LEC17 - The Chain Rule

* Note about TT1: The cutoff for content is section 3.4

Warm Up Problem:

Suppose a measuring device is lowered down into the ocean to measure pressure. The pressure (in pascals) is $P = 9800h$, where h is the depth in the water (in m). Suppose the device is lowered down at a velocity of $h'(t) = 2$ m/s: How fast is the pressure increasing?

→ Pressure change is $9800^{Pa}/_{m}$

→ Pressure change for 1 m/s is $9800^{Pa}/_{s}$

→ Pressure change for 2 m/s is double (19600)

The Chain Rule:

Let f and g be differentiable functions. The chain rule states that the derivative of $f(g(x))$ is:

$$f'(g(x)) \cdot g'(x)$$

Example:

Find the derivative of $f(x) = \tan(x^2+3)$ ($f(x)$: $\tan$, $g(x)$: x^2+3)

$\Rightarrow f'(x) = \sec^2(x^2+3) \cdot (2x)$ ($f'(x)$: $\sec^2$, $g(x)$: x^2+3, $g'(x)$: $2x$)

Example:

Differentiate each of the following:

$f(x) = (x^4+3)^{50}$

→ $50(x^4+3)^{49} \cdot (4x^3)$

→ $200x^3(x^4+3)^{49}$

$g(t) = \sqrt{t^2 \sin t}$

→ $\frac{1}{2}(t^2 \sin t)^{-\frac{1}{2}}(2t\sin t + t^2 \cos t)$

→ $\frac{2t\sin t + t^2 \cos t}{2\sqrt{t^2 \sin t}}$

$h(x) = \tan(\cos x^2)$

→ $\sec^2(\cos x^2) \cdot (-\sin x^2) \cdot (2x)$

→ $-2x\sin(x^2)\sec^2(\cos x^2)$

$k(x) = \sec(5x^2+1)$

→ $\sec(5x^2+1)\tan(5x^2+1) \cdot (10x)$

→ $10x\sec(5x^2+1)\tan(5x^2+1)$

Example:

A furniture company is selling a certain chair at a price P dollars. Suppose that $P = 2C - \frac{23}{C}$, where C is the cost of producing one chair. The cost to produce one chair is given by $C = 45 + 7t$. What is the rate of change of the price as a function of time?

→ $P = 2(45+7t) + \frac{23}{45+7t}$ Substitute $C = 45+7t$

→ $P' = 14 - \frac{23}{(45-7t)^2}$