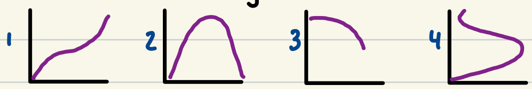
LEC18 - Derivatives of Inverses

*Recall that the HLT can be used to determine whether a function is invertible

Warm Up Problem:

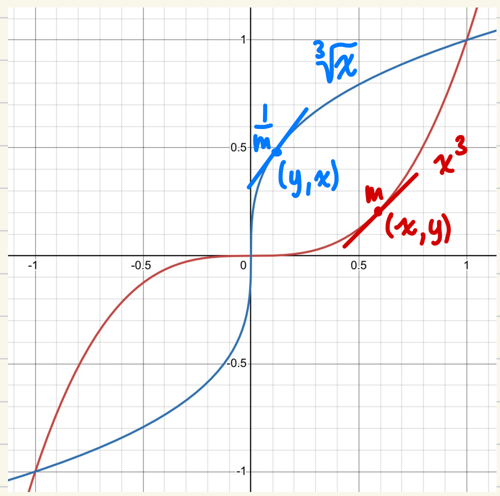
Which of the following are invertible functions? Use HLT to determine which are invertible



1: Yes 2: No 3: Yes 4: No ← Not a function (Fails VLT)

Derivatives of Inverse Functions:

- If f is invertible, we can find $(f^{-1})'$, the derivative of f^{-1}, in 2 different ways
  1. Using the original function
     - → Find $f'(x)$, swap (x,y) to get (y,x). This is the coordinates of the tangent on $f^{-1}(x)$
     - → The slope of the tangent to $f^{-1}(x)$ is $\frac{1}{m}$
  2. Using the formula from theorem 3.11



Theorem 3.11

If f is both invertible and differentiable then

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

Example 1:

Let $f(x) = 3x^5 + 4x^3 + 2x$

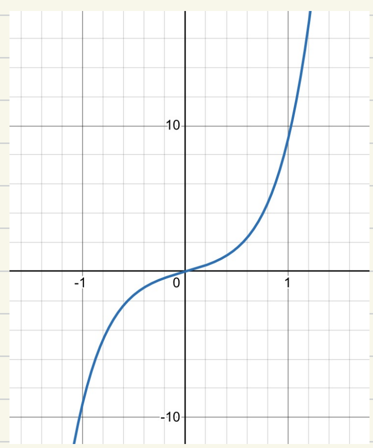
1. Find a formula for $f'(x)$

   $15x^4 + 12x^2 + 2$

2. Is f invertible?

   $15x^4 + 12x^2 + 2 = 0 \Rightarrow$ No solutions

   - Therefore, $f(x)$ is always either increasing or decreasing
   - It is impossible for f to fail the HLT $\Rightarrow f$ is invertible



3. Compute $f(1)$ and $f'(1)$

   $f(1) = 3(1)^5 + 4(1)^3 + 2(1)$
   $= 3 + 4 + 2 = 9$

   $f'(1) = 15(1)^4 + 12(1)^2 + 2$
   $= 29$

4. Find $(f^{-1})'(1)$

   $$(f^{-1})'(9) = \frac{1}{f'(\underbrace{f^{-1}(9)})} = \frac{1}{f'(1)} = \frac{1}{29}$$

   from 3. we know $f(1) = 9$, so $f^{-1}(9) = 1$

   we know this equals 29 from 3.

- For a relation to be an invertible function, it must pass both the HLT and the VLT
- We can find $(f^{-1})'(x)$ by inverting the coordinates and slope of $f'(x)$ at a point, or we can use the formula from 3.11