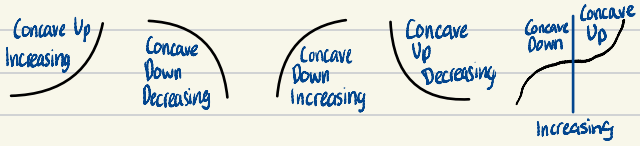
LEC29 - Derivatives and Graph Shape

Wednesday, November 20, 2024

Section 4.5

Warm Up Problem

Which of the graphs are concave up/down?



Is it true that a concave up graph must be increasing?

⇒ No, as just shown, a concave up graph may be decreasing

How can we tell CU from CD by looking at first derivatives? What about second derivatives?

⇒ First Derivative
- Increasing = Concave up
- Decreasing = Concave down

⇒ Second Derivative
- Positive = Concave Up
- Negative = Concave Down

Circle the correct options in the table below:

If f is concave up then	If f is concave down then
f lies above/below its tangent lines	f lies above/below its tangent lines
f' is increasing/decreasing	f' is increasing/decreasing
f'' is positive/negative	f'' is positive/negative

Example

*Inflection points occur when $f''(x)=0$

Find the intervals where $f(x)=2x^4-12x^2-5x$ is concave up and concave down, and find all its inflection points

1. Find where $f''(x)=0$

$f'(x)=8x^3-24x-5$

$f''(x)=24x^2-24$

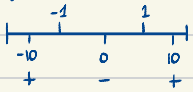
$0=24x^2-24$

$24=24x^2$

$1=x^2$

$x=\pm 1$

2. Sign line and test points



3. Conclusion

CU on $(-\infty,-1)\cup(10,1)$

CD on $(-1,1)$

Inflection points at $x=\pm 1$

Theorem 4.10: Test for continuity

Let f be a function that is twice differentiable over an interval I

(i) If $f''(x)>0$ for all $x\in I$, then f is concave up over I

(ii) If $f''(x)<0$ for all $x\in I$, then f is concave down over I

Inflection Points

If f is continuous at a and f changes concavity at a (different concavity on either side of a), then $(a,f(a))$ is an inflection point of f

Example

Find all intervals where $f(x)=2x^4-12x^2-5x$ is concave up/down, and find all its inflection points

1. Find $f''(x)$

⇒ $f'(x)=8x^3-24x-5$

⇒ $f''(x)=24x^2-24$

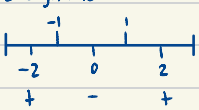
2. Find where $f''(x)=0$

⇒ $0=24x^2-24$

⇒ $0=x^2-1$

⇒ $x=-1,1$

3. Sign line



$f(-1)=-5$ $f(1)=-15$

4. Conclusion

⇒ f is concave up over $(-\infty,-1)\cup(1,\infty)$

⇒ f is concave down over $(-1,1)$

⇒ f has inflection points $(-1,-5)$ and $(1,-15)$

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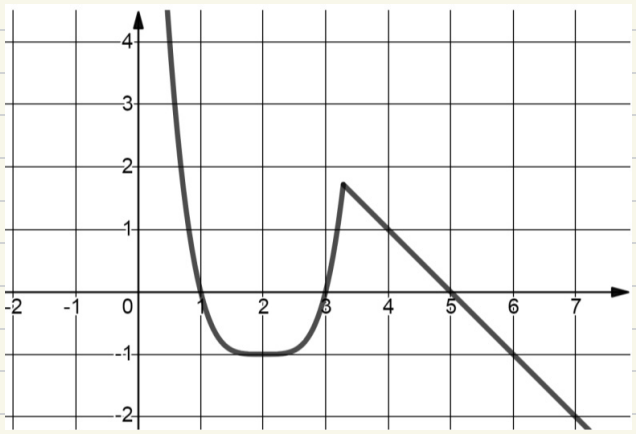
Example

Consider this graph of the derivative f' of a function f



1. For what values does f have a local maximum/minimum
   - ⇒ Local min (f' goes from - to +) at $x = 3$
   - ⇒ Local max (f' goes from + to -) at $x = 1,5$
2. On what intervals is f concave up
   - ⇒ f is concave up when f' is increasing
   - ⇒ f is concave up on $(2, 3.3)$

Example

These are graphs of f, f', f''. Which is which and why?

$f = A$ $f' = C$ $f'' = B$