LEC19 - Implicit Differentiation

Wednesday, Oct 23, 2024
Section 3.8

* When differentiating y, we treat y as a function. Therefore, we apply the chain rule

ex. The derivative of $2y^2 = 4y \cdot \frac{dy}{dx}$

Warm Up Problem

$y = \tan(xy) \Rightarrow \frac{dy}{dx} = y\sec^2(xy)$

Is this true?

No, this is not true because the derivative incorrectly applied the product rule. Applying it correctly gives

$$\frac{dy}{dx} = \sec^2(xy)\left(1 \cdot y + x \cdot \frac{dy}{dx}\right) \neq y\sec^2(xy)$$

Example:

Consider the equation $x^2 + y + 3e^{2y} = 6xy^3$. Find $\frac{dy}{dx}$

$\Rightarrow 2x + \frac{dy}{dx} + 6\frac{dy}{dx}e^{2y} = 6y^3 + 6x3y^2\frac{dy}{dx}$ Implicity differentiate

$\Rightarrow \frac{dy}{dx} + 6\frac{dy}{dx}e^{2y} - 6x3y^2\frac{dy}{dx} = 6y^3 - 2x$ Separate terms containing $\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}(1 + 6e^{2y} - 6x3y^2) = 6y^3 - 2x$ Factor out $\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{6y^3 - 2x}{1 + 6e^{2y} - 18xy}$ Divide to isolate $\frac{dy}{dx}$

* In this example, $4x$ was subtracted from one side, and added to the other, which is not allowed

Example:

Find the mistake for $2x^2 + y^2 = 3\sin y$

$\Rightarrow 4x + 2y\frac{dy}{dx} = 3\cos y\frac{dy}{dx}$ Differentiate

$\Rightarrow 2y\frac{dy}{dx} - 3\cos y\frac{dy}{dx} = 4x$ Move common terms This line is wrong because $4x$ should be $-4x$

$\Rightarrow \frac{dy}{dx}(2y - 3\cos y) = 4x$ Factor out dy/dx

$\Rightarrow \frac{dy}{dx} = \frac{4x}{2y - 3\cos y}$ Divide