Overview

I. Butterfly vs. wasps.

II. Continuous parameters ~~&~~ population.

- ~~Sp~~ Social spiders.
- ~~Bacteria~~ Parasitism.

behavioral phenotype

III. Modelling Butterfly vs. wasps.

- The model
- Stability of the ODE
- Approx. the PDE w/ Laguerre fcns.

I. Butterfly vs. wasps. (Huigens et al)

Butterfly – 2 species.

a) lays eggs in a few large clutches

b) lays eggs in ~~few~~ many small clutches

~~Mate – Male marks the female~~

Mating – Female emits a pheromone to attract a male.

After mating – Male might emit an "anti-aphrodesiac" to dissuade other males.

Male Pro: ~~[illegible]~~ decrease chance other male will mate w/ a female. – dec. competition.

Female Pro: Reduces amount of time & energy associated w/ being pestered by male. Can concentrate on laying eggs.

Con The wasps.

The wasps can detect the anti-~~pheromone~~ aphrodisiac & can tell if a female has mated
⇒ hitch a ride on the female
wait till egg laid.
jump off & lay ~~parasitic~~ eggs (parasitic)

2 species used – behavior seems to be built in
behavior is learned.

Paper – there is selection pressure on the butterfly to ~~st~~ reduce use of anti-~~pheromone~~ aphrodisiac.

anti-aphrodesiac ↑ great ~~[illegible]~~ ~~personal~~ success w.r.t. butterfly interact.

dec. success w.r.t. wasp interactions

Q/ what is the right balance.
is there a balance?

existing
Models

(Look up)

compartmentalize between butterflies that engage in the ~~behaviour~~ behaviour & those that do not.

$(b_1)_t = \cdots$

$(b_2)_{ot} = \cdots$

$w_t = \cdots$

Citation.

motivate — this is a genetic behaviour & it can be switched on or off.

Problem Does not seem to be so cut & dry!
Q — Has anybody tackled this as a cont. dist?

II Continuous ~~[illegible]~~ dist. on a parameter

Carl ~~Kestra~~ ~~Kesse~~. Kerr and Colin Wright.

Social spider
~~[illegible]~~ ~~pop. [illegible]~~ parasitism

Keiser also wright.
describe the spider.

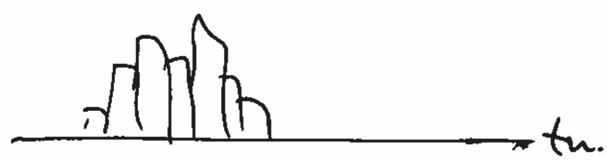
ex/ Social spiders.

"puff" test.

~~test~~
hit the w/ a "puff" of air.

how long to recover.
A mean of assessing a petulance.

Results



It is a cont. dist!

why? multiple genes + combination of genes are responsible for this complex behaviour.

⇒ variation in the colony ~~important~~ necessary!

ex/ ~~Barbara~~ Kortet et al.

~~took this up~~

"Animal Personality"

Various traits + personality vary

genetic traits - also situation dependent

ex: scarcity of food changes boldness level of some animals.

+ other factors.

⇒ parasites alter hosts behaviours

+ impacts vary by individual.

⇒ variation in a pop. can create a pos. feedback loop for the broader pop. include citations for modelling (?)

(4)

Models for moths:

$$\hat{b}_s = \alpha \hat{\theta} \hat{b}(k-\hat{b}) - \gamma \hat{\theta} \hat{\omega} \frac{\hat{b}}{c+\hat{b}} + \mu \hat{b}_{\hat{\theta}\hat{\theta}}$$

$b = b(\theta, \vec{x})$

$$\hat{\omega}_s = -d\hat{\omega} + \int_0^\infty a\hat{\theta}\hat{\omega} \frac{\hat{b}}{c+\hat{b}} d\hat{\theta}$$

$\omega = \omega(s)$

$\hat{b} = \bar{B} b$

$\hat{\omega} = \bar{\omega}\omega$

$s = \bar{T} t$

$\hat{\theta} = \bar{\theta}\theta$

$$\frac{1}{T}\bar{B} b_t = \alpha\bar{\theta}\theta\bar{B} b(k-\bar{B}b) - \gamma\bar{\theta}\theta\bar{\omega}\bar{B}\frac{\omega b}{c+\bar{B}b} + \frac{\mu}{\bar{\theta}^2}\bar{B} b_{\theta\theta}$$

$$\frac{1}{T}\bar{\omega}\omega_t = -d\bar{\omega}\omega + a\bar{\theta}\bar{\omega}\bar{B}\int_0^\infty \theta \frac{\omega b}{c+\bar{B}b}\bar{\theta}\, d\theta$$

set $\bar{B} = k$

$$b_t = T\alpha\bar{\theta}k\,\theta b(1-b) - \gamma\bar{\theta}T\bar{\omega}\theta\frac{\omega b}{c+kb} + \frac{\mu}{\bar{\theta}^2}T b_{\theta\theta}$$

$$\omega_t = -dT\omega + a\bar{\theta}^2 T k\int_0^\infty \theta\frac{\omega b}{c+kb} d\theta$$

$T = \frac{1}{d}$

$\bar{\theta} = \frac{d}{\alpha k}$

$$b_t = \theta b(1-b) - \frac{\gamma d}{\alpha k}\cdot\frac{1}{d}\bar{\omega}\theta\frac{\omega b}{\frac{c}{k}+b}\cdot\frac{1}{k} + \frac{\mu\alpha^2k^2}{d^2 d} b_{\theta\theta}$$

$$\omega_t = -\omega + \left(\frac{ad}{\alpha k}\right)^2\cdot\frac{1}{d}\frac{k}{k}\int_0^\infty \theta\frac{\omega b}{\frac{c}{k}+b} d\theta$$

(2).

$$\frac{\gamma d}{\alpha k}\cdot\frac{1}{d}\cdot\frac{1}{k}\bar{\omega} = 1$$

$$\bar{\omega} = \frac{k\alpha k}{\gamma} = \frac{\alpha k^2}{\gamma}$$

$$b_t = b\theta(1-b) - \theta\frac{\omega b}{\hat{c}+b} + \hat{\mu} b_{\theta\theta}$$

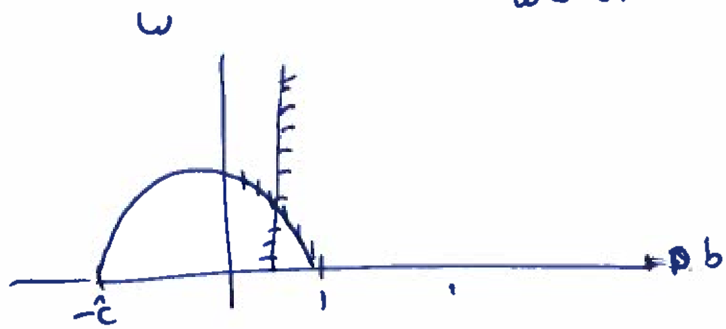
$$\omega_t = -\omega + \hat{a}\int_0^\infty \theta\frac{\omega b}{\hat{c}+b} d\theta$$

$$\theta b\left[1-b-\frac{w}{\hat{c}+b}\right]=0$$

$\theta=0$

$b=0$

$w=(1-b)(\hat{c}+b)$



$$w\left\{-1+\theta\hat{a}\frac{b}{\hat{c}+b}\right\}=0$$

$w=0$ or $b=\dfrac{\hat{c}}{\theta\hat{a}-1}$

$$\theta\hat{a}\frac{b}{\hat{c}+b}=1$$

$$\theta\hat{a}b=\hat{c}+b$$

$$b=\frac{\hat{c}}{\theta\hat{c}-1}$$

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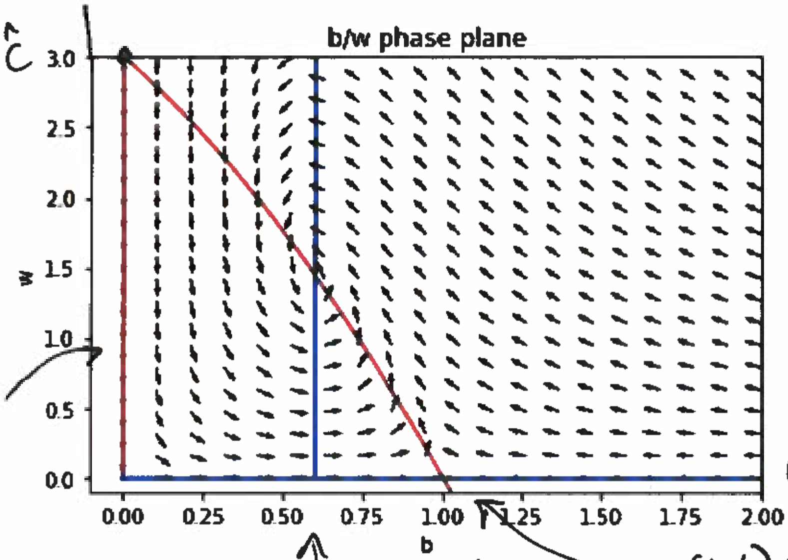
$\hat{c}$



$b=0$

$w=0$

$w = (1-b)(\hat{c}+b)$

$b = \dfrac{\hat{c}}{\theta\hat{a}-1}$

as θ inc. ← shifts left.

modeling Quest?

What about Boundary Conditions? $0 \le \theta < \infty$ (semi-inf. interval.)

should... $b(0,t) = 0$? or $b_\theta(0,t) = 0$?

$\lim_{\theta \to \infty} b(\theta,t) = 0$ (?) well posed?

Approximating?

$$b(\theta,t) = \sum_{n=0}^{\infty} \hat{b}_n(t) \underbrace{\varphi_n(\theta)}_{\text{Laguerre fcn.}}$$

$$\varphi_n(\theta) = \theta^{\alpha/2} \cdot e^{-\theta/2} L_n(\theta)$$

$$\theta L_n''(\theta) + (\alpha - \theta + 1) L_n'(\theta) + n L_n(\theta) = 0$$

L_n is a poly. of deg. n

then...
$$\int_0^\infty \varphi_n(\theta)\varphi_m(\theta)\, d\theta = \begin{cases} 0 & \text{if } n \ne m \\ \# & \text{if } n = m. \end{cases}$$

if $\alpha > 0$ then

$$\lim_{\theta \to \infty} \varphi_n(\theta) = 0$$

and, can use the recurrence relationship

~~$\theta L_n(\theta) = (n+\alpha) L_{n-1}(\theta) - \theta L_{n+1}$~~

$$L_n(\theta) = \left(2 + \frac{\alpha - 1 - \theta}{n}\right) L_{n-1}(\theta) - \left(1 + \frac{\alpha - 1}{n}\right) L_{n-2}(\theta)$$

to approx. Gauss - Quadrature + Gauss-Radau Quadrature.

So...

$$\int_0^\infty P_{m}(\theta) \cdot \theta^\alpha e^{-\theta} \, d\theta = \sum_{i=0}^{n} P_m(\theta_i) w_i$$

is exact for $m \le 2n$

and $\theta_i = 0$.

Let $\psi_i(\theta)$ = Lagrange interpolant on θ_i

Construct variational form $\Rightarrow$ collocation scheme.

Issue...

$$\int_0^\infty f(\theta) \, \underline{\underline{\theta \, d\theta}}$$

wrong weight

$$\frac{w_N b_N}{\hat{c} + b_N}$$

$$b_N = \sum_{n} \hat{b}_n(t) \, \underline{Q_n(\theta)}$$

$$\underline{w_N = w_N(t)}$$

$$Q_{n_i}(\theta)^2 \, \psi_i(\theta) \cdot \theta^{\alpha/2} e^{-\theta/2}$$

use $\underline{\alpha = 1}, 1^2 (?)$

$$w_N(t) \int_0^\infty \frac{1}{\hat{c} + b_N} \cdot \sum \hat{b}_n(t) \, \theta^{\alpha/2} e^{-\theta/2} \, d\theta$$

$$= w_N \sum \hat{b}_n(t) \int_0^\infty \underbrace{\frac{1}{\hat{c} + b_N} \cdot \theta}^{} \theta^{\alpha/2} e^{-\theta/2} \, d\theta$$

Transf to be... $\int \_\_\_ \, \alpha e^{-\theta} d\theta$

but need same Gauss-Radau pts.