LEC05 - Fundamental Theorem of Calculus 2

Friday, January 17, 2025

Section 1.3

Warm Up Problem

What is the main concept of the Fundamental Theorem of Calculus?

$\Rightarrow$ States that derivatives and integrals are inverse processes

Example

Differentiate $F(x) = \int_1^x (t^2 - t)\,dt$

$F'(x) = x^2 - x$ Use the Fundamental Theorem of Calculus (Part 1)

Differentiate $F(x) = \int_{-10}^{x^2} \sqrt[3]{t-2}\,dt$

$F(x) = \int_{-10}^{u} (t-2)^{1/3}\,dt$

$G(y) = \int_{-10}^{y} (t-2)^{1/3}\,dt \Rightarrow G'(y) = (y-2)^{1/3}$

$F(x) = G(u)$

$F'(x) = G'(u) \cdot u'$

$= (u-2)^{1/3} \cdot u'$

$= (x^2-2)^{1/3} \cdot 2x$

Differentiate $F(x) = \int_{2x+1}^{0} \sin t\,dt$

$F(u) = -\int_0^u \sin t\,dt$, where $u = 2x+1$

$G(y) = -\int_0^y \sin t\,dt \Rightarrow G'(y) = -\sin y$

$F(x) = G(u)$

$F'(x) = G'(u) \cdot u'$

$= -\sin(2x+1) \cdot 2$

Differentiate $F(x) = \int_{x+1}^{x^3} e^{2t}\,dt$

$F(x) = \int_0^{x^3} e^{2t}\,dt - \int_0^{x+1} e^{2t}\,dt$

$F'(x) = e^{2x^3} \cdot 3x^2 - e^{2x+2}$

Example

Evaluate $\int_0^{\pi/2} \cos x\,dx$

$\Rightarrow \int \cos x\,dx = \sin x + C$

$\Rightarrow \int_0^{\pi/2} \cos x\,dx = \sin(\frac{\pi}{2}) - \sin(0)$

$= 1 - 0 = 1$

Theorem 1.2 - Comparison Theorem:

1. If $f(x) \geq 0$ for all $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq 0$
2. If $f(x) \geq g(x)$ for all $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$
3. If m and M are constants such that $m \leq f(x) \leq M$ for all $a \leq x \leq b$, then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

Example

Let f be some integrable function such that $f(x) \geq 1-x$ for all $x \geq 0$. Find the lower bound for $\int_0^1 f(x)\,dx$

$\int_0^1 f(x) \geq \int_0^1 (1-x)\,dx \Rightarrow \int_0^1 f(x) \geq x - \frac{1}{2}x^2 \Big|_0^1 \Rightarrow \int_0^1 f(x) \geq 1 - \frac{1}{2} = \frac{1}{2}$