LEC20 - Derivatives of Exponentials and Logarithms

Friday, October 25, 2024

Section 3.9

Important Ideas

$\frac{d}{dx}[\ln x] = \frac{1}{x}$

$\frac{d}{dx}[\log_b x] = \frac{1}{\ln b \cdot x}$

$\frac{d}{dx}[e^x] = e^x$

$\frac{d}{dx}[b^x] = \ln b \cdot b^x$

Warm Up Problem:

Find as many function-derivatives pairs as possible

$f_1(x) = \ln(x^3+4x)$

$f_2(x) = (\sqrt{2})^x \xrightarrow{\frac{d}{dx}}$

$f_3(x) = \ln(x^3+4x)\cdot(3x^2+4)$

$f_4(x) = e^{6x}$

$f_5(x) = 2^{x/2} \cdot \frac{\ln}{2}$

$f_6(x) = \frac{1}{6}e^{6x}$

$\frac{d}{dx}$

Problem:

Let $f(x) = ae^{x-1} + x^2 + b$

Find values of a, b such that the tangent line to (1, 2) has a slope of 5

a

1. Find $f'(x)$

   $f'(x) = ae^{x-1} + 2x$

2. Substitute $x=1, m=5$

   $5 = ae^0 + 2$

   $3 = a$

b

1. Substitute $x=1, a=3, y=2$ into $f(x)$

   $2 = 3e^0 + 1^2 + b$

   $b = -2$

Problem:

Suppose you invest \$1000 in a very good investment which doubles your money every year. If the interest is being compounded continuously, then the value of the investment after t years is given by: $P(t) = 1000\cdot 2^t$

The effective interest rate is given by the relative rate of change: $r = \frac{P'(t)}{P(t)}$

Find the interest r of this investment?

1. Find $P'(t)$

   $P'(t) = 1000\ln 2\cdot 2^t$

2. Substitute $P(t)$ and $P'(t)$ into the relative rate of change formula

   $r = \frac{P'(t)}{P(t)} = \frac{\cancel{1000}\cdot\ln 2\cdot\cancel{2^t}}{\cancel{1000}\cdot\cancel{2^t}} = \ln 2 = 0.69 = 69\%$

*We can use logarithmic differentiation to simplify a problem that has a variable as an exponent

Power property of logarithms:

$\log_b(x^a) = a\log_b(x)$

Example:

Suppose $y = (\cos(2x))^{3x+4}$

Find $\frac{dy}{dx}$.

1. Take the natural logarithm of both sides, and use the power property of logarithms

   $\ln y = (3x+4)\cdot\ln(\cos(2x))$

2. Use implicit differentiation to find $\frac{dy}{dx}$

   $\frac{1}{y}\cdot\frac{dy}{dx} = 3\ln(\cos(2x)) + (3x+4)\left(\frac{-\sin 2x}{\cos 2x}\right)\cdot 2$ Use chain rule and product rule

   $\frac{1}{y}\cdot\frac{dy}{dx} = 3\ln(\cos(2x)) - (6x+8)\tan 2x$ Distribute the 2 into $3x+4$, aswell as the negative sign. Then replace $\frac{\sin 2x}{\cos 2x}$ with $\tan 2x$

   $\frac{dy}{dx} = y[3\ln(\cos(2x)) - (6x+8)\tan 2x]$ Multiply both sides by y

   $\frac{dy}{dx} = (\cos(2x))^{3x+4}[3\ln(\cos(2x)) - (6x+8)\tan 2x]$ Replace y with $(\cos(2x))^{3x+4}$

• All practice problems for derivatives of exponentials, logarithms, and logarithmic differentiation