LEC14 - Growth and Decay

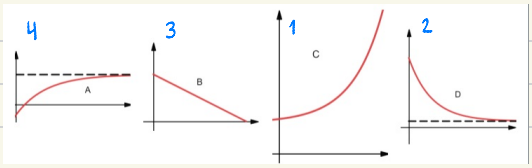
Thursday, February 6, 2025

Section 2.3

Warm Up Problem

Which graph represents which scenario

1) Money in a bank accounts with continuously compounded interest, and initial deposit of $100
2) The temperature of a pizza taken out of a hot oven and left on the counter
3) The speed of a bus slowing with constant deceleration at a stop light
4) The temperature of a cold slushie left outside on a warm day



Exponential Growth/Decay Model

Suppose that f is a function such that $f'(t) = k \cdot f(t)$, where k is a constant. This means that the rate of change of f is proportional to the current value of $f(t)$

What functions have this behaviour?

- If $y = y_0 e^{kt}$, then $\frac{dy}{dt} = k y_0 e^{kt} = k f(t)$
- It can be proven (chapter 4) that this is the only function with this property
- y_0 is the initial value at $t = 0$

Example

Suppose a bacteria population grows exponentially according to $P(t) = 150e^{1.4t}$, where t is in hours. When will the population reach 9000?

$9000 = 150e^{1.4t}$

$60 = e^{1.4t}$

$\frac{\ln 60}{1.4} = t \approx 2.92$

Questions

1. In real life, is the bacteria population function P continuous? No
2. Can there be a non-integer number of bacteria at any time? No
3. Is it possible to differentiate a discontinuous function? No

Example

In which of the following situations do you think we can use an exponential growth model?

1. Population of the world ✓
2. Population of people living in a certain house ✗
3. Population of rabbits that are introduced to a new island with no natural predators ✓
4. Population of bears in a small forest ✗

Example

An archaeologist wants to estimate the age of a dinosaur bone

1. Carbon-14 has a half life of 5730 years. If the minimum detectable amount is 0.05% of the carbon in a living bone, what are the oldest bones that can be detected?

$\Rightarrow 5730 = \frac{\ln 2}{k} \Rightarrow k = \frac{\ln 2}{5730}$

$\Rightarrow 0.0005 = e^{\frac{\ln 2}{5730} t}$

$\Rightarrow \frac{\ln 0.0005 \cdot 5730}{\ln 2} = t \approx 62800$

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2. Dinosaurs went extinct about 65 million years ago. Can a dinosaur be dated using this method?

No, they can't be detected using this method since it is only effective until ~62,800 years

3. Potassium-40 can be used instead of carbon dating. After 5 million years, 99.7% of potassium 40 remains. What is the half life of potassium-40?

$\Rightarrow 0.997 = e^{-k(5000000)} \Rightarrow \frac{\ln 0.997}{5000000} = -k$

$\Rightarrow \frac{\ln 2}{k} = t_{\frac{1}{2}} = 1150$ million years

Example

Suppose you invest your money in an account whose interest is compounded continuously, and suppose your money triples every 5 years. What is the interest rate? ($M(t) = M_0 e^{kt}$, where k is the interest rate)

$\Rightarrow 3 = e^{5k}$

$\Rightarrow \frac{\ln 3}{5} = k = 22\%$