$$\log x = \int_1^x \frac{1}{t}\,dt.$$

$$\frac{d}{dx}\log x = \frac{1}{x}$$

The inverse function of $\log x$ is

$$e^x = \exp(x).$$

$$\frac{d}{dx}(e^x) = e^x.$$

$$e = 2.718\cdots$$

If $a \in \mathbb{R}^+$, then we define

$$a^x = e^{x\log a} = \exp(\log a \cdot x).$$

Theorem: $\frac{d}{dx} e^x = e^x$

If $a \in \mathbb{R}^+$, then $\frac{d}{dx} a^x = \log a \cdot a^x$.

---

proof: $\frac{d}{dx}(a^x) = \frac{d}{dx}\left(e^{\log a \cdot x}\right) = e^{\log a \cdot x} \cdot \log a$

$= \log a \cdot a^x.$

$$\frac{d}{dx}\left(5^x\right) = \log 5 \cdot 5^x.$$

Set $f(x) = x^x$. $x > 0$.

$$\lim_{x \to 0^+} x^x = ?$$

Find any stationary pt. of $f(x)$.

$y = x^x$. $y' = ?$

$$y = x^x = \exp(\log x \cdot x)$$

$$y' = \exp(\log x \cdot x)\left(\frac{1}{x} \cdot x + \log x \cdot 1\right)$$

$$= x^x (1 + \log x)$$

$y' = 0$ when $1 + \log x = 0$.

or $x = e^{-1} \fallingdotseq 0.3678\cdots$

$$\lim_{x \to 0^+} x^x$$

$$= \exp\left(\lim_{x \to 0^+} x \cdot \log x \right)$$

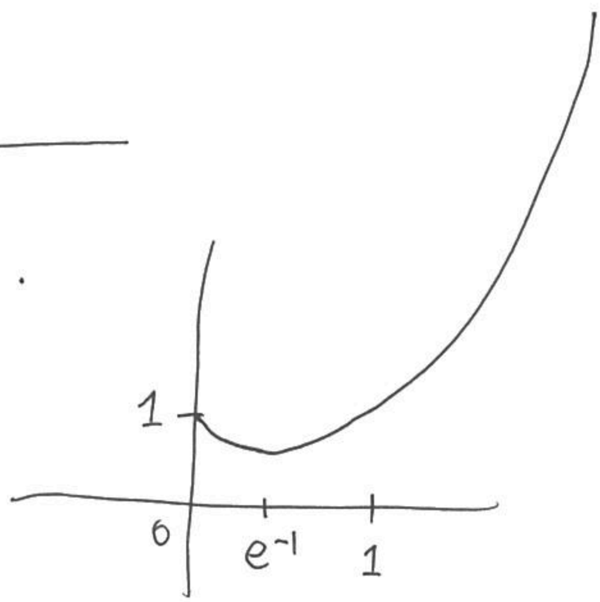
$$\lim_{x \to 0^+} x \cdot \log x = \lim_{x \to 0^+} \frac{\log x}{\frac{1}{x}}$$

$$\overset{L'H}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}}$$

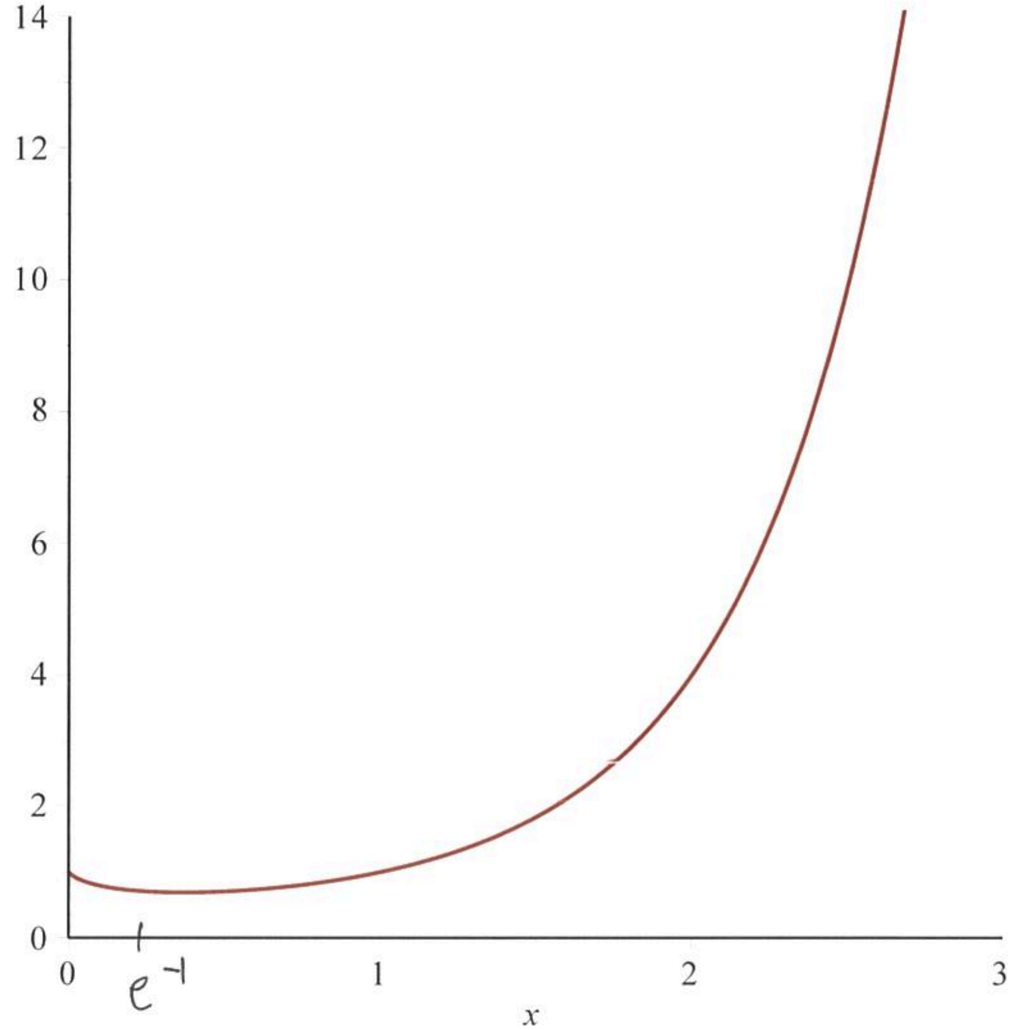
$$= \lim_{x \to 0^+} (-x)$$

$$= 0$$

$$\lim_{x \to 0^+} x^x = 1.$$



$plot\left(x^x, x = 0..3\right)$



Integrals.

$$\int x e^{5x^2}\, dx ,$$

$u = 5x^2$

$du = 10x\, dx$

$\frac{1}{10} du = x\, dx$

$$= \frac{1}{10}\int e^{u}\, du = \frac{1}{10} e^{u} + C$$

$$= \frac{1}{10} e^{5x^2} + C.$$

$$\int \tan\theta \, d\theta$$

$$= \int \frac{\sin\theta}{\cos\theta}\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$= - \int \frac{1}{u}\, du$$

$$= -\ln|u| + C$$

$$= -\ln|\cos\theta| + C.$$

$$\int \cot x \, dx = \ln|\sin x| + C.$$

$$\int \sec\theta \, d\theta = \int \frac{1}{\cos\theta} \, d\theta$$

$$= \int \sec\theta \, \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} \, d\theta$$

$$= \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta} \, d\theta.$$

set $u = \sec\theta + \tan\theta$

$du = (\sec^2\theta + \sec\theta\tan\theta) \, d\theta$

$$= \int \frac{1}{u} \, du$$

$$= \cancel{\ln|u|} \; \log|u| + C$$

$$= \log|\sec\theta + \tan\theta| + C.$$

Similarly, we can show that

$$\int \csc\theta \, d\theta$$

$$= \ln|\csc\theta - \cot\theta| + C.$$

$$\int_0^\infty e^{-x} \cos x \, dx.$$

$$= \lim_{N\to\infty} \int_0^N e^{-x} \cos x \, dx.$$

$$\int e^{-x}\cos x \, dx = \frac{1}{2} e^{-x}(\sin x - \cos x) + C.$$

$$= \lim_{N\to\infty} \frac{1}{2}\left(e^{-N}(\sin N - \cos N) - (-1) \right)$$

$$= \frac{1}{2} \lim_{N\to\infty} \left(e^{-N}(\sin N - \cos N) + 1 \right)$$

$$= \frac{1}{2}$$

logarithmic differentiation.

$$y = f(x)$$

$$\log y = \log f(x)$$

$$\frac{y'}{y} = \frac{d}{dx}(\log y) = \frac{d}{dx}(\log f(x))$$

$$\Rightarrow \quad y' = y \cdot \frac{d}{dx}(\log f(x))$$

---

example:

$$y = \frac{\sqrt{x^2 + x - 1}}{\sqrt[3]{x^4 + 1}}$$

$$y' = ?$$

$$\log y = \frac{1}{2}\log(x^2 + x - 1) - \frac{1}{3}\log(x^4 + 1)$$

$$\frac{d}{dx}(\log y)$$

$$= \frac{1}{2} \cdot \frac{2x+1}{x^2+x-1} - \frac{1}{3} \frac{4x^3}{x^4+1}$$

$$y' = y \cdot \frac{d}{dx}(\log y)$$

$$= \frac{\sqrt{x^2+x-1}}{\sqrt[3]{x^4+1}} \left(\frac{1}{2} \frac{2x+1}{x^2+x-1} - \frac{4}{3} \frac{x^3}{x^4+1} \right).$$