LEC2 - Basic Classes of Functions

Composite Functions

Let f and g such that the range of f is a subset of the domain of g. Then their composition is the function $g \circ f$ defined by $(g \circ f)(x) = g(f(x))$

Example: Suppose f is defined by the table below, and $g(x) = x^2 - 3$.
What is $(g \circ f)(2)$?

x	$f(x)$
0	7
1	6
2	3
3	-1
4	2

$$g(f(2)) = g(3)$$
$$\Rightarrow 3^2 - 3$$
$$\Rightarrow 9 - 3$$
$$\boxed{\Rightarrow 6}$$

Even/Odd Functions

Let f be a function:

If $f(-x) = f(x)$ for all x in the domain of f, then f is an even function
even functions are symmetric about y-axis

If $f(-x) = -f(x)$ for all x in the domain of f, then f is an odd function
odd functions are symmetric about origin $(0,0)$

Even	Odd
$f(x) = x^2$	$f(x) = x^3$
$f(x) = \cos x$	$f(x) = \sin x$
$f(x) = x^4$	$f(x) = x^5$
$f(x) = \lvert x \rvert$	$f(x) = x^{27}$

There are 4 main ways to represent functions. You should be able to use each type, and be able to switch between them freely.

- Table of values
- Graph
- Formula
- Word description

Piecewise-Defined Functions

A piecewise-defined function is a function defined by different formulas on different parts of its domain

Example:

$$f(x) = \begin{cases} x^2, & \text{if } x < 1 \\ 2 - x, & \text{if } 1 \le x < 3 \\ x - 1, & \text{if } 3 \le x \end{cases}$$

$f(-3) = (-3)^2$
$= 9$

$f(1.5) = 2 - 1.5$
$= 0.5$

$f(3) = 3 - 1$
$= 2$

$f(7) = 7 - 1$
$= 6$