# LEC04 - Fundamental Theorem of Calculus

Tuesday, January 14, 2025

Section 1.2-1.3



## Warm Up Problem

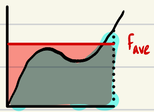
What is the average of 2,4,1,5

Average $= \frac{2+4+1+5}{4} = 3$

## Average Value of a Function

We often want to know the average value of a function on an interval $[a,b]$ (average cost, temperature, etc.)

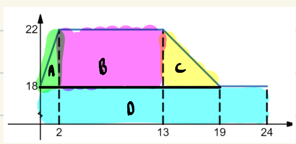
$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$




- $f_{ave}$ is defined such that the red area equals the blue area

## Example

The graph shows the room temperature (°C) in an office building over a 24 hour period. Find the average temperature over the interval $[0,24]$




$A = \frac{2 \cdot 4}{2} = 4$

$B = 11 \cdot 4 = 44$

$C = \frac{6 \cdot 4}{2} = 12$

$D = 24 \cdot 18 = 432$

$\int_0^{24} f(x)\,dx = 4+44+12+432 = 492$

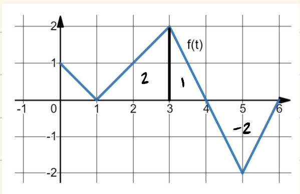
$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{24}\int_0^{24} f(x)\,dx = \frac{1}{24}(492) = 20.5°C$$

## Example

Given a function $f$ whose graph is below, define a new function $F(x) = \int_1^x f(t)\,dt$, where $f(x)$ is the function below. Compute $F(3)$

$\Rightarrow F(x) = \int_1^x f(t)\,dt$

$\Rightarrow F(3) = \int_1^3 f(t)\,dt = \frac{2 \cdot 2}{2} = 2$




Arrange $F(1), F(3), F(4), F(6)$ from smallest to largest

$\Rightarrow F(1) = 0$, so it must be the smallest since $f(x)$ is always positive

$\Rightarrow F(3) = 2$

$\Rightarrow F(4) = 3$

$\Rightarrow F(6) = 1$

$\Rightarrow$ Therefore, $F(1) < F(6) < F(3) < F(4)$

## Fundamental Theorem of Calculus (Part 1)

If $f$ is continuous over $[a,b]$ and the function $F$ is defined as $F(x) = \int_a^x f(t)\,dt$, then $F'(x) = f(x)$ over $[a,b]$

## Fundamental Theorem of Calculus (Part 2)

If $f$ is continuous over $[a,b]$, and $F$ is any antiderivative of $f$, then $\int_a^b f(x)\,dx = F(b) - F(a)$

- Part 2 allows us to easily evaluate definite integrals
- Part 1 and 2 together say that differentiation and integration are inverse operations