LEC15 – Rates of Change

Model Rocket Part 1:

The height (h) of a rocket as a function of time (t) is given by $h(t) = 20t^2$, Find $v(t)$ & $a(t)$.

$v(t) = h'(t) = 40t$ Use power rule

$a(t) = v'(t) = h''(t) = 40$ Use power rule

Model Rocket Part 2:

Consider again the model rocket that is blasting off straight up. Now suppose the formula for height is given by $h(t)$ below:

$$h(t) = \begin{cases} 20t^2 & 0 \le t \le 2 \\ -100 + 100t - 5t^2 & t > 2 \end{cases}$$

Find $v(t)$ & $a(t)$:

$$v(t) = h'(t) = \begin{cases} 40t & 0 \le t \le 2 \\ 100 - 10t & t > 2 \end{cases}$$

$$a(t) = v'(t) = h''(t) = \begin{cases} 40 & 0 \le t \le 2 \\ -10 & t > 2 \end{cases}$$

Model Rocket Part 3:

1) What type of discontinuity does $a(t)$ have at $t = 2$?
   Jump Discontinuity because the left and right limits do not agree
2) What happened to the model rocket at $t = 2$ seconds?
   It started decelerating, although it is still going up at $t = 2$
3) Is it appropriate to model a rocket engine turning off with a jump discontinuity in acceleration?
   Yes, it is generally ok. In real life models we can connect the 2 points with a steep line to make it continuous.

Model Rocket Part 4:

What is the maximum height of the rocket?

$100 - 10t = 0 \Rightarrow t = 10$ Solve $v(t) = 0$ when $t > 2$ (we know since it is still rising at $t = 2$ that the max height is after $t = 2$.

$-100 + 100 - 5(10)^2 \Rightarrow 400$ Sub $t = 10$ into $h(t)$ to get max height

Model Rocket Part 5:

Now, suppose the same upwards acceleration is the same, but the rocket burns for c seconds. The height (h) of the rocket is

$$h(t) = \begin{cases} 20t^2 & 0 \le t \le c \\ -25c^2 + 50ct - kt^2 & t > c \end{cases}$$

What is the most appropriate value for k? Best value of k will make $h(t)$ continuous (L & R $\lim_{t \to c}$ equal to eachother)

Left	Right	Solve for k
$\lim_{t \to c^-} = 20c^2$	$\lim_{t \to c^+} = -25c^2 + 50c^2 - kc^2$	$20c^2 = 25c^2 - kc^2$
	$= 25c^2 - kc^2$	$5c^2 = kc^2$
		$5 = k$

• All practice problems for derivatives