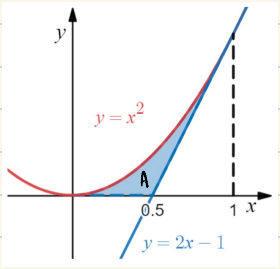
LEC10 - Areas Between Curves

Tuesday, January 28, 2025

Section 2.1

Warm-Up Problem

What expressions equal the area of the shaded region A



a) $\int_0^1 (x^2 - (2x-1))dx$

b) $\int_0^{0.5} (x^2)dx + \int_{0.5}^1 (x^2 - (2x-1))dx$

c) $-\int_0^{0.5} (2x-1)dx + \int_{0.5}^1 (x^2 - (2x-1))dx$

Theorem 2.3 - Areas by Integrating Along y

If $u(y)$ and $v(y)$ are continuous and $v(y) \leq u(y)$ for all $c \leq y \leq d$ then the area between the curves from $y=c$ to $y=d$ is given by:

$$A = \int_c^d [u(y) - v(y)]dy$$

Example

Find the area of region A from the warm-up problem

$u(x) = 2x - 1 \Rightarrow u(y) = \frac{1}{2}y + \frac{1}{2}$

$v(x) = x^2 \Rightarrow v(y) = \sqrt{y}$

Example

Find the area of the region between $x = 2y^2$ and $x = 1 - y$

1. Find bounds in terms of y

$\Rightarrow 2y^2 = 1 - y$

$\Rightarrow 2y^2 + y - 1 = 0$

$\Rightarrow 2y^2 + 2y - y - 1 = 0$

$\Rightarrow 2y(y+1) - (y+1) = 0$

$\Rightarrow (2y-1)(y+1) = 0 \Rightarrow y = \frac{1}{2}, -1$

2. Evaluate the definite integral

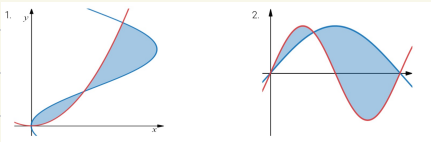
$= \int_{-1}^{\frac{1}{2}} [(1-y) - 2y^2]dy$

$= y - \frac{1}{2}y^2 - \frac{2}{3}y^3 \Big|_{-1}^{\frac{1}{2}}$

$= 9/8$

Example

To find the areas of the shaded regions, should we integrate WRT y or x?



1. There is no clear integral WRT x after the first intersection, since the blue function doubles back.

2. There is no clear integral WRT y for the same reasons as (1).

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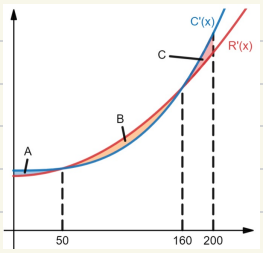
Example

Suppose a tech company makes portable speakers

$C(x)$ = cost of producing x speakers

$R(x)$ = revenue from selling x speakers

The graphs of C' and R' are given. Assume $C(0) = 0$



1. What does the area A represent?

   It represents the total loss from producing and selling the first 50 speakers

2. What does the area C represent?

   It represents the total loss from producing and selling speakers 50-160

3. What does $-A+B-C$ represent?

   It represents the total profit from producing and selling the first 200 speakers

4. How many speakers would maximize profits?

   The first 160 since we know that anything past B would result in a loss