LEC01 - Approximating Areas

Tuesday, January 7, 2025

Section 1.1

Warm-up Problem

What is the total area of these rectangles?

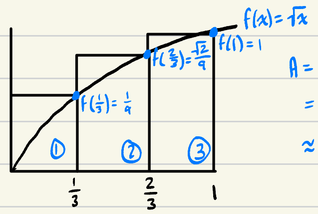
a) 2.394

b) 0.798

c) 0.671

d) 1.562

e) 0.859



$A = ① + ② + ③$

$= \frac{1}{3}\left(\frac{1}{9}\right) + \frac{1}{3}\left(\frac{\sqrt{2}}{9}\right) + \frac{1}{3}(1)$

≈ 0.798

- This is an approximation of the area under the curve on the interval $[0,1]$

Practice Problem

Are each of the following statements True or False

1. Using more (thinner) rectangles generally gives a better approximation of the area True
2. Using more (thinner) rectangles generally gives a worse approximation of the area False
3. Using 1000 rectangles would give the exact area False

Rough Notation

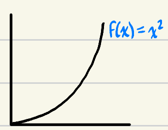
Suppose (for now) that f is continuous and non-negative on $[a,b]$

- The sum of the areas of the rectangles is called a Riemann Sum
- If we use right endpoints for the heights, it's called a right-endpoint approximation
- If we use left endpoints for the heights, it's called a left-endpoint approximation

Example

Note: Left Riemann Sums are underestimates for increasing functions, and overestimates for decreasing functions. Right Riemann Sums are

Find Riemann Sums for $f(x)=x^2$ on $[1,3]$ using a left and right endpoint approximation with 3 subintervals of equal width



Left

$A \approx \frac{2}{3}f(1) + \frac{2}{3}f\left(\frac{5}{3}\right) + \frac{2}{3}f\left(\frac{7}{3}\right) \approx 6.1$

Right

$A \approx \frac{2}{3}f\left(\frac{5}{3}\right) + \frac{2}{3}f\left(\frac{7}{3}\right) + \frac{2}{3}f(3) \approx 11.5$

$\Delta x = \frac{b-a}{n} = \frac{3-1}{3} = \frac{2}{3}$

Recap

- Suppose f is continuous and non-negative on $[a,b]$
- Partition this interval into n subintervals (we always use equal width)
- The height of each rectangle will be $f(x_i^*)$
- The width of each subinterval is $\Delta x = \frac{b-a}{n}$
- Now choose a number in each interval x_i^* in $[x_{i-1}, x_i]$ (left or right endpoint)
- The total area of the rectangles is called a Riemann sum $\sum_{i=1}^{n} f(x_i^*)\Delta x$

Example

Consider a Riemann sum of $f(x) = x^3 + 5x$ on the interval $[2,10]$ using 16 subintervals, what is Δx?

$\Delta x = \frac{b-a}{n} = \frac{10-2}{16} = \frac{8}{16} = \frac{1}{2}$