LEC06 - Net Change

Friday, January 17, 2025

Section 1.4

Example

Let $P(t)$ be the population (in thousands) of a honeybee colony at time t (in weeks)

Suppose $P(0) = 30$, and $P'(t) = 0.09t^2 + 0.7$

1. Evaluate $\int_1^2 p(t)dt$ and interpret the result

$= \int_1^2 (0.09t^2 + 0.7)dt = \int \left(\frac{0.09}{3}t^3 + 0.7t\right)\Big|_1^2 = 0.03(2)^3 + 0.7(2) - 0.03(1)^3 - 0.7(1) = 0.91$, which represents the total change in population from weeks 1 to 2

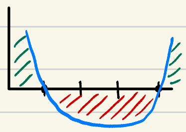
2. What does $30 + \int_0^7 P'(t)$ represent?

It represents the total number of bees (in thousands) after 7 weeks has elapsed

Example

A person jogs along a straight road with velocity $v(t) = 2t^2 - 8t + 6$ (in km/h) for $0 \le t \le 3$, where t is in hours

1. Sketch the graph of $v(t) = 2t^2 - 8t + 6 = 2(t-1)(t-3)$



2. Find the net displacement of the jogger over $[0, 3]$

$= \int_0^3 (2t^2 - 8t + 6)dt$

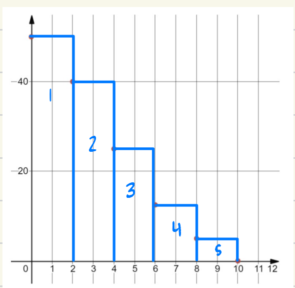
$= \frac{2}{3}t^3 - 4t^2 + 6t\Big|_0^3 = \frac{2}{3}(3)^3 - 4(3)^2 + 6(3) - 0 = 0$

3. Find the distance travelled in the same interval

$= \int_0^1 (2t^2 - 8t + 6)dt - \underbrace{\int_1^3 (2t^2 - 8t + 6)dt}_{\text{area is under graph so we add a negative to make it positive}} = \left(\frac{2}{3}t^3 - 4t^2 + 6t\right)\Big|_0^1 - \left(\frac{2}{3}t^3 - 4t^2 + 6t\right)\Big|_1^3 = \left(\frac{8}{3} - 0\right) - \left(0 - \frac{8}{3}\right) = \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$

Example

You are driving down a country road late at night, and a skunk runs out on the road 300m away, you slam on the brakes but it takes 10s to stop. Using the following table, estimate the total distance travelled. Will you hit the skunk?



t [s]	0	2	4	6	8	10
$v(t)$ [m/s]	50	40	25	12.5	5	0

$\Rightarrow$ To be safe, we will use an overestimate Riemann sum (left since $v(t)$ is decreasing)

$\Rightarrow A = ① + ② + ③ + ④ + ⑤$

$= 50\Delta x + 40\Delta x + 25\Delta x + 12.5\Delta x + 5\Delta x$

$= \Delta x(50 + 40 + 25 + 12.5 + 5)$

$= 132.5\Delta x \Rightarrow \Delta x = \frac{b-a}{n} = \frac{10-0}{5} = 2$

$= 132.5(2) = 165$m

Therefore, even in the worst case, the skunk will be safe, since the car will stop before 300m