LEC09 - Integrals Resulting In Inverse Trig Functions

Friday, January 24, 2025

Section 1.7

Warm-Up Problem

Evaluate the following indefinite integrals:

$$\int \frac{7}{1+x^2} = 7\tan^{-1}(x)+C$$

$$\int \frac{1}{\sqrt{3-t^2}}\,dt = \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)+C$$

Example

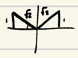
What method should be used to evaluate $\int_0^{1/\sqrt{2}} \frac{\cos(\sin^{-1}(x))}{\sqrt{1-x^2}}\,dx$?

Substitute $u=\sin^{-1}(x)$

Now evaluate the integral

$u=\sin^{-1}(x)$

$du=\frac{1}{\sqrt{1+x^2}}\,dx$



$du\cdot\sqrt{1+x^2}=dx$

$$=\int_{\sin^{-1}(0)}^{\sin^{-1}(1/\sqrt{2})} \frac{\cos(u)}{\cancel{\sqrt{1+x^2}}}\cdot\cancel{\sqrt{1+x^2}}\,du = \int_0^{\pi/4}\cos(u)du = \sin(u)\Big|_0^{\pi/4} = \sin\left(\frac{\pi}{4}\right)-\sin(0)=\frac{1}{\sqrt{2}}$$

Can the following integral be evaluated in a similar way? $\int_0^2 \frac{\cos(\sin^{-1}(x))}{\sqrt{1-x^2}}\,dx$

No because when we get to the part where we adjust the limits of integration, we need to evaluate $\sin^{-1}(2)$, which is not defined at $x=2$

Example

Is there anything wrong with the following argument? If so, on what line?

1 $\Rightarrow$ We know $\int\frac{1}{1+x^2}dx = \tan^{-1}(x)+C$

2 $\Rightarrow$ Also $\frac{d}{dx}[\cot^{-1}(x)] = \frac{-1}{1+x^2}+C$

3 $\Rightarrow$ This means that $\int\frac{1}{1+x^2}dx = \cot^{-1}(x)+C$

4 $\Rightarrow$ Also, it is true that $\int\frac{1}{1+x^2}dx = \tan^{-1}(x)+C$

5 $\Rightarrow$ The domains of both $\cot^{-1}(x)$ and $\tan^{-1}(x)$ are $(-\infty,\infty)$

6 $\Rightarrow$ So therefore $-\cot^{-1}(x)+C=\tan^{-1}(x)+C$ for all $x\in(-\infty,\infty)$

7 $\Rightarrow$ And therefore $-\cot^{-1}(x)=\tan^{-1}(x)$ for all $x\in(-\infty,\infty)$

The error is on line 3. Antiderivatives do not account for constant factors (since $\tan^{-1}(x)=-\cot^{-1}(x)-\frac{\pi}{2}$), so we can't say that $-\int\frac{1}{1+x^2}=\cot^{-1}(x)$