LEC21 - Related Rates

Friday, October 25, 2024
Section 4.1

*We must use the chain rule on r, c since the question states that both variables change with respect to t, meaning they are both functions

Related Rates Strategy

1. Draw a diagram if applicable
2. Assign variables to all values
3. Find an equation relating all variables
4. Differentiate both sides using the chain rule with respect to the independent variable (usually t)
5. Substitute all known values into the derivative
6. Solve for the missing value

Similar Triangles: Two triangles that have the same shape but not necessarily the same size (scalar multiples). If 2 triangles are similar, the angles are equal, and the corresponding sides are proportional (have equal ratios)

Warm Up Problem:

Suppose that both r, c change with respect to t and that $r = c^3 - 2c$

Differentiate both sides with respect to t

$$r = c^3 - 2c$$
$$\frac{dr}{dt} = 3c^2\frac{dc}{dt} - 2\frac{dc}{dt}$$

Example 1:

Suppose that both r, c change with time and that $r = c^3 - 2c$

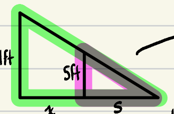
If $\frac{dr}{dt} = 5$, what is $\frac{dc}{dt}$ when $c = 2$?

$$\frac{dr}{dt} = 3c^2\frac{dc}{dt} - 2\frac{dc}{dt}$$
$$5 = \frac{dc}{dt}(3(2)^2 - 2)$$
$$5 = \frac{dc}{dt}(10)$$
$$\frac{5}{10} = \frac{dc}{dt} = \frac{1}{2}$$

Example 2: Similar triangles problem

A street light is at the top of a light post of height 14ft. A 5ft tall girl is walking away from the light at a rate of 3ft/s. At what rate is the length of the girls shadow increasing, when she is 25ft from the pole?

1/2. Draw a diagram, assign variables



The green are purple triangles are similar triangles

distance from light pole

length of shadow

3. Find an equation

$$\frac{x+s}{14} = \frac{s}{5} \Rightarrow 5x + 5s = 14s \Rightarrow 5x = 9s$$

4. Differentiate

$$5\frac{dx}{dt} = 9\frac{ds}{dt}$$

5. Substitute

$$5(3) = 9\frac{ds}{dt}$$

6. Solve for missing value

$$\frac{ds}{dt} = \frac{15}{9}$$

•All example related rates problems

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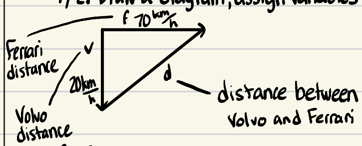
Example 3: Pythagorean Theorem

Two cars start from the same location. A Volvo drives South at 40km/h. A Ferrari drives East at 70km/h. How fast is the distance between the cars changing when the Volvo is 20 km from the starting point?

1/2. Draw a diagram, assign variables



3. Find an equation

$v^2 + f^2 = d^2$

4. Differentiate

$2v\frac{dv}{dt} + 2f\frac{df}{dt} = 2d\frac{dd}{dt} \Rightarrow v\frac{dv}{dt} + f\frac{df}{dt} = d\frac{dd}{dt}$

5. Substitute

We know that the Volvo is 20km away at $t=0.5$, and at $t=0.5$, the Ferrari is 35km away. At $t=0.5$, the 2 cars are $\sqrt{20^2+35^2} = 40.3113$km apart

$20(40) + 35(70) = 40.3113\frac{dd}{dt}$

6. Solve for missing value

$\frac{dd}{dt} = 80.62 \text{ km/h}$

• All example related rates problems