LEC9 - Continuity

3 point Continuity Test:

This test states a function $f(x)$ is continuous at a if the following 3 conditions are met:

1. $f(a)$ exists
2. $\lim_{x\to a} f(x)$ exists
3. $f(a) = \lim_{x\to a} f(x)$

Removable Discontinuity:

A discontinuity that occurs when the limit exists, even though the function might not

Jump Discontinuity:

A discontinuity that occurs when the left and right limits do not equal each other

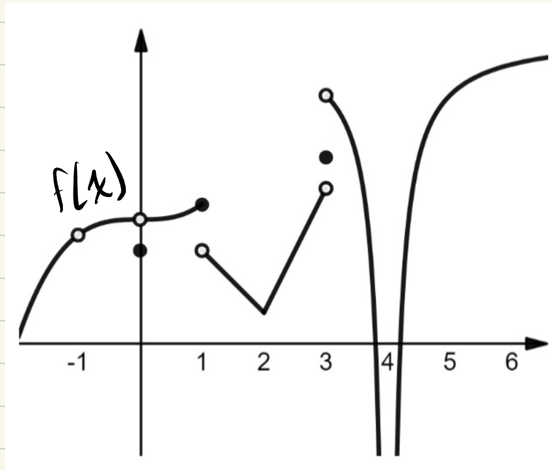
Infinite Discontinuity:

A discontinuity that occurs when the left or right limits approach positive or negative infinity (vertical asymptote)

Warm Up Problem:

For what values of x does $f(x)$ have...

- Removable discontinuity $\rightarrow x = -1, 0$
- Jump Discontinuity $\rightarrow x = 1, 3$
- Infinite Discontinuity $\rightarrow x = 4$



Jumping back to Section 2.3:

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

1. Is L smaller or larger than L_0?

   L is larger or equal to L_0 because L_0 can be at most $1-1$ with L (if $v=0$)

2. If v gets larger, how does that affect L?

   If v gets larger, then L gets smaller because the square root gets smaller

3. What is $\lim_{v\to c^-} L_0\sqrt{1-\frac{v^2}{c^2}}$?

   $\Rightarrow \lim_{v\to c^-} L_0\sqrt{1-\frac{c^2}{c^2}}$

   $\Rightarrow \lim_{v\to c^-} L_0\sqrt{1-1}$

   $\Rightarrow = 0$

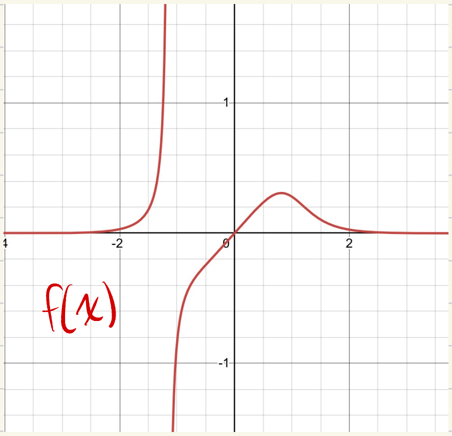
Example 1:

Consider $f(x) = \frac{\sin x}{x^5+2}$, is $f(x)$ continuous or discontinuous at $x = \pi/2$?

$$f\left(\tfrac{\pi}{2}\right) = \frac{\sin\frac{\pi}{2}}{\frac{\pi^5}{32}+2} = \frac{1}{\frac{\pi^5}{32}+2}$$

$$\lim_{x\to\frac{\pi}{2}} = f\left(\tfrac{\pi}{2}\right) = \frac{1}{\frac{\pi^5}{32}+2}$$

Since $\lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$, the function is continuous



Example 2:

Consider $f(x) = \cos\left(\frac{\pi}{2} + \frac{\pi x}{4|x|}\right)$, is $f(x)$ continuous or discontinuous at $x=0$

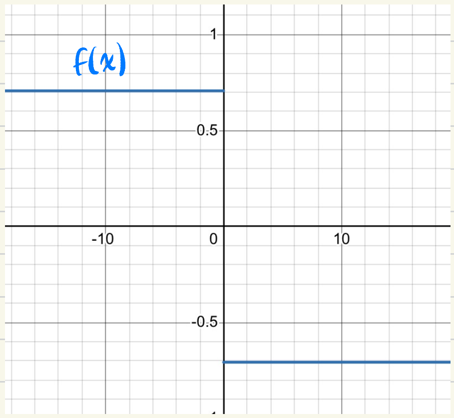
$f(0) = \cos\left(\frac{\pi}{2} + \frac{0}{0}\right) = \text{Undefined}$

Since $f(0)$ is undefined, it is impossible for it to pass the 3 point continuity test

$$\lim_{x\to 0^-} = \cos\left(\tfrac{\pi}{2}+\tfrac{\pi}{4}\right) = -\tfrac{\sqrt{2}}{2}$$

$$\lim_{x\to 0^+} = \cos\left(\tfrac{\pi}{2}-\tfrac{\pi}{4}\right) = \tfrac{\sqrt{2}}{2}$$

Since the left and right limits do not agree, it must be a jump discontinuity



- A function is continuous if it meets all the criteria for the 3 point continuity test
- A function can be either continuous or discontinuous
- If a function is discontinuous, it can either have a removable, jump, or infinite discontinuity