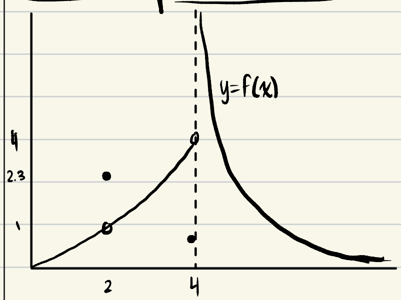
LEC6 – Intro to Limits

Warm Up Problem



$\lim_{x\to 2} f(x) = 1$ } Graph approaches 1, even though $f(2) = 2.3$

$\lim_{x\to 4^-} f(x) = 4$ } left and right limits do not agree ⇒ limit does not exist

$\lim_{x\to 4^+} f(x) = +\infty$

$\lim_{x\to 4} f(x) = \text{DNE}$

Note: an infinite limit is a type of limit that DOES NOT EXIST

Limits

- Limits are the foundation of calculus
  - Everything in calculus is about limits or defined primarily in terms of limits
- Limits allow us to define derivatives (MAT135), integrals (MAT136), and much more
- Limits describe the value, if any, that a functions outputs tend towards as its inputs approach a certain x-value, but do not equal that x-value
- Limits can be illustrated through graphs, but the most rigorous way to find a limit is through computations (we will not generally use tables)
- There are many strategies for computing limits

Select Definitions

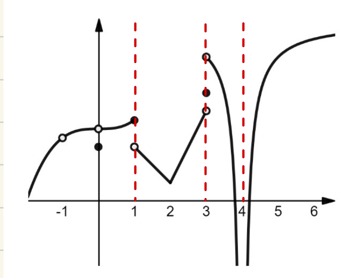
$\lim_{x\to a} f(x) = L$ ⇒ As x gets closer to a, y approaches L

$\lim_{x\to a} f(x) = \infty$ ⇒ As x approaches a, y grows without bound

$\lim_{x\to a^+} f(x) = L$ ⇒ As x-values larger than a approach a, y approaches L

Example:

For which values of a does $\lim_{x\to a} f(x)$ NOT exist?



1 & 3 do not exist because the left hand and right hand limits do not agree. 4 does not exist because infinite limits do not exist

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*For a limit to exist at a specific value a, the left and right hand limits must equal each other

Derivative: Will be taught formally in later lectures, but is basically a way to find the slope of the line tangent to any point on a function

Integral: Will be taught during MAT136, but is basically the inverse function of a derivative

Tend towards: A graph tends towards a specific y-value if the previous y-values close by approach the specific y-value

*The main 3 ways to evaluate a limit are graphically, algebraically, and using a table of values

*Limits only exist if they approach a real, finite number

- We can use limits to figure out what a functions y-values get closer to as the x-values get closer to a particular value
- We can take left and right hand limits to see what a function approaches from the left or right, the limit exists if left limit = right limit
- Limits exist when they approach a real, finite value
- We can evaluate limits using graphs, algebra, and tables of values

Indeterminate Form: When directly substituting into a limit, if the result simplifies to $\frac{0}{0}$, then the limit is of indeterminate form, and we can use various techniques to evaluate it

Multiplying by the conjugate: A strategy when a limit in indeterminate form contains a sum / difference involving a square root, swap the signs and multiply the numerator and denominator by the new sum / difference

Example: Evaluate the following limit

$$\lim_{x \to 16} \frac{4-\sqrt{x}}{16-x}$$

plugging in 16, we get $\frac{0}{0}$, indeterminate form

the presence of the square root suggests we should try multiplying by the conjugate

When do we multiply by the conjugate?

- Function is in indeterminate form
- Numerator or denominator is the sum / difference with a square root

SOLUTION

$$\Rightarrow \lim_{x \to 16} \frac{(4-\sqrt{x})(4+\sqrt{x})}{(16-x)(4+\sqrt{x})}$$

$$\Rightarrow \lim_{x \to 16} \frac{\cancel{(16-x)}}{\cancel{(16-x)}(4+\sqrt{x})}$$

$$\Rightarrow \lim_{x \to 16} \frac{1}{4+\sqrt{x}}$$

$$\Rightarrow \frac{1}{4+\sqrt{16}} \Rightarrow \frac{1}{4+4} \Rightarrow \frac{1}{8}$$

Example: Evaluate the following limit

$$\lim_{x \to 5} \frac{|x-5|}{x^2-25}$$

Split absolute value into piecewise

$$\Rightarrow |x-5| = \begin{cases} -x-5 & \text{if } x \le 5 \\ x-5 & \text{if } x > 5 \end{cases}$$

$$\Rightarrow \frac{|x-5|}{x^2-25} = \begin{cases} \frac{-x-5}{x^2-25} & \text{if } x \le 5 \\ \frac{x-5}{x^2-25} & \text{if } x > 5 \end{cases}$$

SOLUTION

Left Hand

$$\Rightarrow \lim_{x \to 5^-} \frac{-x-5}{x^2-25}$$

$$\Rightarrow \lim_{x \to 5^-} \frac{-\cancel{(x+5)}}{\cancel{(x+5)}(x-5)}$$

$$\Rightarrow \lim_{x \to 5^-} \frac{-1}{x-5}$$

$$\Rightarrow -\infty$$

Right Hand

$$\Rightarrow \lim_{x \to 5^+} \frac{x-5}{x^2-25}$$

$$\Rightarrow \lim_{x \to 5^+} \frac{\cancel{x-5}}{(x+5)\cancel{(x-5)}}$$

$$\Rightarrow \lim_{x \to 5^+} \frac{1}{x+5}$$

$$\Rightarrow \frac{1}{10}$$

- When a limit in indeterminate form contains a sum / difference involving a square root in the numerator / denominator, we can try multiplying by the conjugate