$$y = x^2 e^x$$

Domain: $\mathbb{R}$.

Range: $[0, \infty)$.

x- intercept. $x = 0$.

y- intercept: $y = 0$.

no symmetry.

no vertical asymptote.

$$\lim_{x \to \infty} x^2 e^x = \infty.$$

$$\lim_{x \to -\infty} x^2 e^x = \lim_{x \to -\infty} \frac{x^2}{e^{-x}}$$

$$= \lim_{x \to -\infty} \frac{2x}{-e^{-x}}$$

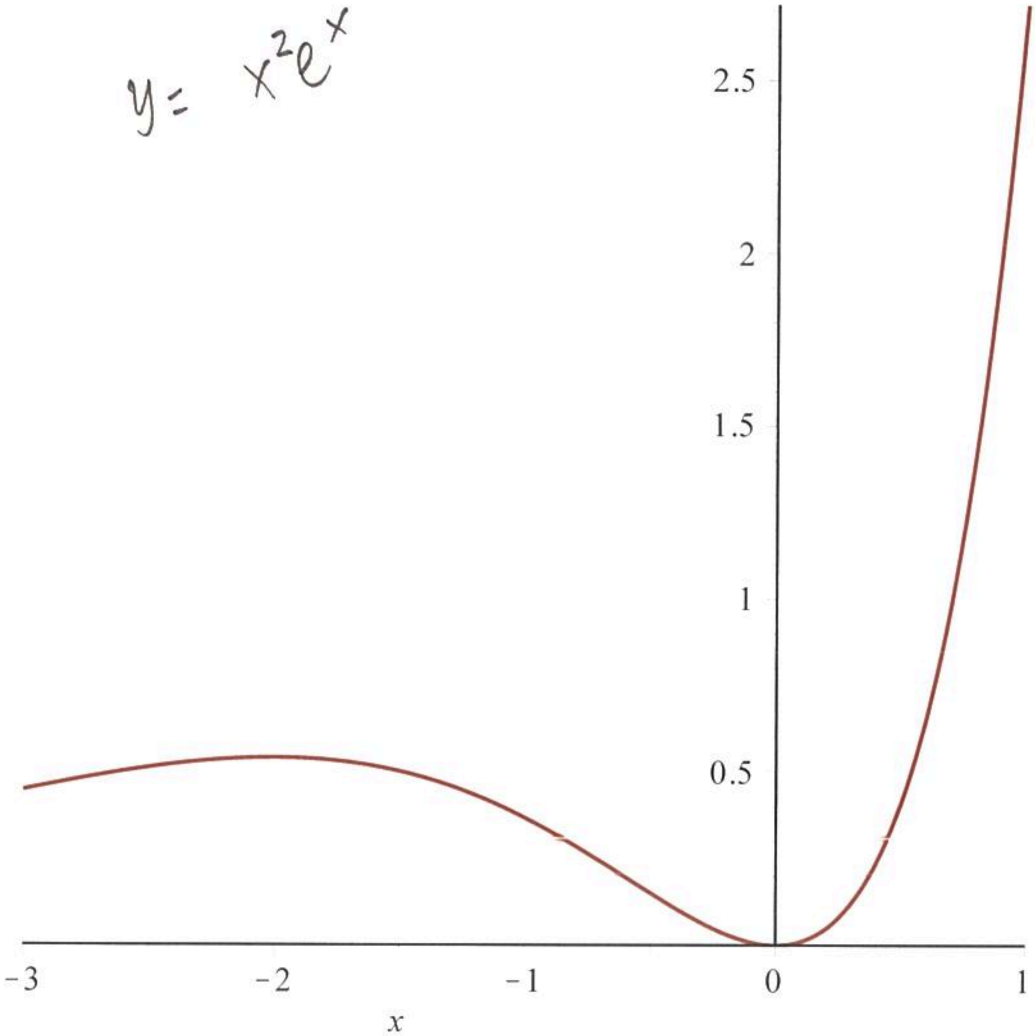
$$= \lim_{x \to \infty} \frac{2}{e^{-x}}$$

$$= 0$$

Horizontal asymptote $y = 0$.

$plot\left(x^2 \exp(x), x = -3 \,..1\right)$

$y = x^2 e^x$

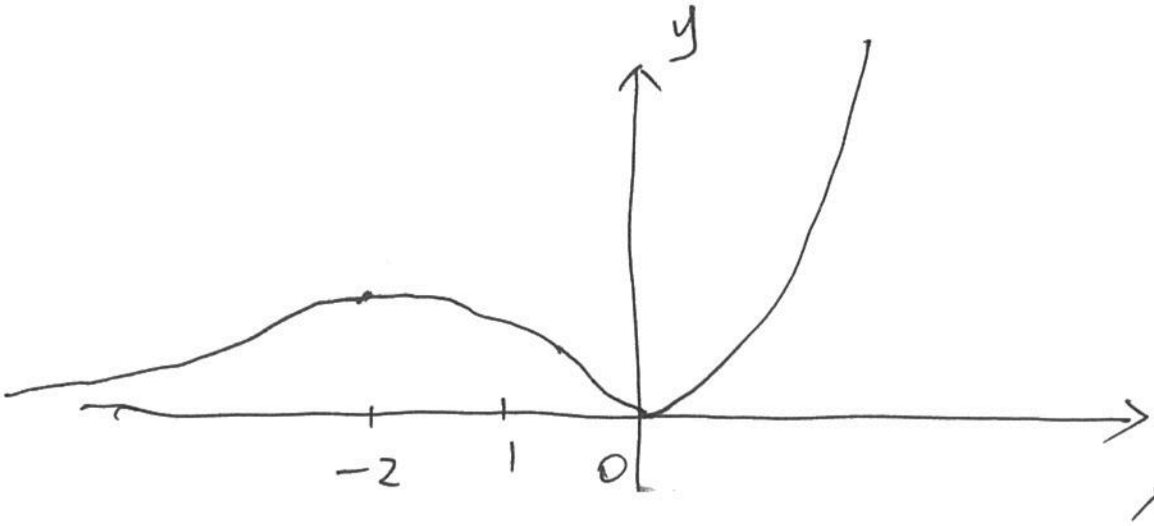


~~$with(plots)$~~ :

~~$polarplot(2\,\theta, \theta = 0 \,..4\,\pi)$~~

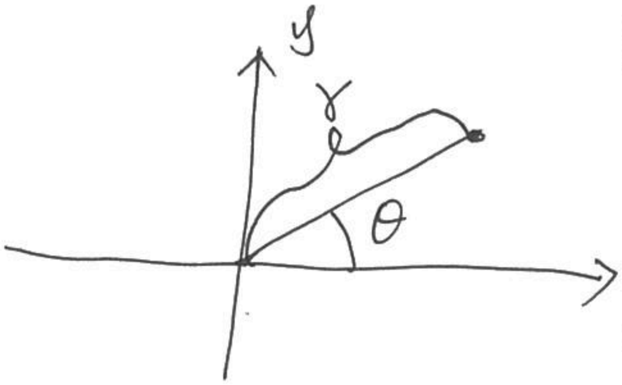
$$y' = 2xe^x + x^2e^x = e^x \cdot x(2+x).$$

$y' = 0$ when $x = 0$ or $x = -2$.



Polar coordinates.

On $\mathbb{R}^2$, we can specify a point by specifying a direction and a distance from the origin.

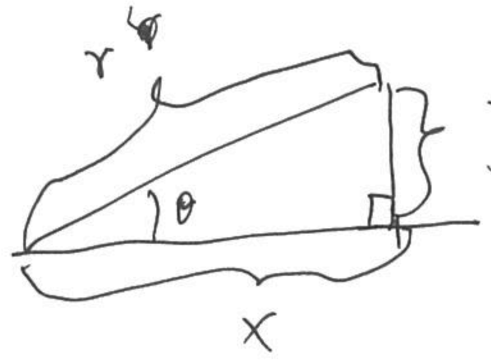


θ - direction

r - distance.

θ - the angle that the direction makes with the positive x-axis.

r - distance from the origin.

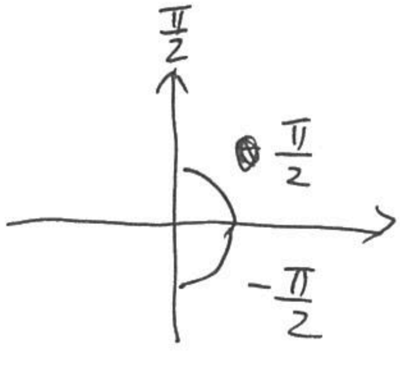


$x = r\cos\theta, \qquad y = r\sin\theta.$

$r = \sqrt{x^2 + y^2} \qquad \tan\theta = \frac{y}{x}.$

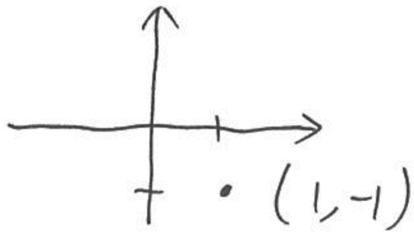
$\theta = \arctan\frac{y}{x}.$

$\arctan : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



need to be careful when using $\theta = \arctan\frac{y}{x}$

example: convert $(1, -1)$ to polar coordinates.

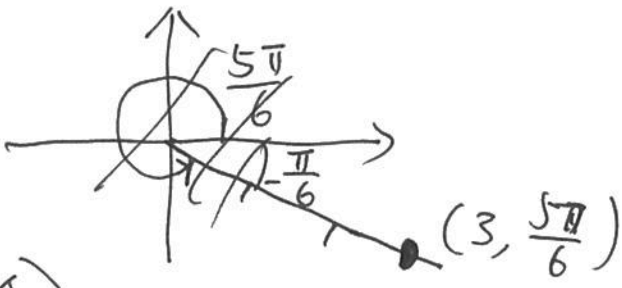


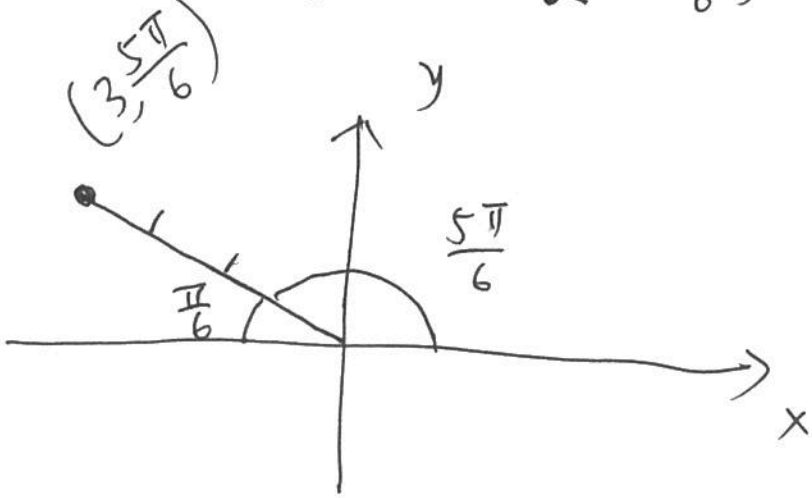
$$r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\theta = \arctan\left(\frac{-1}{1}\right) = \arctan(-1) = -\frac{\pi}{4}.$$

Convert $\left(3, \frac{5\pi}{6}\right)$ to Cartesian Coordinates.

$r = 3, \quad \theta = \frac{5\pi}{6},$





$$x = r\cos\theta = 3 \cdot \cos\frac{5\pi}{6} = 3 \cdot \frac{-\sqrt{3}}{2} = -\frac{3}{2}\sqrt{3}.$$

$$y = r\sin\theta = 3 \cdot \sin\frac{5\pi}{6} = 3 \cdot \frac{1}{2} = \frac{3}{2}.$$

$$(r, \theta) = (r, \theta + 2\pi)$$

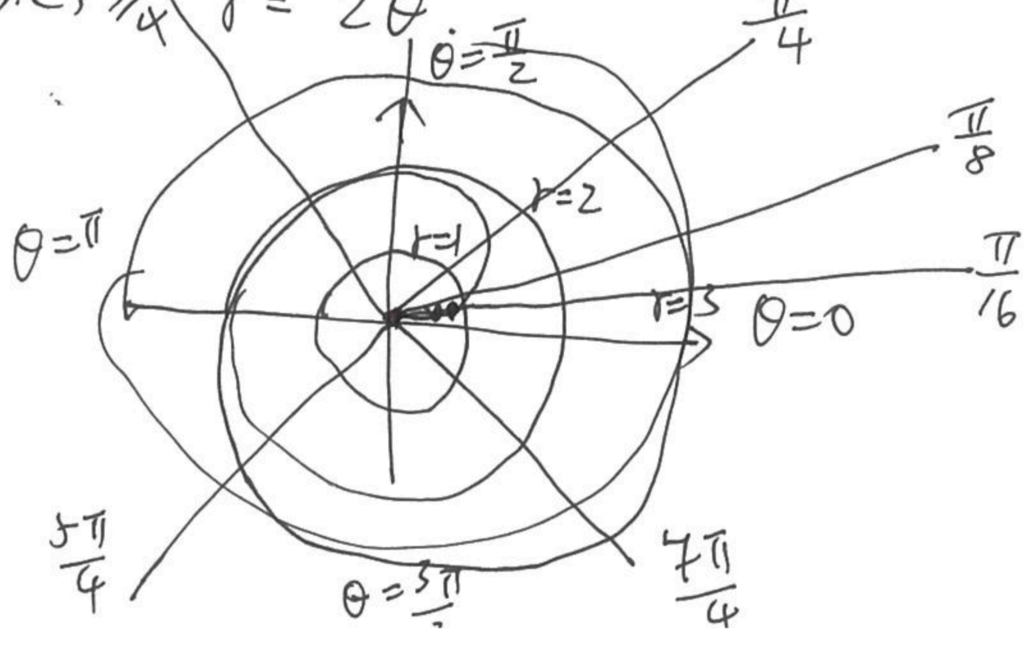
$$= (r, \theta + 2k\pi), \quad k \in \mathbb{Z}$$

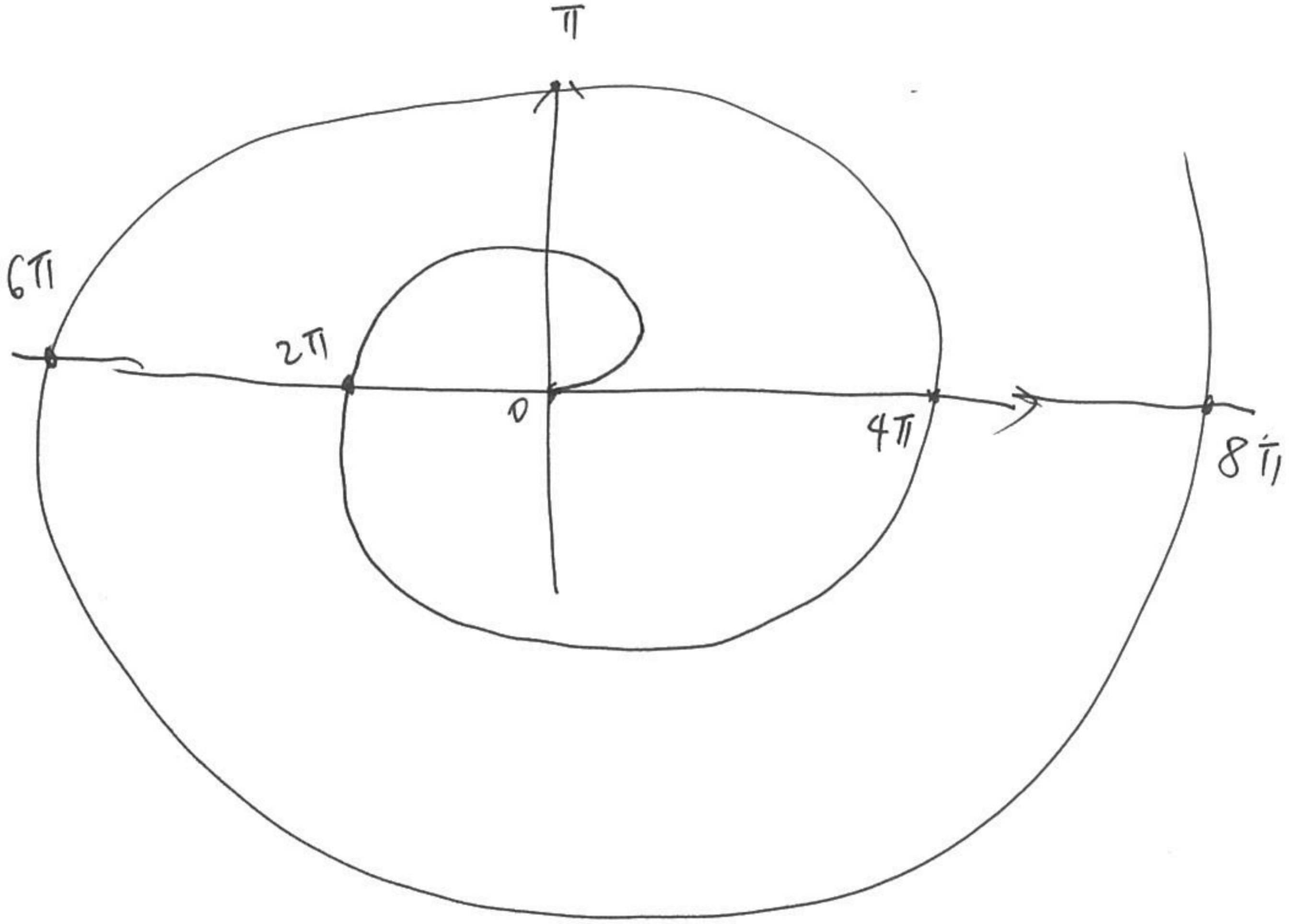
k is ~~a~~ an integer.

It is not possible to define θ for the origin.

Graphing using polar coordinates.

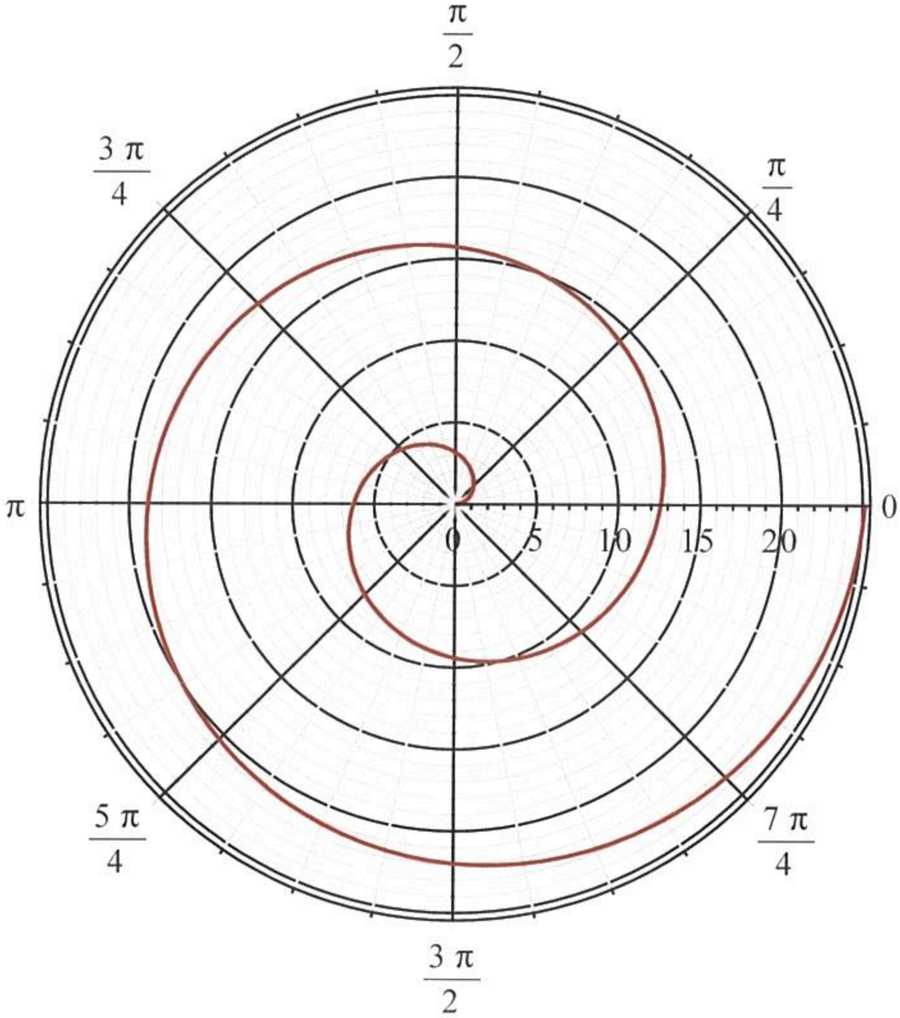
Example, $r = 2\theta$





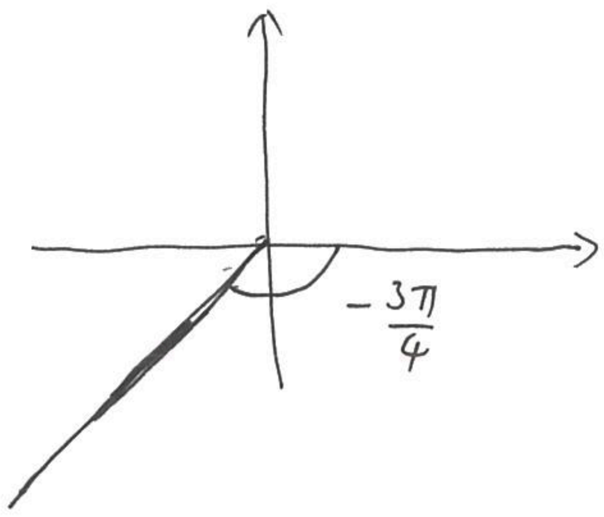
π
6π
2π
0
4π
8π

$r = 2\theta.$



~~*polarplot*(2·cos(θ), θ = 0..2 π)~~

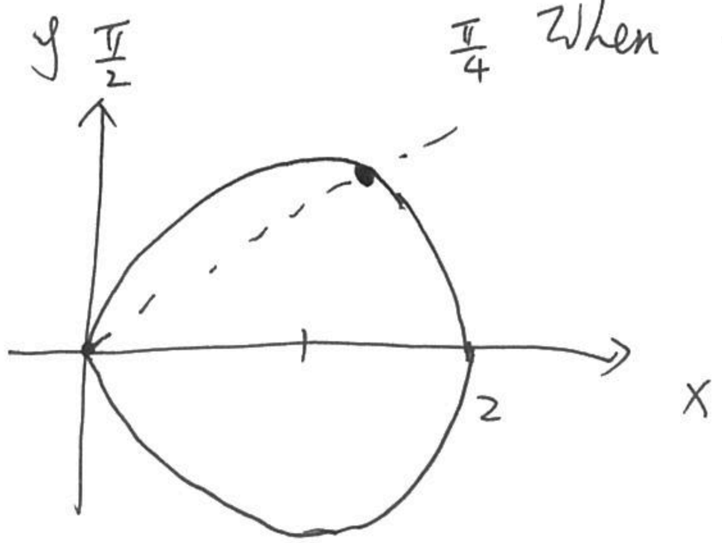
$\theta = -\frac{3\pi}{4}$



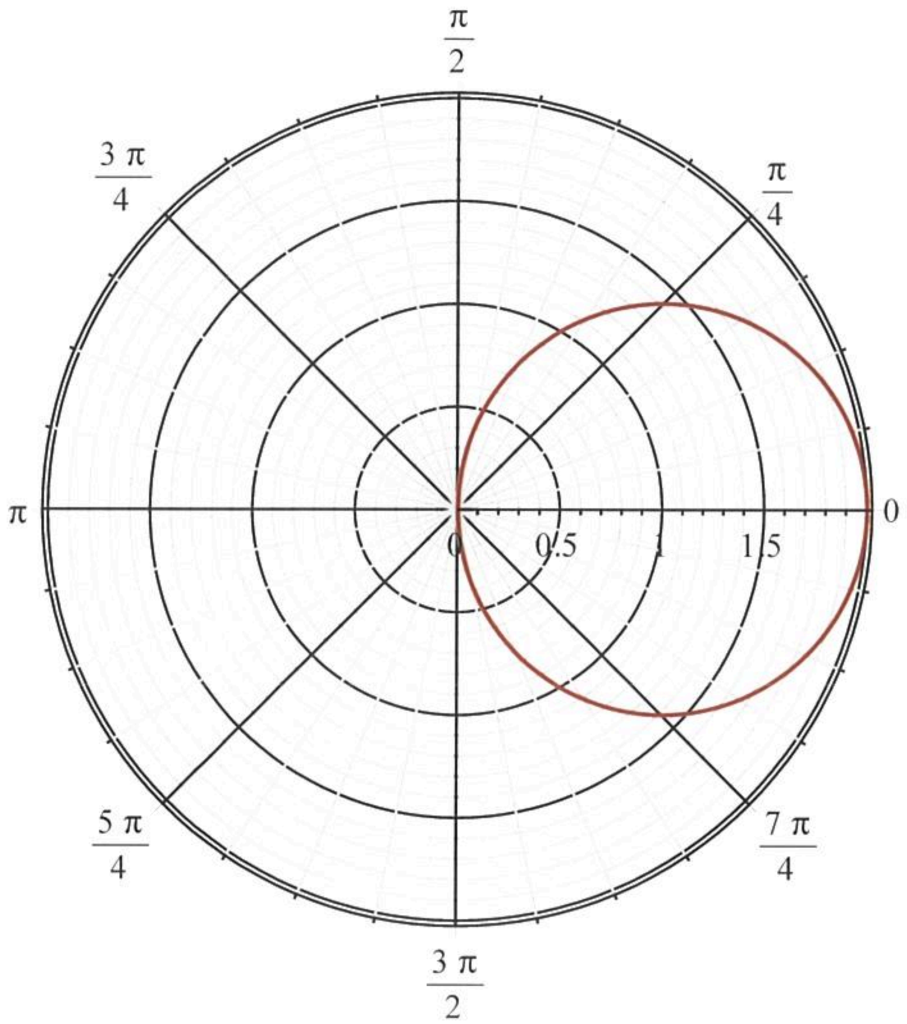
$r = 2\cos\theta.$

$\cos\theta \geq 0$

when $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}.$

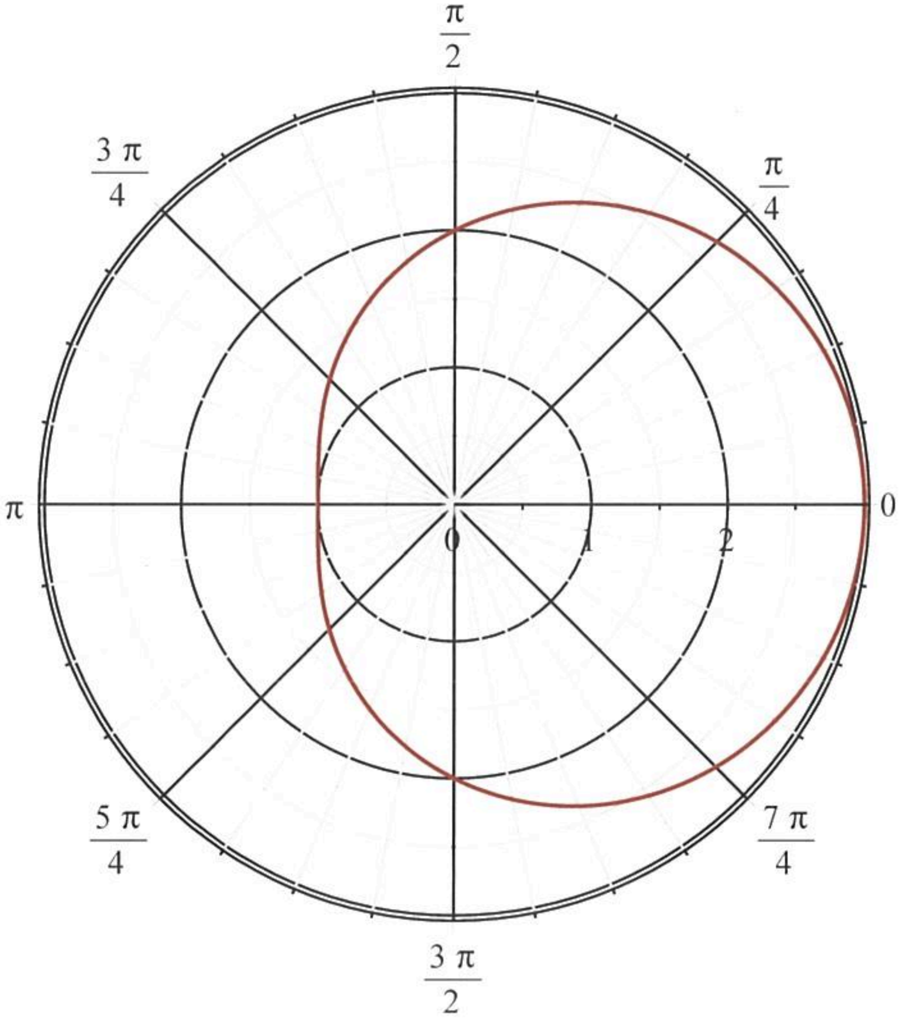


$r = 2\cos\theta$.



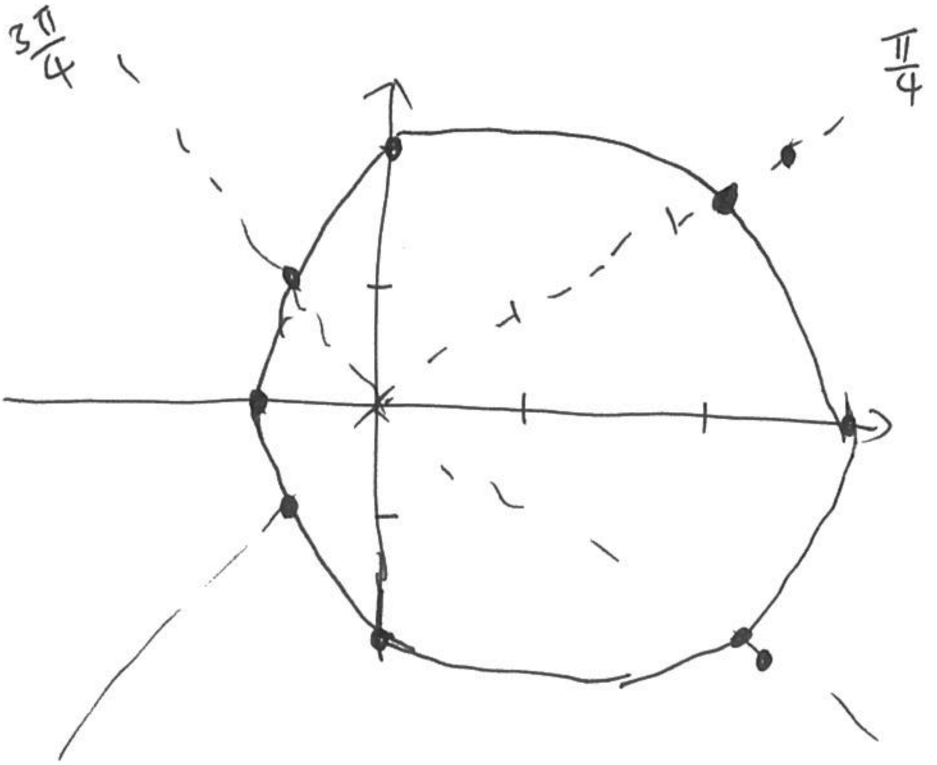
~~*polarplot*$(2 + \cos(\theta), \theta = 0..2\pi)$~~

$r = 2 + \cos\theta$

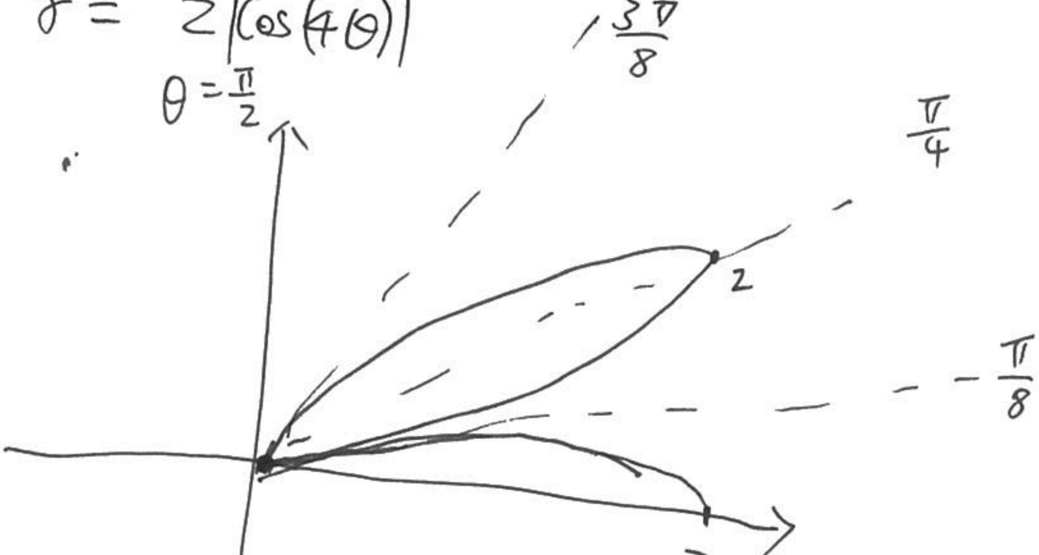


~~polarplot(2·|cos(4·θ)|, θ = 0 ..2 π)~~

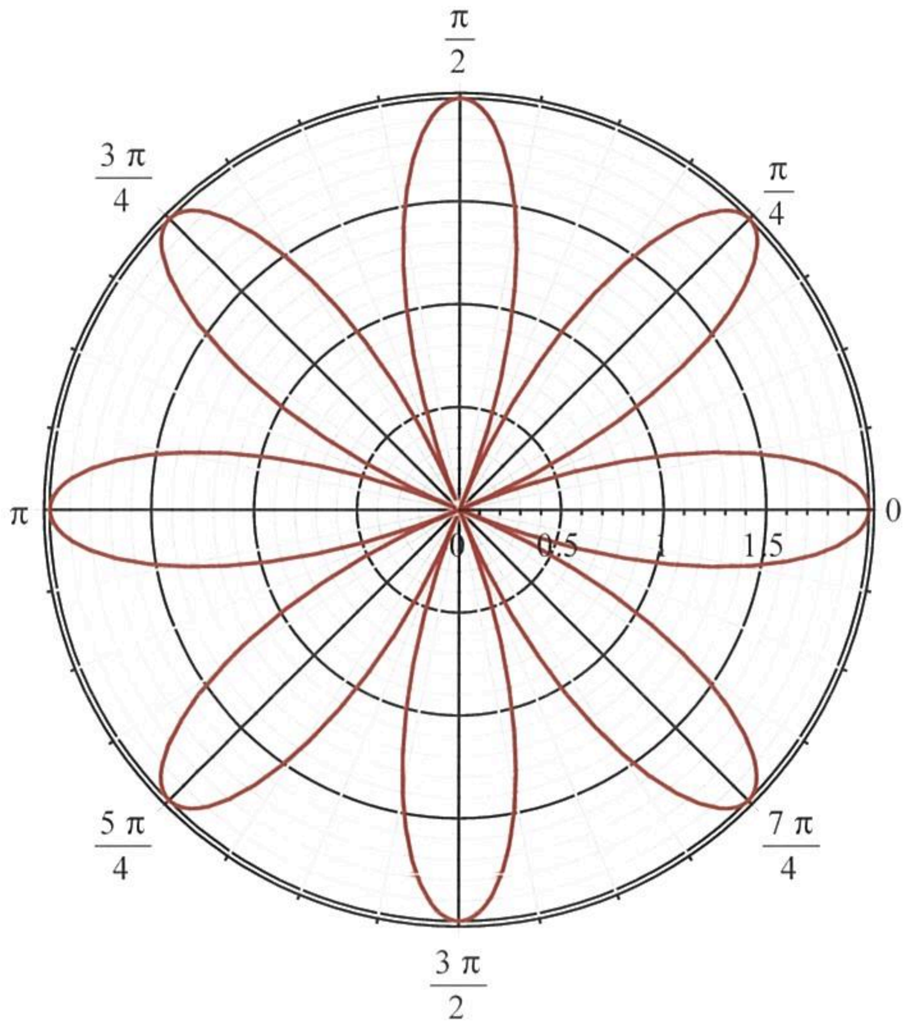
Example. $r = 2 + \cos\theta$.



$r = 2|\cos(4\theta)|$



$r = 2\left|\cos(4\cdot\theta)\right|.$



~~*plots*[*implicitplot*]($r^2 = 4\cdot \sin(2\cdot\theta)$, $r = 0..2$, $\theta = 0..2\pi$, *coords* = polar, *axes* = *boxed*, *outlines*)~~

Theorem: If $x = r(\theta)\cos\theta$,
$y = r(\theta)\sin\theta$,

where $r(\theta)$ is a function in θ, then

$$\frac{dy}{dx} = \frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cos\theta}.$$

---

proof:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{\frac{dr}{d\theta}\sin\theta + r(\theta)\cdot\cos\theta}{\frac{dr}{d\theta}\cos\theta + r(\theta)(-\sin\theta)}.$$

$\boxtimes$