$$y = \frac{x^2+3}{x-1} \qquad y' = \frac{2x(x-1) - (x^2+3)\cdot 1}{(x-1)^2}$$

$$= \frac{x^2 - 2x - 3}{(x-1)^2}$$

$$= \frac{(x-3)(x+1)}{(x-1)^2}$$

Domain: $\mathbb{R} \setminus \{1\}$.

Range: $\mathbb{R} \setminus (-2, 6)$.

no x- intercept.

y- intercept. $y = -3$.

not even or odd.

Vertical ~~ay~~ asymptote at $x=1$.

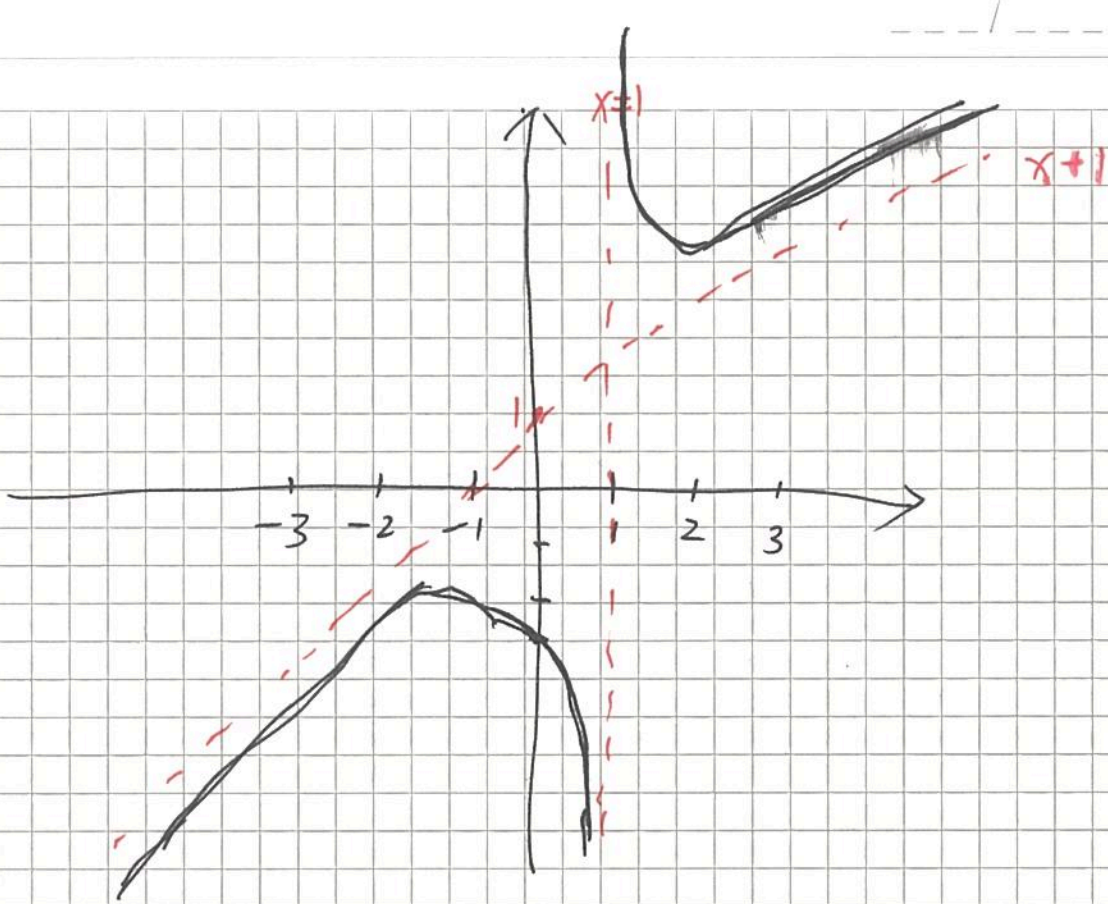
no horizontal asymptote.

$$y = \frac{x^2+3}{x-1} = x+1 + \frac{4}{x-1}$$

$$\begin{array}{r|l} & x+1 \\ \hline x-1 & x^2+3 \\ & \underline{x^2-x} \\ & \quad x+3 \\ & \quad \underline{x-1} \\ & \qquad 4 \end{array}$$

oblique asymptote $y = x+1$.

Stationary pt's at $x = -1$, $x = 3$.

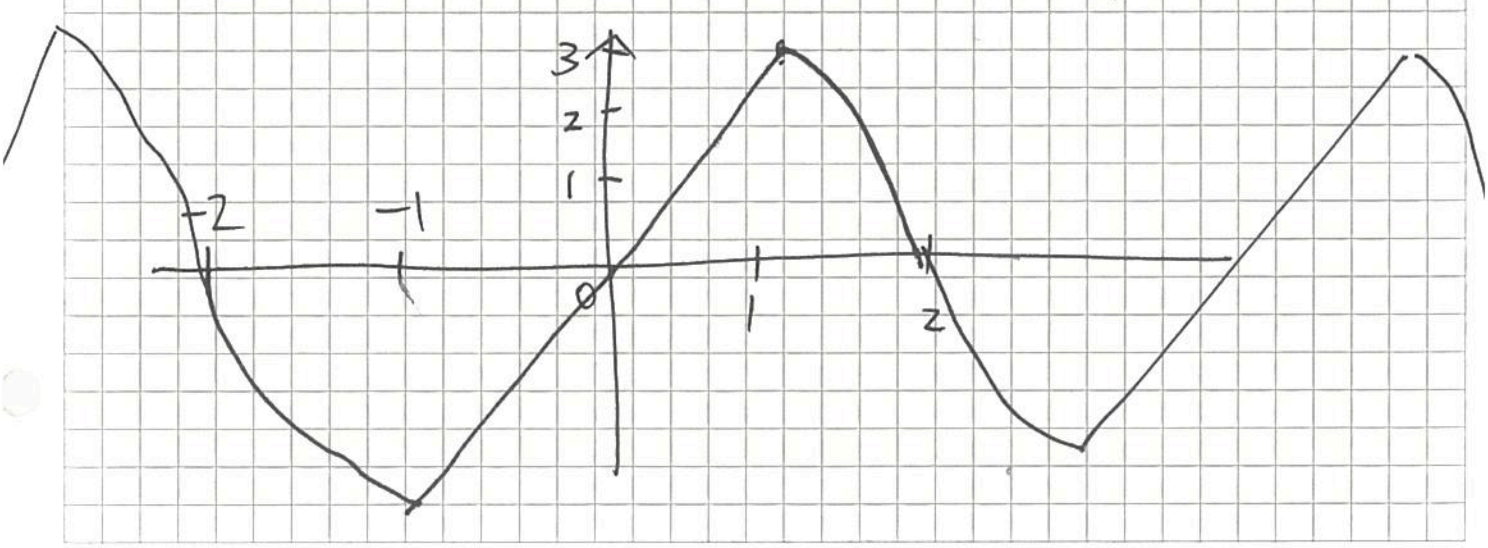


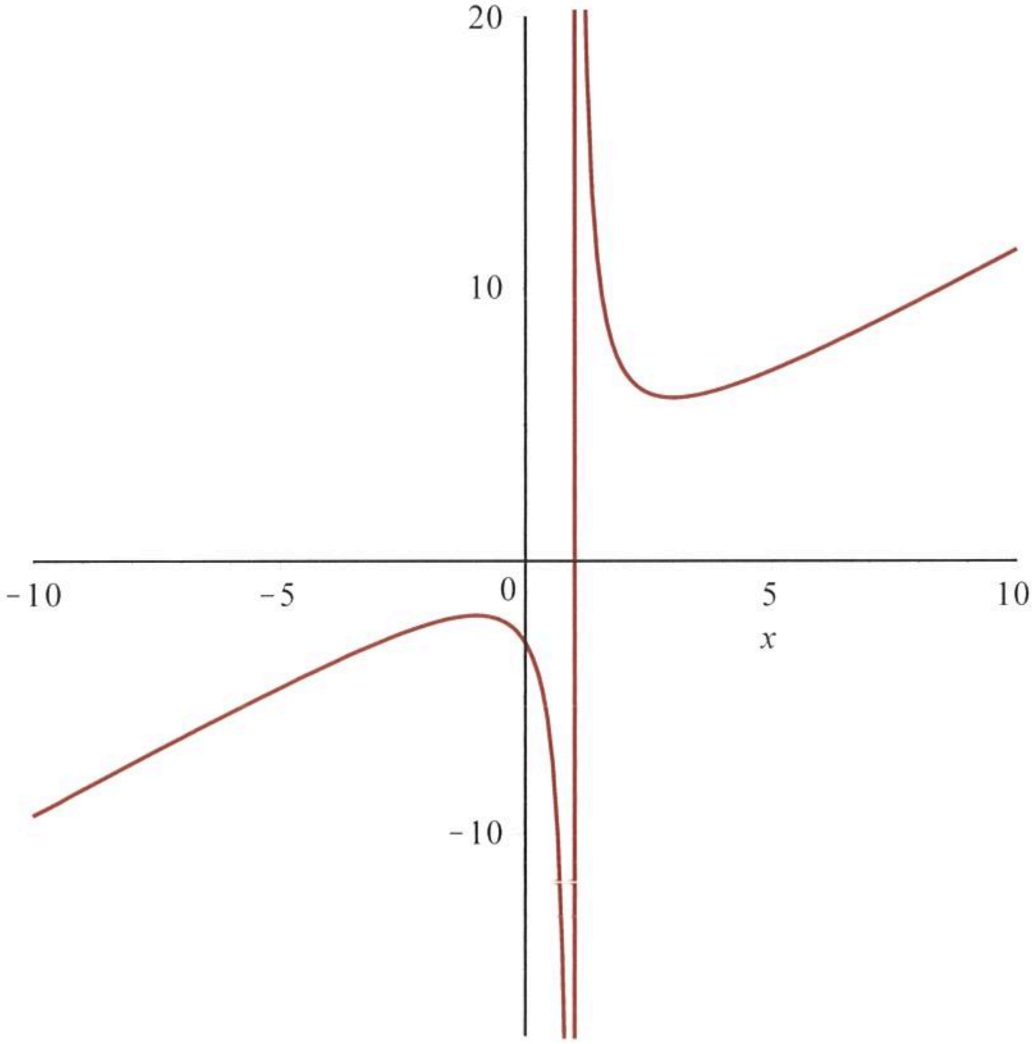
when $x=3$, $y = \dfrac{3^2+3}{3-1} = 6$ local min

when $x=-1$, $y = \dfrac{(-1)^2+3}{(-1)-1} = -2$. local max

$$f(x) = \begin{cases} 3x, & 0 \le x < 1 \\ 4-x^2, & 1 \le x < 2. \end{cases}$$

f is odd and $f(x+4) = f(x)$.





$implicitplot\left(y^2 = x(x-1)^2, x=-2..2, y=-2..2\right)$

$$implicitplot\left(y^2 = x(x-1)^2, x = -2..2, y = -2..2\right) \tag{5}$$

$with(plots, implicitplot)$

$$[implicitplot] \tag{6}$$

$implicitplot\left(y^2 = x \cdot (x-1)^2, x = 0..2, y=-2..2\right)$

Implicitly defined functions.

$$y^2 = x(x-1)^2$$

$$y = \sqrt{x}\,(x-1)$$

The graph is symmetric about the x-axis.

Domain $x \geq 0$,

Range : $[-\frac{1}{3}, \infty)$

x-intercept, $x=0$, $x=1$

y-intercept, $y=0$.

no asymptotes.

no oblique asymptotes.

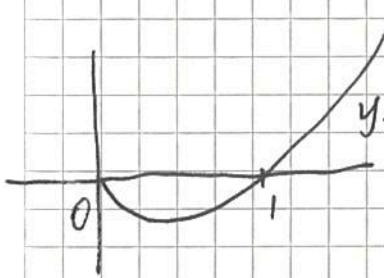
$$y^2 = x(x-1)^2$$

$$2y \cdot y' = (x-1)^2 + 2x(x-1)$$

$$= 3x^2 - 4x + 1 = (x-1)(3x-1).$$

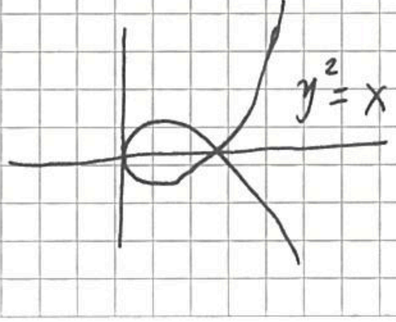
~~Note when $x=1$, $y=0$,~~

$$\sqrt{x}\,(x-1)\,y' = (x-1)(3x-1)$$

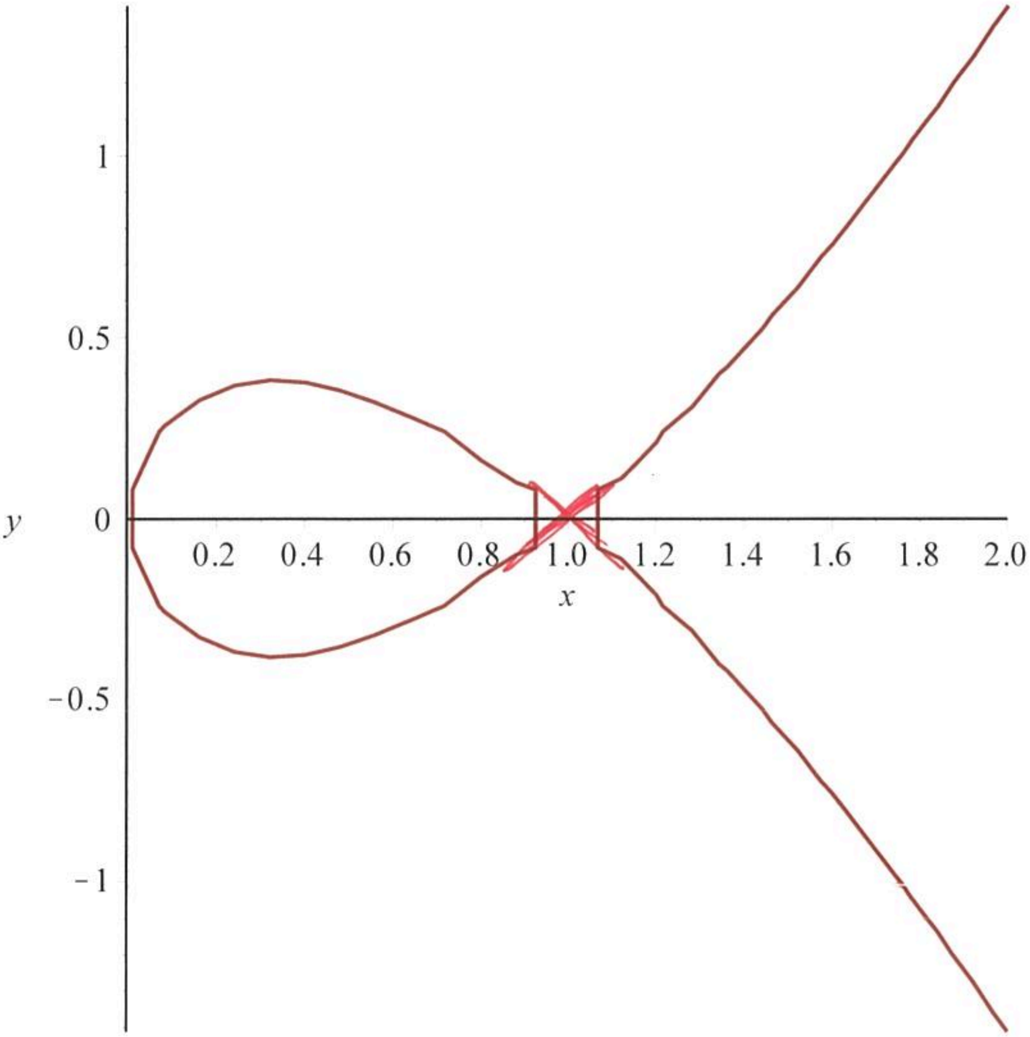


$$y' = \frac{3x-1}{\sqrt{x}}.$$

Stationary pt at $x=\frac{1}{3}$



local min.



plot([$t - \sin(t)$, $1 - \cos(t)$, $t = -20..20$])

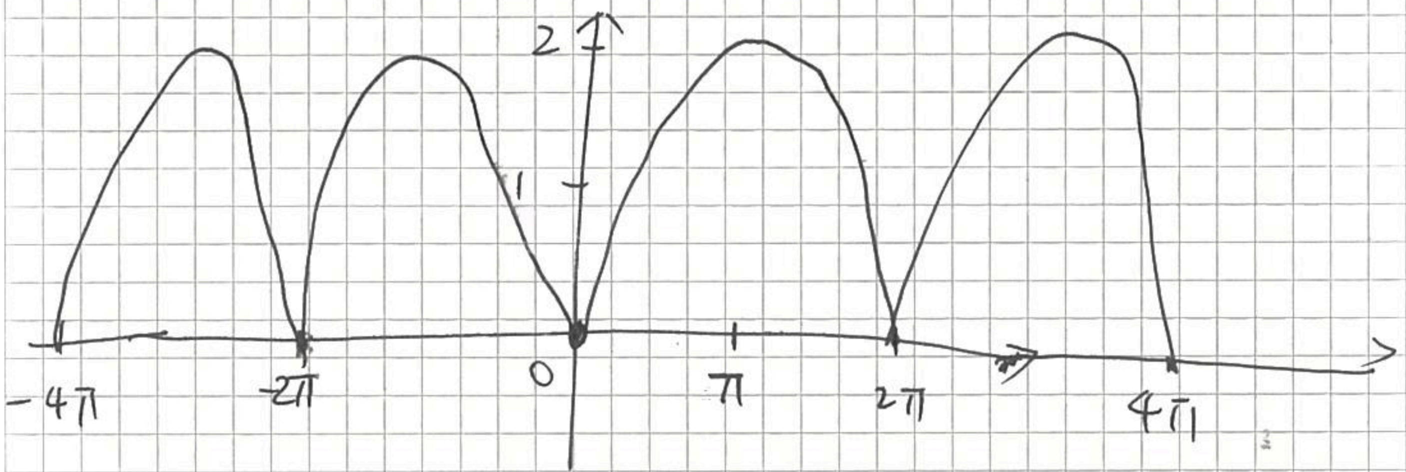
Parametrized functions.

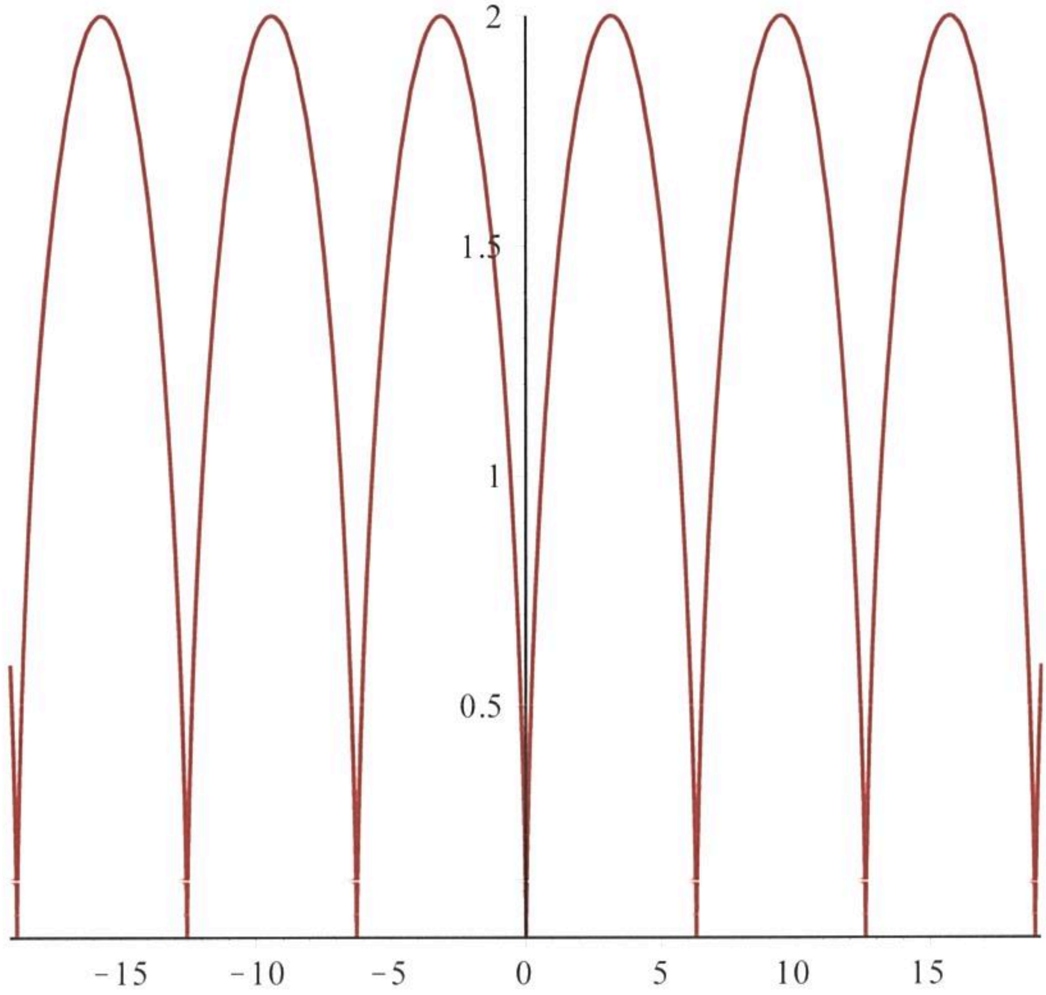
$x = f(t), \qquad y = f(t).$

Example: $x = \theta - \sin\theta, \qquad y = 1 - \cos\theta.$

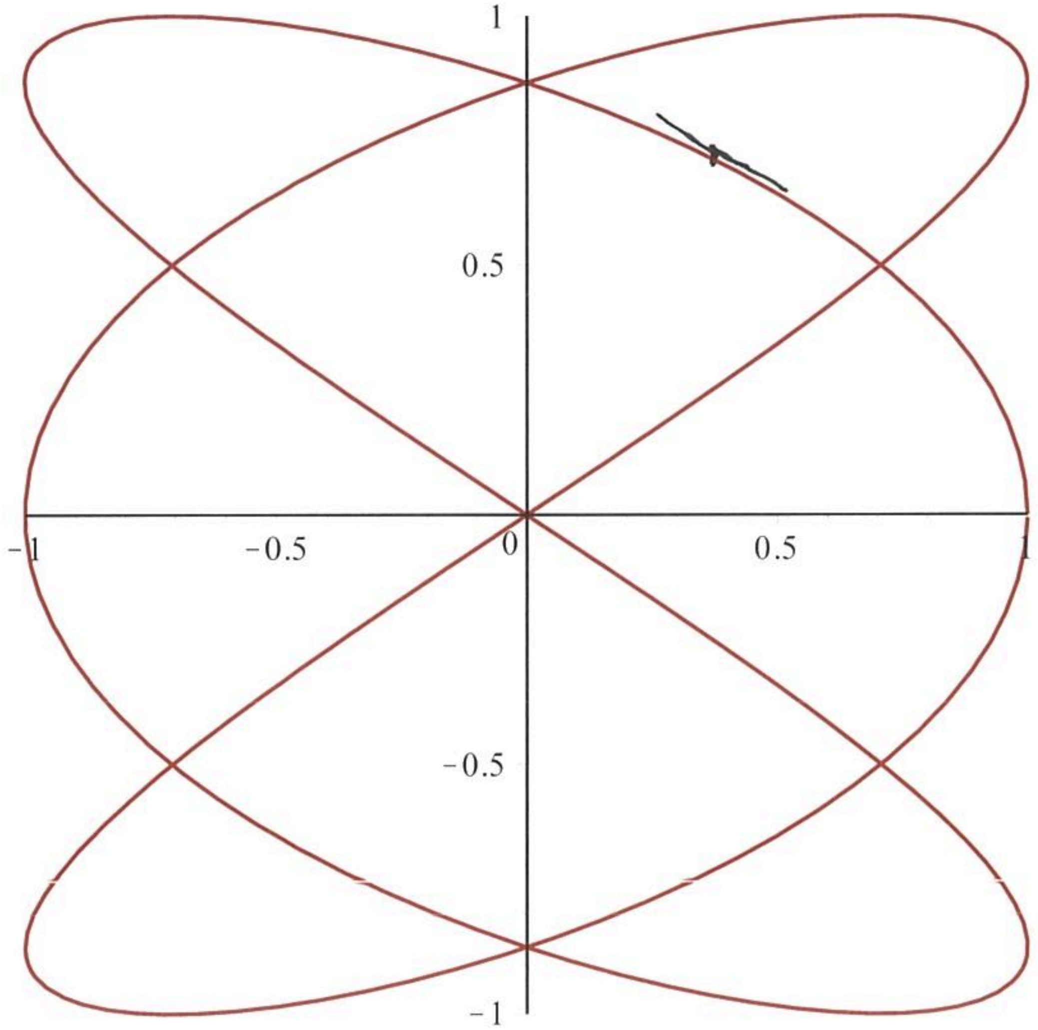
y is periodic in θ with period 2π.

$0 \le y \le 2$





plot([cos(3*t*), sin(2 *t*), *t* = 0 ..2·3.14])



$x = \cos 3t$

$y = \sin 2t.$

$$\begin{cases} x = \cos 3t \\ y = \sin 2t. \end{cases} \qquad \begin{cases} x = \theta - \sin\theta \\ y = 1 - \cos\theta \end{cases}$$

$$\frac{dy}{dx} = ?$$

To find $\frac{dy}{dx}$ in a parametrized function.

$$x = f(t), \quad y = g(t).$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

If $\begin{cases} x = \theta - \sin\theta \\ y = 1 - \cos\theta. \end{cases}$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\sin\theta}{1 - \cos\theta}.$$

Find $\frac{dy}{dx}$ at $t=1$, for the curve.

$$x = \frac{3t}{1+t^3}, \qquad y = \frac{3t^2}{1+t^3}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{6t(1+t^3) - 3t^2(3t^2)}{(1+t^3)^2}}{\frac{3(1+t^3) - 3t \cdot 3t^2}{(1+t^3)^2}}$$

$$= \frac{-3t^4 + 6t}{-6t^3 + 3}$$

When $t=1$, $\frac{dy}{dx} = \frac{-3+6}{-6+3} = -1$.