**Project 4**

Attachments:

*Stable\_Marriage\_v1\_Lists\_Hashmaps* - verification and matching

*Stable\_Marriage\_v2\_2D\_Stack* - matching

**1. Stable Checking:** *(Stable\_Marriage\_v1\_Lists\_Hashmaps)*

The matching algorithm for the Gale Shapely implementation of the stable marriage problem falls into an asymptotic time complexity of at least n2.

Theoretically for verifying stable pairings, it is sufficient to check each member of one sex, for example the males, as a potential member of a blocking pair. For each male, only the females that he prefers to his partner need to be checked. Since there are n males in an instance of size n, and for each, at most n-1 females need to be examined; the algorithm has a theoretical time complexity of O(n2).

The list/hashmap implementation that I chose for my verification method shows a complexity of slightly more than n2. The total complexity of the method is represented by n2+ 2n. A nested for loop represents the n2, while 2 additional for loops represent the 2n. Using n2 as the dominant growth factor gives an asymptotic rate of Θ(n2) rather than big O(n2). This can be accounted for by an extra for loop used to map sets into a tree used later by for each loops to iterate through potential block pairs of both the males and females.

The time complexity of the method was investigated by not only assessing the code but by also strategically placing counters throughout the method and running various experiments.

**2. Stable Generalization Writing:**

The stable marriage problem along with the Gale Shapely algorithm can be used to generalize other problems derivative of stable matching. One such problem is the college admission problem also known as the hospital/resident problem.

The college admission problem differs from the stable marriage problem based on the fact that a college can take an incoming class of more than one student. Comparatively speaking in the context of the stable marriage problem, the “females” can accept “proposals” from more than one “male”. This invariably creates a new type of problem with multiple markets or sides that require two-sided matching from a many-to-one perspective.

Although the stable marriage problem and the college admission problem are different, they are similar enough that the algorithmic mechanisms used to solve the stable marriage problem can be applied to that of the college admissions problem. Instead of using deferred acceptance from a one sided perspective, it is applied to the two-sided situation encountered in the college admission problem.

The following is a simplified walkthrough of how the Gale Shapely algorithm would be applied to the college admissions problem:

The two-sided matching algorithm uses the preferences expressed in the rank order lists submitted by applicants and colleges to place individuals. The process begins with an attempt to place an applicant into the college indicated as most preferred on that applicant's list. If the applicant cannot be matched to his/her first college choice, an attempt is then made to place the applicant into the applicant’s second choice. This is repeated until the applicant obtains a tentativematch, or all the applicant's choices have been exhausted.

An applicant can be tentatively matched to a college with this algorithm if the college also ranks the applicant on its rank order list, and either:

* The college has an unfilled position. In this case, there is room in the college to make a tentative match between the applicant and college.
* The college does not have an unfilled position, but the applicant is more attractive to the college than another applicant who is already tentatively matched to the college. In this case, the applicant who is the least preferred current match in the college is removed, to make room for a tentative match with the more preferred applicant.

Matches are deemed "tentative" because an applicant who is matched to a college at one point in the matching process may be removed from the college at some later point, to make room for an applicant more preferred by the college, as described in the second case above. When an applicant is removed from a previously made tentative match, an attempt is made to re-match that applicant, starting from the top of his/her list. This process is carried out for all applicants, until each applicant has either been tentatively matched to the most preferred college possible, or all choices submitted by the applicant have been exhausted. When all applicants have been considered, The Match is complete and all tentative matches become final. (extracted and modified from nrmp.org)

Based on the application of the Gale Shapely deferred algorithm to the college admissions problem, it can be shown that there is at least one valid solution. There is a great deal of debate surrounding this problem and its algorithmic solution dealing with fairness, additional weighted variables, gaming (collusion), etc... This problem along with its derivatives has and will continue to be a focus of study as we move forward in the fields of mathematics and computer science.