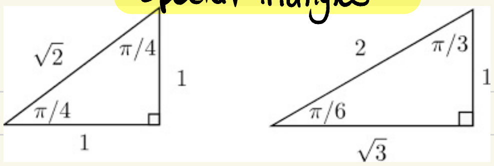
Special Triangles



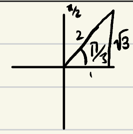
LEC4 – Trigonometry & Exponential Functions

We can evaluate trig ratios by hand by using the unit circle and special triangles

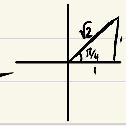
Quick Review

Solve each of the following by hand

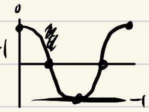
a) $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$



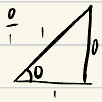
b) $\tan\frac{\pi}{4} = 1$



c) $\cos\pi = -1$



d) $\tan 0 = \frac{0}{1}$

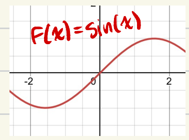


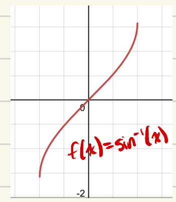
Recall: Functions are invertible if they are one-to-one (Pass HLT), otherwise their domains must be restricted

Question ⇒ Is the sin function invertible

No, because it is not one to one because it fails the horizontal line test. For it to be invertible, we must restrict its domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$

These values are by convention, many other sets also work





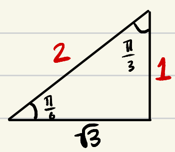
$\sin^{-1}(x)$

$\{x \mid -1 \le x \le 1\}$

$\{y \mid -\frac{\pi}{2} \le x \le \frac{\pi}{2}\}$

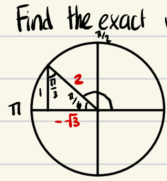
Example: Find the exact value of $\sin^{-1}(\frac{1}{2})$

$\therefore, \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$



$\sin\theta = \frac{O}{H}$

Find the exact value of $\cos^{-1}(-\frac{\sqrt{3}}{2})$



$\cos\theta = \frac{A}{H}$

$\cos^{-1}(-\frac{\sqrt{3}}{2}) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Find the exact value of $\sin^{-1}(-1)$

$\sin^{-1}(-1) = \theta$

$\sin(\theta) = -1$

$\sin(-\frac{\pi}{2}) = -1$

Use these identities to assist in solving equations, proofs, etc. There are many more, but these are the base ones.

Trig Identities

$\sin^2 x + \cos^2 x = 1$ $\quad$ $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$1 + \tan^2 x = \sec^2 x$ $\quad$ $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

Section 1.5 - Exponential and Logarithmic

Recall: An exponential function is of the form $y = b^x$, where b is constant. Logarithmic equations are of the form $x = \log_b y$, and are inverse functions of exponentials

The natural exponential function is $f(x) = e^x$, where $e \approx 2.71$

The inverse of $f(x) = e^x$ is called the natural logarithm, and is written $f^{-1}(x) = \ln(x)$

TRUE or FALSE: $\ln(x)$ is the same as $\log_e x$ TRUE

Solve for x: $\ln(x) = y \Rightarrow x = e^y$

Simplify: $e^{\ln x} = x$

Simplify: $\ln(e^x)$

↳ $x = 1 \Rightarrow 1$

↳ $x = 0 \Rightarrow 0$

Evaluate

↳ $\ln(0) =$ Undefined

↳ $\ln(-1) =$ Undefined

- We can evaluate trig functions by hand using the unit circle and special triangles
- We can use various trig identities to simplify our equations
- The natural number $e \approx 2.71$
- The inverse of an exponential function is a logarithm

LEC4 - Trigonometry & Exponential Functions

Reminder
"∪" means union, or join the 2 sets together.

Example 1: Find the domain of $f(x) = \underbrace{\sqrt{5-x}} + \underbrace{\ln(x+3)}$

$$\{x \mid -3 < x \leq 5\} \xLeftarrow{\text{combine}} \{x \leq 5\} \cup \{x > -3\}$$

Solve for the inverse function by solving for x.

Example 2: Find $f^{-1}(x)$ for $f(x) = 7e^{-4x} + 2$

$$y = 7e^{-4x} + 2$$

$$\Rightarrow \frac{y-2}{7} = e^{-4x}$$

$$\Rightarrow \ln\left(\frac{y-2}{7}\right) = -4x$$

$$\Rightarrow -\frac{1}{4}\ln\left(\frac{y-2}{7}\right) = x$$

$$\Rightarrow f^{-1}(x) = -\frac{1}{4}\ln\left(\frac{x-2}{7}\right)$$

Solving for the inverse algebraically is impossible here. Instead, we can recall the property of functions & their inverses that states:

$$f(x) = y \Leftrightarrow f^{-1}(y) = x$$

Using this property, we can manually check each choice until we find one such that LS=RS when substituting the given x and y values

Example 3: Find $f^{-1}(2)$, $f(x) = x^3 + \ln(x) + 1$

a) $f^{-1}(2) = e$
$f(e) = 2$
$e^3 + \ln(e) + 1 \neq 2$

b) $f^{-1}(2) = 0$
$\ln(0)$ is undefined

c) $f^{-1}(2) = 1$
$f(1) = 2$
$1^3 + \ln(1) + 1 = 2$
$1 + 0 + 1 = 2$

d) $f^{-1}(2) = 2$

- This page contains only examples, all content is on page 1/1