$n^2 c_1 + n \cdot c_2$

8) $T(n) = c_1 + \sum_{i=1}^{n-1} \cdot \left(\sum_{j=i+1}^{n} \cdot \left(\sum_{k=1}^{j} \cdot c_2 \right) \right) = c_1 + \sum_{i=1}^{n-1} \cdot \left[\sum_{j=i+1}^{n} \cdot j \cdot c_2 \right] \Rightarrow$

$= c_1 + \sum_{i=1}^{n-1} \cdot \left(c_2 \cdot \left(\sum_{j=1}^{n} \cdot j - \sum_{j=1}^{i} \cdot j \right) \right) = c_1 + \sum_{i=1}^{n-1} \cdot \left(c_2 \cdot \left(\frac{n(n+1)}{2} - \frac{i \cdot (i+1)}{2} \right) \right) \Rightarrow$

$= c_1 + \sum_{i=1}^{n-1} \cdot \left(\frac{c_2}{2} \cdot (n^2+n) - \frac{c_2}{2} \cdot (i^2+i) \right) = c_1 + \left(\frac{c_2}{2} \cdot \sum_{i=1}^{n-1} \cdot (n^2+n) - \sum_{i=1}^{n-1} \cdot \frac{c_2}{2} \cdot (i^2+i) \right) \Rightarrow$

$= c_1 + \left(\frac{c_2}{2} \cdot \left(\sum_{i=1}^{n-1} \cdot n^2 + \sum_{i=1}^{n-1} \cdot n \right) - \frac{c_2}{2} \cdot \left(\sum_{i=1}^{n-1} \cdot i^2 + \sum_{i=1}^{n-1} i \right) \right) \Rightarrow$

$= c_1 + \left(\frac{c_2}{2} \cdot \left((n-1) \cdot n^2 + (n-1) \cdot n \right) - \frac{c_2}{2} \cdot \left(\frac{n(n+1)(2n+1)}{6} - \frac{n \cdot (n+1)}{2} \right) \right) \Rightarrow$

$= c_1 + \left(\frac{c_2}{2} \cdot (n^3 - n^2 + n^2 - n) - \frac{c_2}{2} \cdot \left(\frac{(n^2+n) \cdot (2n+1)}{6} - \frac{3 \cdot (n^2+n)}{6} \right) \right) \Rightarrow$

$= c_1 + \left(\frac{c_2}{2} \cdot (n^3 - n) - \frac{c_2}{2} \cdot \left(\frac{2n^3 + n^2 + 2n^2 + n - 3n^2 - 3n}{6} \right) \right) \Rightarrow$

$= c_1 + \left(\frac{c_2}{2} \cdot n^3 - \frac{c_2}{2} n - \frac{c_2}{2} \left(\frac{2n^3 - 2n}{6} \right) \right)$

$\Downarrow$

$\frac{c_2 \cdot (2n^3 - 2n)}{2 \cdot 6} = \frac{c_2 2n^3 - c_2 2n}{12}$

$$c_1+\left(\frac{c_2 n^3}{2}-\frac{c_2 n}{2}-\frac{c_2 2n^3-c_2 2n}{12}\right)$$

$$c_1+\left(\frac{6c_2 n^3}{12}-\frac{6c_2 n}{12}-\frac{c_2 2n^3-c_2 2n}{12}\right)$$

$$c_1+\left(\frac{4c_2 n^3-4c_2 n}{12}\right)=c_1\left(\frac{4\cdot(c_2 n^3-1c_2 n)}{12}\right)=c_1+\frac{c_2 n^3-c_2 n}{3}$$

$$=c_1+\frac{c_2 n^3}{2}-\frac{c_2 n}{2}=T(n)$$

2. $c_1+\sum_{i=1}^{n}\cdot\sum_{j=1}^{i^2}\cdot\sum_{k=1}^{j}\cdot c_2$

$$c_1+\sum_{i=1}^{n}\cdot\sum_{j=1}^{i^2}\cdot(j\cdot c_2)$$

$$c_1+\sum_{i=1}^{n}\cdot\left(c_2\sum_{j=1}^{i^2}\cdot j\right)$$

$$c_1+\sum_{i=1}^{n}\cdot\left(c_2\cdot\left(\frac{i^2\cdot(i^2+1)}{2}\right)\right)$$

$$\cdot\left(c_2\cdot\left(\frac{i^4+i^2}{2}\right)\right)$$

$$c_1+\sum_{i=1}^{n}\cdot\frac{c_2}{2}\cdot(i^4+i^2)$$

$$c_1+\left(\frac{c_2}{2}\cdot\sum_{i=1}^{n}\cdot(i^4+i^2)\right)$$

$$c_1+\left(\frac{c_2}{2}\cdot\left(\sum_{i=1}^{n}\cdot i^4+\sum_{i=1}^{n} i^2\right)\right)$$

$$c_1+\left(\frac{c_2}{2}\cdot\left(\frac{n(n+1)(6n^3+9n^2+n-1)}{30}+\left(\frac{n(n+1)(2n+1)}{6}\right)\right)\right)$$

$$c_1+\frac{c_2}{2}\cdot\left(\frac{(n^2+n)\cdot(6n^3+9n^2+n-1)}{30}+\frac{(n^2+n)\cdot(2n+1)}{6}\right)$$

$$c_1+\frac{c_2}{2}\left(\frac{6n^5+9n^4+n^3-n^2+6n^4+9n^3+n^2-n}{30}+\frac{2n^3+n^2+2n^2+n}{6}\right)$$

X^1

$$*\ c_1+c_2\cdot\left(\frac{6n^5+15n^4+10n^3-n}{30}+\frac{2n^3+n^2+2n+1}{6}\right)$$

$$c_1+c_2\cdot\left(\frac{6n^5+15n^4+20n^3+15n^2+4n}{60}\right)$$

$$c_1+c_2\cdot\left(\frac{6n^5}{60}+\frac{15n^4}{60}+\frac{20n^3}{60}+\frac{15n^2}{60}+\frac{4n}{60}\right)$$

$$c_1+\frac{c_2}{10}n^5+n^4\frac{c_2}{4}+n^3\frac{c_2}{3}+n^2\frac{c_2}{4}+n\frac{c_2}{15}$$

$$T(n)=\frac{c_2}{10}n^5+n^4\frac{c_2}{4}+n^3\frac{c_2}{3}+n^2\frac{c_2}{4}+n\frac{c_2}{4}+c_1$$

b) 1. $c_1 + \frac{c_2 n^3}{3} - \frac{c_2 n}{3}$ es de $O(n^3)$

Es negativo, se acota con 0

$$\left.\begin{array}{l} c_1 \leq n^3 \\ c_1 \leq c_1 n^3 \end{array}\right\} \begin{array}{l} c = c_1 \\ n_0 = 1 \end{array} \;\Bigg|\; \left.\begin{array}{l} \frac{c_2}{3} n^3 \leq n^3 \\ \frac{c_2}{3} n^3 \leq \frac{c_2}{3} n^3 \end{array}\right\} \begin{array}{l} c = \frac{c_2}{3} \\ n_0 = 0 \end{array} \;\Bigg|\; \left.\begin{array}{l} -\frac{c_2}{3} n \leq n^3 \\ -\frac{c_2}{3} n \leq 0 n^3 \end{array}\right\} \begin{array}{l} c = 0 \\ n_0 = 0 \end{array}$$

$T(n) \leq c_1 n^3 + \frac{c_2}{3} n^3 + 0 n^3$

$T(n) \leq n^3 \left(c_1 + \frac{c_2}{3} + 0 \right)$

$\leq c \cdot n^3$

$T(n) \leq c n^3$, con $c = \left(c_1 + \frac{c_2}{3} \right)$ $\forall\, n \geq n_0$ con $n_0 = 1$

2. $T(n) = \frac{c_2}{16} n^5 + \frac{c_2}{4} n^4 + n^3 \frac{c_2}{3} + n^2 \frac{c_2}{3} + n \frac{c_2}{4} + c_1$ tiene $O(n^5)$

regla del polinomio