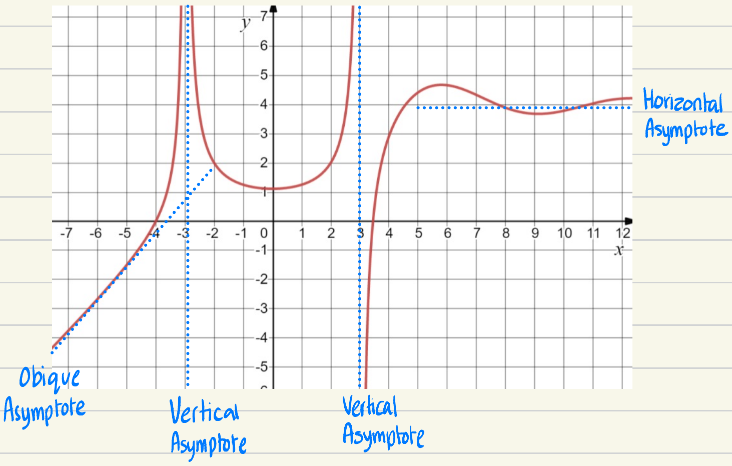
LEC31 - Limits at Infinity and Asymptotes

Friday, November 22, 2024
Section 4.6

Warm Up Problem

How many asymptotes does this function have? Of what types?



Definition: Horizontal Asymptote

The line $y=L$ is a horizontal asymptote of f if either

$$\lim_{x\to\infty} f(x) = L \quad \text{OR} \quad \lim_{x\to-\infty} f(x) = L$$

Example 1

Find all horizontal asymptotes of $f(x) = \arctan(e^x)$

∞

$$\lim_{x\to\infty} \arctan(e^x) = \arctan(e^\infty) = \arctan(\infty) = \frac{\pi}{2}$$

$-\infty$

$$\lim_{x\to-\infty} \arctan(e^x) = \arctan(0) = 0$$

Conclusion

HA at $y = \frac{\pi}{2}, 0$

Definition: Slant/Oblique Asymptote

The line $y=mx+b$ (where $m\neq 0$) is an oblique asymptote of f if either

$$\lim_{x\to\infty} (f(x)-(mx+b)) = 0 \quad \text{and/or} \quad \lim_{x\to-\infty} (f(x)-(mx+b)) = 0$$

This means that the curve $y=f(x)$ gets closer and closer to the line $y=mx+b$

Notes:

1. For rational functions, this happens when the degree of the numerator is exactly one more than the degree of the numerator
2. In that case, we use long division to find the oblique asymptote

Example 2

Find the oblique asymptote of $f(x) = \frac{x^2+12}{x-2}$

$\Rightarrow$ We know the degree of the numerator is one more than the denominator, so an oblique asymptote exists

$\Rightarrow$ Use long division:

$$\begin{array}{r|l}
 & x + 2 \\
\hline
x-2 & x^2 + 0x + 12 \\
 & -(x^2 - 2x) \\
\hline
 & 0 + 2x + 12 \\
 & -(2x - 4) \\
\hline
 & 16
\end{array}$$

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$\Rightarrow$ Therefore, $\frac{x^2+12}{x-2} = x+2+\frac{16}{x-2}$

$\Rightarrow$ Therefore, the oblique asymptote is $y=x+2$

<u>Example</u>

Evaluate the limit $\lim\limits_{x\to\infty} \frac{3x}{\sqrt{x^2+1}-x}$

$$\Rightarrow \lim_{x\to\infty} \left(\frac{3}{\frac{\sqrt{x^2+1}}{x}-1}\right)$$

$$\Rightarrow \lim_{x\to-\infty} \left(\frac{3x}{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}-x}\right)$$

$$\Rightarrow \lim_{x\to\infty} \left(\frac{3x}{-x\sqrt{1+\frac{1}{x^2}}-x}\right)$$

$$\Rightarrow \lim_{x\to\infty} \left(\frac{3}{-\sqrt{1+\frac{1}{x^2}}-1}\right) \Rightarrow \frac{3}{-1(1)-1} = -\frac{3}{2}$$