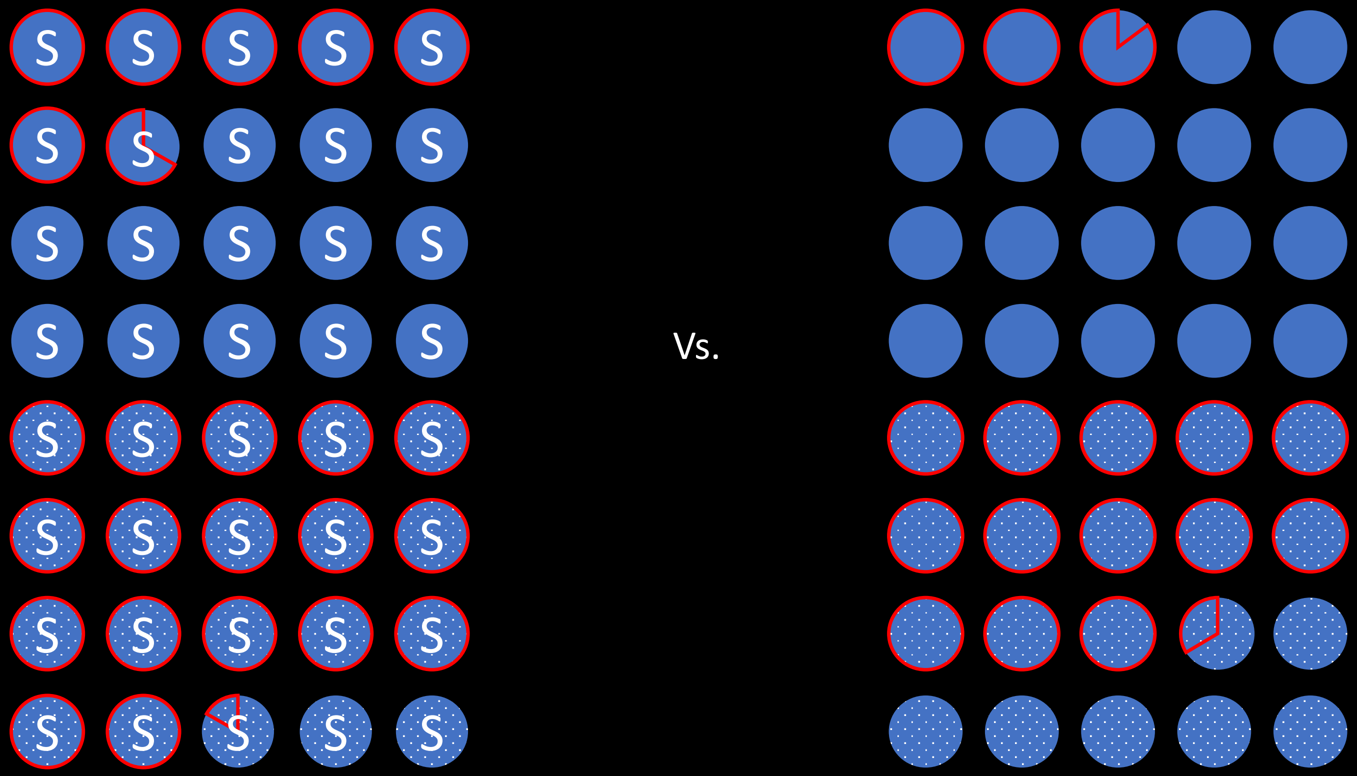


Effect of smoking among the population:

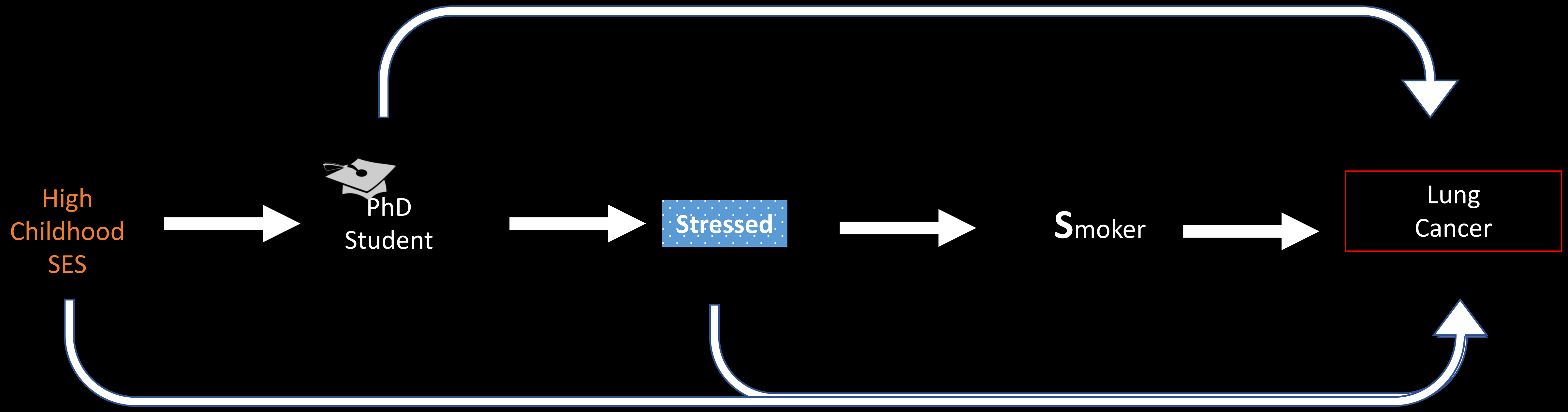
$$\frac{23.8095}{40} - \frac{16.1904}{40} = \frac{4}{21}$$

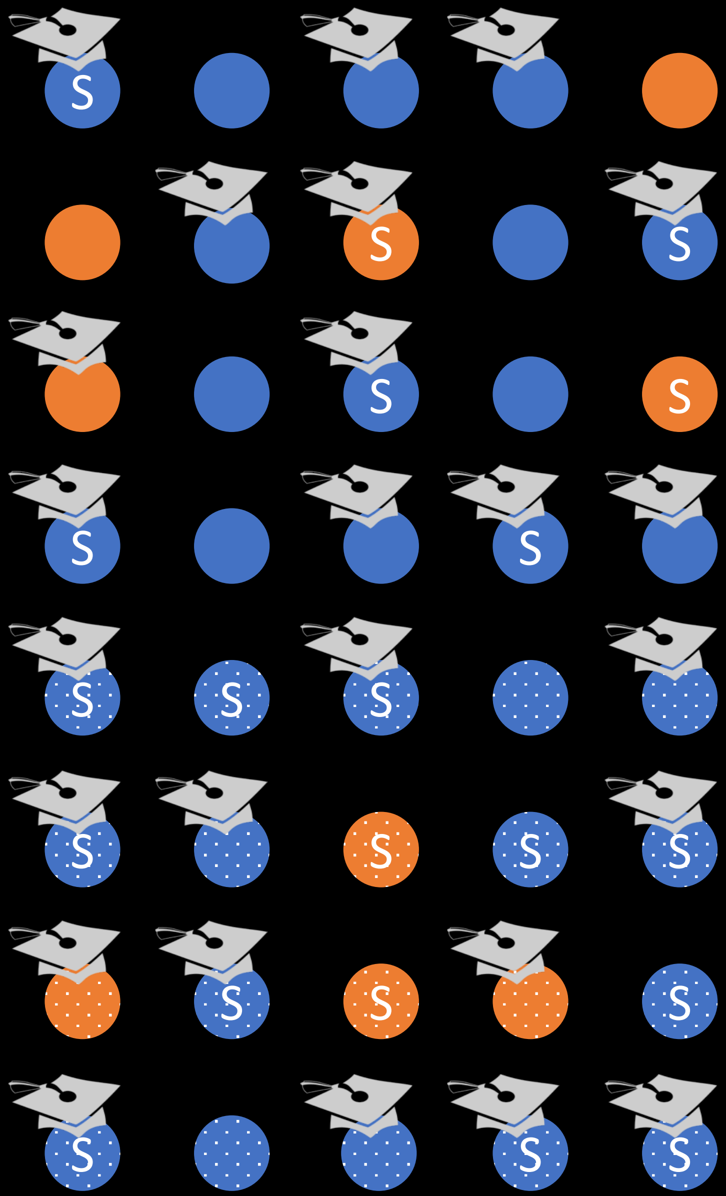
MSM is a model for the counterfactual mean of the outcome

$$\hat{E}[Y_{smoking}] = \frac{16.1904}{40} + \frac{4}{21}Smoking$$

Weighting for Treatment at Multiple Times

We are interested in the joint effect of smoking and PhD studies on lung cancer





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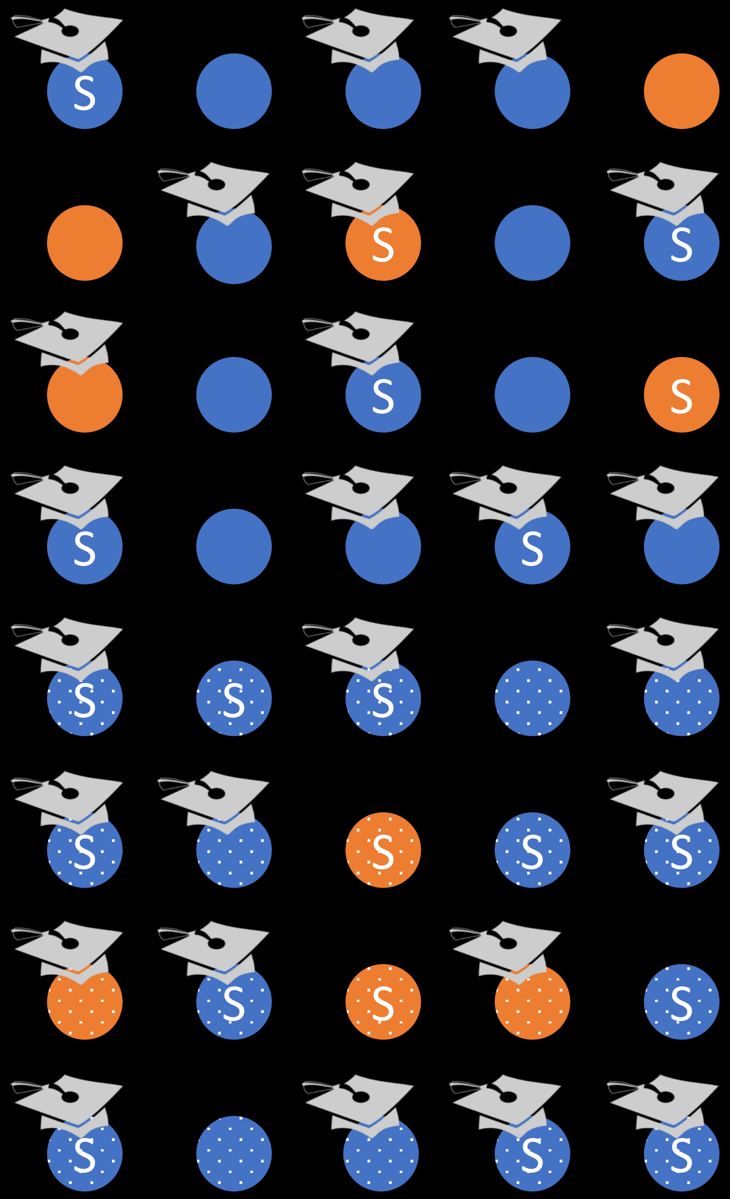
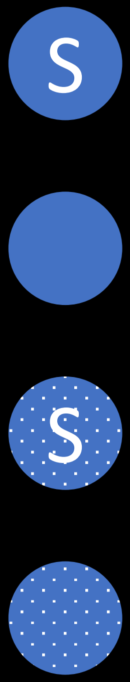


We imagine that Smoking was assigned as follows:

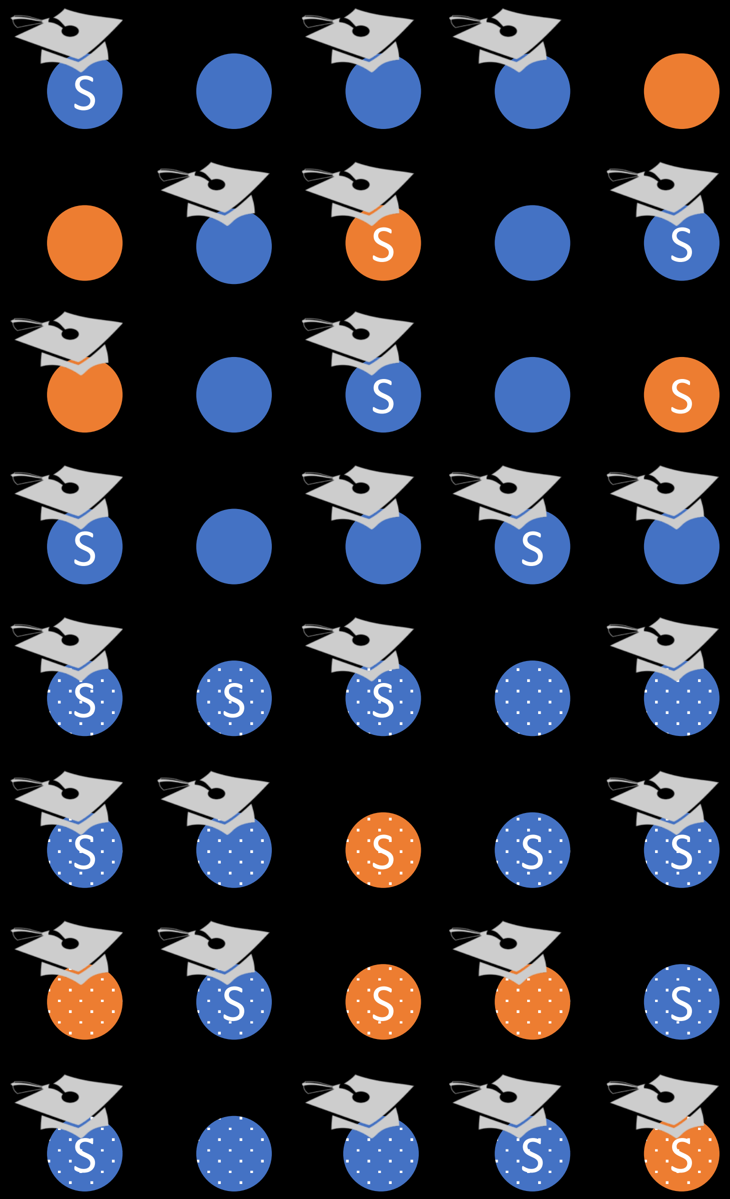


$$P[Smoker = 1 \mid Stress = 0, PhD, cSES]$$

$$P[Smoker = 0 \mid Stress = 0, PhD, cSES]$$

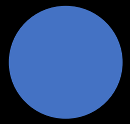
$$P[Smoker = 1 \mid Stress = 1, PhD, cSES]$$

$$P[Smoker = 0 \mid Stress = 1, PhD, cSES]$$



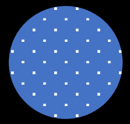
We undo this portion of the sequential randomization scheme using weights

$$w = \frac{1}{P[Smoker = 1 \mid Stress = 0, PhD, cSES]}$$

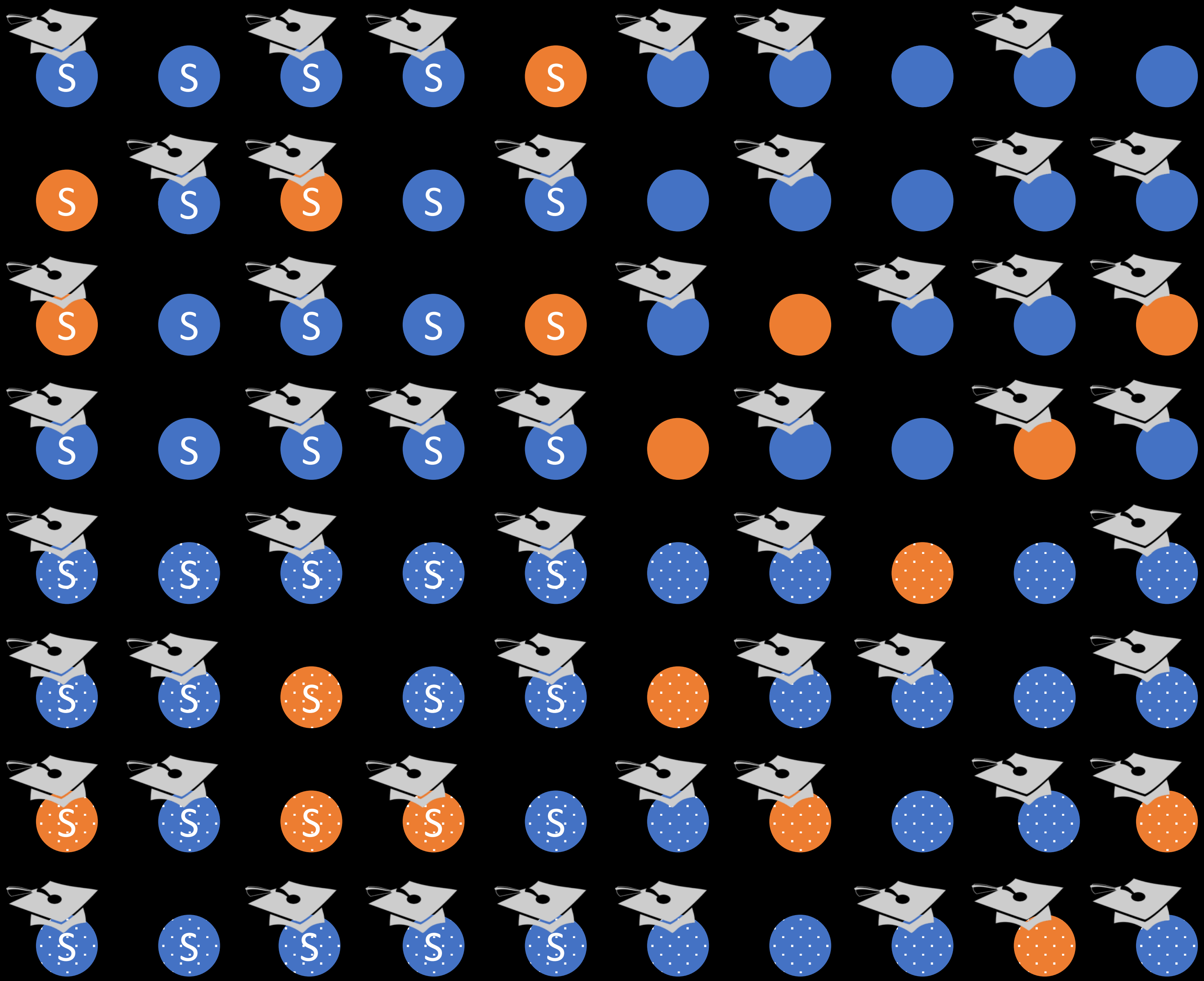


$$w = \frac{1}{P[Smoker = 0 \mid Stress = 0, PhD, cSES]}$$

$$w = \frac{1}{P[Smoker = 1 \mid Stress = 1, PhD, cSES]}$$



$$w = \frac{1}{P[Smoker = 0 \mid Stress = 1, PhD, cSES]}$$



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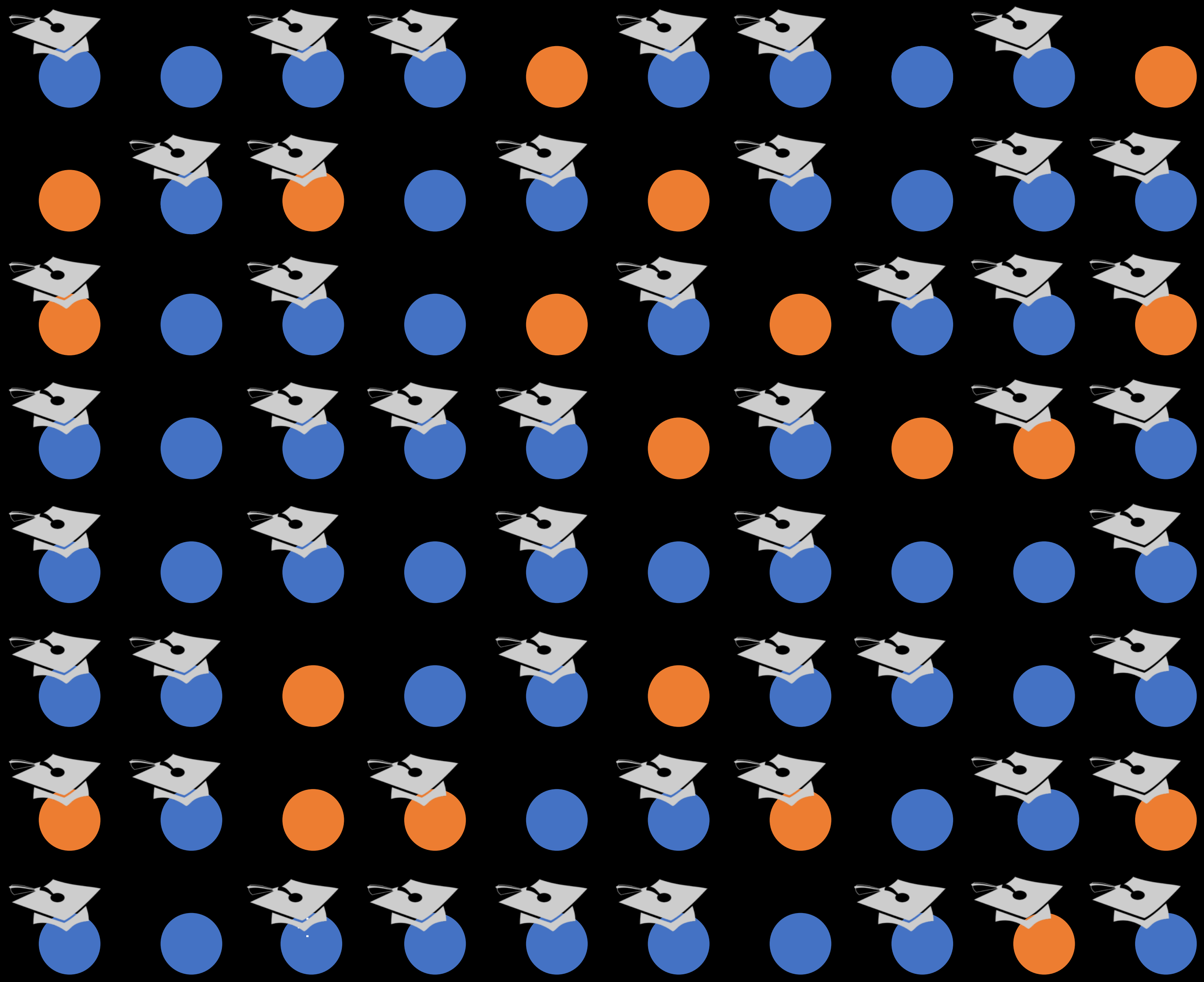


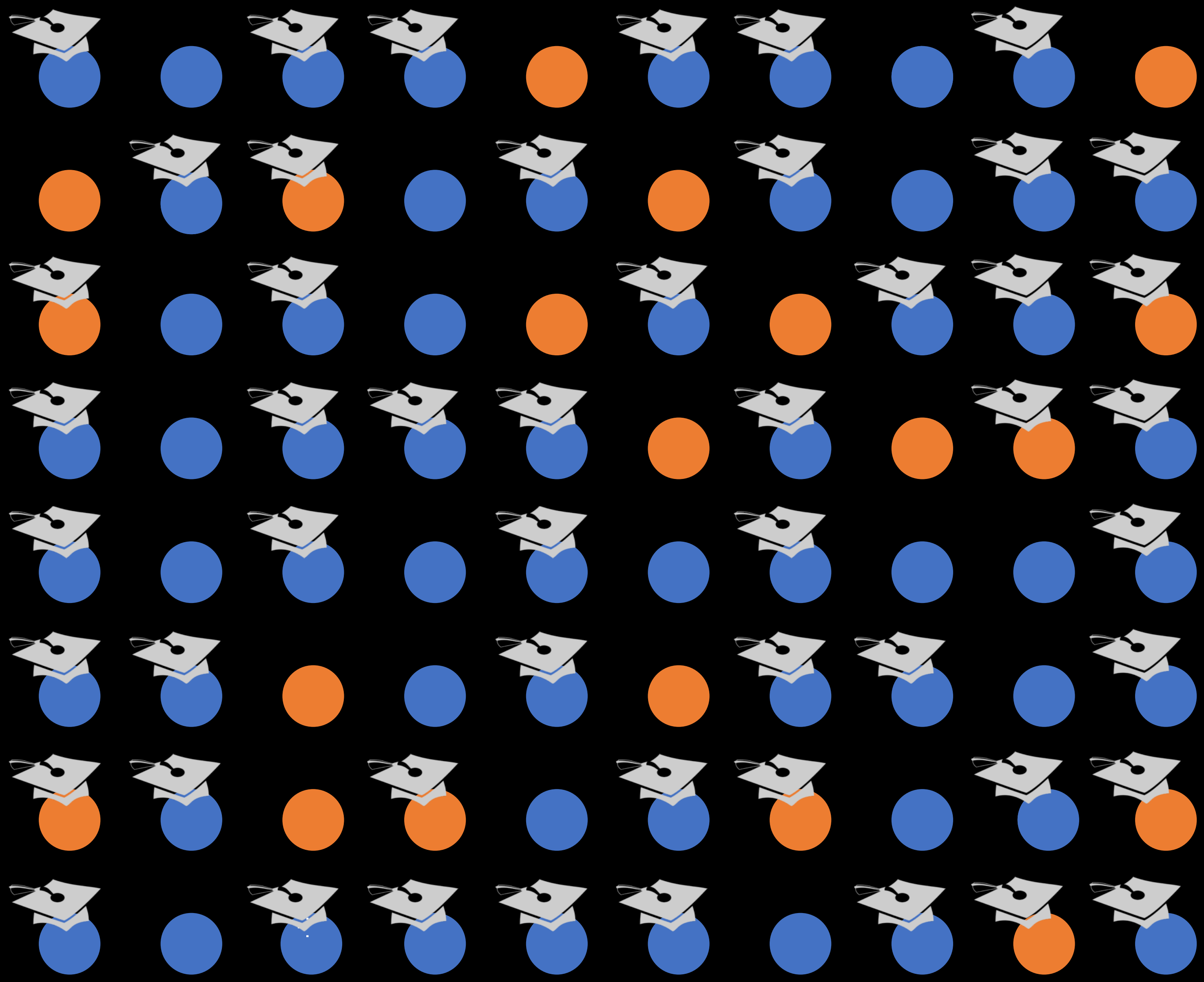
We imagine that PhD Status was assigned as follows:

$P[PhD = 1 \mid cSES = 0]$

$P[PhD = 0 \mid cSES = 0]$

$P[PhD = 1 \mid cSES = 1]$

$P[PhD = 0 \mid cSES = 1]$



We undo this portion of the sequential randomization scheme using weights

$$w = \frac{1}{P[PhD = 1 \mid cSES = 0]}$$

$$w = \frac{1}{P[PhD = 0 \mid cSES = 0]}$$

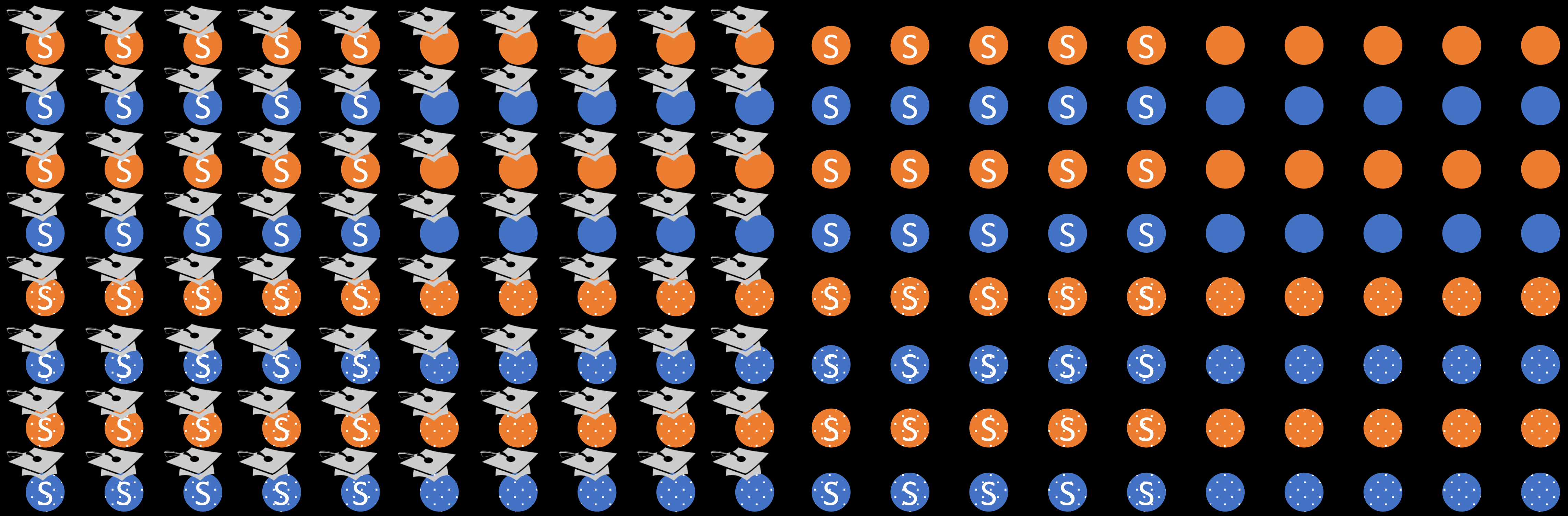
$$w = \frac{1}{P[PhD = 1 \mid cSES = 1]}$$

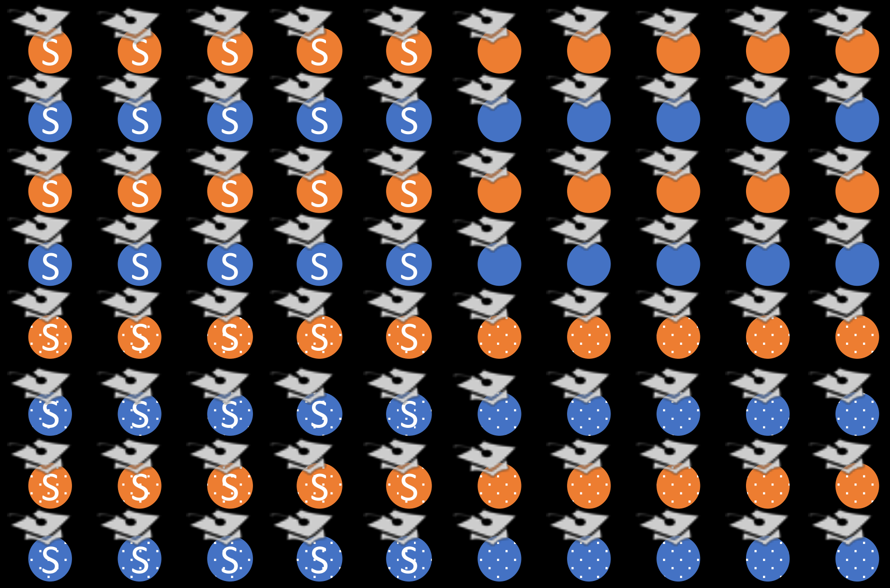
$$w = \frac{1}{P[PhD = 0 \mid cSES = 1]}$$

In our pseudo population:

a) the distribution of stress and cSES is the same among smokers as it is among non-smokers

b) the distribution of stress and cSES is the same among PhD students as it is among non-PhD students





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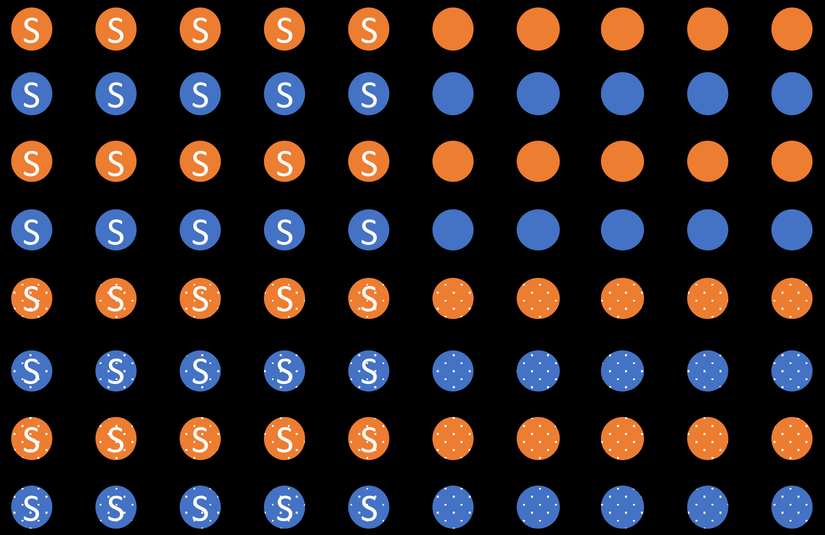
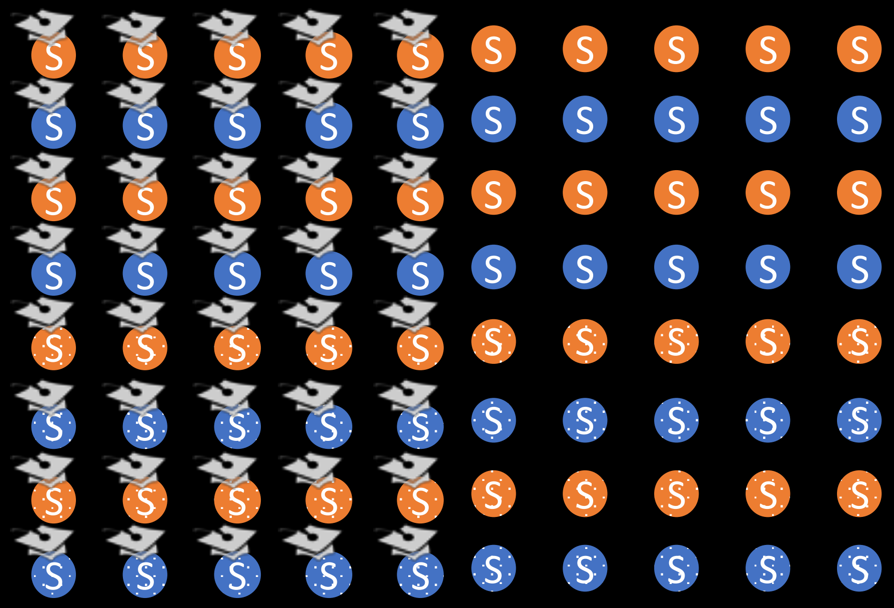


Effect of PhD Studies among the population:

$$\text{Cumulative Incidence of Lung Cancer}_{\text{LHS}} - \text{Cumulative Incidence of Lung Cancer}_{\text{RHS}} = \hat{\beta}_{PhD}$$



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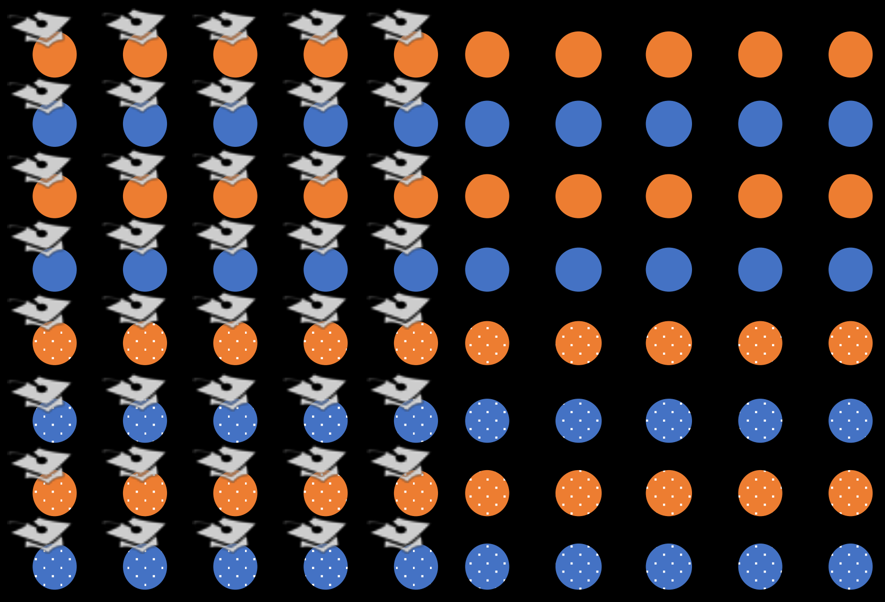

Effect of Smoking among the population:

$$\text{Cumulative Incidence of Lung Cancer}_{\text{LHS}} - \text{Cumulative Incidence of Lung Cancer}_{\text{RHS}} = \hat{\beta}_{Smoking}$$

MSM is a model for the counterfactual mean of the outcome

$$\hat{E}\left[Y_{smoking,PhD}\right] = \hat{\beta}_0 + \hat{\beta}_{smoking} Smoking + \hat{\beta}_{PhD} PhDStatus$$