

# LEC32 - Limits at Infinity and Asymptotes Continued

Friday, November 22, 2024  
Section 4.6

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## Example 1

Sketch the graph of  $f(x) = \frac{x^3-1}{x^2+1}$  by first finding:

• Domain  $\rightarrow \{x \in \mathbb{R} | x \neq 1\}$

• x, y intercepts  $\rightarrow$

y-intercept:

$$\Rightarrow \frac{x^3-1}{x^2+1}$$

$$\Rightarrow \frac{0^3-1}{0^2+1} \Rightarrow -1$$

x-intercept:

$$\Rightarrow 0 = \frac{x^3-1}{x^2+1}$$

$$\Rightarrow 0 = x^3 - 1$$

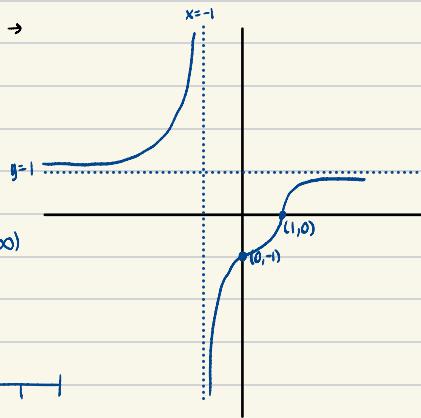
$$\Rightarrow 1 = x^3 \Rightarrow x = 1$$

• Min/max and intervals where  $f$  is increasing/decreasing  $\rightarrow$

### Critical Points

$$\begin{array}{c|ccc} & -1 & 0 & 1 \\ \hline -2 & + & + & + \\ 0 & + & + & + \end{array}$$

Increasing for  $(-\infty, -1) \cup (-1, \infty)$



• All asymptotes  $\rightarrow$

Vertical Asymptotes:  $x = -1$

Horizontal Asymptotes:

$$\lim_{x \rightarrow \infty} \frac{x^3-1}{x^2+1} = \infty$$

$$\lim_{x \rightarrow -\infty} \frac{x^3-1}{x^2+1} = -\infty$$

$$\lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x^2}}{1 + \frac{1}{x^2}} \Rightarrow 1$$

$$\lim_{x \rightarrow -\infty} \frac{1 - \frac{1}{x^2}}{1 + \frac{1}{x^2}} \Rightarrow 1$$

• Intervals of concavity

$$0 = \frac{12x(1-2x^3)}{(x^2+1)^3}$$

$$0 = 12x(1-2x^3)$$

$$x=0$$

$$0 = 1-2x^3$$

$$1 = 2x^3$$

$$x = \sqrt[3]{\frac{1}{2}}$$

$$\begin{array}{c|ccc} & -1 & 0 & \sqrt[3]{\frac{1}{2}} \\ \hline -2 & - & - & - \\ 0 & - & - & - \\ 1 & - & - & - \end{array}$$

$$\frac{(-)(+)\cdot(-)}{(-)\cdot(+)} = (+)$$

$$\frac{(-)(+)\cdot(+)}{(-)\cdot(+)} = (-)$$