LEC13 - Solids of Revolution
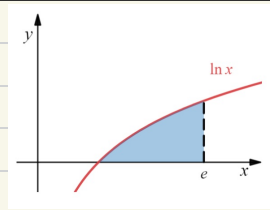
Warm Up Problem

The shaded region is rotated about the...

  1) x-axis

  2) y-axis

When slicing the result, will we get disks or washers?

  1) disks   2) washers

To compute the volume, should we integrate $\int \cdots dy$ or $\int \cdots dx$

  1) $\int \cdots dx$ 2) $\int \cdots dy$

What integral gives the resulting solid?

  1) $\int \pi (1 - \ln x)^2 dx$ 2) $\int \pi e^{2y} dy$

Example

The base of a solid S is the right triangle enclosed by $y = 2 - 2x$ and both axes. The cross-sections of S when sliced perpendicular to the base and the x-axis are squares. Find the volume of S.

$A = (2 - 2x)^2$

$V = \int_0^1 (2 - 2x)^2 dx$

$= -\frac{1}{6}(2 - 2x)^3 \Big|_0^1 = -\frac{1}{6}(2-2)^3 - \left(-\frac{1}{6}(2-0)^3\right)$

$= 0 + \frac{1}{6}(2)^3 = \frac{8}{6} = \frac{4}{3}$

Example

Find an integral which represents the volume of a sphere of radius r.

$\Rightarrow$ we want to rotate this shape about the x-axis

$\Rightarrow A = \pi \left(\sqrt{r^2 - x^2}\right)^2 = \pi (r^2 - x^2)$

$\Rightarrow V = \int_{-r}^{r} \pi (r^2 - x^2) dx$