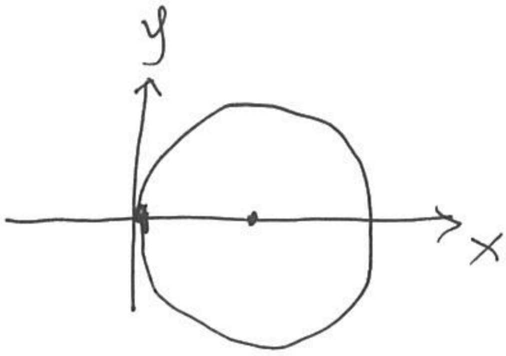
$r = 2\cos\theta$



$\Rightarrow r = 2 \cdot \frac{x}{r}$

$\Rightarrow r^2 = 2x$

$x = r\cos\theta \Rightarrow \cos\theta = \frac{x}{r}$.

$\Rightarrow x^2 + y^2 = 2x$

$\Rightarrow x^2 - 2x + 1 + y^2 = 1$

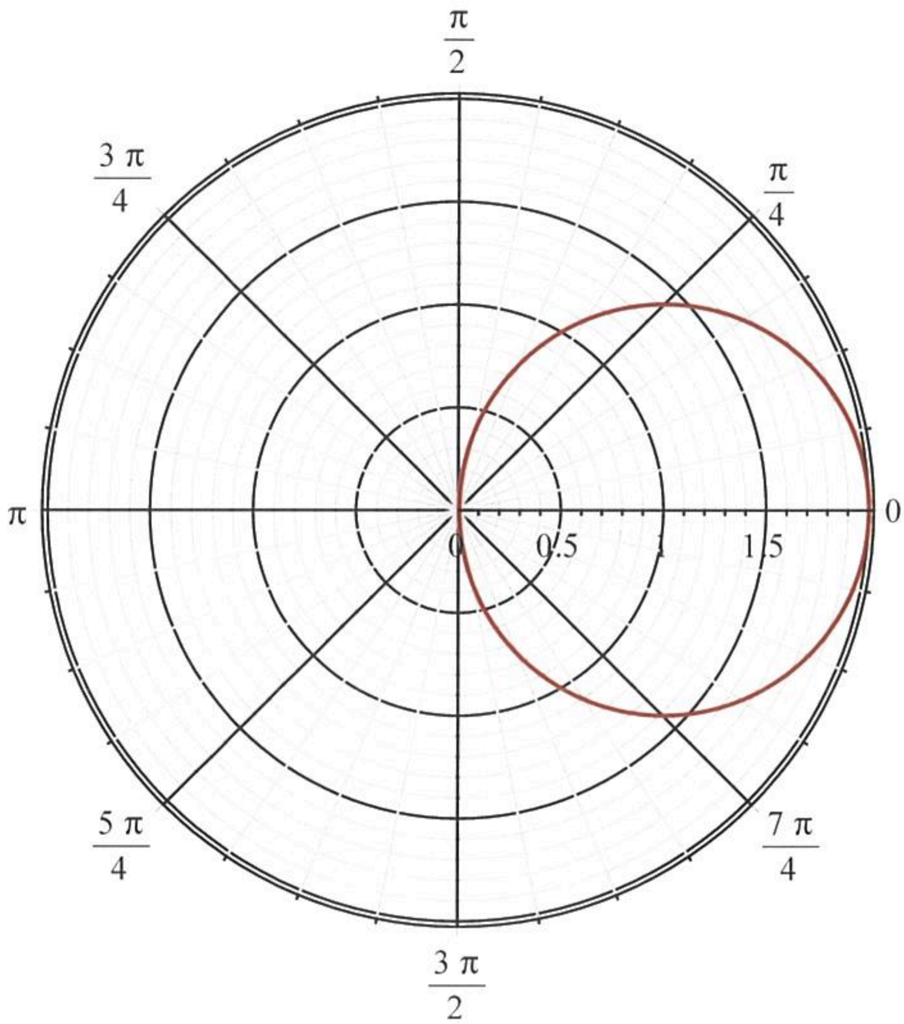
$\Rightarrow (x-1)^2 + y^2 = 1$. (A circle centered at ~~to~~ $(1,0)$ with radius 1)

$x = r(\theta)\cos\theta, \quad y = r(\theta)\sin\theta.$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cos\theta}.$$

Example. $r = 1 + \sin\theta$. Where does this curve have horizontal and vertical tangent lines.

$r = 2\cos\theta$.



~~$polarplot(2 + \cos(\theta), \theta = 0..2\pi)$~~

$$r = 1 + \sin\theta .$$

$$x = r\cos\theta , \qquad y = r\sin\theta$$

$$\frac{dr}{d\theta} = \cos\theta$$

$$\frac{dy}{dx} = \frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cos\theta}$$

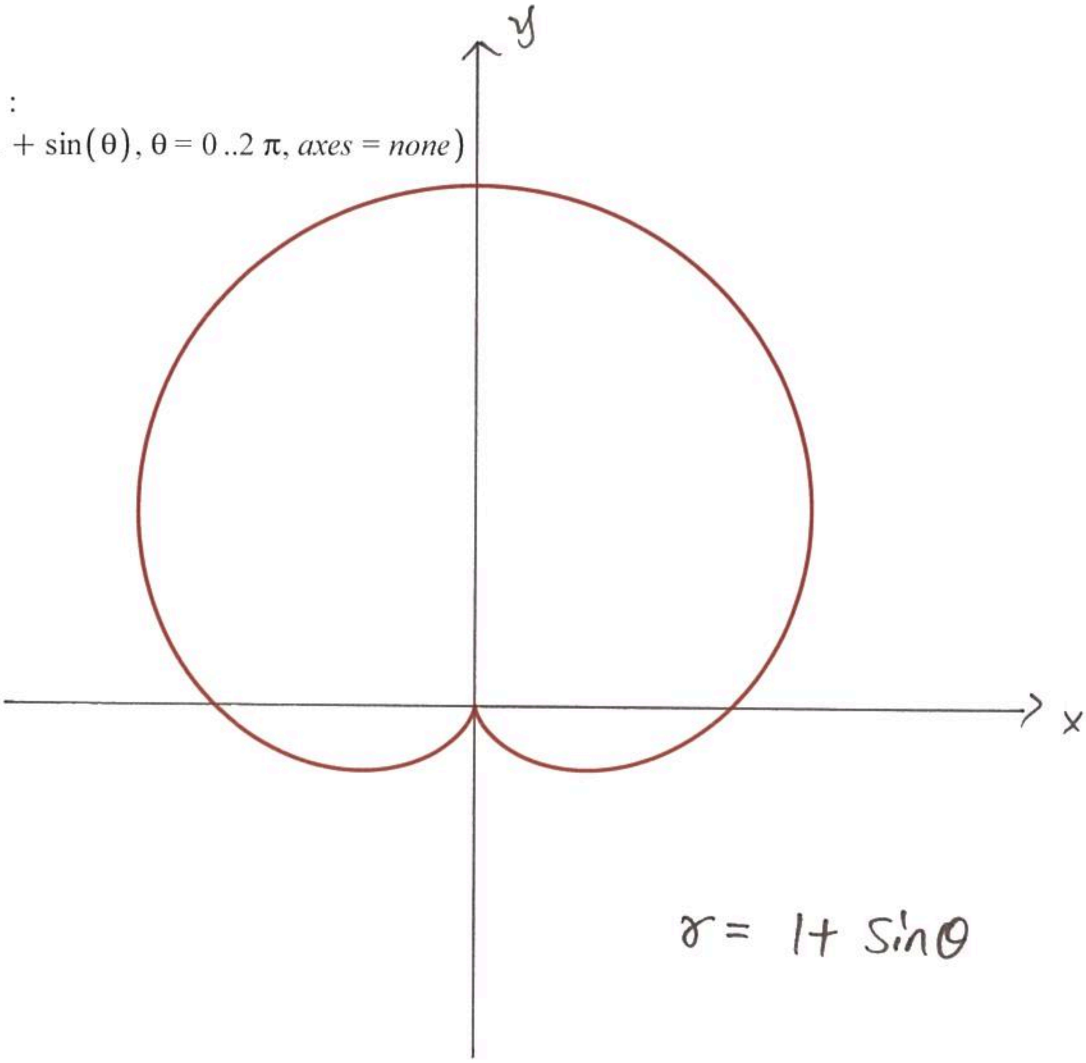
$$= \frac{(1+\sin\theta)\cos\theta + \cos\theta\sin\theta}{-(1+\sin\theta)\sin\theta + \cos^2\theta}$$

$$= \frac{\cos\theta\,(1 + 2\sin\theta)}{-\sin\theta - \sin^2\theta\ \cancel{2\sin^2\theta} + \cos^2\theta}$$

$$= \frac{\cos\theta\,(1 + 2\sin\theta)}{1 - \sin\theta - 2\sin^2\theta}$$

$$= \frac{\cos\theta\,(1 + 2\sin\theta)}{-(\sin\theta + 1)(2\sin\theta - 1)}$$

with(*plots*) :
polarplot(1 + sin(θ), θ = 0 ..2 π, *axes* = *none*)



polarplot(1 + sin(θ), θ = 0 ..2 π) horizontal tangent line. if

$$\cos\theta = 0 \quad \text{or} \quad \sin\theta = -\frac{1}{2}$$

$$\theta = \frac{\pi}{2}, \ \frac{3\pi}{2}, \qquad \theta = \frac{7\pi}{6}, \ \frac{11\pi}{6}.$$

vertical tangent line if.

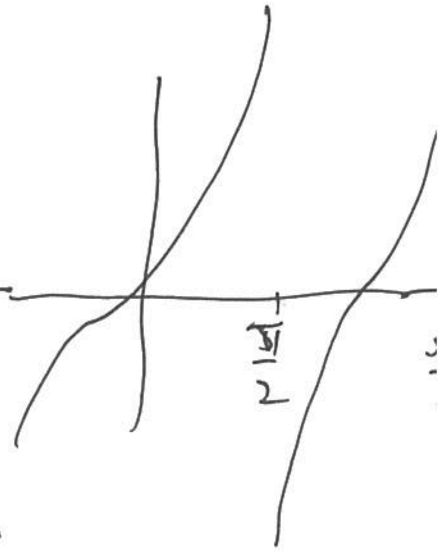
$$\sin\theta = -1 \quad \text{or} \quad \sin\theta = \frac{1}{2}$$

$$\theta = \frac{3\pi}{2}. \qquad \theta = \frac{\pi}{6}, \ \frac{5\pi}{6}.$$

$r = 1+\sin\theta$ has horizontal tangent lines at $\theta = \frac{\pi}{2}, \frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

$r = 1+\sin\theta$ has vertical tangent lines at $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$.

$$\lim_{\theta \to \frac{3\pi}{2}} \frac{\cos\theta}{\sin\theta + 1} = \lim_{\theta \to \frac{3\pi}{2}} \frac{-\sin\theta}{\cos\theta}$$

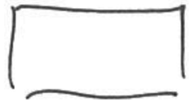


$$= -\lim_{\theta \to \frac{3\pi}{2}} \tan\theta.$$

vertical tangent line at $\frac{3\pi}{2}$.

Integration.

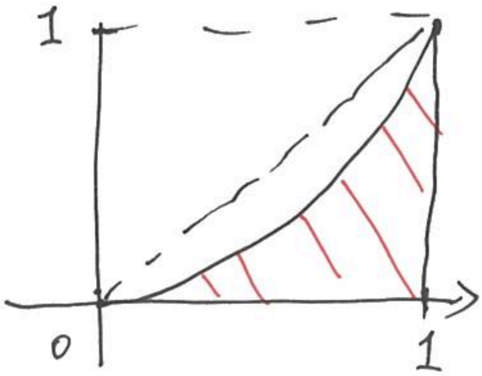
Area = $\frac{1}{2} b \cdot h$



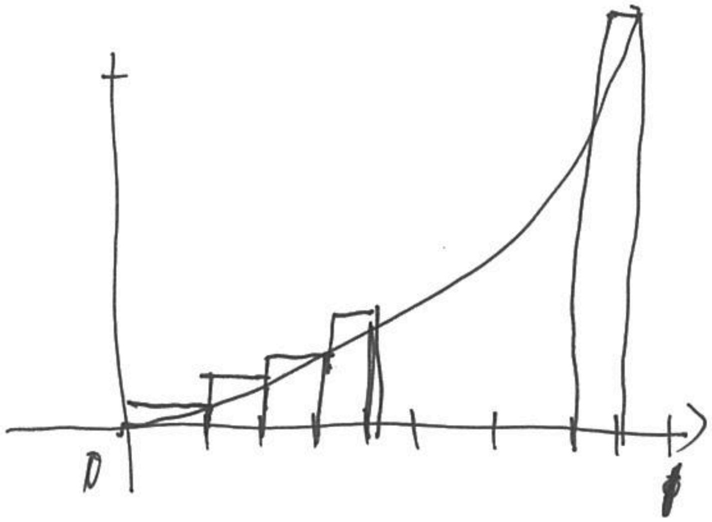
Area = $b \cdot h$.

Area = Sum of the areas of the smaller triangles.

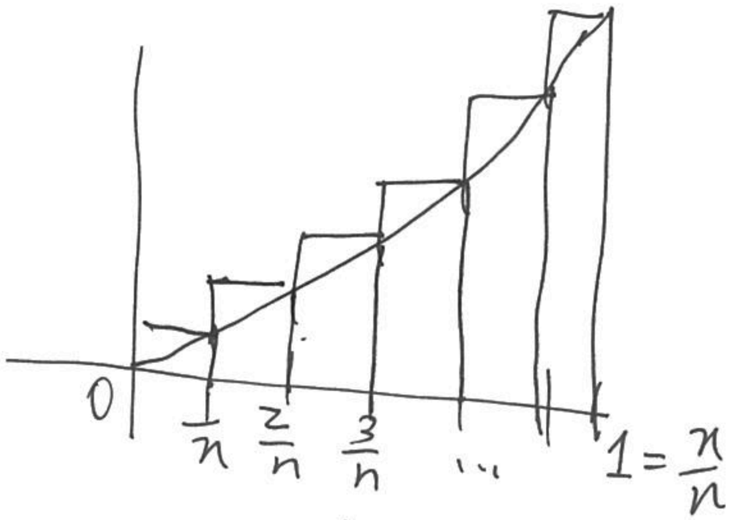
$f(x) = x^2$.

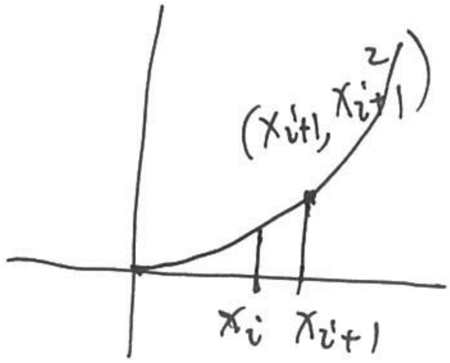


What if we want to know the area of shaded region?



We divide the interval $[0, 1]$ into n equal subintervals of width $\frac{1}{n}$.





Area under the curve $y = x^2$ between 0 and 1

$\leq$ Sum of the areas of the rectangles.

$$= \underbrace{\frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \left(\frac{3}{n}\right)^2 + \cdots + \left(\frac{n}{n}\right)^2\right]}$$

$\overline{S_P}$ = upper Riemann sum of f for this partition.

$$= \frac{1}{n^3}\left(1^2+2^2+3^2+\cdots+n^2\right)$$

$$= \frac{1}{n^3}\left(\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n\right)$$

$$\sum_{k=1}^{n} k^2 = \frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n.$$

$$= \frac{1}{3}+\frac{1}{2n}+\frac{1}{6n^2}$$

Area of interest

$$\geq \frac{1}{n}\left(0^2+\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\cdots+\left(\frac{n-1}{n}\right)^2\right)$$

$$= \underbrace{\frac{1}{n^3}\left(\frac{1}{3}(n-1)^3+\frac{1}{2}(n-1)^2+\frac{1}{6}(n-1)\right)}$$

$$= \frac{1}{3}-\frac{1}{2n}+\frac{1}{6n^2}$$

lower Riemann sum of f for the partition.

$$\shortparallel$$

$$\underline{S_P}$$

$$\frac{1}{3} - \frac{1}{2n} + \frac{1}{6n^2} \leq \text{Area} \leq \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}.$$

Taking $n \to \infty$, we must have that the area under the curve $y = x^2$ between 0 and 1 is $\frac{1}{3}$.

$$\underline{S_P} \leq \text{Area} \leq \overline{S_P}$$