LEC11 - Defining Derivatives

*There are 2 possible limits that are correct, only one is shown

Warm Up Problem:

Let $f(x) = 3\sqrt{x} + x$

If you are using the limit definition of the derivative to find $f'(2)$, what is the correct limit?

$$f'(2) = \frac{\sqrt{(2+h)} + (2+h) - 3\sqrt{2} - 2}{h}$$

Derivatives and tangent lines:

Let f be a function defined on an open interval containing a

The tangent line to the graph of f at $x=a$ is the line passing through the point $(a, f(a))$ with slope $f'(a)$ where

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{or equivalently} \quad f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

*Take Aways: The average rate of change across a small interval can be used to estimate the instantaneous rate of change at a point

Investigation Example:

In an experiment, the location p (in meters) of a running cheetah is recorded at certain times (in seconds)

t	2.7	2.8	2.9	3.0	3.1	3.2	3.3
$p(t)$	49.55	52.43	55.36	58.37	61.41	64.43	67.32

$$\frac{61.41 - 58.37}{3.1 - 3.0} = \frac{3.04}{0.1} \approx 30.4 \text{ m/s}$$

Use the data to estimate the velocity of the cheetah at time $t=3$

Example: What is $f'(0)$, $f(x) = |x|$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{|0+h| - |0|}{h} = \lim_{h \to 0} \frac{|h|}{h}$$

$$|x| = \begin{cases} -x & x \le 0 \\ x & x > 0 \end{cases}$$

$$\lim_{h \to 0^-} \frac{-h}{h} = -1 \qquad \lim_{h \to 0^+} \frac{h}{h} = 1$$

The left and right limits do not agree. Therefore, $f(x)$ is not differentiable at $x=0$

- There are 2 definitions for the derivative of a function $f(x)$
- To find the slope of the tangent for a function, we find to slope of the secant with our desired point and a sample point as the sample point approaches our desired value