Math 1B Maple Lab Test

Question 1: Score 1/1

Before attempting this question you should work through the self-directed learning Module "Lesson 10 - Further Linear Algebra" on UNSW Blackboard.

Use Maple to find the eigenvalues of

$$A = \begin{pmatrix} -59469 & -29005 & 13340 & -13520 & -9269 \\ 60056 & 23370 & -13070 & 47960 & -2296 \\ -11468 & -14188 & -20632 & -4176 & -1260 \\ 43500 & 36462 & -3014 & 8308 & -820 \\ 6019 & 7277 & -650 & 11420 & -28757 \end{pmatrix}.$$

Enter the eigenvalues in the box below as a Maple set. (Eg, if the eigenvalues were 1, 2, 3, 4, you should enter $\{1,2,3,4\}$.)

To avoid typing errors, you may copy and paste the following sequence of entries of A: -59469, 60056, -11468, 43500, 6019, -29005, 23370, -14188, 36462, 7277, 13340, -13070, -20632, -3014, -650, -13520, 47960, -4176, 8308, 11420, -9269, -2296, -1260, -820, -28757 and edit it appropriately to create a matrix.

Solution:

- ⇒ A:=<<-59469, 60056, -11468, 43500, 6019>|< -29005, 23370, -14188, 36462, 7277>|< 13340, -13070, -20632, -3014, -650>|< -13520, 47960, -4176, 8308, 11420>|< -9269, -2296, -1260, -820, -28757>>;
- \Rightarrow with(LinearAlgebra):
- \Rightarrow Eigenvalues(A)

Answer:

(Do not copy from Maple, take note of formatting, order of eigenvalues does not matter) {-38590, -30872, -23154, -15436, 30872}



Question 2: Score 1/1

Before attempting this question you should work through the self-directed learning Module "Lesson 10 - Further Linear Algebra" on UNSW Blackboard.

Use Maple to find the rank of

$$A = \begin{pmatrix} 11572 & 11287 & -41749 & -18340 & 44806 \\ -17744 & 27466 & -36022 & -9400 & 33148 \\ 8104 & -25076 & 38372 & 25520 & -26288 \\ -7108 & 29447 & -18029 & -12980 & 30686 \\ 26860 & -26435 & 28985 & 22100 & -15470 \end{pmatrix}.$$

To avoid typing errors, you can copy and paste the following sequence of entries of *A*: 11572, -17744, 8104, -7108, 26860, 11287, 27466, -25076, 29447, -26435, -41749, -36022, 38372, -18029, 28985, -18340, -9400, 25520, -12980, 22100, 44806, 33148, -26288, 30686, -15470

and edit it appropriately to make a matrix. Enter the rank of A in the box below.

Solution:

- ⇒ A:=<<11572, -17744, 8104, -7108, 26860>|< 11287, 27466, -25076, 29447, -26435>|< -41749, -36022, 38372, -18029, 28985>|< -18340, -9400, 25520, -12980, 22100>|< 44806, 33148, -26288, 30686, -15470>>;
- \Rightarrow with(LinearAlgebra):
- \Rightarrow Rank(A);

Answer:

5



Question 3: Score 1/1

Consider the sum

$$\sum_{k=100}^{1300} (6 x - 3 k)^k =$$

$$(6 x - 300)^{100} + (6 x - 303)^{101} + (6 x - 306)^{102} + (6 x - 309)^{103}$$

$$+ ... + (6 x - 3891)^{1297} + (6 x - 3894)^{1298} + (6 x - 3897)^{1299} + (6 x - 3900)^{1300}.$$

Create a Maple expression using "add" or "sum" that produces this sum and enter it in the box below. (Your answer should be of the form "sum(___,__)" and not include and assignment ":=" or semi-colon ";".)

Answer:

$$sum((6*x-3*k)^k,k=100..1300)$$

Question 4: Score 1/1

Consider the product

$$\prod_{k=190}^{1500} (x^4 + 3x - k) = (x^4 + 3x - 190) (x^4 + 3x - 191) (x^4 + 3x - 192) (x^4 + 3x - 193)$$
... $(x^4 + 3x - 1497) (x^4 + 3x - 1498) (x^4 + 3x - 1499) (x^4 + 3x - 1500)$.

Create a Maple expression using "product" or "mul" that produces this product and enter it in the box below. (Your answer should be of the form "product(___,___)" and not include and assignment ":=" or semi-colon ";".)

$$product(x^4+3*x-k,k=190..1500)$$





Question 5: Score 1/1

Use Maple to find the cross product, $\mathbf{u} \times \mathbf{v}$, of the vectors \mathbf{u} and \mathbf{v} where

$$\mathbf{u} = \begin{pmatrix} 25 \\ 42 \\ -5 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} 43 \\ -7 \\ -14 \end{pmatrix}.$$

Type in your answer, using the Maple syntax "Vector([, ,])" or "< , , >", or copy (Ctrl-C) from your Maple worksheet and paste (Ctrl-V) the answer into the box below.

Solution:

- \Rightarrow u:=<25,42,-5>;
- \Rightarrow v:=<43,-7,-14>;
- ⇒ with(LinearAlgebra):
- \Rightarrow CrossProduct(u,v);

(Copy from Maple)
Vector(3,
$$\{(1) = -623, (2) = 135, (3) = -1981\}$$
)



Question 6: Score 1/1

Find the partial fraction decomposition of

$$\frac{p(x)}{q(x)}$$

where

$$p(x) = 1241499 - 1286573 x^2 - 1871081 x^3 + 688866 x^4 - 254338 x^6 + 619220 x^5 - 9 x^{12} + 203 x^{11} - 1433 x^{10} + 1113 x^9 + 23220 x^8 - 39284 x^7 + 1722565 x$$

and

$$q(x) = 388962 + 9261 x^{2} - 1228416 x^{3} - 69539 x^{4} - 120990 x^{6} + 658619 x^{5} - 27 x^{4} + 232 x^{11} - 31 x^{10} - 11307 x^{9} + 68844 x^{8} - 133576 x^{7} + x^{13} + 990927 x^{10}$$

and enter it in the box below. To prevent typing errors, you can copy and paste p := 1241499-1286573*x^2-1871081*x^3+688866*x^4-254338*x^6+619220*x^5-9*x^12+203*x^11-1433*x^10+1113*x^9+23220*x^8-39284*x^7+1722565*x; q := 388962+9261*x^2-1228416*x^3-69539*x^4-120990*x^6+658619*x^5-27*x^12+232*x^11-31*x^10-11307*x^9+68844*x^8-133576*x^7+x^13+990927*x; into your Maple worksheet and copy and paste the Maple output into the answer box.

Solution:

- \Rightarrow p := 1241499-1286573*x^2-1871081*x^3+688866*x^4-254338*x^6+619220*x^5-9*x^12+203*x^11-1433*x^10+1113*x^9+23220*x^8-39284*x^7+1722565*x;
- \Rightarrow q := 388962+9261*x^2-1228416*x^3-69539*x^4-120990*x^6+658619*x^5-27*x^12+232*x^11-31*x^10-11307*x^9+68844*x^8-133576*x^7+x^13+990927*x;
- \Rightarrow convert(p/q,parfrac,x);

Answer:

(Copy from Maple)

 $-13/(256*(x-3)^2)-2/(x+7)+7/(64*(x-3)^3)-5/(x-7)^3-9/(512*(x+1))-9/(x-6)-5/(256*(x+1)^2)+9/(512*(x-3))+(2*x-2)/(x^2-x-1)+1/(64*(x+1)^3)$



Question 7: Score 2/2

A function f is defined by

$$f(x, y) = x^7 y^4 \sin(4 x + 5 y)$$
.

(a) Enter a Maple expression for the function f using the arrow operator "->" in the box below. (Do not enter "f :=" at the beginning or ";" at the end of your answer.)

Answer:

$$(x,y)->x^7*y^4*\sin(4*x+5*y)$$

(b) If the Maple variable f is assigned to be the function in part (a), select the correct Maple syntax for evaluating $\frac{\partial^{10}}{\partial y^5 \partial x^5} f(x, y)$ at (x, y) = (1, 4).

Answer:

$$#1 = 1^{st}$$
 variable (x)

$$#2 = 2^{nd}$$
 variable (y)

Question 8: Score 2/2

Use Maple to find the solution of the initial value problem

$$y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$
 with initial conditions $y(0) = 4$ and $y'(0) = 3$.

Using Maple syntax, type in your answer in the box below, or copy (Ctrl-C) from your Maple worksheet and paste (Ctrl-V) in the answer box the solution. Do **NOT** enter the y(x)= part of the Maple output.

Solution:

$$\Rightarrow$$
 ODE:= $y(x)*diff(y(x),x$2)+(diff(y(x),x))^2=0;$

$$\Rightarrow$$
 dsolve({ODE,y(0)=4,D(y)(0)=3},y(x));

Answer:

(Copy from Maple) (2/3)*sqrt(54*x+36)



Question 9: Score 2/2

Use Maple to find the solution of the initial value problem

$$y \frac{d^2y}{dx^2} - \frac{1}{2} \left(\frac{dy}{dx}\right)^2 = 0$$
 with initial conditions $y(0) = 5$ and $y'(0) = 5$.

Using Maple syntax, type in your answer in the box below, or copy (Ctrl-C) from your Maple worksheet and paste (Ctrl-V) in the answer box the solution. Do **NOT** enter the y(x)= part of the Maple output.

Solution:

- \Rightarrow ODE:= $y(x)*diff(y(x),x$2)-1/2*(diff(y(x),x))^2=0$
- \Rightarrow dsolve({ODE,y(0)=5,D(y)(0)=5},y(x));

Answer:

(Copy from Maple)
$$(5/4)*x^2+5*x+5$$

Question 10: Score 2/2

Use Maple to find the solution of the initial value problem

$$y \frac{d^2y}{dx^2} - \frac{2}{3} \left(\frac{dy}{dx}\right)^2 = 0$$
 with initial conditions $y(0) = 4$ and $y'(0) = 6$.

Using Maple syntax, type in your answer in the box below, or copy (Ctrl-C) from your Maple worksheet and paste (Ctrl-V) in the answer box the solution. Do **NOT** enter the y(x)= part of the Maple output.

Solution:

- \Rightarrow ODE:=y(x)*diff(y(x),x\$2)-2/3*(diff(y(x),x))^2=0;
- \Rightarrow dsolve({ODE,y(0)=4,D(y)(0)=6},y(x));



Question 11: Score 2/2

Use Maple to find the solution of the initial value problem

$$y \frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^2 = 0$$
 with initial conditions $y(0) = 2$ and $y'(0) = 8$.

Using Maple syntax, type in your answer in the box below, or copy (Ctrl-C) from your Maple worksheet and paste (Ctrl-V) in the answer box the solution. Do **NOT** enter the y(x)= part of the Maple output.

Solution:

- \Rightarrow ODE:= $y(x)*diff(y(x),x$2)+5*(diff(y(x),x))^2=0;$
- \Rightarrow dsolve({ODE,y(0)=2,D(y)(0)=8},y(x));

Answer:

(Copy from Maple)
$$2*(24*x+1)^{(1/6)}$$

Question 12: Score 1/1

Which of the following is a correct Maple command that will define the point A to be (1,2,3) for use with the geom3d package? You may assume that the geom3d package has already been loaded.

Answer:

point(A, [1,2,3])

Question 13: Score 1/1

Which of the following is a correct Maple command that will define the line L, for use with the geom3d package, to be the line passing through the two points A and B? You may assume that the geom3d package has already been loaded and the points A and B have already been defined.

Answer:

line(L, [A, B])

Question 14: Score 1/1

Which of the following is a correct Maple command that will define the line L, for use with the geom3d package, to be the line passing through the point A and in the direction of the

vector
$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$
? You may assume that the geom3d package has already been loaded and the

point A has already been defined.

Answer:

Question 15: Score 1/1

Which of the following is a correct Maple command that will define the plane P, for use with the geom3d package, to be the plane passing through the three points A, B and C? You may assume that the geom3d package has already been loaded and the points A, B and C have already been defined.

Answer:

Question 16: Score 1/1

Which of the following is a correct Maple command that will define the plane P, for use with

the geom3d package, to be the line passing through the point A with normal vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

? You may assume that the geom3d package has already been loaded and the point A has already been defined.



Question 17: Score 1/1

Which of the following is a correct Maple command that will define the sphere S, for use with the geom3d package, to be the sphere with centre A and radius r? You may assume that the geom3d package has already been loaded and the point A and radius r have already been defined.

Answer:

sphere(S, [A, r])

Question 18: Score 1/1

Which of the following is a correct Maple command that will define the sphere S, for use with the geom3d package, to be the sphere with a diameter being the line segment joining the two points A and B? You may assume that the geom3d package has already been loaded and the points A and B have already been defined.

Answer:

sphere(S, [A, B])

Question 19: Score 1/1

Which of the following is a correct Maple command that will define the sphere S, for use with the geom3d package, to be the sphere passing through the four points A, B, C and E? You may assume that the geom3d package has already been loaded and the points A, B, C and E have already been defined.

(Note that the 4th point is called E to avoid using D which is used by Maple for differentiating functions.)

Answer:

sphere(S, [A, B, C, E])

Question 20: Score 1/1

Which of the following is a correct Maple command from the geom3d package that will define T to be the intersection of the two spheres S1 and S2? You may assume that the geom3d package has already been loaded and the spheres S1 and S2 have already been defined.

(Note that although the help page for "geom3d,intersection" in Maple does not include the case of two spheres, "intersection" works for two spheres using a form similar to the intersection of two planes.)

Answer:

intersection(T, S1, S2)



Question 21: Score 1/1

Which of the following is a correct Maple command that will define the point A, for use with the geom3d package, to be the centre of the circle or sphere S? You may assume that the geom3d package has already been loaded and the circle or sphere S has already been defined.

(Note that Maple uses the American spelling "center" not the Australian spelling "centre".)

Answer:

center(A, S)

Question 22: Score 1/1

Which of the following is a correct Maple command from the geom3d package that will define L to be the intersection of the two planes P1 and P2? You may assume that the geom3d package has already been loaded and the planes P1 and P2 have already been defined.

Answer:

intersection(L, P1, P2)

Question 23: Score 1/1

Which of the following is a correct Maple command from the geom3d package that will define A to be the intersection of a line L and a plane P? You may assume that the geom3d package has already been loaded and the line L and plane P have already been defined

Answer:

intersection(A, L, P)



Question 24: Score 3/3

Given the three points A(2, -5, -1), B(0, 1, 3), C(2, 0, 1), let:

- L1 be the line through A parallel to $\begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}$,
- P be the plane through B with normal $\begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$,
- E be the point of intersection of L1 and P,
- S be the sphere through A, B, C, E,
- F be the centre of S,
- L2 be the line through C and F.

Using the geom3d package, or otherwise:

Solution:

- \Rightarrow with(geom3d):
- \Rightarrow point(A,[2,-5,-1]);
- \Rightarrow point(B,[0,1,3]);
- \Rightarrow point(C,[2,0,1]);
- \Rightarrow line(L1,[A,[2,5,-4]]);
- \Rightarrow plane(P,[B,[0,4,1]]);
- \Rightarrow intersection(E,L1,P);
- \Rightarrow sphere(S,[A,B,C,E]);
- \Rightarrow center(F,S);
- \Rightarrow line(L2,[C,F]);
- (i) Find a decimal approximation to the angle between L1 and P, correct to 10 significant figures. Enter your answer in the box below. To prevent typing errors you can copy and paste the answer from your Maple worksheet.

Solution:

- \Rightarrow FindAngle(L1,P);
- \Rightarrow evalf(%,10);

Answer:

(Copy from Maple) 0.6168655153



(ii) Find the coordinates of F and enter them in the box below. You should enclose the coordinates with square brackets, eg [1,2,3], and your answer should be exact, ie **not** a decimal approximation. To prevent typing errors you can copy and paste the answer from your Maple worksheet.

Solution:

 \Rightarrow coordinates(F);

Answer:

(Copy from Maple) [-39/4, 9/8, -145/16]

(iii) Find the distance between A and L2. Your answer should be exact, not a decimal approximation. Enter your answer in the box below using Maple syntax. To prevent typing errors you can copy and paste the answer from your Maple worksheet.

Solution:

 \Rightarrow distance(A,L2);

Answer:

(Copy from Maple) (3/5599)*sqrt(192473)*sqrt(509)





Question 25: Score 3/3

Given the three points A(-5, 2, 1), B(-1, 4, 4), C(16, 13, 15), let:

- S1 be the sphere with centre A and radius 8,
- S2 be the sphere which has the line segment BC as a diameter,
- T be the circle of intersection of S1 and S2,
- E be the centre of T,
- L1 be the line through B and E,
- L2 be the line through A parallel to $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$.

Using the geom3d package, or otherwise:

Solution:

- \Rightarrow with(geom3d):
- \Rightarrow point(A,[-5,2,1]);
- \Rightarrow point(B,[-1,4,4]);
- \Rightarrow point(C,[16,13,15]);
- \Rightarrow sphere(S1,[A,8]);
- \Rightarrow sphere(S2,[B,C]);
- \Rightarrow intersection(T,S1,S2);
- \Rightarrow center(E,T);
- \Rightarrow line(L1,[B,E]);
- \Rightarrow line(L2,[A,[2,1,1]]);

(i) Find the coordinates of E and enter them in the box below. You should enclose the coordinates with square brackets, eg [1,2,3], and your answer should be exact, ie **not** a decimal approximation. To prevent typing errors you can copy and paste the answer from your Maple worksheet.

Solution:

 \Rightarrow coordinates(E):

Answer:

(Copy from Maple) [-115/1083, 4922/1083, 4687/1083]



(ii) Find a decimal approximation to the angle (in radians) between L1 and L2. Your answer should be correct to 10 significant figures. Enter your answer in the box below.

Solution:

 \Rightarrow evalf(FindAngle(L1,L2),10);

Answer:

(Copy from Maple) 0.1407941166

(iii) Find the distance between L1 and L2. Your answer should be exact, not a decimal approximation. Enter your answer in the box below using Maple syntax. To prevent typing errors you can copy and paste the answer from your Maple worksheet.

Solution:

 \Rightarrow distance(L1,L2);

Answer:

(Copy from Maple)

(212/166733)*sqrt(166733)

Question 26: Score 3/3

(i)

The skeleton "for loop" below is used to evaluate the sum

$$\sum_{n=10}^{27} \sin\left(\frac{k}{n}\right) \text{ for k from 1 to 70}$$

Select the correct item from each drop down box so that the output from the loop contains exactly 70 lines and displays the sum for each integer value of k from 1 to 70.

Answer:

(Green highlight = multiple choice)

⇒ for k from 1 to 70 do # shift + enter evalf(add(sin(k/n), n=10..27) # shift + enter end do;



(ii)

The terms of a sequence a_n are generated by the recurrence relation

$$a_{n+1} = a_n - 4 a_{n-1} + a_{n-2}$$
 for $n = 3, 4, 5,...$

Using your Maple worksheet, write a for loop to find the value of a_{70} given that

$$a_1 = 5$$
, $a_2 = 2$ and $a_3 = -1$

Copy (Ctrl-C) the correct value of a_{70} from your Maple worksheet and paste (Crtl-V) it in th answer box.

Solution:

```
⇒ a := proc(n) # shift + enter
local a,i; # shift + enter
a[1]:=5; # shift + enter
a[2]:=2; # shift + enter
a[3]:=-1; # shift + enter
for i from 3 to n-1 do # shift + enter
a[i+1]:=a[i]-4*a[i-1]+a[i-2] # shift + enter
end do; # shift + enter
return a[n] # shift + enter
end proc; # shift + enter
⇒ a(70);
```

Answer:

57998811649237237229



Question 27: Score 3/3

A simple iteration procedure with $a_0 = 0$ and

$$a_{n+1} = \sin\left(\left(1 + \frac{1}{6} a_n\right)^2\right), \ n \ge 0,$$

is being used to find an approximate solution to the equation $x = \sin\left(\left(1 + \frac{1}{6}x\right)^2\right)$.

Select the correct expressions from the drop down menus to define a procedure which takes a positive integer m and uses a for loop to calculate a_m . The procedure should

return a_m if $|a_m - a_{m-1}| < 10^{-16}$ and -1 otherwise. All calculations are done using 30 significant figures.

Answer: (Green highlight = multiple choice)

```
⇒ Digits := 30; # shift + enter

f := proc(m) # shift + enter

local a,i; # shift + enter

a[0] := 0; # shift + enter

for i from 1 to m do # shift + enter

a[i] := evalf(sin((1+a[i-1]/6)^2)) # shift + enter

end do; # shift + enter

if abs(a[m]-a[m-1]) <10^(-16) then # shift + enter

a[m] # shift + enter

else # shift + enter

else # shift + enter

end if # shift + enter

end proc;
```

Use this procedure to calculate f(9) and f(20) and enter your answers in the box below as decimal approximations correct to 30 significant figures. To prevent typing errors, you can copy and paste your answers from your Maple worksheet.

The value of f(9) is

Solution:

 \Rightarrow f(9);

Answer:

-1

The value of f(20) is

Solution:

 \Rightarrow f(20);

Answer:

0.976125175779941224727247391125



Good Luck!

Minkshap Ooch Manoni

