arcsin x is the inverse function of sin x.

~~arcsin x~~ $\arcsin x = \sin^{-1} x$.

$\arcsin : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

~~Exam~~ examples:

$$\sin^{-1}\left(\sin \frac{5\pi}{3}\right) = \frac{5\pi}{3}$$

$$\sin\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = -\frac{1}{2}$$

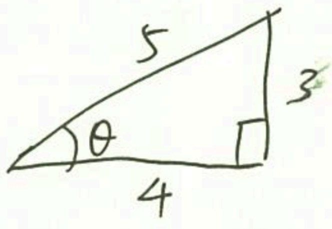
$$\sin\left(2\cos^{-1}\left(\frac{4}{5}\right)\right)$$

$\sin 2x$
$= 2\sin x \cdot \cos x$

$$= 2\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)\cos\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$$

$$= \frac{8}{5}\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$$

$$= \frac{8}{5}\cdot\frac{3}{5} = \frac{24}{25}$$



$\cos\theta = \frac{4}{5}$

$\theta = \cos^{-1}\frac{4}{5}$

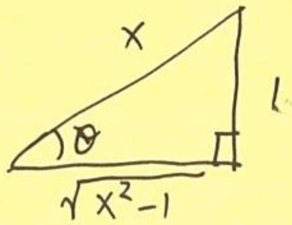
$$\frac{d}{dx}\csc^{-1}x = ?$$

$$\frac{d}{dx}\csc^{-1}x = \frac{1}{\csc'(\csc^{-1}x)}$$

$$= \frac{-1}{\csc(\csc^{-1}x)\cdot\cot(\csc^{-1}x)}$$

$$= \frac{-1}{|x|\cot(\csc^{-1}x)}$$

$$= \frac{-1}{|x|\sqrt{x^2-1}}$$



$\csc\theta = x$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

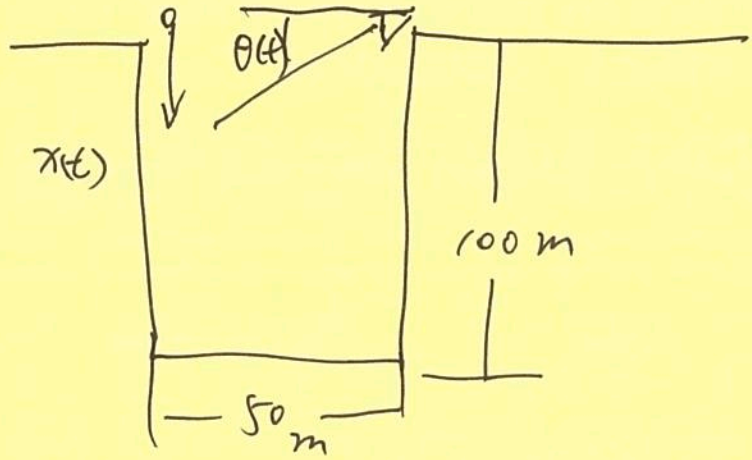
$$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)' = \frac{-1}{1+x^2}$$

$$(\operatorname{arcsec} x)' = \frac{1}{|x|\sqrt{x^2-1}}$$

example, 6.5.3. on page 120 in the book.



$x(t)$ – position of a rock dropped from rest on one side of the hole.

$\theta(t)$ is the angle between the line of sight and the horizontal direction.

When is $\theta(t)$ changing the fastest?

When is $\theta'(t)$ the greatest?

$x(0) = 0.$ $\cancel{v(t)}$ $v(0) = 0.$

$$\frac{dx}{dt} = v(t) \qquad \frac{dv}{dt} = g$$

$$v(t) = gt + C_0$$

$$v(0) = 0 = 0 + C_0 \Rightarrow C_0 = 0.$$

$$v(t) = gt.$$

Similarly, we get

$$x(t) = \frac{1}{2} g t^2.$$

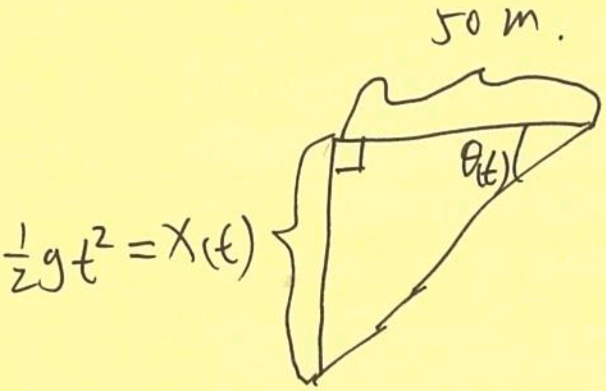
The rock falls ~~at~~ to the bottom when

$$100 = \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{200}{g}} \approx 4.5.$$

We need to find the max of

$\theta'(t)$ $\cancel{\theta'(t)}$ on $[0, \sqrt{\frac{200}{g}}]$.



$$\theta(t) = \arctan\left(\frac{gt^2}{100}\right)$$

$$\frac{d\,\theta(t)}{dt} = \frac{1}{1+\left(\frac{gt^2}{100}\right)^2} \cdot \frac{gt}{50}.$$

~~d~~ ~~Wh~~ We need to find the max of $\frac{d\theta}{dt}$ on $[0, \sqrt{\frac{200}{g}}]$.

$$\frac{d^2\theta}{dt^2} = \frac{(10^4)\,200g - 600\,g^3t^4}{(10^4 + g^2t^4)^2}$$

$\frac{d^2\theta}{dt^2} = 0$ when

$$10^4 \cdot 200g - 600\,g^3 t^4 = 0.$$

$$\Rightarrow t^4 = \frac{10000}{3g^2} \Rightarrow t = \frac{10}{\sqrt[4]{3g^2}} \approx 2.42.$$

$\theta'(0) = 0$

$\theta'\left(\frac{10}{\sqrt[4]{3g^2}}\right) \approx 1.1$

$\theta'\left(\sqrt{\frac{200}{g}}\right) \approx 0.26$

θ' is the largest when $t = \frac{10}{\sqrt[4]{3g^2}}$.

Curve sketching.

$y = f(x)$

We would like to sketch the graph $y = f(x)$.

A. Domain & range of f.

B. x - intercept(s) $(y=0)$, y - intersept $(x=0)$

C. symmetries of the graph.
even, odd, periodic.

D. Horizontal and vertical asyptotes.

E. oblique asyptotes $\quad \frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$

$$y = \frac{x^2+3}{x-1} = (x+1) + \frac{4}{x-1}$$

This function has an oblique asymptote

$y = x+1$.

F. Stationary pt's and inflection pts ($f''_{(x)} = 0$)

$$y = \frac{x^2}{x^2-1} = 1 + \frac{1}{x^2-1}$$

A. Domain : $x \neq \pm 1$,

Range : $y \neq 1$.

B. y-intercept. $y = 0$.

x-intercept. $x = 0$.

C. even.

D. horizontal asymptote $y = 1$

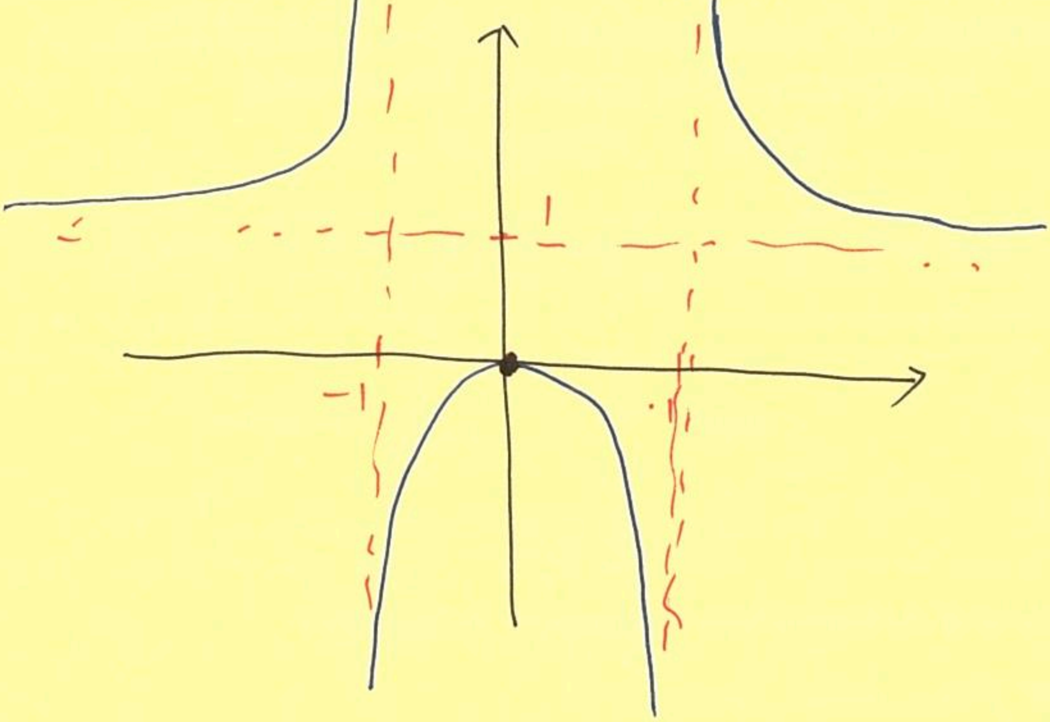
vertical asymptote. $x = \pm 1$.

E. no oblique ~~as~~ asymptote.

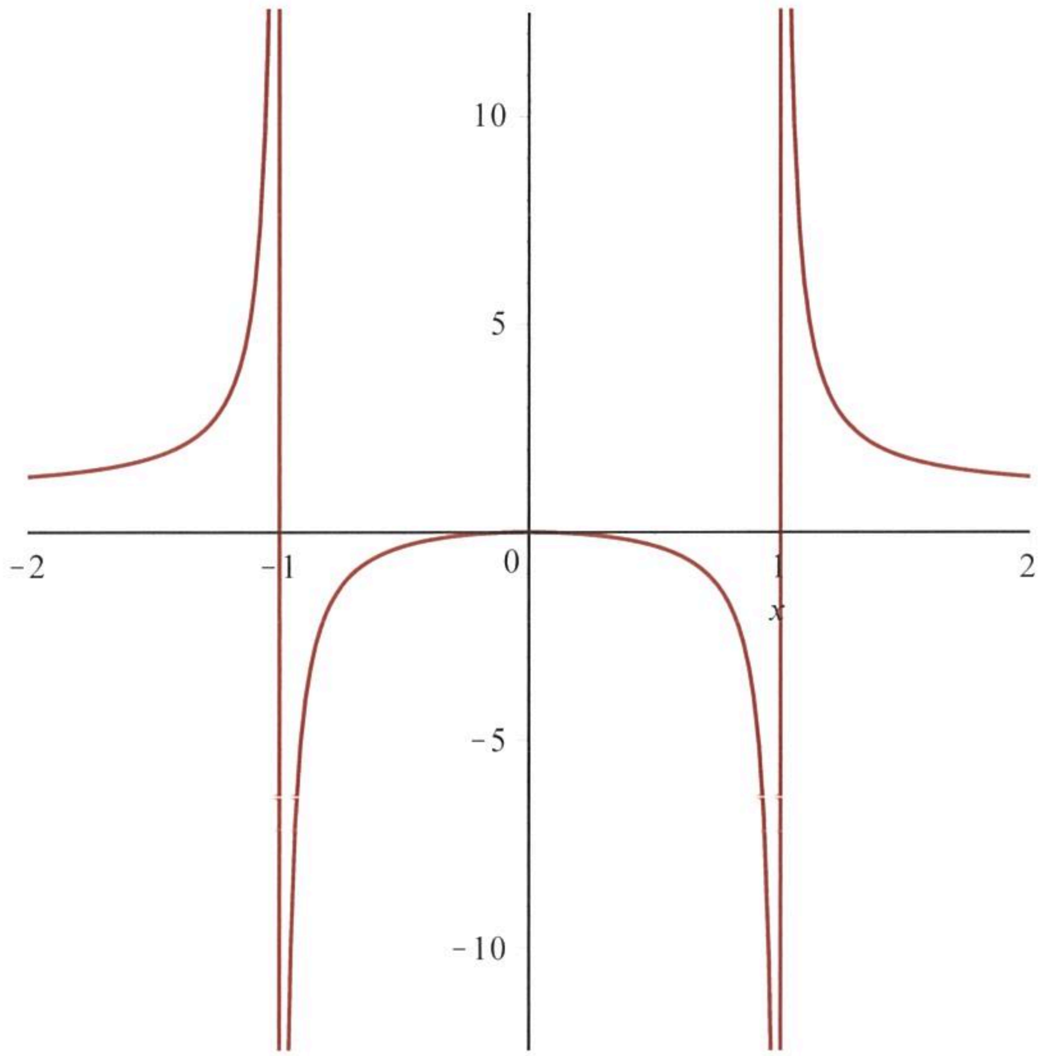
F. $y' = \frac{x^3 - 2x}{(x^2-1)^2}$, Stationary pts, $x = 0$, $x = \pm\sqrt{2}$.

$y'' \neq 0$. no pts of inflection.

$$\frac{x^2}{x^2-1} = 1 + \frac{1}{x^2-1}$$



$$plot\left(\frac{x^2}{x^2-1}, x = -2 ..2\right)$$



$$diff\left(\frac{x^2}{x^2-1}, x\right)$$

$$\frac{2\,x}{x^2-1} - \frac{2\,x^3}{\left(x^2-1\right)^2} \tag{1}$$

$$diff\left(\frac{x^2}{x^2-1}, x\$2\right)$$

$$\frac{2}{x^2-1} - \frac{10\,x^2}{\left(x^2-1\right)^2} + \frac{8\,x^4}{\left(x^2-1\right)^3} \tag{2}$$

$$plot\left(x^2 \cdot \exp(x), x = -2 ..1\right)$$