LEC08 - Substitution Continued

Friday, January 24, 2025

Sections 1.5, 1.6

Warm Up Problem

Which of the following indefinite integrals can be evaluated using substitution?

1) $\int x^2 e^{x^3+2}dx$ 2) $\int x^3 e^{x^2+1}$ 3) $\int \frac{\cos(\ln(x))}{x}dx$ 4) $\int \cos(x^2)dx$

$u = x^3+2$ $\quad u = x^2+1$ $\quad u = \ln(x)$ $\quad u = x^2$

$du = 3x^2$ $\quad du = 2x\,dx$ $\quad du = \frac{1}{x}dx$ $\quad du = 2x$

Therefore, only 1) and 3) can be solved using u-substitution

Challenge Problem

Evaluate $\int \frac{x^3}{\sqrt{3+x^2}}dx$

$u = 3+x^2 \quad \frac{du}{2x} = dx$

$$= \int \frac{x^2 \cdot x}{\sqrt{3+x^2}}dx = \int \frac{(u-3)\cdot \cancel{x}}{\sqrt{u}} \cdot \frac{1}{2\cancel{x}}du = \frac{1}{2}\int (u-3)\cdot u^{-1/2}du = \int \frac{1}{2}(u^{1/2} - 3u^{-1/2})du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} - \frac{3}{2}\cdot 2u^{1/2} + C = \frac{1}{3}(3+x^2)^{3/2} - 3(3+x^2)^{1/2} + C$$

Example

Suppose the population P of a certain country is projected to be

$P(t) = 5.3e^{0.022t}$

million people, where t is the number of years after 2020

1. What computation do we do to find the average population between 2025 and 2030, then compute it

$$\frac{1}{10-5}\int_5^{10} 5.3e^{0.022t}dt = \frac{5.3}{5}\left(e^{0.022t}\Big|_5^{10}\right) = 0.138 \text{ million people}$$

2. What does $\int_5^{10} P'(t)dt$ represent?

It represents the amount of change from 2025 to 2030

Example

Evaluate the indefinite integral of $f(x) = \frac{e^x+1}{e^x+x}$

$\Rightarrow \int \frac{e^x+1}{e^x+x}dx$, let $u = e^x+x$, then $du = (e^x+1)dx$, so $\frac{du}{e^x+1} = dx$

$\Rightarrow \int \frac{\cancel{e^x+1}}{u}\cdot\frac{1}{\cancel{e^x+1}}du = \int \frac{1}{u}du = \ln|u| + C = \ln|e^x+x| + C$