LEC16 - Trigonometric Integrals

Tuesday, February 11, 2025

Section 3.2

Example

If the substitution $u=\cos x$ was used for the following integral, what would the resulting integral be?

$\int \cos^4 x \sin^5 x \, dx$, $u = \cos x$ so $\frac{du}{-\sin x} = dx$

$= \int -u^4(1-u^2)^2 du$ $(\cos^2 x + \sin^2 x = 1)$

Integrating $\cos^n x \sin^m x$, where n and m are non-negative integers

Case 1: m is odd (ex. $\int \cos^2 x \sin^3 x \, dx$)

Separate one $\sin x$ and write the rest in terms of $\cos x$ using the pythagorean identity $(\cos^2 x + \sin^2 x = 1)$ $[u = \cos x]$

Case 2: n is odd (ex. $\int \cos^3 x \sin^2 x \, dx$)

Separate one $\cos x$ and write the rest in terms of $\sin x$ using the pythagorean identity $[u = \sin x]$

Case 3: n and m are even (ex. $\int \cos^2 x \sin^2 x \, dx$)

Try repeatedly using the double angle identities

$\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$ & $\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$