

LEC15 - Integration By Parts

Thursday, February 6, 2025

Section 3.1

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Warm Up Problem

Suppose we want to use integration by parts to evaluate $\int 3x^2 \sin x dx$. What should we choose as u and dv ?

$$\int 3x^2 \sin x dx \quad \text{LIATE} \quad \text{Therefore, you should choose } u = 3x^2, dv = \sin x$$

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Integration By Parts

Let $u = f(x)$ and $v = g(x)$ be functions with continuous derivatives. Then

$$\int u dv = uv - \int v du \quad \text{and so} \quad \int u dv = [uv]_a^b - \int v du$$

Example

Evaluate $\int 3x^2 \sin x dx$

$$u = 3x^2 \quad dv = \sin x \Rightarrow \int 3x^2 \sin x dx = -3x^2 \cos x + \int (\cos x)(6x) dx$$

$$du = 6x \quad v = -\cos x$$

$$u = 6x \quad dv = \cos x \Rightarrow \int 3x^2 \sin x dx = -3x^2 \cos x + 6x \sin x - \int 6 \sin x dx \Rightarrow \int 3x^2 \sin x dx = -3x^2 \cos x + 6x \sin x + 6 \cos x + C$$

$$du = 6 \quad v = \sin x$$

Example

Evaluate $\int_{0}^{\pi/4} \cos(4x) \cdot 2x dx$

$$u = 2x \quad dv = \cos(4x) \Rightarrow \int_0^{\pi/4} \cos(4x) \cdot 2x dx = \left[\frac{1}{2} \sin(4x) \right]_0^{\pi/4} - \int_0^{\pi/4} \frac{1}{2} \sin(4x) dx$$

$$du = 2 \quad v = \frac{1}{4} \sin(4x)$$

$$\Rightarrow \int_0^{\pi/4} \cos(4x) \cdot 2x dx = \frac{1}{16} \sin(\pi/2) + \left[\frac{1}{8} \cos(4x) \right]_0^{\pi/4} = \frac{1}{16} + \left[0 - \frac{1}{8} \right] = \frac{7}{16}$$

Example

Evaluate $\int x \arcsin x dx$

$$u = \arcsin x \quad dv = 1 \Rightarrow \int x \arcsin x dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$$

$$du = \frac{1}{\sqrt{1-x^2}} \quad v = x$$

$$\text{let } u = 1-x^2, \text{ so } du = 2x dx \Rightarrow \int x \arcsin x dx = x \arcsin x - \frac{1}{2} \int \frac{1}{1-x^2} du = x \arcsin x - \sqrt{1-x^2} + C$$

$$\text{then } \frac{du}{dx} = 2x \Rightarrow du = 2x dx$$

When to use integration by parts

Case 1: Two different types of functions multiplied together, such as...

$$\int x^2 \sin x dx \quad \int (x^2 - 3x) e^{5x} dx \quad \int 3x^2 \ln(3x) dx \quad \int e^x \cos(2x) dx$$

Case 2: Some integrals involving only one part (usually an inverse function), such as...

$$\int \arcsin x dx \quad \int \arccos(4x) dx \quad \int \ln x dx \quad \int \sin(\ln x) dx$$

Example

For which of the following will integration by parts be suitable?

$$\int (x^2 + 4) \cos x dx \quad \int \arccos(3x) dx \quad \int x^2 \sin(x^3) dx \quad \int 3x e^{5x} dx$$

u - substitution