Seasonal variations in surface temperature: Modeling a *Game of Thrones* style-planet

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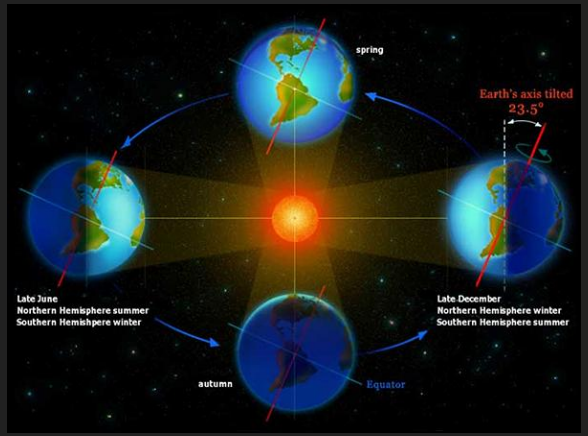
Motivation

- *Start: 1-D Energy Balance Model (EBM)*
  - Single layer grey atmosphere
  - Latitudinal parameterization of solar radiation and albedo
  - Heat diffusion
- *Aim: Add seasonal variations in surface temperature by latitude*
  - How does seasonal variation in solar radiation by latitude impact the modeled surface temperature over time?



Matt Williams, *Universe Today*

- *Application: Adapt our Earth like base model to the Known World in Game of Thrones*
  - How does extending the length of the orbital period impact the seasonal cycle in surface temperature by latitude?

Assumptions about Earth

We first developed a model for Earth…

Radius	$6.37 * 10^6$ m
Solar Constant	$S0 = 1361$ W m^{-2}
Atmospheric emissivity fraction	0.82
Depth of ocean mixed layer and land active layer	100 m
Heat capacity per unit area	$3.37 * 10^8$ J m^{-2} K^{-1}
Latitudinal grid	N = 300

Methods: Incoming Solar Radiation

Idea: Find the distribution of solar radiation by latitude on a given Julian Day

1. Calculate the Julian Day from time input (in days)

   $JD = np.mod(t\_days,365)$

2. Declination angle: latitude at which the Sun is directly overhead for a given Julian Day

   $Delta = 23.45 * \cos(2*\pi* (JD - 172)/365)$

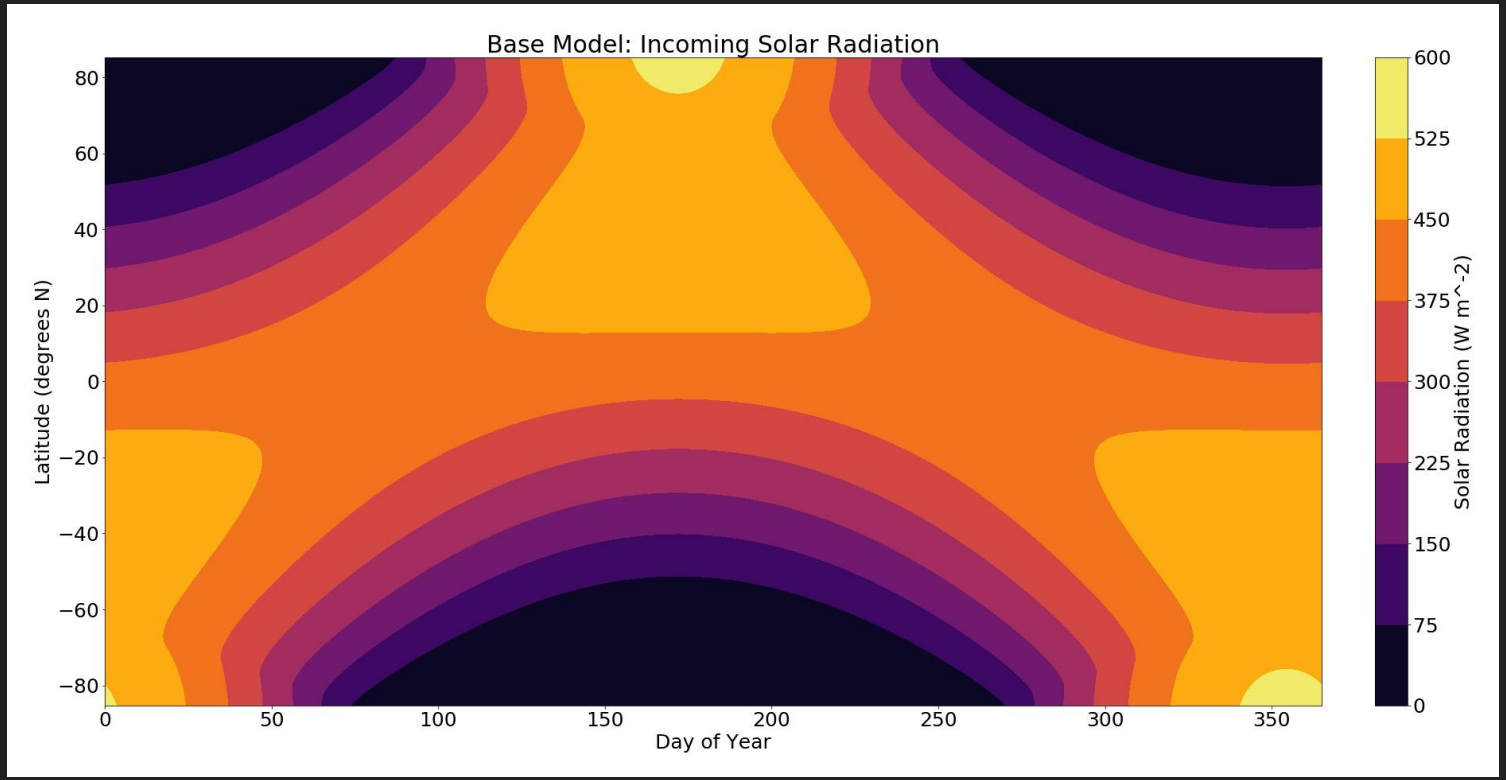
3. Length of daylight: hour angle (convert all angles to radians)

   $ho = \cos^{-1}(-\tan(Delta) * \tan(model\_lat))$

4. Solar radiation

   $Q = (So/\pi) * (ho * \sin(model\_lat) * \sin(Delta) + \cos(model\_lat) * \cos(Delta) * \sin(ho))$

Methods: Incoming Solar Radiation



Methods: Albedo Function

Temperature dependent albedo function from the ATOC 7500 Midterm:

1. Latitudinally changing albedo function
2. Time varying albedo
3. Temperature dependent albedo

```
# Latitudinally changing/temperature and time dependent albedo
# (from 7500 Midterm Solution)
def alpha_t_GoT(t,t_end_GoT,y,T,option=1):
    if option==3:
        a = alb_GoT(t,t_end_GoT,y)
    else:
        a = albedo(y)
    for i in range(N):
        if T[i] <=(273.15-25):
            a[i]=0.55
        else:
            a[i]=a[i]
    type(a)
    return a
```

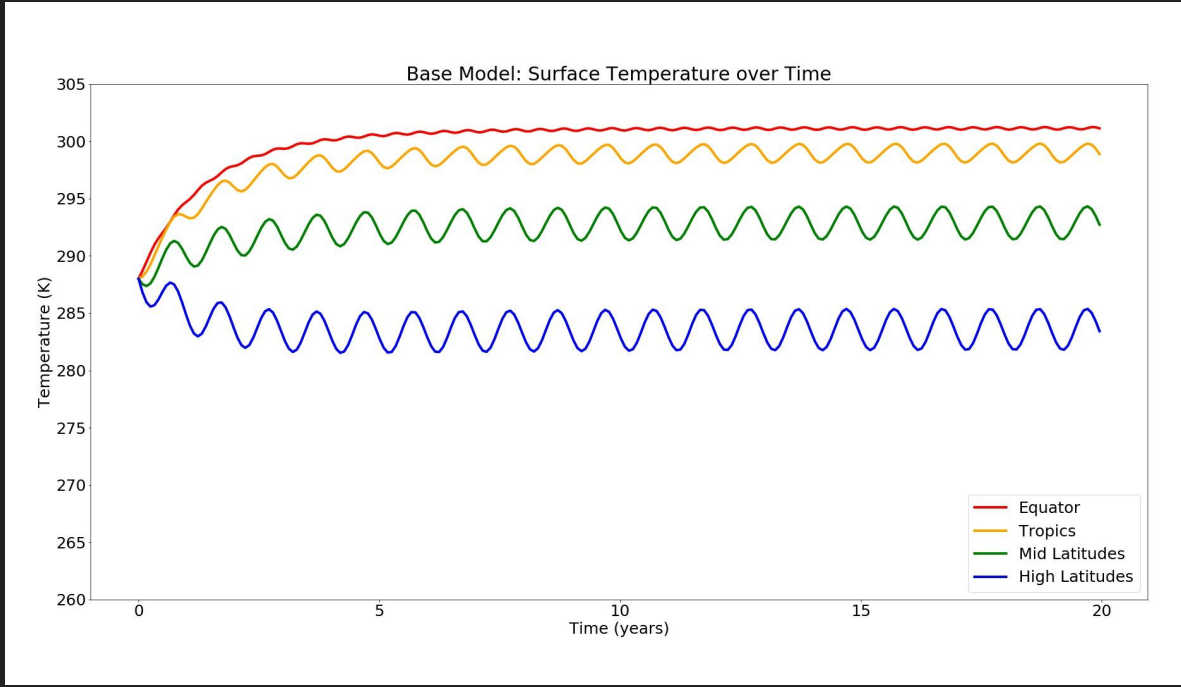
Methods: Time Integration

- Numerical scheme: Radau
  - Implicit 5th order Runge-Kutta method → high accuracy
  - Stiff equation: seasonal cycles in surface temperature occur with a period of 365 days
- No flux boundary conditions at the poles
- Including seasonal solar forcing function and temperature-dependent albedo
- N = 300 latitude segments
- Used scipy and solve_ivp to call the time integrator
  - t_end = 20 years
  - dt = 10 days

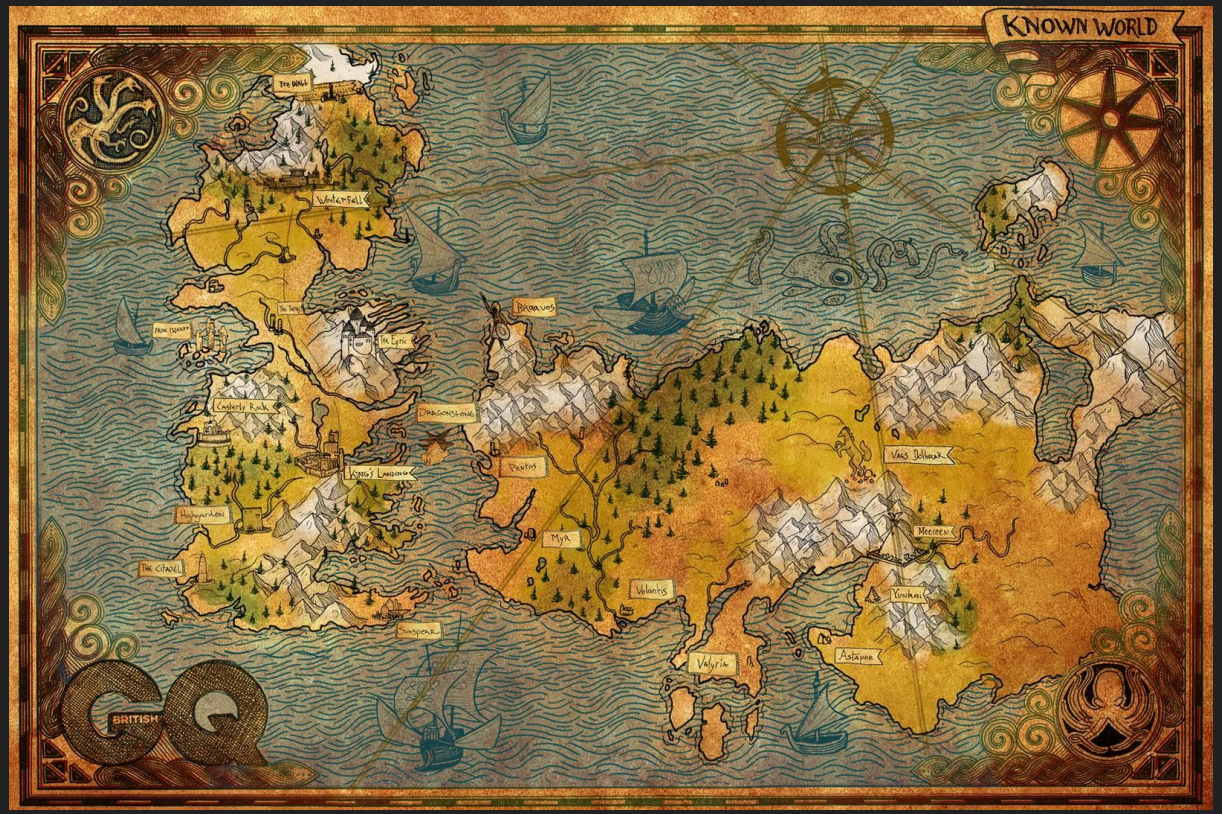
```
solution = solve_ivp(RHS_lat_time_temp, (0,t_end), T0, "Radau", times, max_step=dt )
```

Results: Surface Temperature over Time

- Distinct seasonal cycle in surface temperatures at all latitudes
- Mid and high latitudes experience greater range of temperatures
- Model stabilizes around 5 years



The Experiment: *Game of Thrones*



GQ Magazine

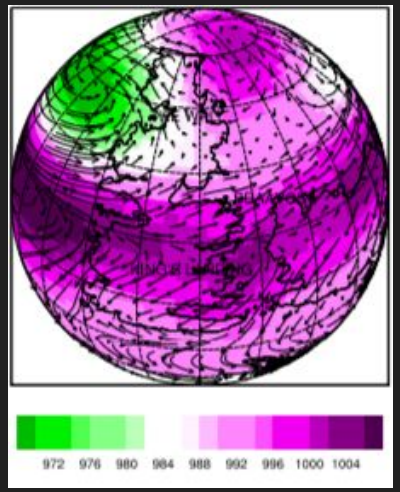
The Known World...

- Supports life
- Cold temperatures in the North, permanently frozen north of the Wall
- Tropical climate in the south
- Large desert to the east
- *Long seasons*

Assumptions about the Known World

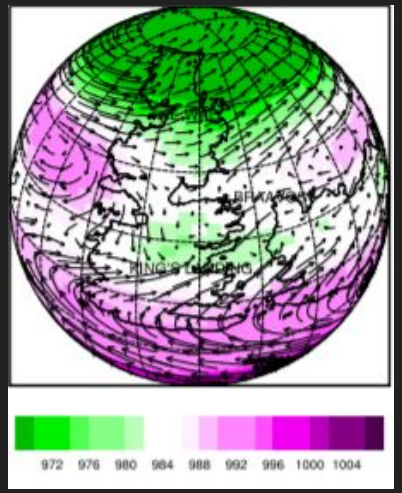
- Driver: Long Seasons
  - Orbital period: 7300 days
- Solar Radiation
  - So = 1361 W m²
- Initially radius and heat capacity same as Earth
- Diffusion
  - No advection term so the primary source of heat transport is wind
  - D=0.9
  - Slightly increased the surface temperature and larger value of D to wind only heat transport

Winter Wind Transport - Primary Heat Source



Tarly, Samwell, "The Climate of the World of Game of Thrones"

Summer Wind Transport - Primary Heat Source



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Methods: Extend seasons

Idea: Set the orbital period to 7300 days from 365 days; this way each season is 5*365 days long

1. Set second-year conversion

   ```
   Years_GoT = 7300 * 24 * 3600
   ```

2. Julian Day

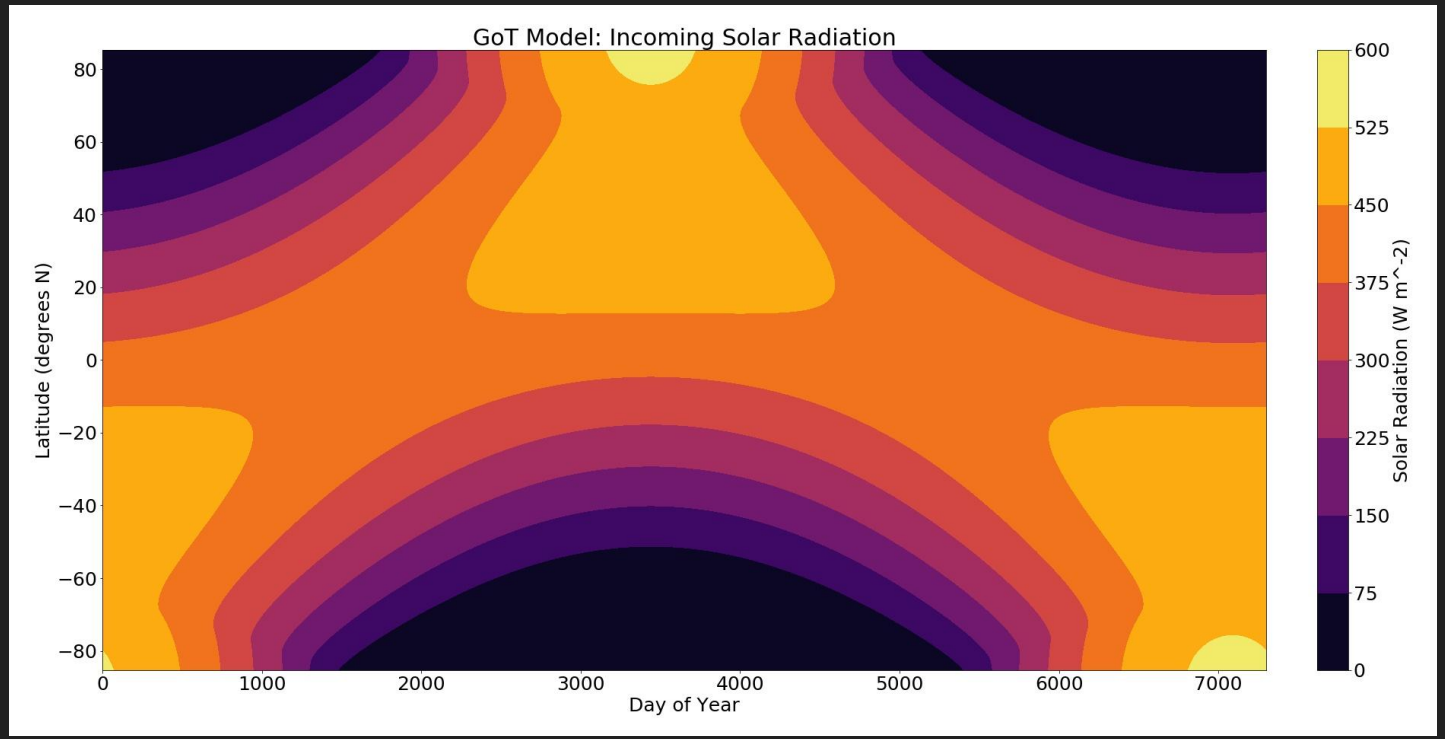
   ```
   JD_GoT = np.mod(t_days,7300)
   ```

3. Declination Angle

   ```
   Delta_GoT = 23.45 * cos(2*π*(JD_GoT-3440)/7300)
   ```

4. Calculations for hour angle and solar radiation remain the same

Methods: Extend seasons

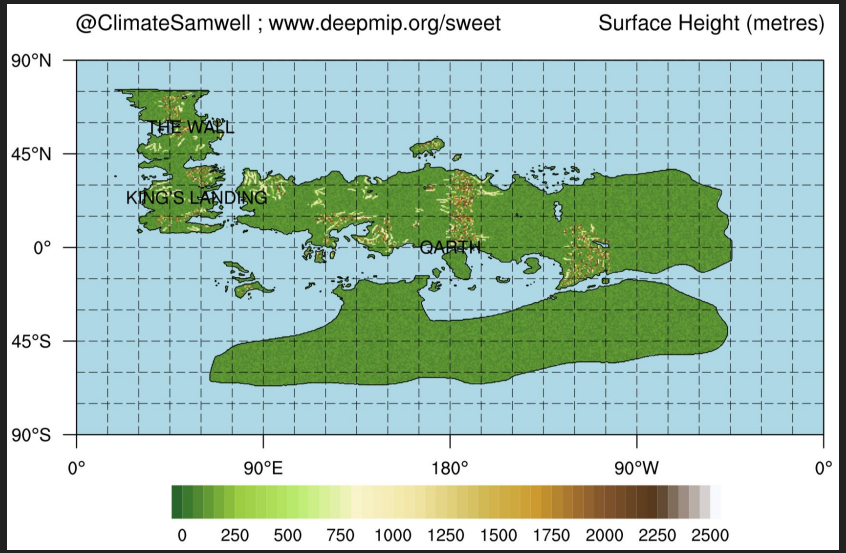


Expectation: Stronger seasonal cycle in surface temperature at all latitudes…

Methods: Radius and Heat Capacity

- Increase radius
  - Re = 6.371 * 10^6 m
  - Re_GoT = 6.915 * 10^6 m

- Heat Capacity
  - 7% Ice and snow (the Wall)
  - 8% Vegetation (Dothraki Sea and jungles)
  - 2% Sand (Red Waste desert and Dorne)
  - 63% Water
  - 20% Silicate Rock



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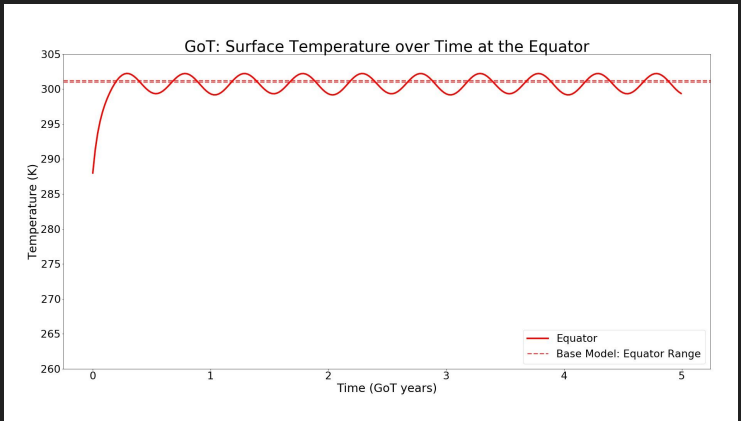
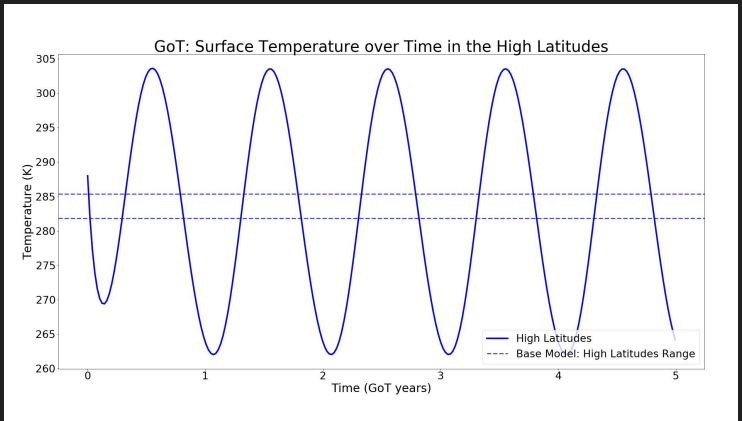
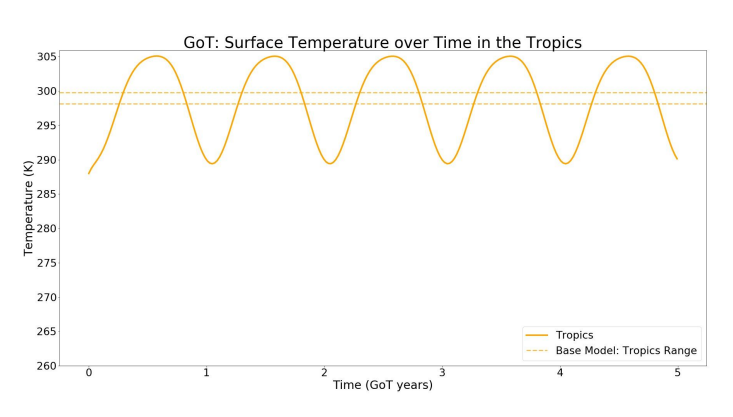
C_bar = 3.366 * 10^8 J K^{-1} m^{-2}
C_bar_GoT = 2.974 * 10^8 J K^{-1} m^{-2}

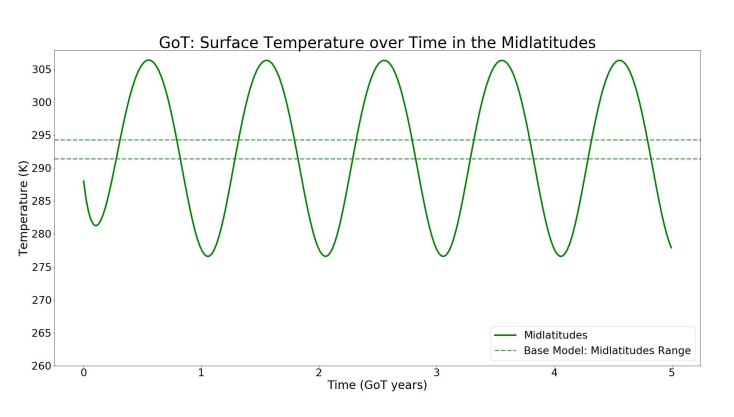
C_e = 1.719 * 10^{23} J K^{-1}
C_e_GoT = 1.788 * 10^{23} J K^{-1}

Results: Surface Temperature over Time



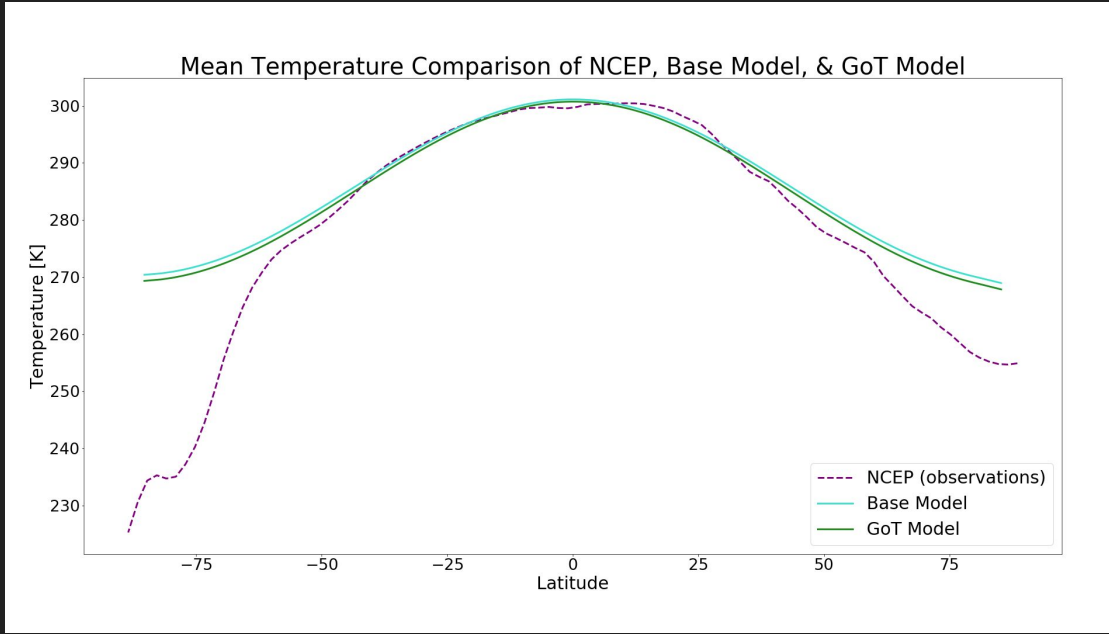






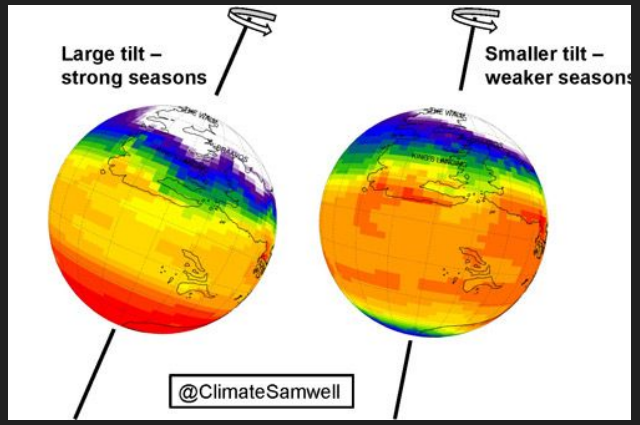
Conclusion and Model Evaluation

- Extending the length of each season disproportionately affects temperatures in the mid and high latitudes
- Surface temperature is well represented between 50° S and 50° N by our models



Further Experiments

- The Greenhouse Effect
  - Change atmospheric emissivity, GHG concentrations
- Parameterize the seasons in *Game of Thrones*
  - Length of seasons is variable
  - There are “winters” and “summers” lasting for *many years*, indicating a longer timeframe for cold and warm periods
- Milankovitch Cycle
  - Axis potentially wobbles over much shorter periods (~100 years) than Earth’s (~23,000 years)



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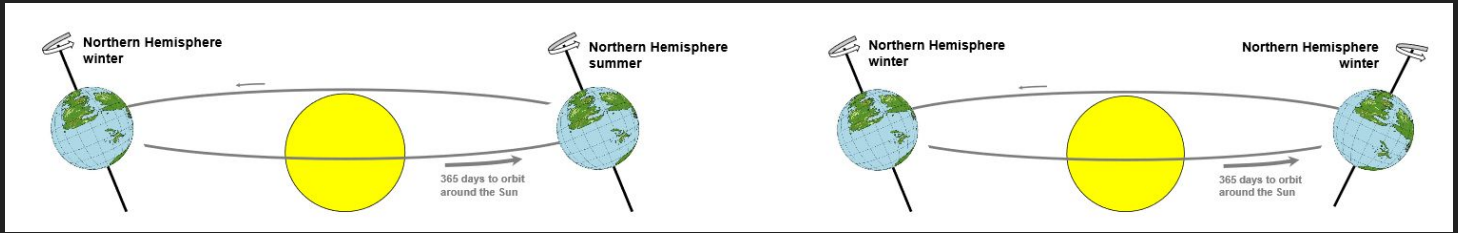

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