LEC26 - Maxima and Minima

Friday, November 8, 2024
Section 4.3

Local extrema: A point where a function is smaller/bigger than all y-values nearby the extrema

Absolute extrema: A point where a function is smaller/bigger that all other y-values on the entire domain of the function

* Absolute and global extrema are interchangeable

* Critical points occur where $f'(x)=0$, or is undefined

* The EVT says nothing about a function if its conditions are not met. The same goes for all theorems, unless otherwise stated

Warm Up Problem

Fill in the blanks for the function to the right

Domain = $(-\infty, \infty)$

Local min → 1 at x = 4

Absolute min → DNE at x = DNE

Local max → 5, 4 at x = 2, 6

Absolute max → 5 at x = 2

Theorem 4.1: The extreme value theorem (EVT)

If f is a continuous function over the closed, bounded interval $[a,b]$ then:

- f has an absolute maximum at some point on the interval
- f has an absolute minimum at some point on the interval

Example 1

What does the EVT say about the function $f(x)=x^{-2}$ on $[-1,2]$

⇒ $f(x)$ is not continuous and differentiable on the interval $[-1,2]$

⇒ Therefore, the EVT says nothing about $f(x)$

Theorem 4.3: Location of absolute extrema

Let f be continuous function over a closed, bounded interval I

The absolute maximums and minimums of f over I occur at endpoints of the interval, or at critical points.

Example 2

What do these theorems say about $g(x)=(x-1)^{2/3}$ on $[-1,2]$

⇒ $g(x)$ is continuous on $[-1,2]$

⇒ The EVT guarantees that there is an absolute max and min on $[-1,2]$

⇒ Theorem 4.3 states that these extrema occur at the endpoints of $[-1,2]$, or on critical points of $g(x)$ on $[-1,2]$

Example 3

Find the locations of the absolute max and min of $f(x)=\frac{x+45}{x^2+1000}$ over the interval $[-50,50]$

$$f'(x)=\frac{x^2+1000-(x+45)(2x)}{(x^2+1000)^2}$$

$$0 = x^2+1000-2x^2-90x$$

$$0=-x^2-90x+1000$$

$$0=x^2+90x-1000$$

$$0=(x+100)(x-10)$$

$$x=-100,10$$

$f(-100) = -0.005$ Absolute minimum

$f(10) = 0.05$ Absolute maximum

$f(-50) = -0.001...$

$f(50) = 0.027...$

- A function has local extrema where the value peaks for all y-values close to the extrema. A global extrema is similar except it is the smallest/largest value on the entire domain. A global extrema may be a local extrema, and a particular extrema (or multiple) may not exist on a function
- The extreme value theorem states that a function that is continuous on a closed interval has an absolute max/min on that interval
- Theorem 4.3 states that the extrema from the EVT occur either at interval endpoints, or critical points on the interval

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Example 4

Which of the following statements are true?

1. If f is defined and continuous on $[0,\infty)$, then f has an absolute minimum True by EVT
2. If $x=a$ is a local maximum of a function g, then $x=a$ is a critical point of g True by EVT
3. If $x=c$ is not a local maximum of a function h, then $x=c$ is not a critical point of h False with counter example of vertical asymptotes. They are critical points ($f'(x)$ is undefined), but they are not local maxima