LEC03 – Definite Integrals

Section 1.2

Warm Up Problem

What does the definite integral $\int_0^8 2x\,dx$ represent?

$\Rightarrow$ The area under the curve $y=2x$ from $x=0$ to $x=8$

Integrals and Areas

If f is negative on part of $[a,b]$, then $\int_a^b f(x)\,dx$ is the sum of the areas below its graph and above its x-axis, minus the sum of the areas above its graph and below the x-axis.

In other words, definite integrals count areas below the x-axis negative.

Example

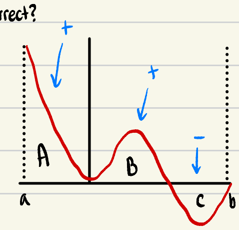
Which of the following is correct?

a) $\int_a^b f(x)\,dx = -A+B+C$

b) $\int_a^b f(x)\,dx = A+B+C$

c) $\int_a^b f(x)\,dx = A+B-C$

d) $\int_a^b f(x)\,dx = A-B+C$

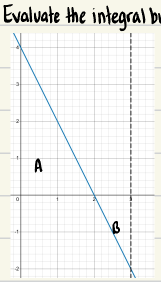


Net Signed Area vs Total Area

- Definite integrals compute the area between f and the x-axis
- If we want total unsigned area we should compute $A+B+C$

Example

Evaluate the integral by computing areas



$$\int_0^3 (4-2x)\,dx = A-B$$
$$= \frac{2\cdot 4}{2} - \frac{1\cdot 2}{2}$$
$$= 4-1$$
$$= 3$$

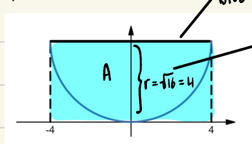
Example

Evaluate the following integral given the graph of the function inside it

$$\int_{-4}^{4} \left(4-\sqrt{16-x^2}\right)dx$$

Blue rectangle is B

Semicircle area $= \frac{\pi r^2}{2}$



$$\text{Area} = B - A = 4(8) - \frac{\pi(4)^2}{2} = 32-8\pi$$

Properties of Definite Integrals

1. $\int_a^a f(x)\,dx$
2. $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
3. $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
4. $\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$, for any constant c
5. $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$, c does not need to be inbetween a and b (f only needs to be integrable over the intervals)

Example

Suppose that f and g are integrable functions and we know the following:

$$\int_2^6 (3f(x)-g(x))\,dx = 10 \quad \text{and} \quad \int_2^6 g(x)\,dx = 2$$

What is $\int_2^6 f(x)\,dx$?

$\Rightarrow 10 = \int_2^6 (3f(x)-g(x))\,dx$

$\Rightarrow 10 = 3\int_2^6 f(x)\,dx - \int_2^6 g(x)\,dx$

$\Rightarrow 10 = 3\int_2^6 f(x)\,dx - 2$

$\Rightarrow \frac{12}{3} = \int_2^6 f(x)\,dx = 4$

Example

Suppose that f and g are integrable and we know the following:

$$\int_{-3}^{2} (f(x)+1)\,dx = 0 \quad \text{and} \quad \int_2^0 f(x)\,dx = 6$$

Wh t is $\int_{-3}^{0} f(x)\,dx$?

$\Rightarrow \int_{-3}^{2} (f(x)+1)\,dx = 0$

$\Rightarrow \int_0^2 (f(x)+1)\,dx + \int_{-3}^{0} (f(x)+1)\,dx = 0$

$\Rightarrow \int_0^2 f(x)\,dx + \int_0^2 1\,dx + \int_{-3}^{0} f(x)\,dx + \int_{-3}^{0} 1\,dx = 0$

$\Rightarrow -\int_2^0 f(x)\,dx + 2 + \int_{-3}^{0} f(x)\,dx + 3 = 0$

$\Rightarrow -6+2+\int_{-3}^{0} f(x)\,dx + 3 = 0$

$\Rightarrow \int_{-3}^{0} f(x)\,dx = 1$