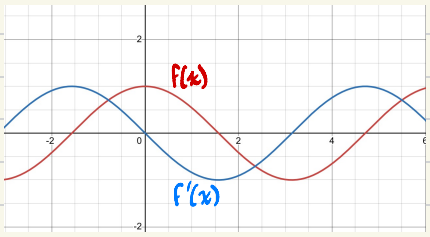
LEC16 - Trig Derivatives

Trig Derivative Chart

$f(x)$	$f'(x)$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\cot x$	$-\csc^2 x$

Warm-Up Problem:

Sketch the derivative of $f(x) = \cos x$



Example 1:

$f(x) = \sin x \tan x$, find where the tangent is horizontal. Find where $f'(x) = 0$

$f'(x) = \cos x \tan x + \sin x \sec^2 x$ Find $f'(x)$

$= \cos x \frac{\sin x}{\cos x} + \sin x \frac{1}{\cos^2 x}$

$= \sin x + \frac{\sin x}{\cos^2 x}$

$0 = \frac{\sin x \cos^2 x + \sin x}{\cos^2 x}$ Solve $f'(x)$ for 0

$0 = \sin x(\cos^2 x + 1)$ Multiply both sides by $\cos^2 x$, then factor out $\sin(x)$

$\sin x = 0$ $\quad$ $\cos^2 x + 1 = 0$

$x = 0$ $\quad$ $\cos^2 x = -1$ Always positive because squared

No solutions

Example 2:

A ball is attached to a spring hanging from the ceiling in a simple harmonic motion, bouncing up & down given by:

$x(t) = x_0 + A\cos(\omega t)$

1. What do x_0, A, ω represent
   - x_0 represents the lowest distance
   - A represents the distance the spring travels
   - ω represents the bouncing speed
2. If $\omega = 1$, find $v(\frac{\pi}{6})$, $a(\frac{\pi}{6})$
   - $x(t) = x_0 + A\cos(t)$
   - $v(t) = A\cos(t) \Rightarrow -A\sin(\frac{\pi}{6}) = -A/2$
   - $a(t) = -A\sin(t) \Rightarrow -A\cos(\frac{\pi}{2}) = -\sqrt{3}/2\, A$
3. At $t = \frac{\pi}{6}$ is the speeding up or slowing down
   - Speeding up because $a(t)$ and $v(t)$ are the same sign

- All practice problems for trig derivatives