Classroom Activities
Math 1113 - Precalculus

University of Georgia
Department of Mathematics

Contents

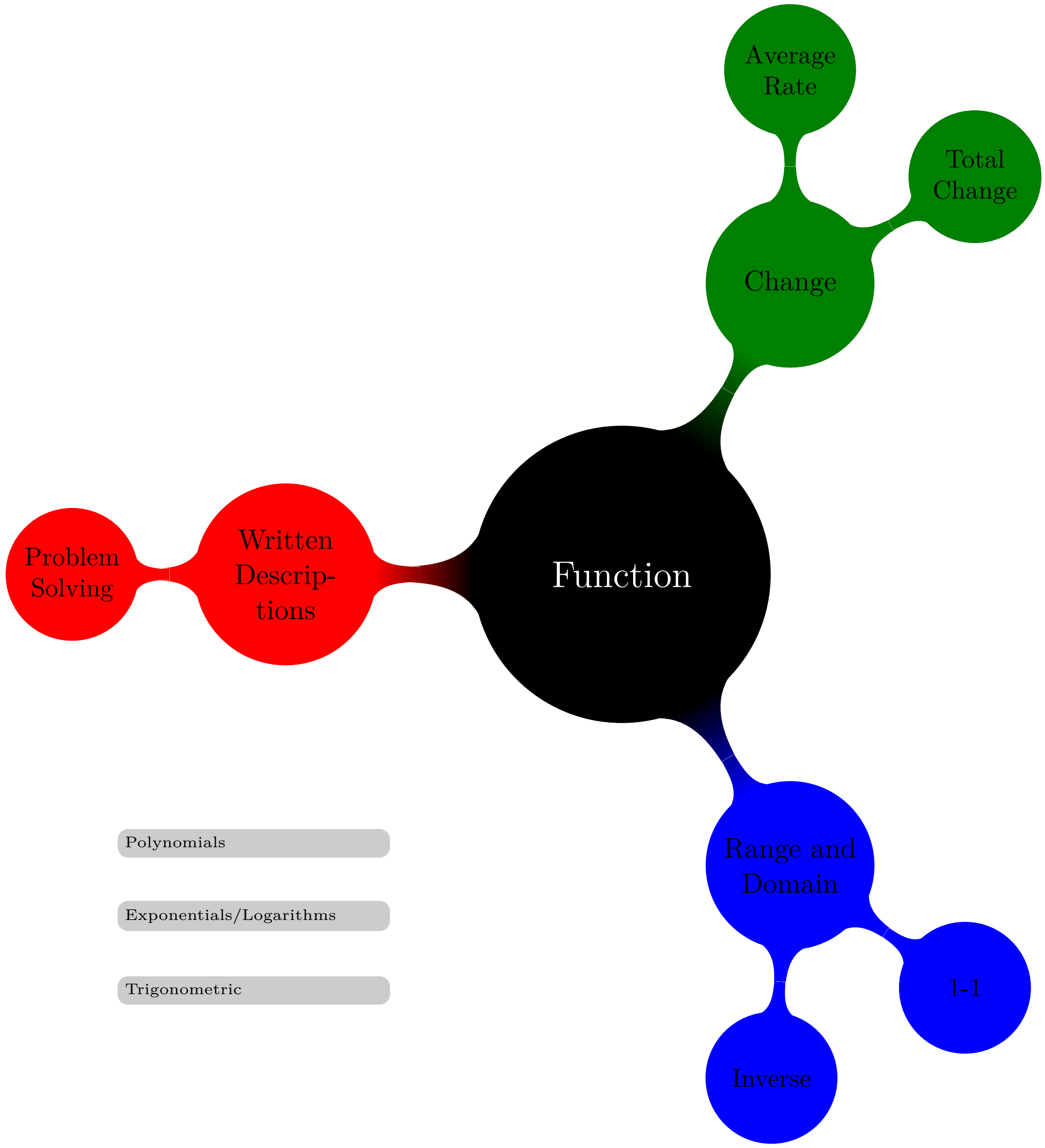


Figure 1: Broad overview of the topics for the full course.

Chapter 1

Functions and Preliminaries

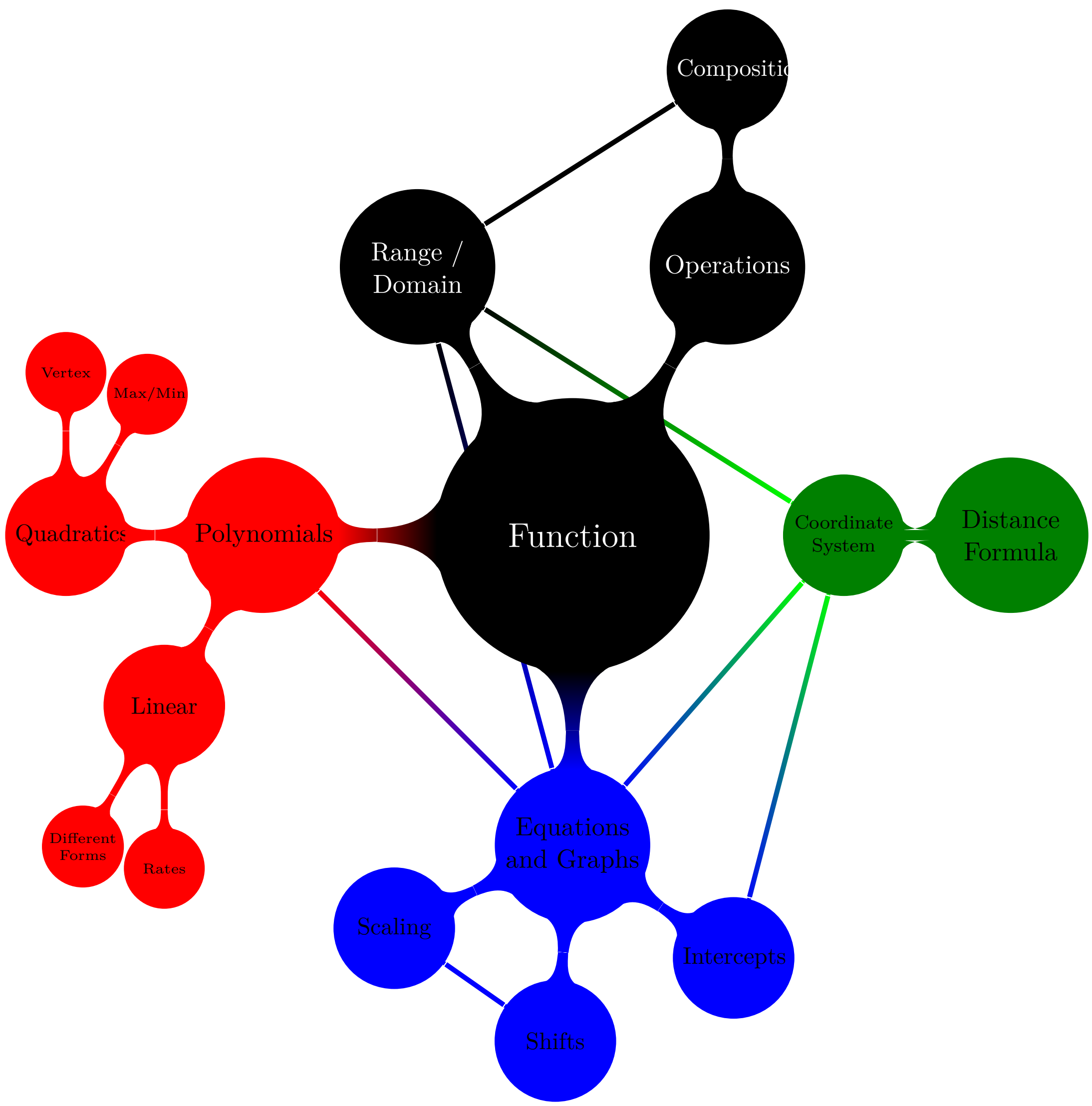
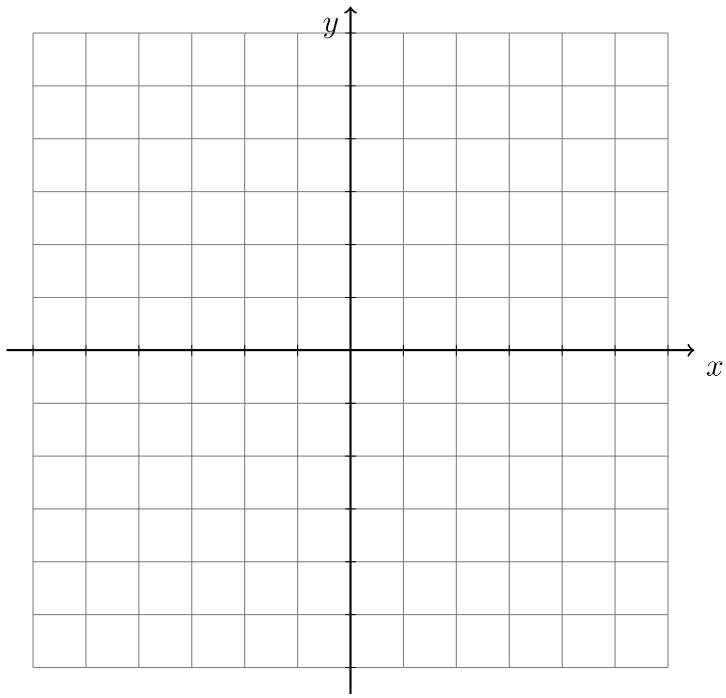


Figure 1.1: Topics for the first section of the course.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. The points ABC form a triangle.

   (a) Plot $A(-1,2), B(3,0), C(4,2)$, and draw the triangle on the graph below.



   (b) Find the perimeter of the triangle.

   (c) Is the triangle a right triangle? Show work to support your answer.

2. Find the x and y intercepts of the following equations.

   (a) $x^2 + y = 9$

   (b) $y = |x + 4| - 3$

3. Use the given problem solving process below to determine all points lying on the y-axis that are 5 units away from the point $(4, -2)$. As a group, write your complete solution on the board.

Problem Solving Process

(a) Re-read the problem.

(b) Determine what the problem is asking for along with the format of that answer.

(c) Circle/Underline the important components of the problem.

(d) Determine the topics/concepts being assessed.

(e) Write down relevant formulas, definitions, and equations.

(f) Discuss your ideas with your group.

(g) Solve the problem and verify that your solution answers the question in the correct format.

4. Use the following parts to find all points on the line $y = 2x$ that are 5 units away from $P(-1, 3)$.

   (a) Find the points (x, y) on the line $y = 2x$ for $x = 1, -2$, and 5.

   (b) What if we didn't know the value of x? Write an ordered pair formula that works for every point on the line $y = 2x$?

   (c) Use part (b) to find all points on the line $y = 2x$ that are 5 units away from $P(-1, 3)$.

Watch the Pre-Class videos for Section 1.3 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

Actor x	**Number of Oscar Nominations** y
Tom Hanks	5
Jack Nicholson	12
Sean Penn	5
Dustin Hoffman	7

1. Use the relation given in the table above to answer the following.

   (a) Write a set of ordered pairs (x, y) that defines the relation.

   (b) Write the domain of the relation.

   (c) Write the range of the relation.

   (d) Determine if the relation defines y as a function of x.

2. Given $f(x) = x^2 + 3x$ and $g(x) = \frac{1}{x}$, evaluate the function at the given value of x.

   (a) $f(-2) =$

   (b) $g(-\frac{1}{2}) =$

3. Write the domain of the function in interval notation.

   (a) $f(x) = \frac{x-3}{x-4}$

   (b) $g(x) = \sqrt{x+9}$

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Consider the relation that is defined by taking an object on your desk and assigning to it its color. For example, if there is a hat on your desk that is green and blue, we would write

$$f(\text{hat}) = \{\text{green, blue}\}.$$

If there is a pen on your desk that is red, we would write

$$f(\text{pen}) = \text{red}.$$

(a) List at least three items on your desk and come up with your own relation of this sort. (You can use imaginary items, if you need to.)

(b) What is the domain and range of your relation?

(c) Is your relation a function? Why or why not?

2. Answer True or False. In either case, verify your answer.

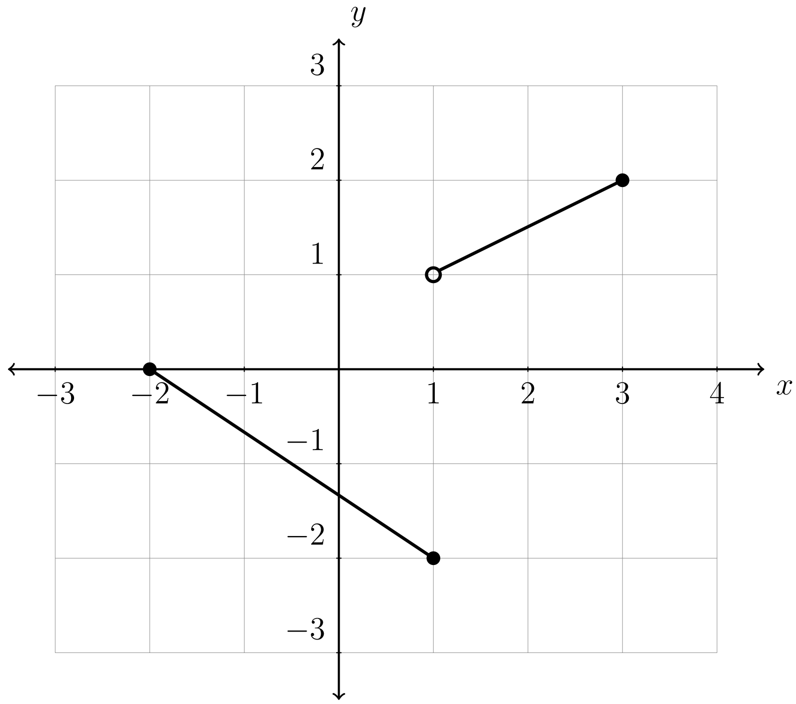
(a) $x = -\frac{1}{3}$ is in the domain of $f(x) = \dfrac{x-2}{3x+1}$.

(b) $x = -3$ is in the domain of $f(x) = \sqrt{x+3}$.

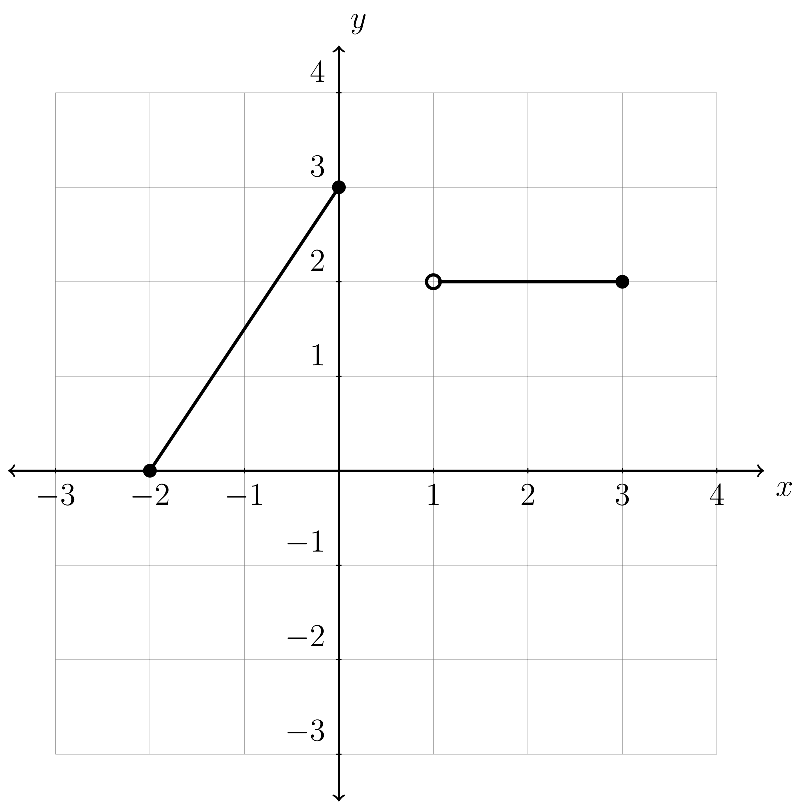
(c) $x = -4$ is in the domain of $f(x) = \sqrt{x+3}$.

3. Determine the domain and range for each of the following functions. Give your answer in interval notation.

(a)



(b)



4. Determine the domain of each of the following functions. Give your answer in interval notation.
   (a) $g(x) = x^2$

   (b) $f(x) = \sqrt{3x - 7}$

   (c) $h(x) = \dfrac{5}{x^2 - 25}$

(d) $f(x) = \sqrt{x^2 - 4x + 3}$

(e) $g(x) = \dfrac{\sqrt{x^2 - 4x + 3}}{x^3 + 8}$

Watch the Pre-Class videos for Section 1.4 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Determine a *point-slope* equation for a line through $(-5, 2)$ and $(6, -3)$.

2. Write the *slope-intercept* equation for a line through the point $(2, 3)$ that is perpendicular to the line $2x + 4y - 7 = 0$.

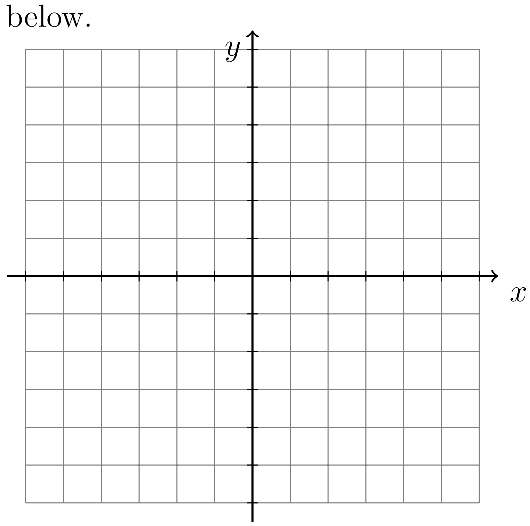
3. Given the function defined $f(x) = x^2 + 3$, determine the average rate of change of $f(x)$ from $x_1 = 2$ to $x_2 = 4$.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Let $f(x)$ be a linear function such that $f(2) = \dfrac{7}{3}$ and the graph of $f(x)$ is parallel to the line $2x + 3y + 4 = 0$.

   (a) Determine $f(x)$ and write your final answer in *slope-intercept* form. (Leave fractions in your answer, no decimals.)

   (b) Graph $f(x)$ and $2x + 3y + 4 = 0$ on the rectangular coordinate system below.



   (c) Determine the domain and range of $f(x)$.

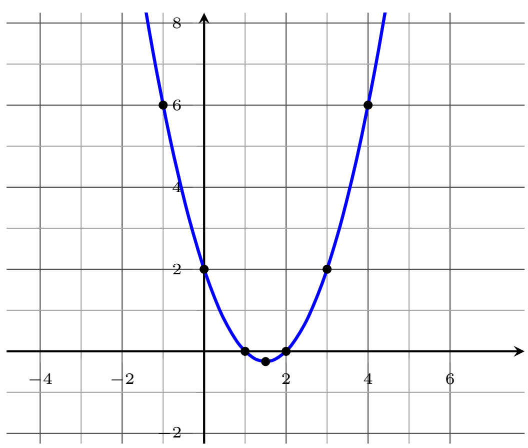
2. Determine whether the lines $y = 2x + 3$ and and $x - 3y - 5 = 0$ are parallel, perpendicular, or neither.

3. Determine an equation of the *vertical* line which passes through the point $(-2, 3)$. Then determine an equation of the *horizontal* line through $(-2, 3)$.

4. Consider the function $f(x) = x^2 - 3x + 2$.

   (a) Algebraically determine the average rate of change of $f(x) = x^2 - 3x + 2$ between $x_1 = 1$ and $x_2 = 3$.

   (b) The graph of $f(x) = x^2 - 3x + 2$ is given below. Draw a line between the points $(1, f(1))$ and $(3, f(3))$.



   (c) Find the slope of the line between the points $(1, f(1))$ and $(3, f(3))$ on $f(x) = x^2 - 3x + 2$.

   (d) What do you notice about the slope of the line between the points $(1, f(1))$ and $(3, f(3))$ on $f(x) = x^2 - 3x + 2$ and the average rate of change of $f(x) = x^2 - 3x + 2$ between $x_1 = 1$ and $x_2 = 3$.

5. Consider the line $x + 2y = 3$.

   (a) Find the x and y intercepts of the line.

   (b) Determine the distance between the x-intercept and the y-intercept.

   (c) Find the slope of the line $x + 2y = 3$.

Watch the Pre-Class videos for Section 1.5 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. A sales person makes a base salary of \$350 per week plus 15% commission on sales.

   (a) Write a linear function to model the sales person's weekly salary $S(x)$ for x dollars in sales.

   (b) Evaluate $S(700)$ and interpret the meaning in the context of this problem.

2. A town's population has been growing linearly. In 2004 the population was 6,200. By 2009 the population had grown to 8,100. Assume this trend continues.

   (a) Use the points $(0, 6200)$ and $(5, 8100)$ to write a linear model for this data.

   (b) Interpret the meaning of the slope in this context.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. A small business makes cookies and sells them at the farmer's market. The fixed monthly cost for use of a Health Department-approved kitchen and rental space at the farmer's market is \$790. The cost of labor, taxes, and ingredients for the cookies amounts to \$0.24 per cookie, and the cookies sell for \$6.00 per dozen.

   (a) Write a linear cost function representing the cost $C(x)$ to produce x dozen cookies per month. Then determine the monthly cost to produce 100 dozen cookies each month.

   (b) Write a linear revenue function representing the revenue $R(x)$ for selling x dozen cookies. Then determine the revenue for for selling 100 dozen cookies.

   (c) Will the business make a profit or lose money if they sell 100 dozen cookies each month?

   (d) Write a linear profit function representing the profit $P(x)$ for producing and selling x dozen cookies in a month. (Profit is revenue minus cost.)

   (e) Determine the least number of cookies (in dozens) that must be produced and sold for a monthly profit. Your answer should be an appropriate whole number.

   (f) If 150 dozen cookies are sold in a given month, how much money will the business make or lose?

2. The boiling point of water is 212° Fahrenheit and 100° Celsius. The freezing point of water is 32° Fahrenheit and 0° Celsius. Fahrenheit and Celsius are linearly related.

   (a) Determine a function that will tell you the temperature in degrees Celsius if you already know the temperature in degrees Fahrenheit.

   (b) If it is 83° Fahrenheit, determine the temperature in degrees Celsius.

3. A car has a 15-gal tank for gasoline and gets 30 mpg on a highway while driving 60 mph. Suppose that the driver starts a trip with a full tank of gas and travels 450 mi on the highway at an average speed of 60 mph.

   (a) Write a linear model representing the amount of gas $G(t)$ left in the tank t hours into the trip.

   (b) Evaluate $G(4.5)$ and interpret the meaning in the context of the problem.

4. The table gives the number of calories and the amount of cholesterol for selected fast food hamburgers.

Hamburger Calories	**Cholesterol (mg)**
220	35
420	50
460	50
480	60
560	70
590	105
610	65
680	80
720	90

(a) Use the data points (480,60) and (720,90) to write a linear function that defines the amount of cholesteral $c(x)$ as a linear function of the number of calories x.

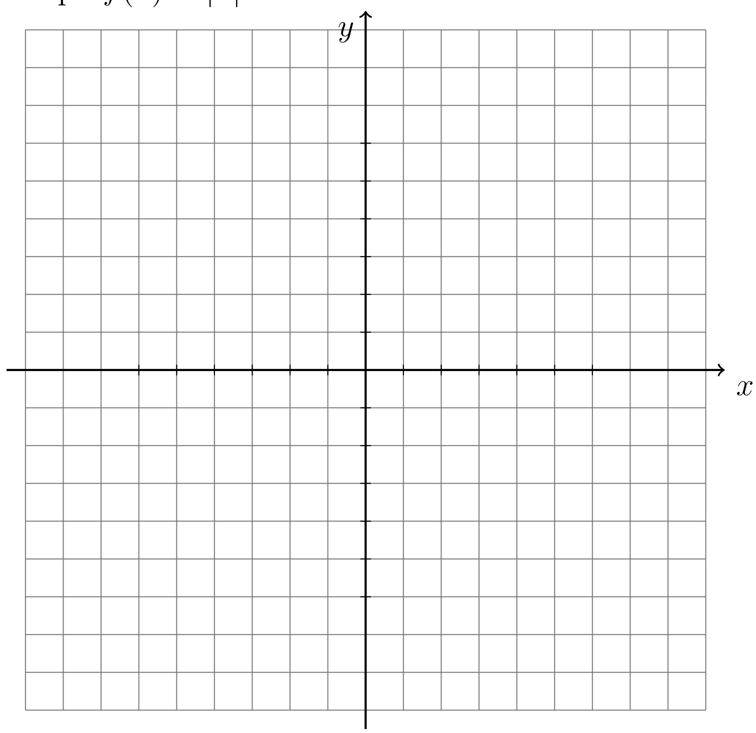
(b) Interpret the meaning of the slope in the context of this problem.

(c) Use the model from part (a) to predict the amount of cholesterol for a hamburger with 650 calories.

Watch the Pre-Class videos for Section 1.6 Day 1 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. For this problem, let $f(x) = |x|$.

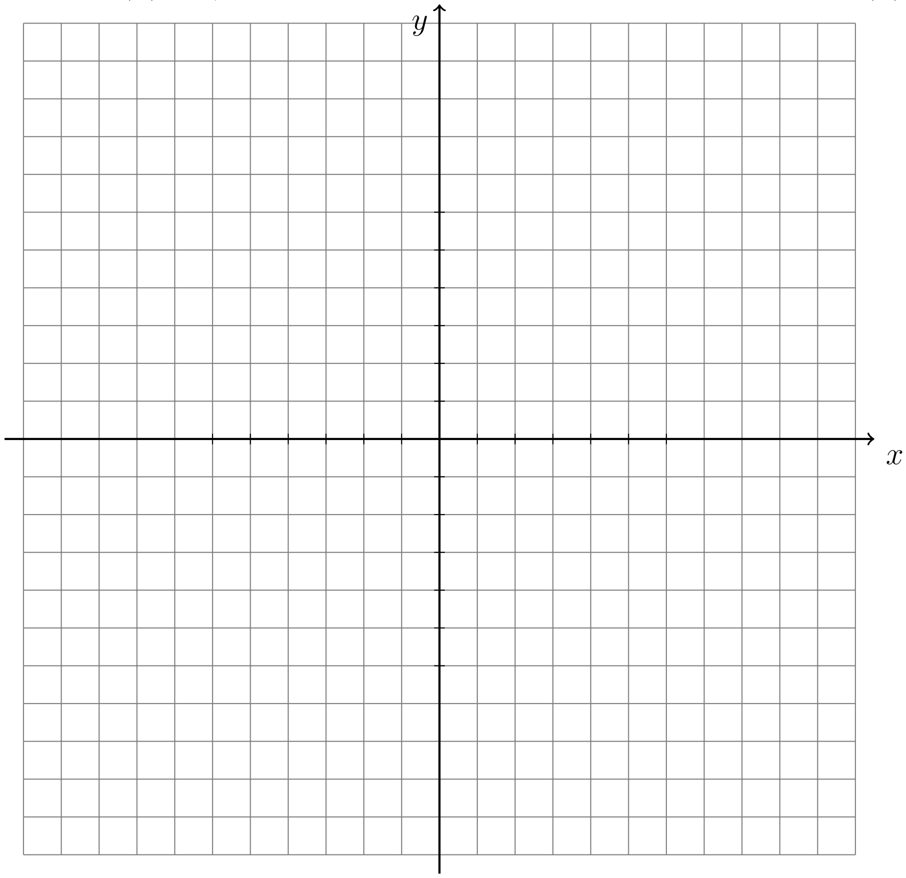
   (a) Graph $f(x) = |x|$. Then determine the domain and range of $f(x)$.



   (b) Graph and label $f(x+2)$ on the coordinate system above. Then determine the domain and range of $f(x+2)$.

2. For this problem, let $f(x) = \sqrt{x}$.

   (a) Graph $f(x) = \sqrt{x}$. Then determine the domain and range of $f(x)$.

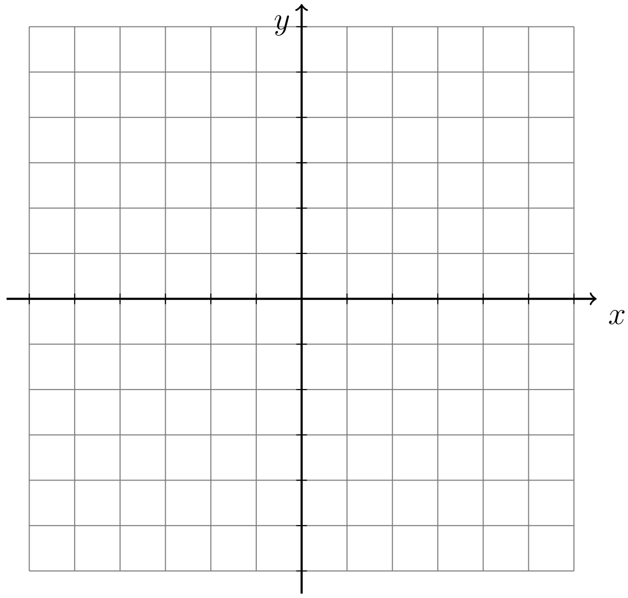


   (b) Graph and label $f(x) - 3$ on the coordinate system above. Then determine the domain and range of $f(x) - 3$.

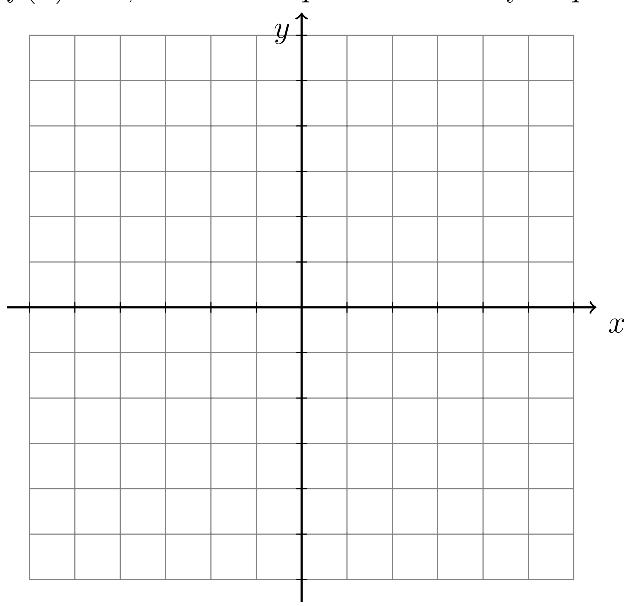
Instructions: Work together in groups of 3 or 4 to complete the following problems. You may be asked to share some of your solutions on the board.

1. Graph the given function and determine how many points need to be plotted to understand the shape of the graph.

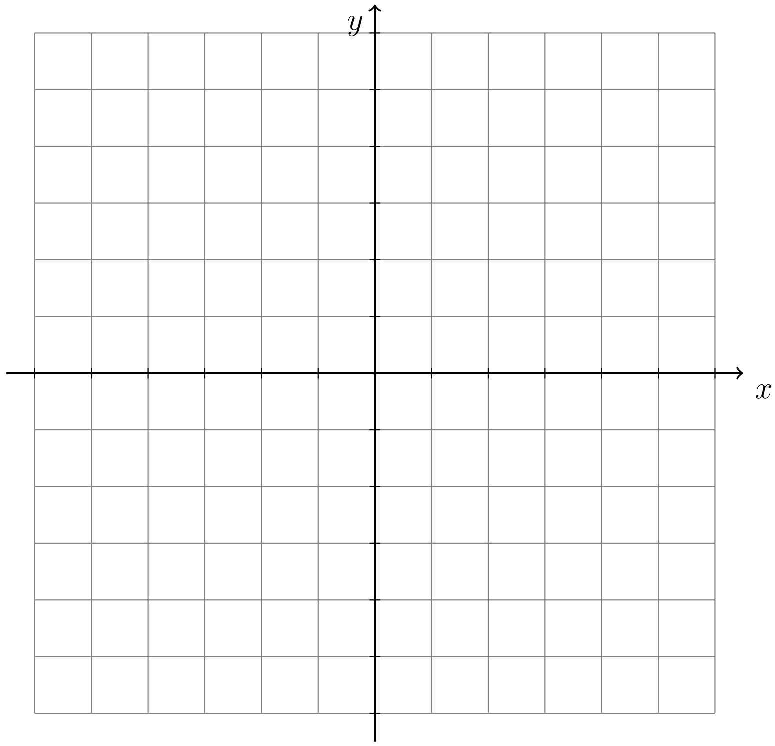
   (a) $f(x) = -3$, Number of points necessary to plot: ___________

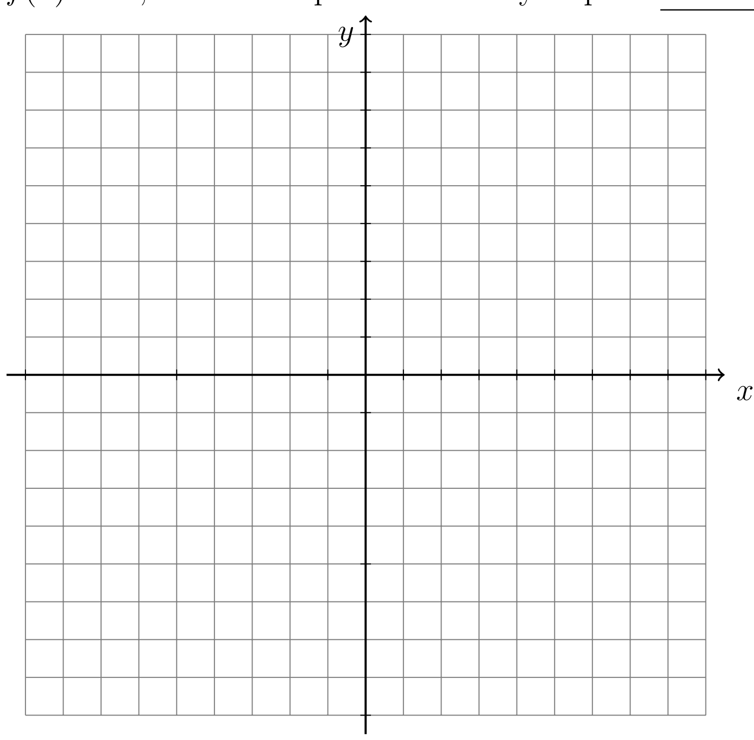
   (b) $f(x) = x$, Number of points necessary to plot: ___________

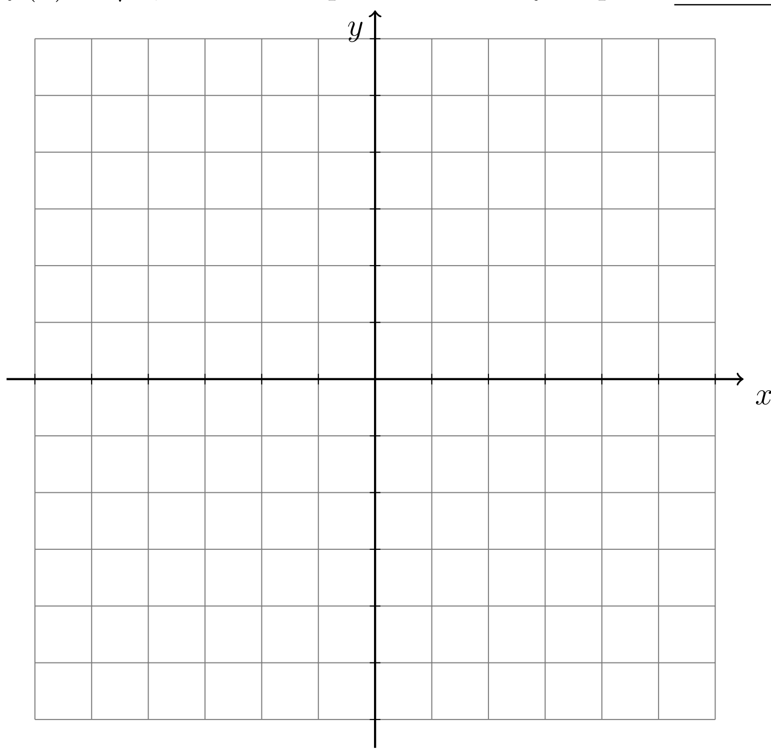
(c) $f(x) = x^2$, Number of points necessary to plot: ____________



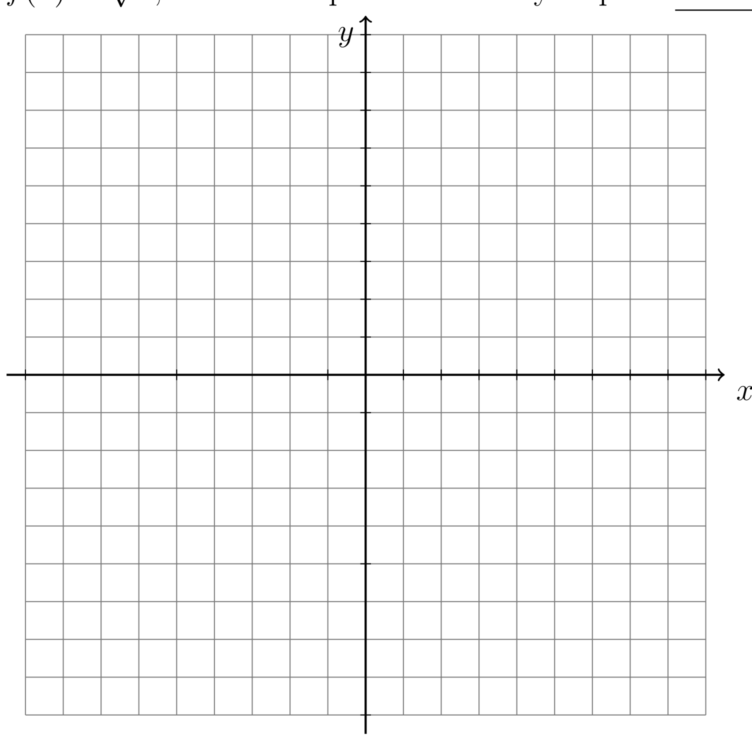
(d) $f(x) = x^3$, Number of points necessary to plot: ____________



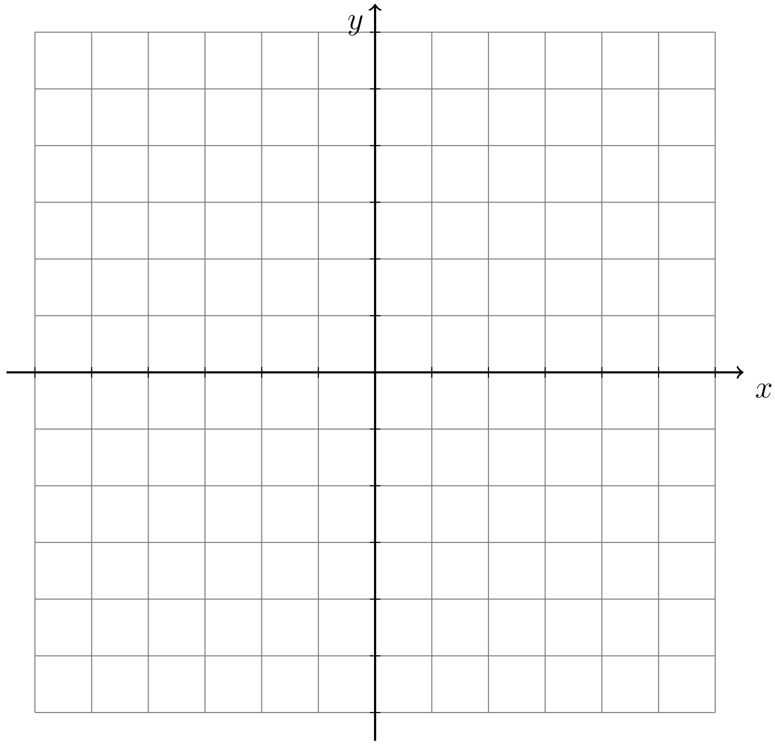
(e) $f(x) = \sqrt{x}$, Number of points necessary to plot: ____________



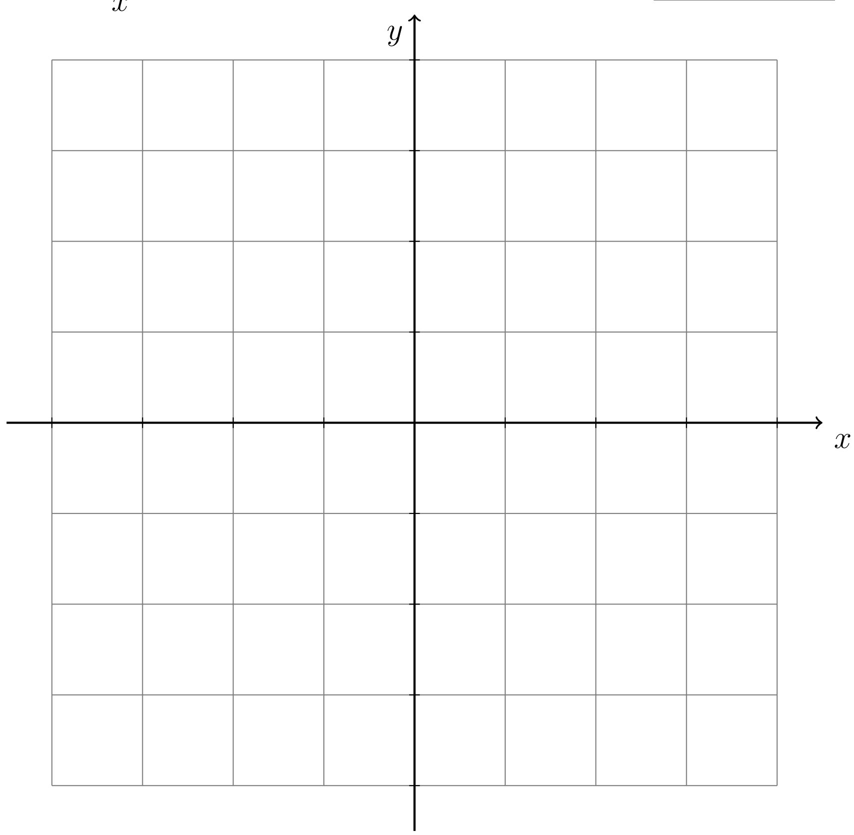
(f) $f(x) = \sqrt[3]{x}$, Number of points necessary to plot: ____________



(g) $f(x) = |x|$, Number of points necessary to plot: ____________

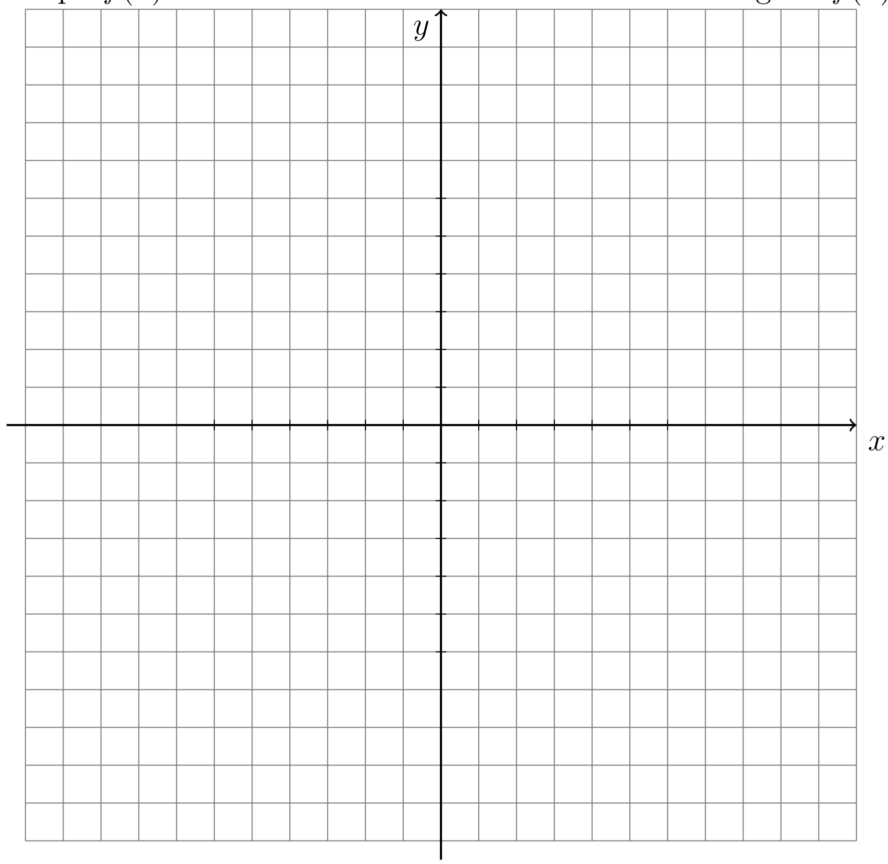


(h) $f(x) = \dfrac{1}{x}$, Number of points necessary to plot: ____________



2. For this problem, let $f(x) = x^3$.

   (a) Graph $f(x) = x^3$. Then determine the domain and range of $f(x)$.



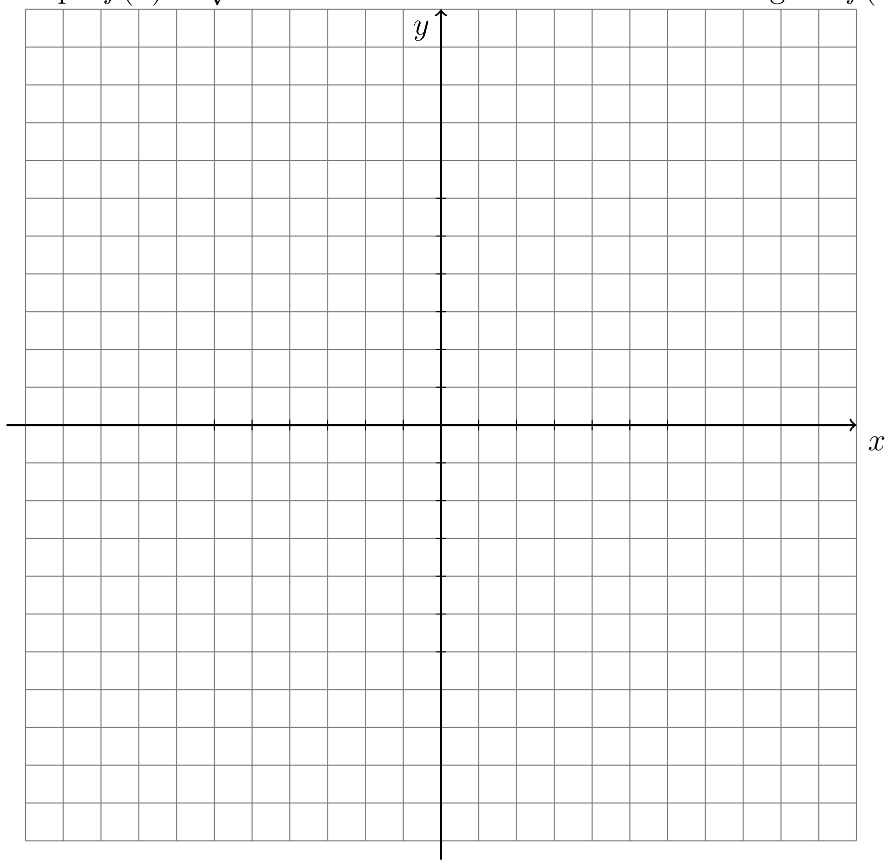
   (b) Use part (a) to graph $g(x) = x^3 + 2$ above. Then determine the domain and range of $g(x)$.

   (c) Use part (a) to graph $h(x) = (x - 2)^3$ above. Then determine the domain and range of $h(x)$.

   (d) Use part (a) to graph $j(x) = (x - 2)^3 + 2$ above. Then determine the domain and range of $j(x)$.

3. For this problem, let $f(x) = \sqrt{x}$.

   (a) Graph $f(x) = \sqrt{x}$. Then determine the domain and range of $f(x)$.



   (b) Use part (a) to graph $g(x) = \sqrt{x} - 5$ above. Then determine the domain and range of $g(x)$.

   (c) Use part (a) to graph $h(x) = \sqrt{x - 1}$ above. Then determine the domain and range of $h(x)$.

   (d) Use part (a) to graph $j(x) = \sqrt{x - 1} - 5$ above. Then determine the domain and range of $j(x)$.

4. In words, explain how the graph of $f(x) = |x - 4| + 37$ is different from the graph of $g(x) = |x|$.

5. Write an equation for the function that has been transformed.

   (a) $f(x)$ looks like $g(x) = x^2$ after it has been transformed by shifting 3 units to the right and 4 units up. Write the equation of $f(x)$.

   (b) $f(x)$ looks like $g(x) = \sqrt{x}$ after it has been transformed by shifting 5 units to the left and 1 unit down. Write the equation of $f(x)$.

   (c) $f(x)$ looks like $g(x) = \dfrac{1}{x}$ after it has been transformed by shifting 2 units to the left and 5 units up. Write the equation of $f(x)$.

6. If the point $(-2, 5)$ is on the graph of $y = f(x)$, find the corresponding point on the graph of $y = f(x + 2) - 4$

7. Determine the domains and ranges of $f(x) = \sqrt{x}$, $g(x) = \sqrt{x+5}$, and $h(x) = \sqrt{x} - 7$.

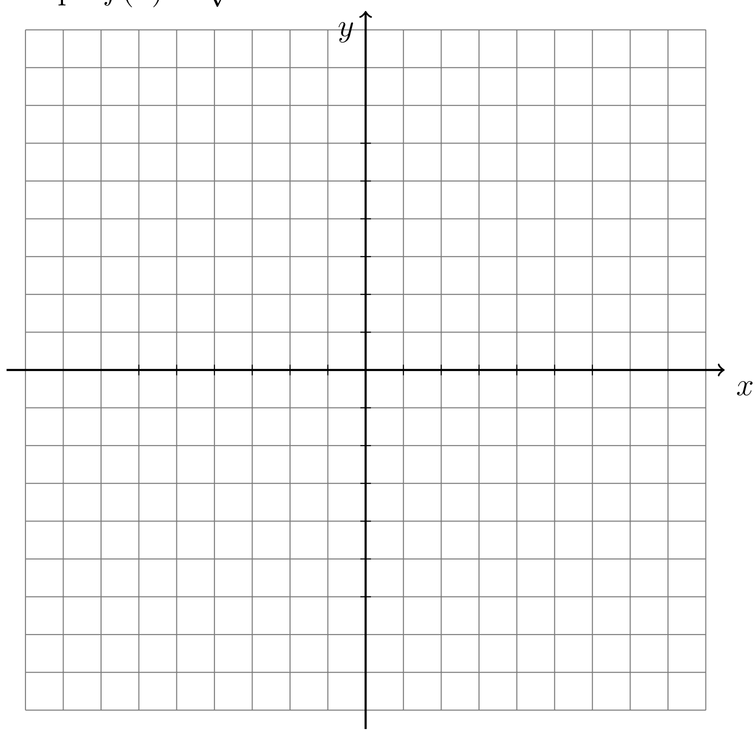
8. Determine the domains and ranges of $f(x) = x^2$, $g(x) = (x-3)^2$, and $h(x) = x^2 - 4$.

9. Determine the domains and ranges of $f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{1}{x+2}$, and $h(x) = \dfrac{1}{x} - 2$.

Watch the Pre-Class videos for Section 1.6B and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. For this problem, let $f(x) = \sqrt{x}$.

   (a) Graph $f(x) = \sqrt{x}$. Then determine the domain and range of $f(x)$.

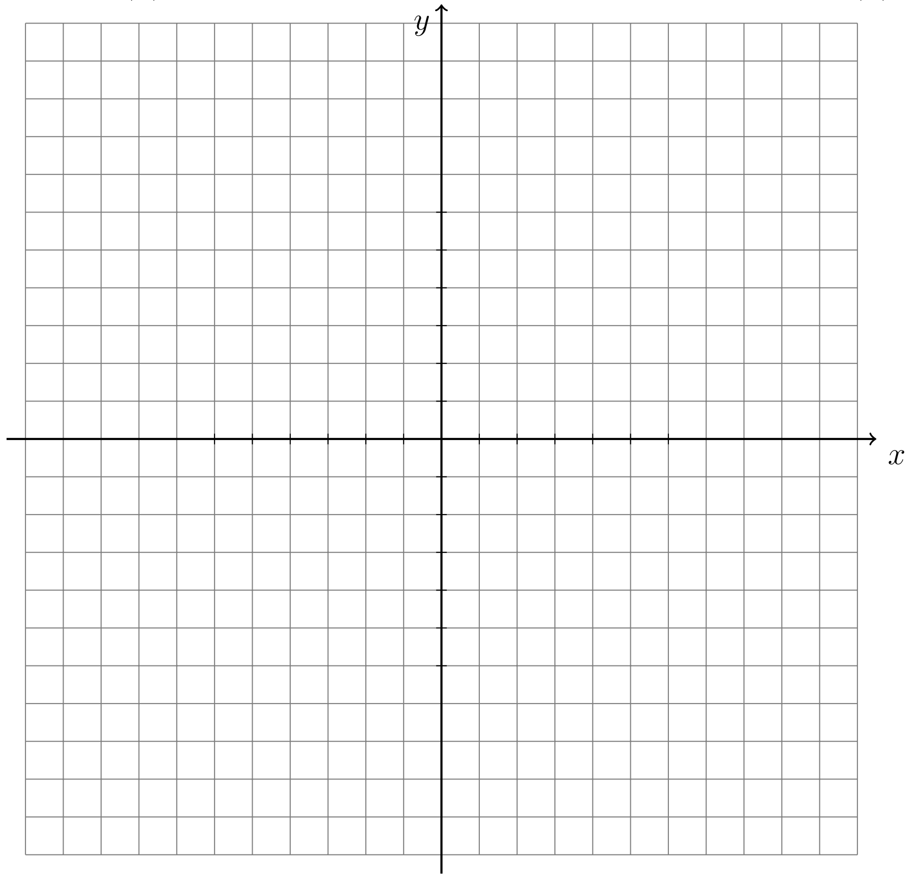
   

   (b) Graph and label $-f(x)$ on the coordinate system above. Then determine the domain and range of $-f(x)$.

   (c) Graph and label $f(-x)$ on the coordinate system above. Then determine the domain and range of $f(-x)$.

2. For this problem, let $f(x) = x^3$.

(a) Graph $f(x) = x^3$. Then determine the domain and range of $f(x)$.



(b) Graph and label $2f(x)$ on the coordinate system above. Then determine the domain and range of $2f(x)$.

(c) Graph and label $f(2x)$ on the coordinate system above. Then determine the domain and range of $f(2x)$.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

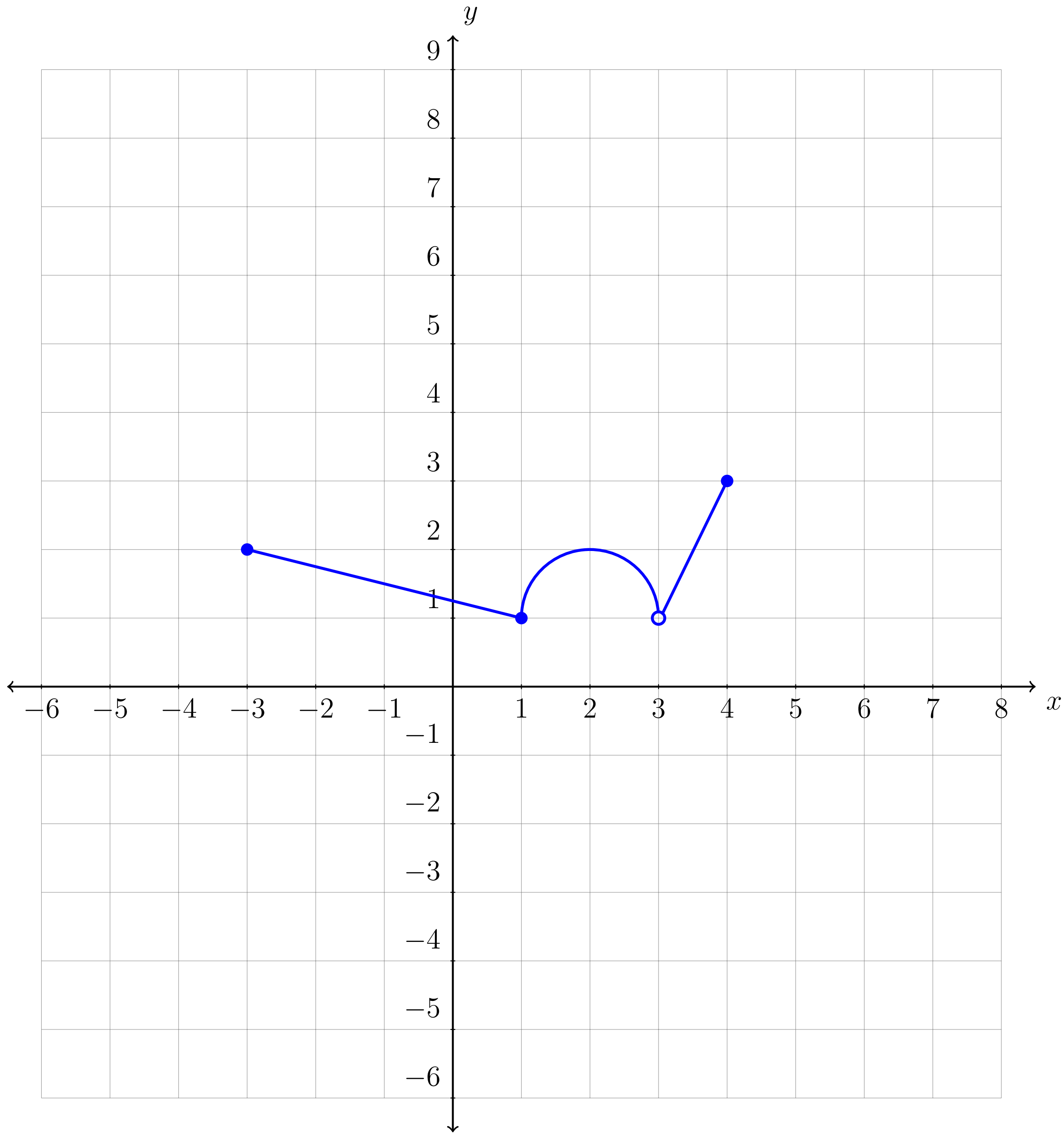
1. Use the graph of $y = f(x)$ below to graph the given functions.

   (a) $y = \frac{1}{3}f(x)$

   (b) $y = 3f(x)$

   (c) $y = f(2x)$

   (d) $y = f(\frac{1}{2}x)$

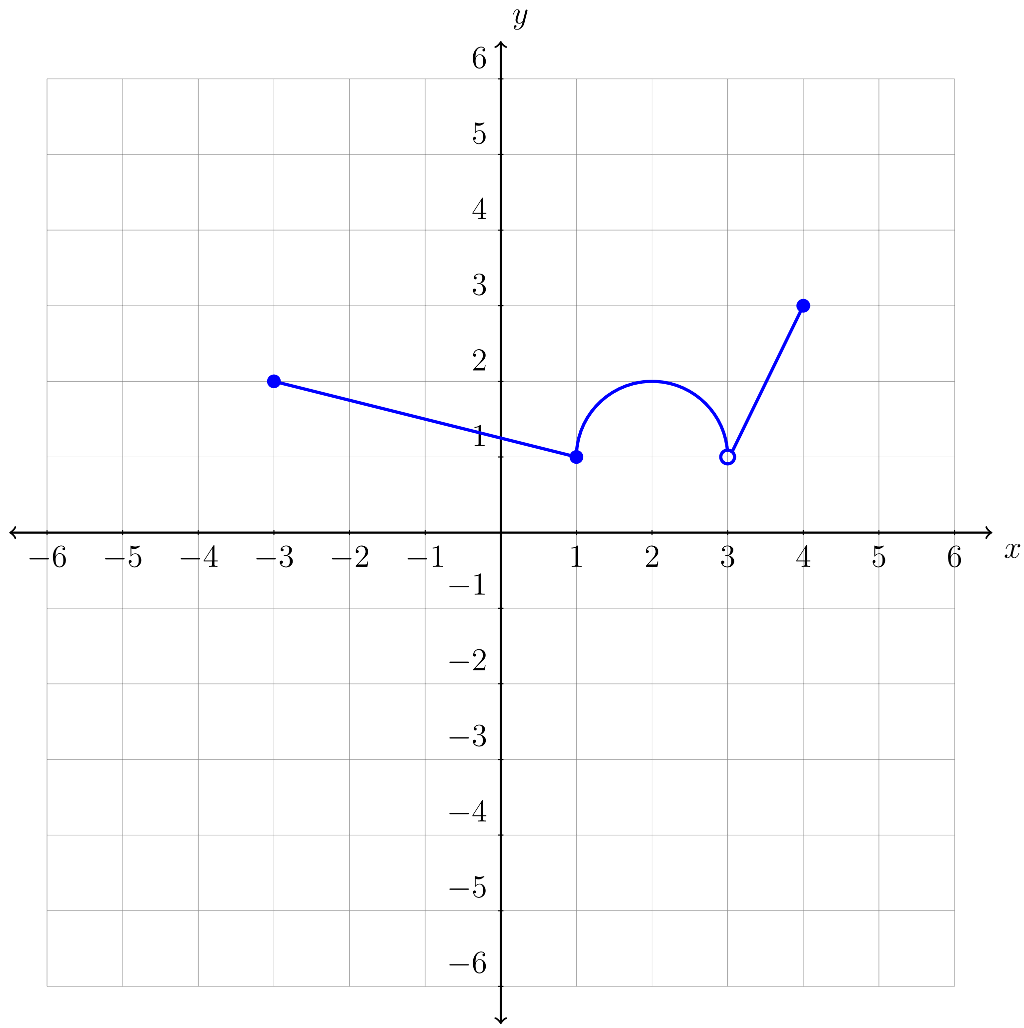


2. Use the graph of $y = f(x)$ below to graph the given functions.

   (a) $y = -f(x)$

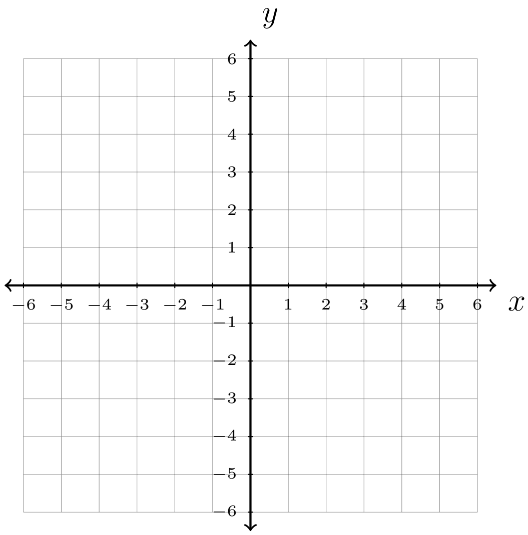
   (b) $y = f(-x)$

   (c) $y = -f(-x)$

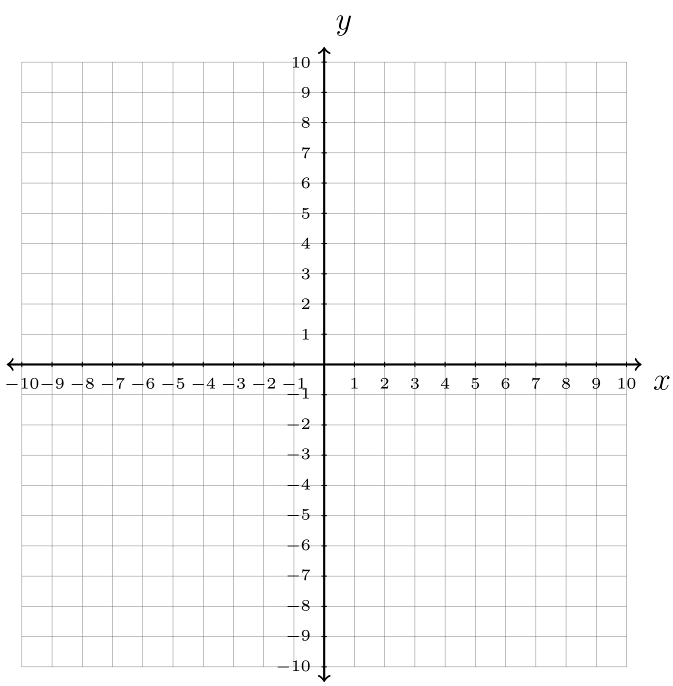


3. Identify and sketch the *parent* function of each of the following functions. Then use the transformation rules to sketch their graphs.

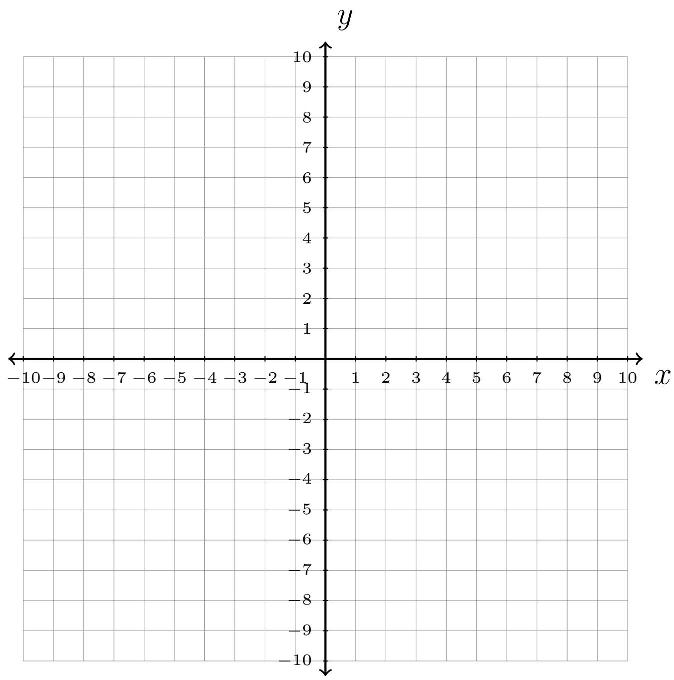
   (a) $f(x) = \sqrt{2x+4} - 1,$ parent function:



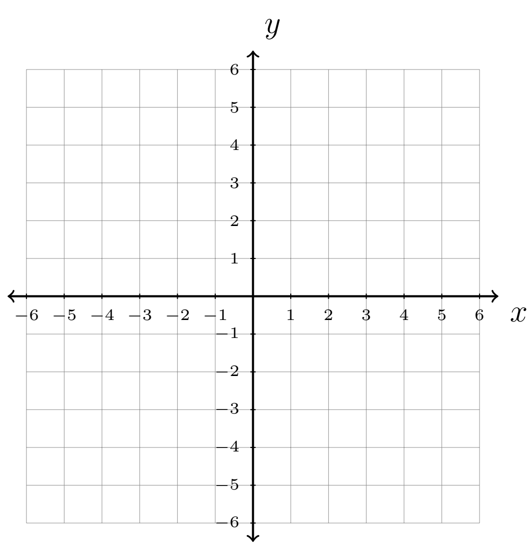
   (b) $g(x) = (-x+1)^3$, parent function:



(c) $h(x) = \sqrt[3]{8x} - 2$, parent function:



(d) $k(x) = 3 - \dfrac{1}{(x+2)}$, parent function:



4. Write a function $f(x)$ based on the given parent function and transformations in the given order.

   (a) $g(x) = x^2$

      i. Shift 4 units to the left.
      ii. Reflect across the y-axis.
      iii. Shift upward 2 units.

   (b) $g(x) = \sqrt{x}$

      i. Shift 1 unit to the left.
      ii. Stretch horizontally by a factor of 4.
      iii. Reflect across the x-axis.

   (c) $g(x) = \dfrac{1}{x}$

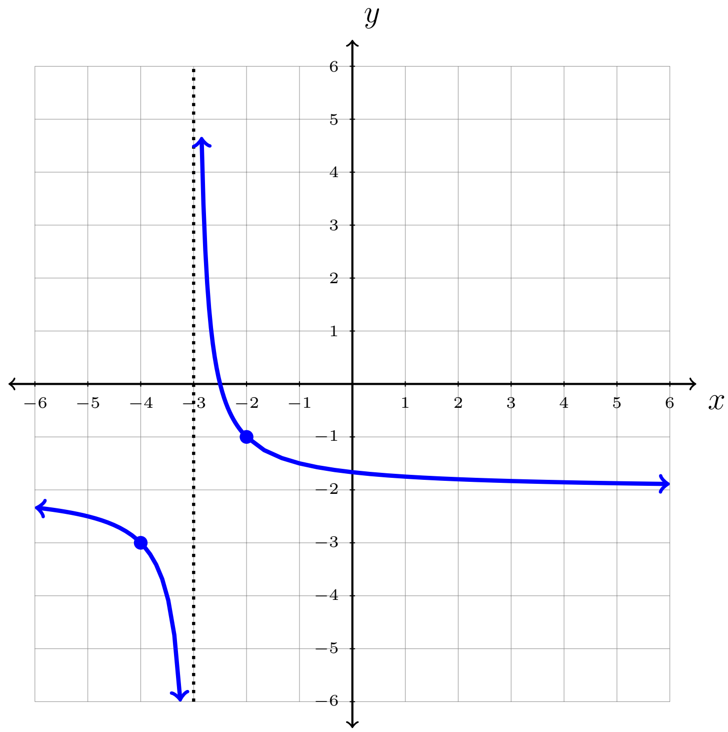
      i. Stretch vertically by a factor of 2.
      ii. Reflect across the x-axis.
      iii. Shift downward 3 units.

   (d) $g(x) = |x|$

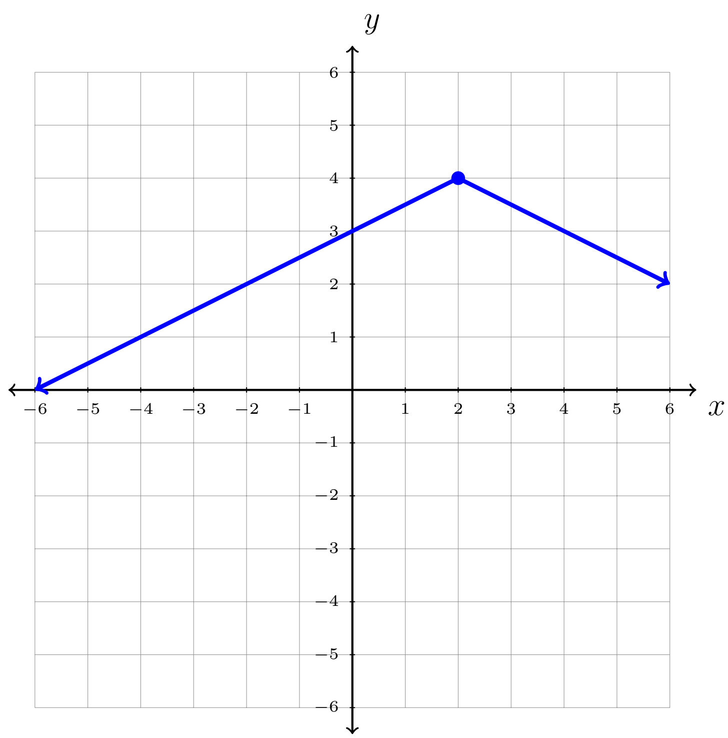
      i. Shift 3 units to the right.
      ii. Shrink horizontally by a factor of $\dfrac{1}{3}$.
      iii. Reflect across the $y - axis$.

5. Use transformations on the basic parent functions to write an equation $y = f(x)$ that would produce the given graph.

(a)



(b)



6. In words, explain how the graph of $f(x) = -\frac{1}{2}(x-4)^2 + 3$ is different from the graph of $g(x) = x^2$. (Note: List the transformations in the correct order.)

7. If the point $(-2, 5)$ is on the graph of $y = f(x)$, find the corresponding point on the graph of $y = -3f(7x) - 4$

Watch the Pre-Class videos for Section 1.7 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Determine whether the function f is even, odd, or neither.

$$f(x) = 4x^3 - x$$

2. Evaluate the function for the given values of x.

$$f(x) = \begin{cases} x + 3 & \text{for } x < -1 \\ x^2 & \text{for } -1 \le x < 2 \end{cases}$$

   (a) $f(-2) =$

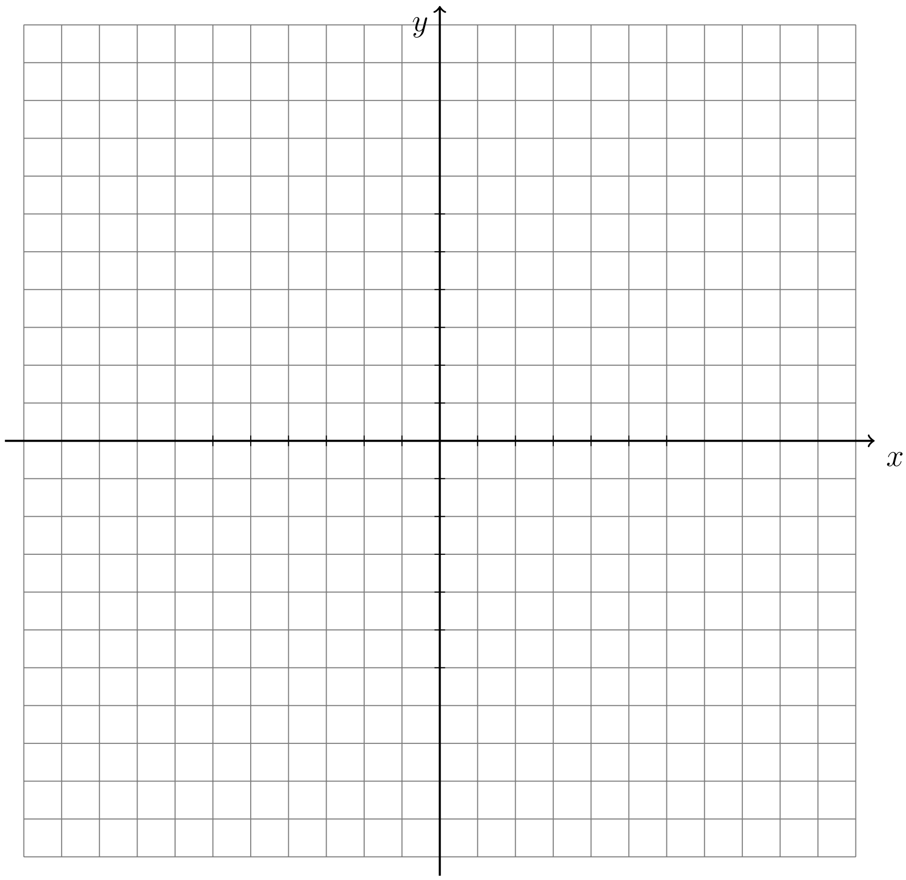
   (b) $f(-1) =$

   (c) $f(0) =$

   (d) $f(-5) =$

3. Graph the piece-wise defined function.

$$f(x) = \begin{cases} 2 & \text{for } x \leq -1 \\ 2x & \text{for } x > -1 \end{cases}$$

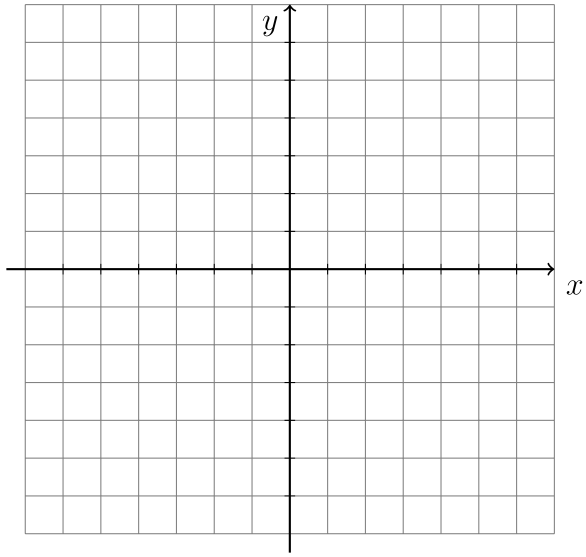


Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Graph each of the following piecewise functions.

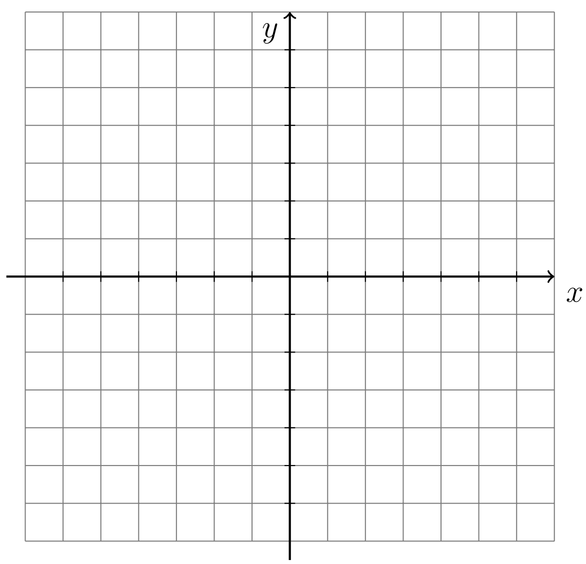
(a)

$$f(x) = \begin{cases} x+5 & \text{if } x < -2 \\ -4 & \text{if } x \geq 2 \end{cases}$$



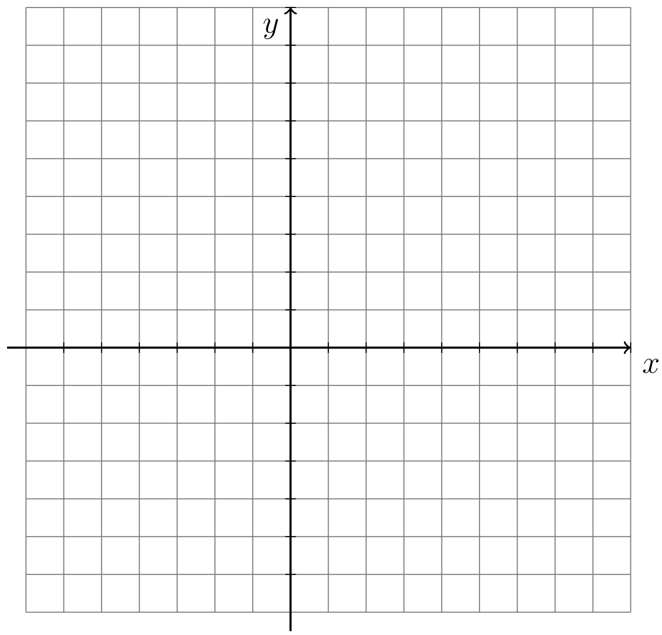
(b)

$$f(x) = \begin{cases} 2x+1 & \text{if } x < 1 \\ -2x+3 & \text{if } x \geq 1 \end{cases}$$



(c)

$$f(x) = \begin{cases} 5 & \text{if } x < -2 \\ \frac{1}{2} & \text{if } -2 \leq x \leq 6 \\ -2x + 10 & \text{if } x > 6 \end{cases}$$



2. Evaluate the piecewise function for the given values of x.

(a)

$$f(x) = \begin{cases} x + 5 & \text{if } x < -2 \\ -4 & \text{if } x \geq 2 \end{cases}$$

$f(3) =$ $f(-4) =$ $f(-2) =$

(b)

$$f(x) = \begin{cases} x - 1 & \text{if } x < -2 \\ 2x - 1 & \text{if } -2 < x \leq 4 \\ -3x + 8 & \text{if } x > 4 \end{cases}$$

$f(-1) =$ $f(-4) =$ $f(5) =$

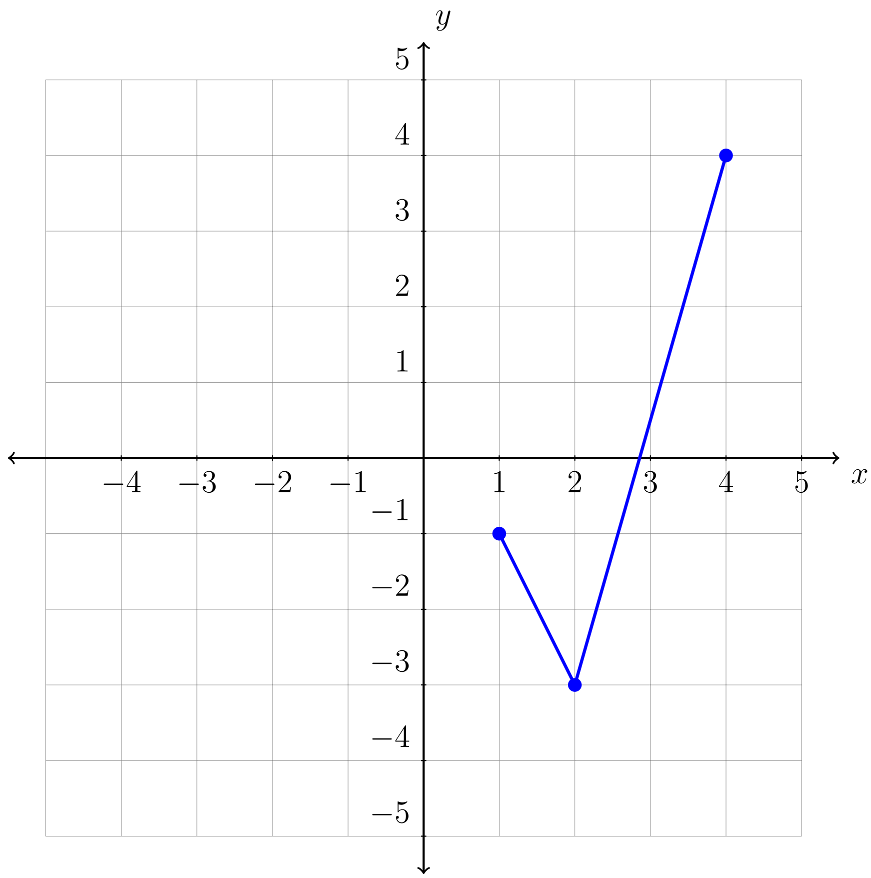
3. Determine if the function is even, odd, or neither.

   (a) $f(x) = 4x^2 - 3|x|$

   (b) $f(x) = 4x^3 - 2x$

   (c) $f(x) = 4x^2 + 2x - 3$

4. Part of the graph of $f(x)$ is shown below. The graph for positive values of x is shown while the portion of the graph for negative values of x is missing.



(a) Sketch the portion of the graph that is missing given that $f(x)$ is an even function.

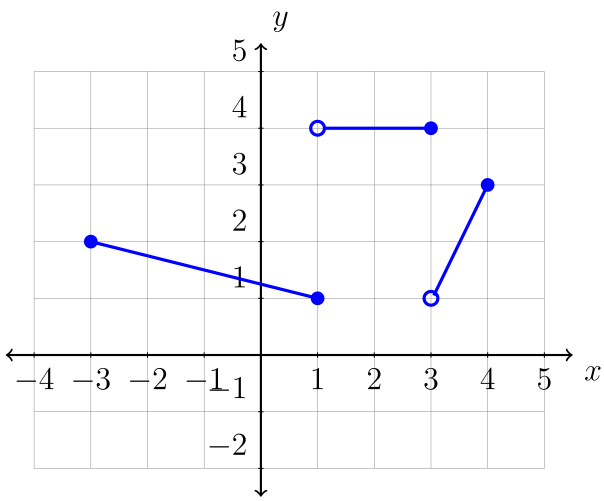
(b) Using your finished graph from part (a), determine the domain and range of the function $f(x)$. Give your answers in interval notation.

Domain: Range:

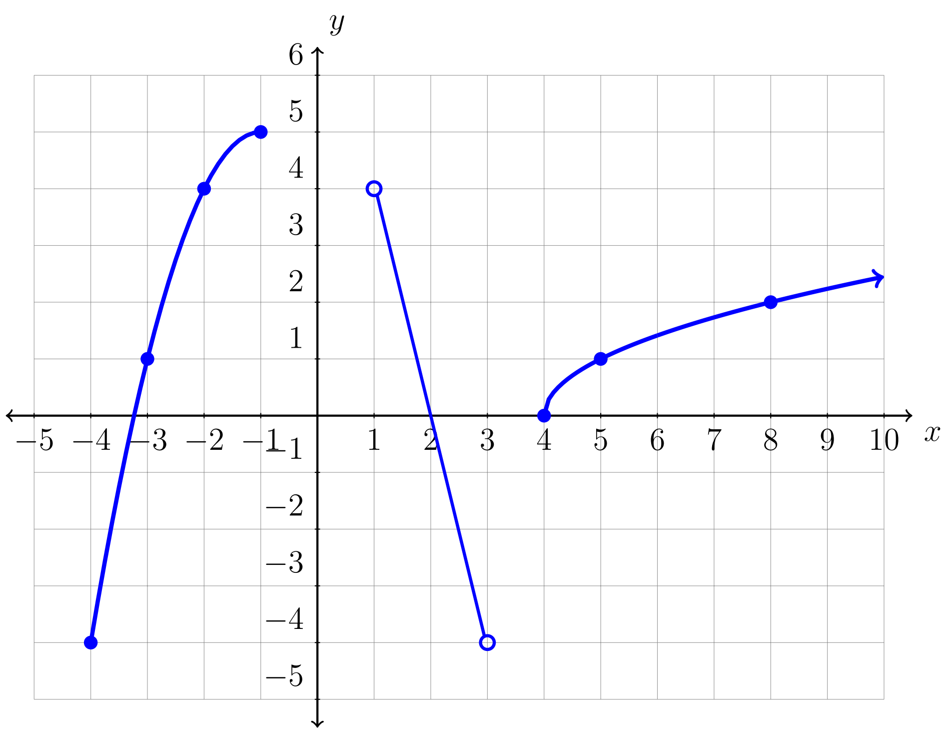
(c) Find the average rate of change of $f(x)$ on the interval $[1, 4]$.

5. For each of the following graphs, give equations determining the piecewise function.

(a)

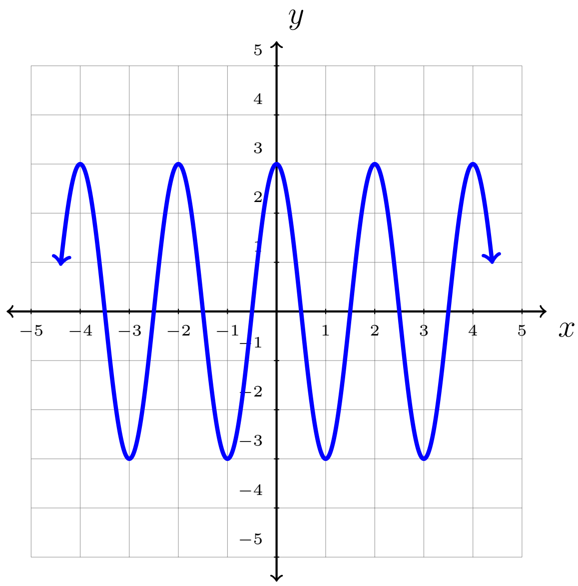


(b)

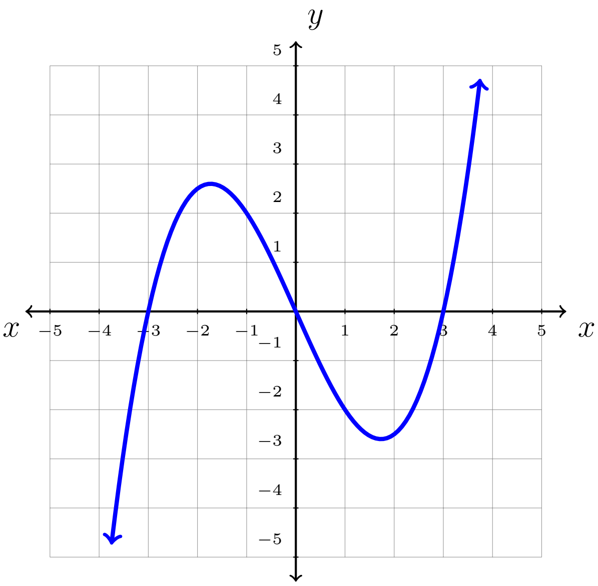


6. Determine whether or not the following graphs display odd symmetry, even symmetry, or neither.

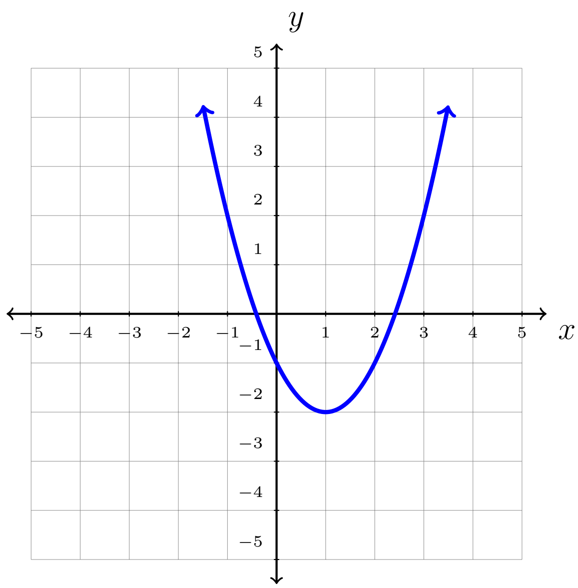
(a)



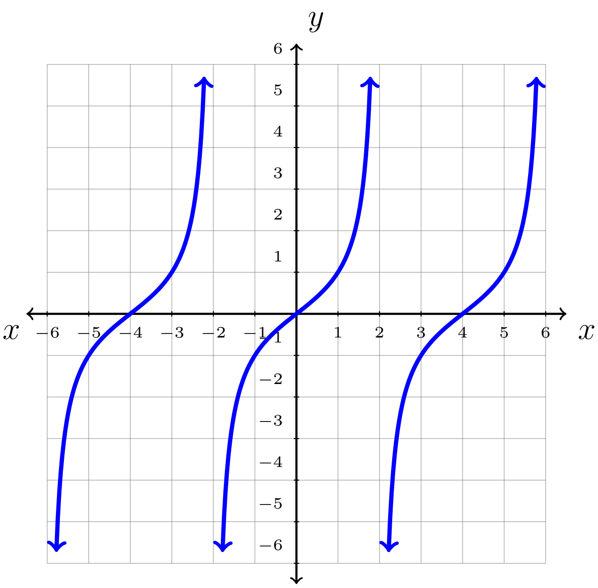
(c)



(b)



(d)



7. Write the function $f(x) = |x|$ as a piecewise defined function with two linear function pieces.

Watch the Pre-Class videos for Section 1.8 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Find the following values for the functions $f(x) = |x-3|$, $g(x) = x^3$, $h(x) = \sqrt{x+1}$.

   (a) $(f+g)(2) =$

   (b) $\dfrac{h}{f}(8) =$

   (c) $g(h(3)) =$

   (d) $g(f(8)) =$

2. Use $f(x) = x^2 + 1$ and $g(x) = x + 5$ to determine $f(g(x))$ and the domain of $f(g(x))$.

3. Given $f(x) = 7x + 2$. Find the difference quotient, $\dfrac{f(x+h) - f(x)}{h}$.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Given $f(x) = 5x^2 + 2x - 3$ and $g(x) = x + 3$.
   (a) Find $(f \circ g)(x)$.

   (b) Find $(g \circ f)(x)$.

   (c) Find $(f \circ g)(1)$.

   (d) Find $(g \circ f)(1)$.

2. Given $f(x) = x^2$ and $g(x) = \sqrt{x}$.

   (a) Determine the domains of $f(x)$ and $g(x)$.

   (b) Find $(f \circ g)(x)$ and simplify completely.

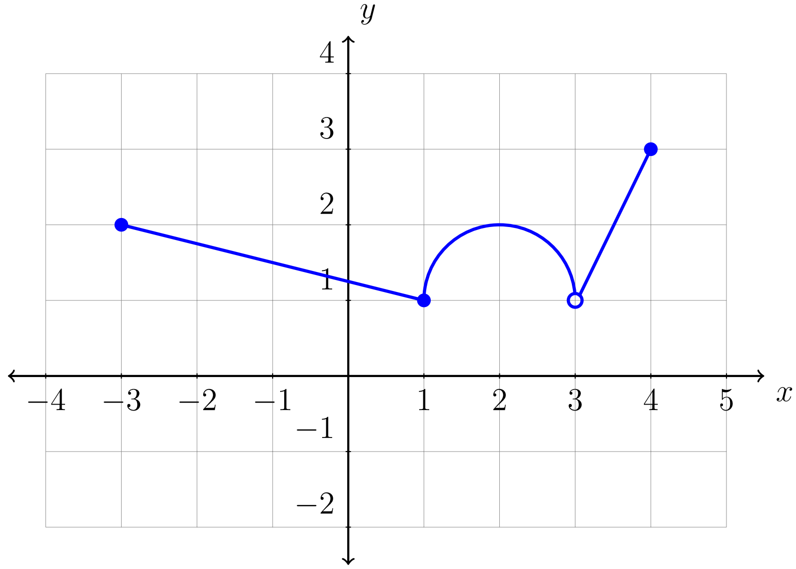
   (c) Determine the domain of $(f \circ g)(x)$. Keep in mind that the domain of a function is the collection of x-values that can be plugged into the function.

   (d) Find $(g \circ f)(x)$ and simplify completely.

   (e) Determine the domain of $(g \circ f)(x)$.

   (f) What do you notice about $(f \circ g)(x)$ and $(g \circ f)(x)$? What do you notice about their domains? Does the domain of the inside function affect the domain of function composition?

3. Let $f(x) = x^2 - 1$, $g(x)$ be given by the graph below, and $h(x)$ be given by the table below.



x	$h(x)$
-3	2
0	4
1	5
3	-6

(a) Determine the $(f \circ g)(4)$.

(b) Determine the $(g \circ h)(-3)$.

(c) Determine the $(h \circ f)(1)$..

(d) Determine the $(g \circ f)(2)$.

4. Given $f(x) = 4x - 9$ and $g(x) = \sqrt{x+6}$

   (a) Find $\left(\frac{f}{g}\right)(x)$ and determine its domain.

   (b) Find $\left(\frac{g}{f}\right)(x)$ and determine its domain.

5. Given $f(x) = 2x^2 - 5x + 1$. Find the difference quotient, $\dfrac{f(x+h) - f(x)}{h}$.

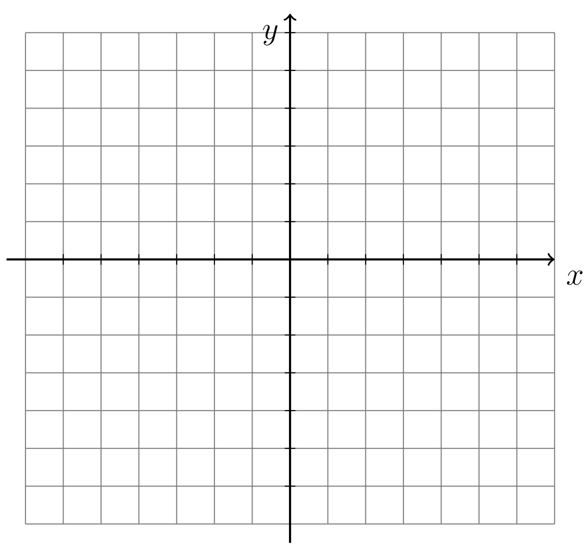
Watch the Pre-Class videos for Section 2.1A and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Given $f(x) = 4(x+2)^2 - 4$.

   (a) Identify the vertex.

   (b) Determine the x-intercept(s).

   (c) Sketch the function $f(x)$.



   (d) Determine an equation for the axis of symmetry.

2. Complete the square and use that work to find the vertex of the graph of the quadratic function.
$$y = 5x^2 - 30x + 49$$

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Given $f(x) = (x+2)^2 - 1$.

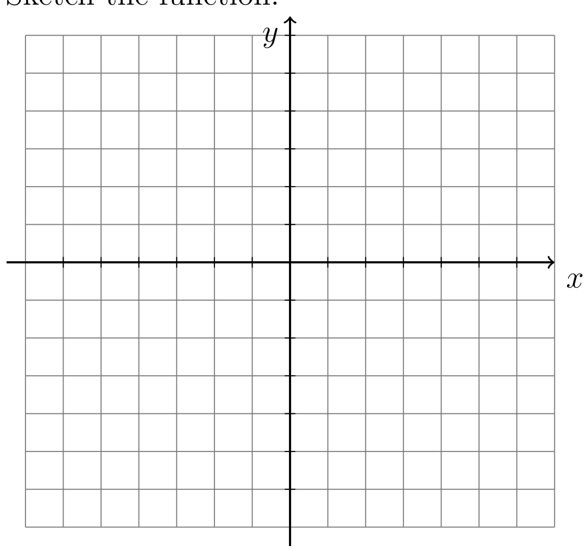
   (a) Determine whether the graph of the parabola opens upward or downward.

   (b) Identify the vertex.

   (c) Determine the x-intercept(s).

   (d) Determine the y-intercept.

   (e) Sketch the function.



   (f) Determine the axis of symmetry.

2. Find the quadratic function with the given vertex and point. Put your answer in standard (vertex) form.

   (a) Vertex $(0, 0)$ passing through $(-2, 8)$.

   (b) Vertex $(2, 0)$ passing through $(1, 3)$.

   (c) Vertex $(2, 5)$ passing through $(3, 7)$.

   (d) Vertex $(-3, 4)$ passing through $(0, 0)$.

3. In each problem below, complete the square and then use that work to find the vertex of the graph of the quadratic function.

   (a) $y = x^2 + 4x$

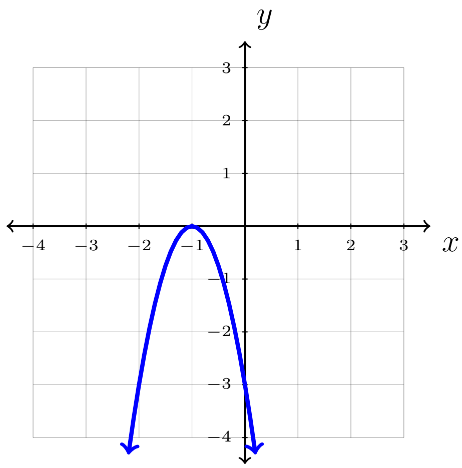
   (b) $y = x^2 - 2x + 2$

   (c) $y = 6x - 10 - x^2$

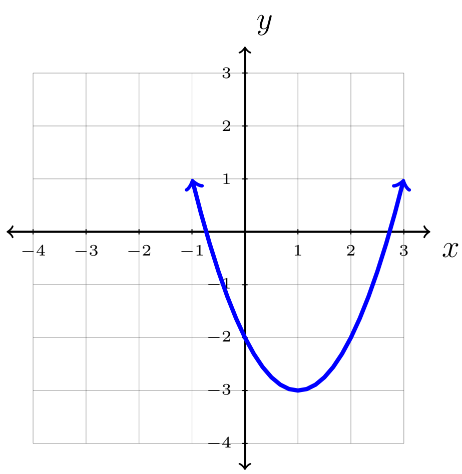
   (d) $y = -2x^2 + 16x - 29$

4. Find the equation for the parabolas below. Put your answers in standard form.

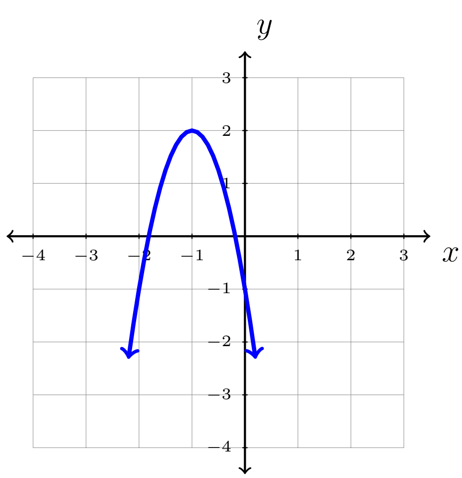
   (a) $y =$



   (b) $y =$



   (c) $y =$



5. Solve the following equations.

   (a) $x^2 - 10x + 8 = 0$

   (b) $3x^2 + 6x = 4$

   (c) $x - 3 = \sqrt{1 + 2x^2}$

Watch the Pre-Class videos for Section 2.1B and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. You have a 500 foot roll of fencing and a large field. You want to construct a rectangular playground area. What are the dimensions of the largest such playground? What is the largest area?

   (a) Draw a picture of the rectangular playground and label the side lengths using your own variables.

   (b) Using your variables, write an equation that represents the area of the playground.

   (c) Your previous answer should have two variables. Use 500 to represent the perimeter of the the playground and solve for one of your variables. (Note: all four sides of the rectangle will be used.)

   (d) Rewrite your area equation in terms of only one variable and simplify. The result should be a quadratic function.

(e) To determine the maximum area possible for the playground, what part of the parabola do you need to locate?

(f) Find that part of the parabola (from your previous answer).

(g) What is the largest area the playground could be?

(h) What are the dimensions of that playground?

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. An object is thrown upward. The height, h, in feet, at time t, in seconds, is given by the formula $h(t) = -16t^2 + 96t$.

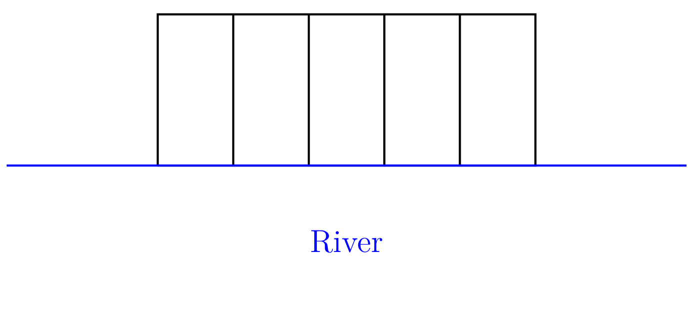
   (a) Determine the number of seconds required for it to hit the ground.

   (b) Determine the maximum height of the object.

   (c) Determine the time required for the object to reach a height of 50 feet on its way up.

2. Farmer Ed has 700 feet of fencing to enclose a rectangular plot that borders on a river. If farmer Ed does not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed? Include units of measurement in your answer.

3. A farmer wants to build a rectangular pen along a straight river. She wants to divide the pen into 5 equal rectangular pieces as shown in the picture. What is the largest area she can enclose with 3,000 feet of fencing?



4. A diver jumps vertically off a diving board at time $t = 0$. The diver's height h above the water (in feet), t seconds later is given by the formula $h(t) = -16t^2 + 6t + 5$.

   (a) Determine the number of seconds t required for the diver to reach the water ($h = 0$)

   (b) How many seconds after jumping is the diver at her maximum height above the water?

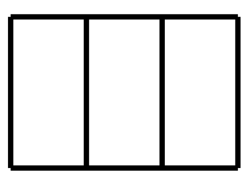
   (c) Determine the height of the diving board above the water.

5. A rectangle is drawn in the first quadrant with two sides on the coordinate axes and the corner opposite the origin on the line $y = -2x + 3$. Answer the following:

   (a) Write the area of the rectangle as a function of x.

   (b) For all first quadrant points on the given line, determine the maximum area enclosed by the rectangle.

6. Farmer Brown has 400 yards of fencing with which to build a rectangular corral. He wants to divide it evenly into three pens, so he adds in two divider fences, as shown below. Answer the following:



   (a) Write the area of the corral as a function of x:

   (b) Determine the maximum area enclosed by the corral.

7. We form two shapes using 28 inches of string. One shape is a square with sides of length z. The other is a rectangle with width x and height $3x$.

   (a) Draw a picture showing these two shapes, with their sides labeled, **and** write a formula for the combined area A of the two shapes. (Your formula should use both x and z in it.)

   (b) We only have a small surface to work with. Determine the dimensions of each shape that will minimize the combined area of both shapes together.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. The value of a newly purchased equipment is a linear function of time. A company purchases a piece of equipment for $40,000. After 5 years, the equipment loses 15% of its value. Answer the following:

   (a) Determine the value of the equipment after 5 years.

   (b) Express the value V (in dollars) of the equipment as a function of time t (in years) since purchase.

   (c) Determine the total time (in years) it will take for the machine to be worth 45% of its original value.

2. Give the coordinates of all of the points that lie on *both* the parabola $y = x^2 + 1$ *and* the line $y = 2x + 4$. Use the following steps to answer the question.

   (a) How can one describe an arbitrary point on the line $y = 2x + 4$ as an ordered pair?

   (b) How can one describe an arbitrary point on the line $y = x^2 + 1$ as an ordered pair?

   (c) How do these descriptions help you find the intersection.

3. You have 50 cm of wire, and you have to use part of this wire to make a rectangle that's twice as long as it is wide, and the rest of the wire (if there is any left) to make a square. What should the dimensions of the shapes be if you want the total area to be as small as possible?

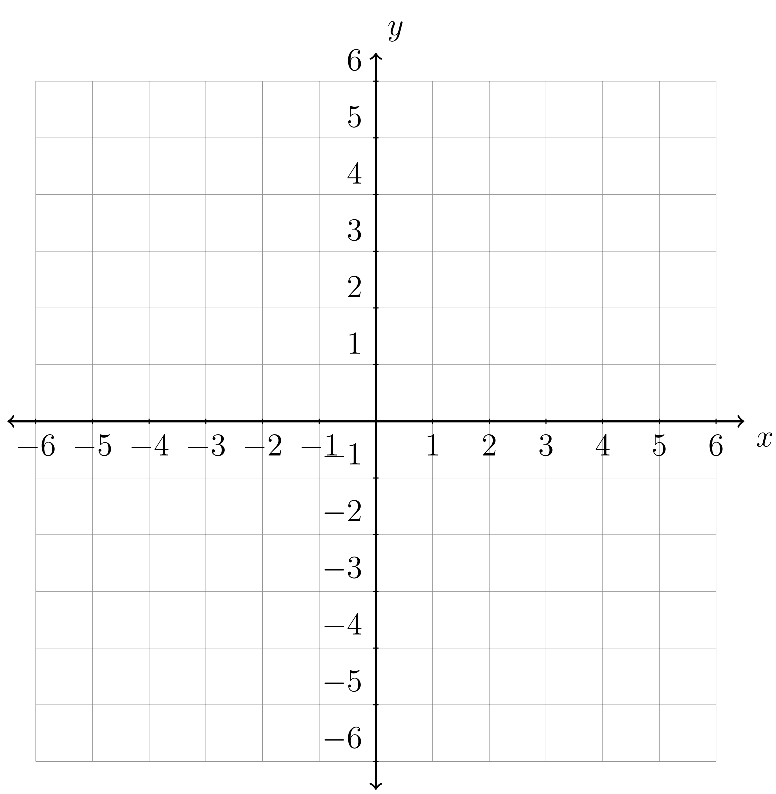
4. Find the point (x, y) lying on the parabola $y = x^2 + 2x + 1$ so that the average rate of change on the interval between x and 2 is zero.

5. On the coordinate axes below, draw a function whose domain is

$$[-2, 1] \cup (2, 3] \cup \{5\}$$

and whose range is

$$[-6, -4) \cup [-1, 1] \cup \{2\} \cup (3, 4].$$



6. Find the point P on the graph of $\sqrt{x}$ such that the line through P and $(1,1)$ has slope $\dfrac{4}{7}$.

7. A business forms a model of its widget sales via a pricing function $p(x) = 400 - \frac{70}{8}x$. Here, x is the number of widgets sold and $p(x)$ is the sales price in dollars per widget.

   (a) Find the revenue function $R(x)$ for this business (revenue is total sales).

   (b) Find the number x sold that will maximize revenue. (This will be an unrealistic fraction, but do not round.)

   (c) What is the maximum revenue?

   (d) What is the price $p(x)$ that yields maximum revenue?

Chapter 2

Exponential and Logarithmic Functions

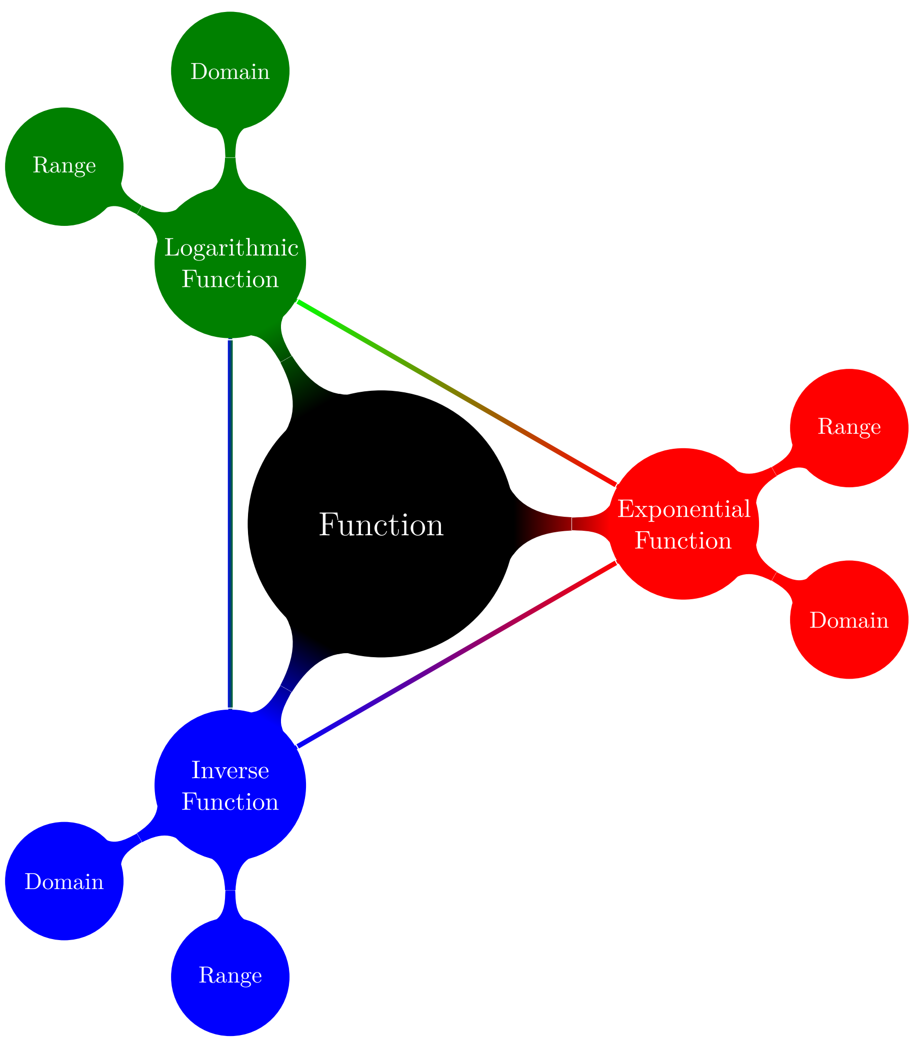
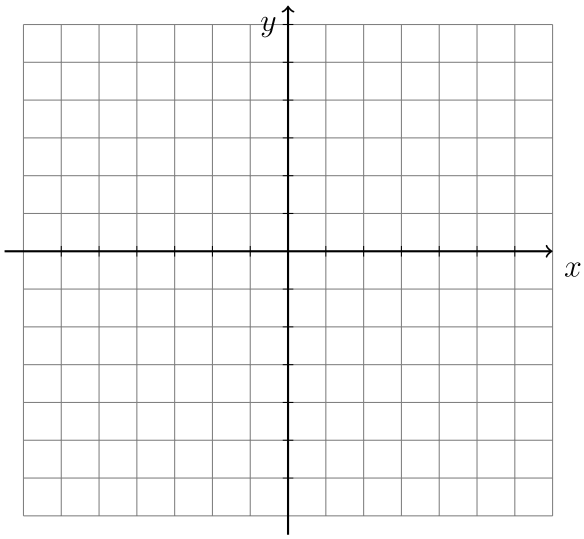


Figure 2.1: Topics for the second section of the course.

Watch the Pre-Class videos for Section 3.1 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Determine if $f(x) = -4x + 1$ is one-to-one both algebraically and graphically.

   (a) Graph $f(x) = -4x+1$ and use the graph to determine if $f(x)$ is one-to-one.

   (b) Algebraically determine if $f(x) = -4x + 1$ is one-to-one.

2. Let $f(x) = 5x + 4$ and $g(x) = \dfrac{x-4}{5}$.
   Use the theorem on inverse functions (function composition) to determine whether f and g are inverses. Show every step.

3. (1 point) Find the inverse function of $f(x) = \dfrac{8-x}{3}$.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Choose 3 of the following functions for your group to work with. You must choose x^2 and two other functions.

$$f(x) = x \qquad g(x) = x^2 \qquad h(x) = x^3 \qquad j(x) = \frac{1}{x} \qquad m(x) = \sqrt{x} \quad p(x) = \sqrt[3]{x}$$

2. Use transformations like shifting, stretching, compressing, and reflecting to transform each of your 3 functions. Write your 3 new functions.

3. For each of the 3 new functions, calculate the domain and range.

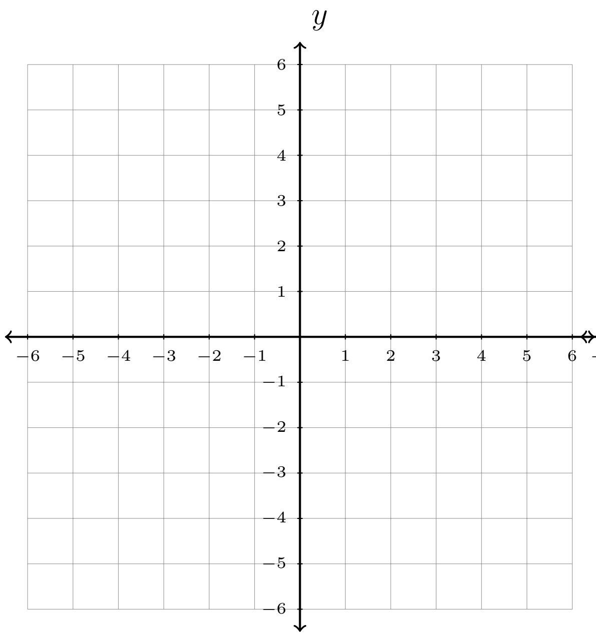
4. Determine *algebraically* if each of your functions are one-to-one.

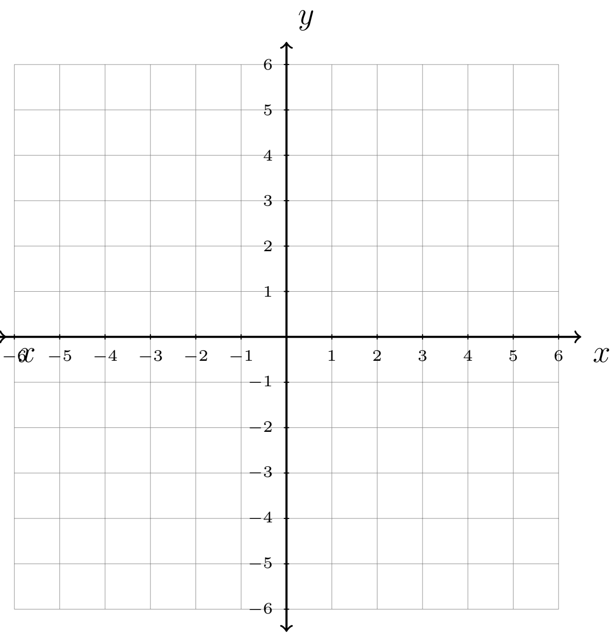
5. For each of your functions that are not one-to-one, make a domain restriction so that the function becomes one-to-one on the new domain.

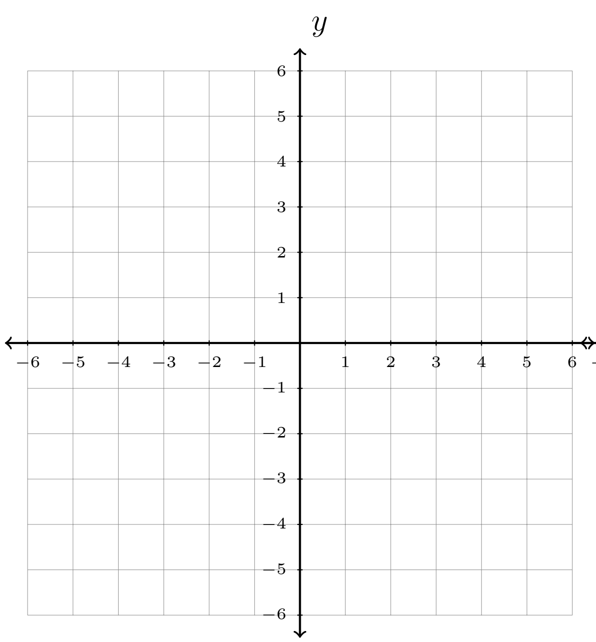
6. For each of your functions (including those with new domain restrictions), find their inverse functions.

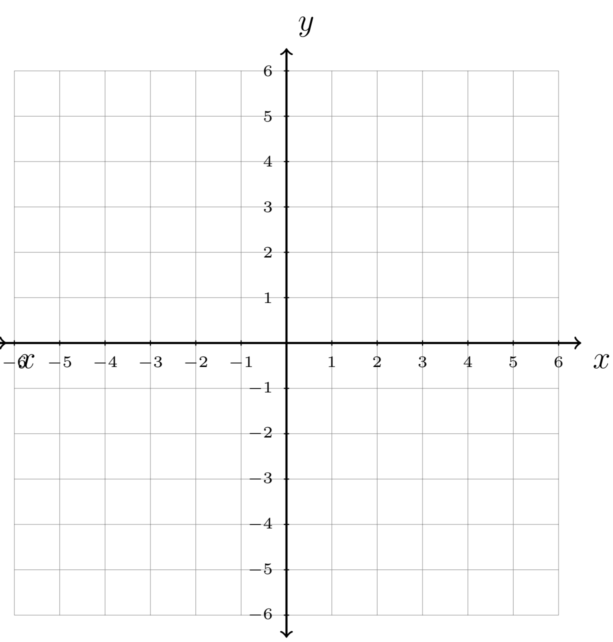
7. Use function composition to verify that the inverse functions you have described are truly inverses to your functions.

8. Graph your function and its inverse on the same coordinate axes. Check that they are properly symmetric to each other.







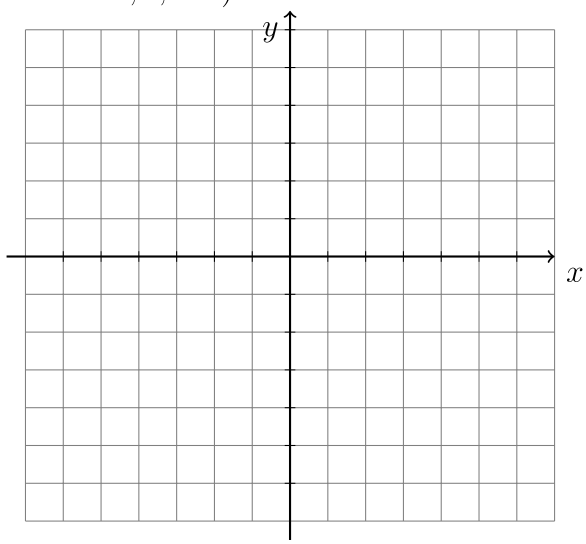


9. Repeat all of the above steps with a fourth, fifth, and even sixth function from the above list.

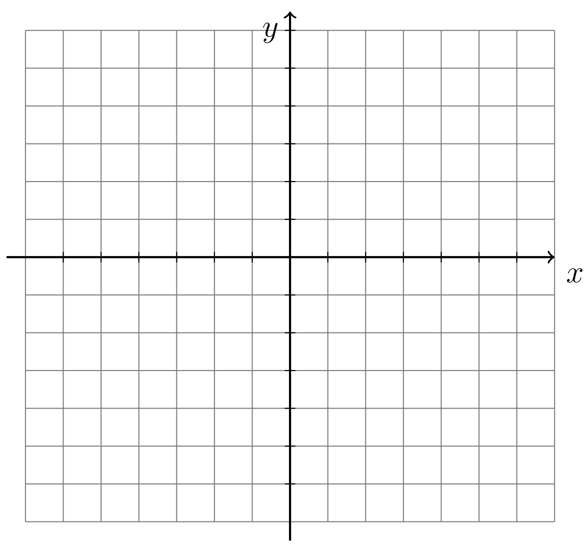
Watch the Pre-Class videos for Section 3.2 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Graph the following functions.

   (a) Graph $f(x) = \left(\frac{1}{4}\right)^x$ along with it's asymptote. (It is helpful to plot values for $x = 0, 1, -1$.)

   (b) Transform part (a) to graph $g(x) = \left(\frac{1}{4}\right)^{x+3} - 2$ along with it's asymptote.

2. Suppose that \$5000 is invested and pays 6.5% per year under the following compounding options. Determine the total amount in the account after 10 years with each option.

   (a) Compounded monthly

   (b) Compounded continuously

Instructions: Work together in groups of 3 or 4 to complete the following problems.

2.2.1 Exponential Functions

1. Which of the following equations represent exponential functions? Circle the exponential functions.

   $f(x) = 2x+1 \quad g(x) = -4^x \quad h(x) = 1^x \quad j(x) = 3(2)^x \quad m(x) = x^2 \quad p(x) = \left(\frac{3}{10}\right)^{x+3}$

2. Write two examples of exponential growth functions.

3. Write two examples of exponential decay functions.

4. Fill in the table of values for the function $f(x) = 3(2)^x$.

x	$f(x)$
-2	
-1	
0	
1	
2	

5. Write a function $f(x)$ based on the given parent function and transformations in the given order.

   (a) $g(x) = 3^x$

      i. Shift 4 units to the left.
      ii. Reflect across the y-axis.
      iii. Shift upward 2 units.

   (b) $g(x) = \left(\frac{1}{3}\right)^x$

      i. Shift 1 unit to the left.
      ii. Stretch horizontally by a factor of 4.
      iii. Reflect across the x-axis.

2.2.2 Solving Exponential Equations Solving an equation means to find the set of values that can be substituted for the variable, creating a true statement.

Example. To solve the equation, $\sqrt[3]{x} = 3$ we need to undo the operations happening to x. Remember, $\sqrt[3]{x} = x^{1/3}$.

$$\begin{aligned} x^{1/3} &= 3 \\ \left(x^{1/3}\right)^{3/1} &= 3^{3/1} \\ x^1 &= 3^3 \\ x &= 27 \end{aligned}$$

6. Use the example above to help you solve the equation $x^{3/2} = 64$.

7. Solve the equation $x^{3/2} = 8$

8. Solve the equation $5x^{1/7} - 2 = 13$

2.2.3 Compound Interest The compound interest formula is given here.

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

9. If \$10,000 is invested at an annual rate of 8%, determine the amount present after 10 years given the following:

   (a) Compounded annually

   (b) Compounded monthly

   (c) Compounded weekly

   (d) Compounded daily

   (e) Compounded hourly

   (f) Compounded every minute

   (g) Compounded continuously

2.2.4 Laws of Exponents

Laws of Exponents

(1) $a^m \cdot a^n = a^{m+n}$	(4) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
(2) $(a^m)^n = a^{mn}$	(5) $\frac{a^m}{a^n} = a^{m-n}$
(3) $(ab)^n = a^n b^n$	(6) $\frac{1}{a^n} = a^{-n}$

10. Simplify the expressions completely (there should only be one instance of each variable and only positive exponents). For each step, identify the rule used to simplify.

    (a) $\left(\frac{x}{y}\right)^{-9} \cdot y^{10}$

    (b) $\frac{x^{8/3} y^{3/5}}{x^2}$

    (c) $\left(\frac{-2x^{-3}}{y^{12}}\right)^{2/5}$

    (d) $(-2x^3y)^5 \left(\frac{x^9}{5y^2}\right)$

Watch the Pre-Class videos for Section 3.3 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Write the following in exponential form.

   (a) $\log_8(1) = 0$

   (b) $\ln(a) = b$

2. Write the following in logarithmic form.

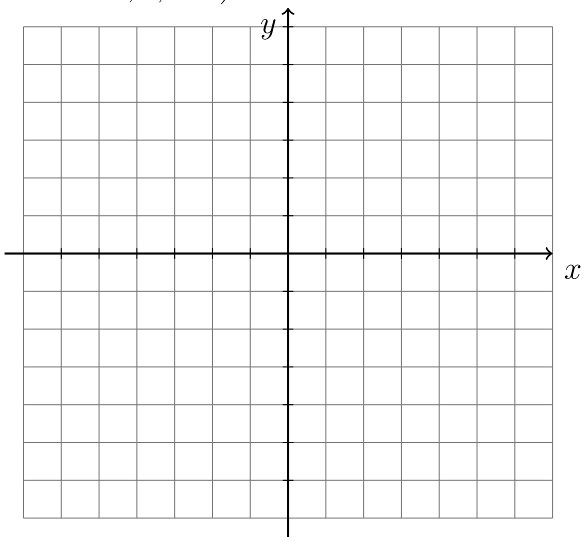
   (a) $7^0 = 1$

   (b) $10^3 = 1000$

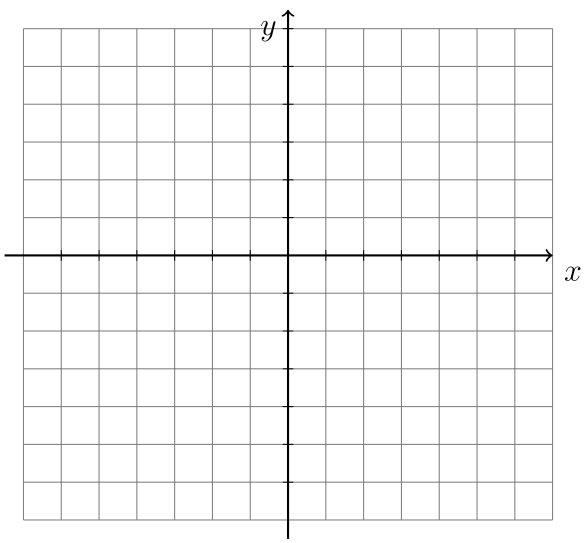
3. Determine the domain of $\log_7(2x + 5)$. Write your answer in interval notation.

4. Graph the following functions.

   (a) Graph $f(x) = e^x$ along with it's asymptote. (It is helpful to plot values for $x = 0, 1, -1$.)



   (b) Use part (a) to graph $g(x) = \ln(x)$ along with it's asymptote.



Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Write the following in exponential form.

   $\log_3(x) = 9$ $\quad\log_2(8) = x$ $\quad\log_2(y) = 5$ $\quad\log_5(y) = 2$

2. Write the following in logarithmic form.

   $y = 3^4$ $\quad m = 4^2$ $\quad 64 = 4^x$ $\quad 32 = x^5$

3. Solve the following by first rewriting the equation in exponential form.

   (a) $\log_3(x) = 4$

   (b) $\log_m(81) = 4$

   (c) $\log_2(\frac{x}{2}) = 5$

   (d) $\log_2(4x) = 5$

4. Determine the inverse of the following functions. (Hint: you will need to rewrite in logarithmic or exponential form after switching x and y.) Then determine the domain and range of the function and its inverse.

   (a) $f(x) = \log_3(x)$ $f^{-1}(x) =$

   Domain of $f(x)$: Range of $f(x)$:

   Domain of $f^{-1}(x)$: Range of $f^{-1}(x)$:

   (b) $f(x) = \log_5(x + 3)$ $f^{-1}(x) =$

   Domain of $f(x)$: Range of $f(x)$:

   Domain of $f^{-1}(x)$: Range of $f^{-1}(x)$:

   (c) $f(x) = \ln(7x - 4)$ $f^{-1}(x) =$

   Domain of $f(x)$: Range of $f(x)$:

   Domain of $f^{-1}(x)$: Range of $f^{-1}(x)$:

(d) $f(x) = 4^{x+3}$ $f^{-1}(x) =$

Domain of $f(x)$: Range of $f(x)$:

Domain of $f^{-1}(x)$: Range of $f^{-1}(x)$:

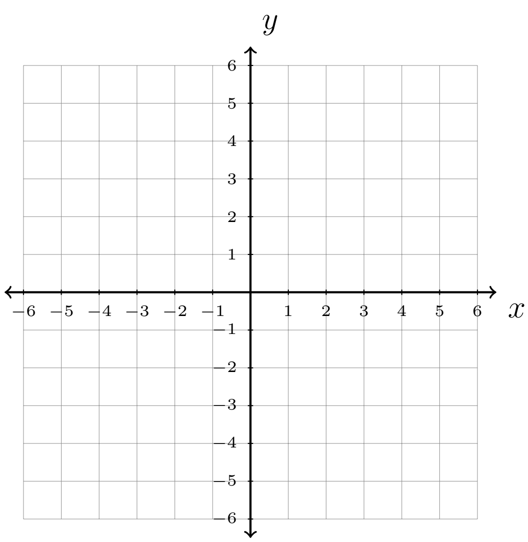
(e) $f(x) = e^{3x}$ $f^{-1}(x) =$

Domain of $f(x)$: Range of $f(x)$:

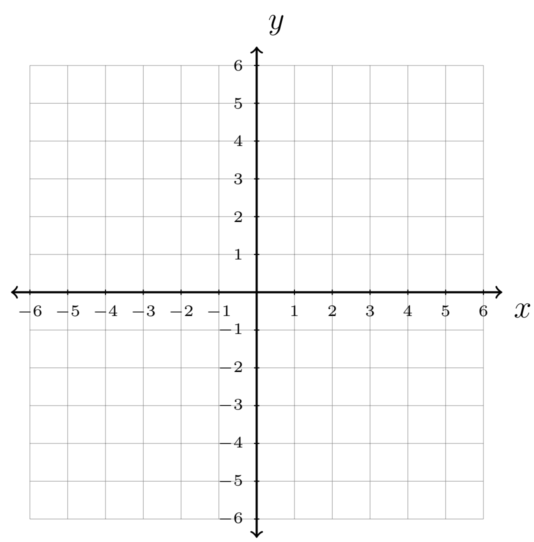
Domain of $f^{-1}(x)$: Range of $f^{-1}(x)$:

5. Graph $f(x) = 2^x$ and $f^{-1}(x) = \log_2(x)$ on the rectangular coordiantes below and include their asymptotes.

(a) $f(x) = 2^x$

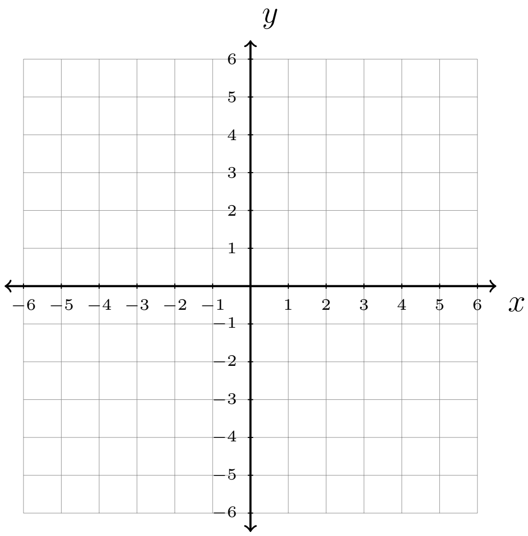


(b) $f^{-1}(x) = \log_2(x)$

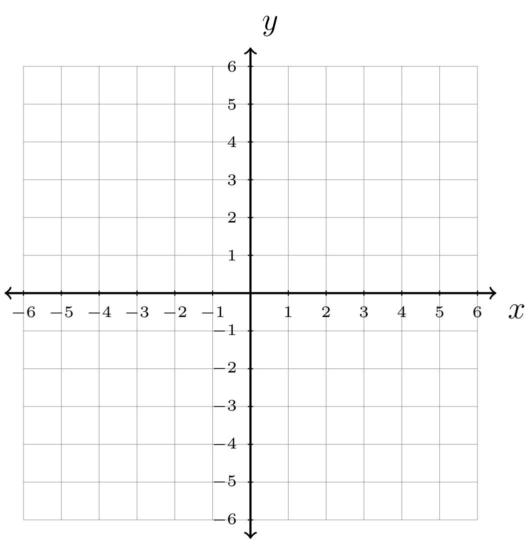


6. Graph each of the following transformed logarithmic functions.

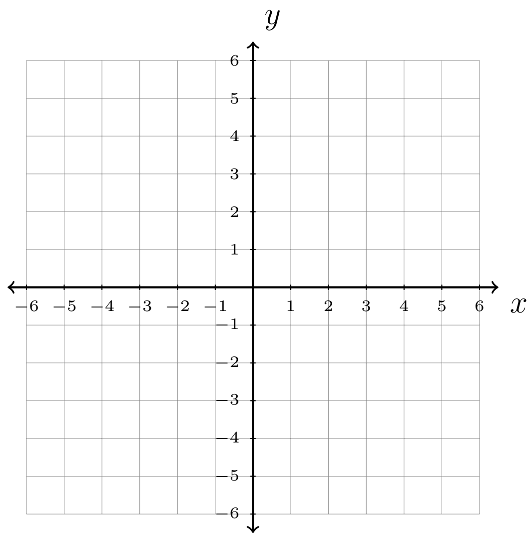
   (a) $f(x) = \log_2(-x+3) + 1$



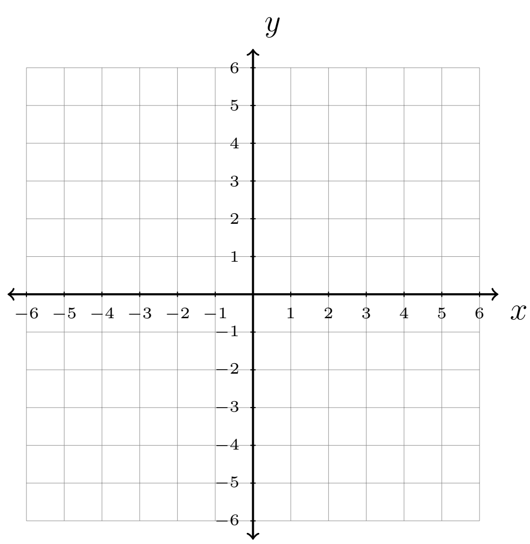
   (b) $f^{-1}(x) = 2\log_3(x+4) - 1$



(c) $f(x) = -3\ln(x-2)$



(d) $f^{-1}(x) = \log_5(5-x)$



Since $f(x) = b^x$ and $g(x) = \log_b(x)$ are inverses,

$$f(g(x)) = b^{\log_b(x)} = x \qquad\qquad g(f(x)) = \log_b(b^x) = x.$$

7. Evaluate the following expressions.

   (a) $\log_{10}(1000) =$

   (b) $\log_3(27) =$

   (c) $\log_4(1) =$

   (d) $\log_2(\frac{1}{4}) =$

   (e) $3^{\log_3(12)} =$

   (f) $e^{\ln(4)} =$

   (g) $64^{\log_4(2)} =$

Watch the Pre-Class videos for Section 3.4 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Use logarithmic properties to simplify and condense the following into one logarithm.

   (a) $\ln(x) + \ln(y) - \ln(z)$

   (b) $\log_2(x^2) + \dfrac{1}{2}\log_2(x-1) - 3\log_2((x+3)^2)$

2. Use logarithmic properties to expand each expression.

   (a) $\log_2\left(\dfrac{x}{y^5}\right)$

   (b) $\ln\left((y^2-16)(y+8)^2\right)$

3. (Use the change of base formula to approximate $\log_{7.5}(98)$ and round your answer to 4 decimal places. Show all work.

Instructions: Work together in groups of 3 or 4 to complete the following problems. CAUTION: There is NO RULE for breaking down $\log_a(u+w)$ or $\log_a(u-w)$.

1. Let $M = \log_b(x)$ and $N = \log_b(y)$.
   (a) Write the given equations in exponential form.

   (b) Show that $xy = b^{M+N}$.

   (c) Write the expression $xy = b^{M+N}$ in logarithmic form.

   (d) Substitute for M and N.

   (e) What did you just find?

2. Let $M = \log_b(x)$ and $N = \log_b(y)$. Use the process above to show the Quotient Property of Logarithms is true.

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

3. Expand the following to express in terms of logarithms of x, y, z, or w.

   (a) $\log_4(xz)$

   (b) $\log_2\left(\frac{x}{y}\right)$

   (c) $\ln\left(\frac{w^2z^3}{y^2\sqrt[5]{x}}\right)$

4. Condense the following to express as a single logarithm.

   (a) $\log_7(4x^5) - \log_7(x^2)$

   (b) $-4\Bigg(\ln(2) - \ln(7)\Bigg) + 5\ln(3)$

   (c) $7\log_8(x) - 3\log_8(2x+3) + 2\log_8(x-3)$

5. Rewrite the function without using any logarithms.

   (a) $f(x) = 10\ln(e^{-7+2x})$

   (b) $f(x) = 3^{\log_3(8) - 5\log_3(x^2+1)}$

   (c) $f(x) = \log(10^{4x+y}100^{y-x})$

6. Use the given approximations to approximate the value of the following logarithms.

   $\log_b(2) \approx 0.356 \qquad \log_b(3) \approx 0.565 \qquad \log_b(5) \approx 0.827$

   (a) $\log_b(15)$

   (b) $\log_b(10)$

   (c) $\log_b(81)$

   (d) $\log_b(\frac{15}{2})$

7. For each of the following functions, determine whether or not its graph is shown below.

   (a) $y = 2^x + 3$

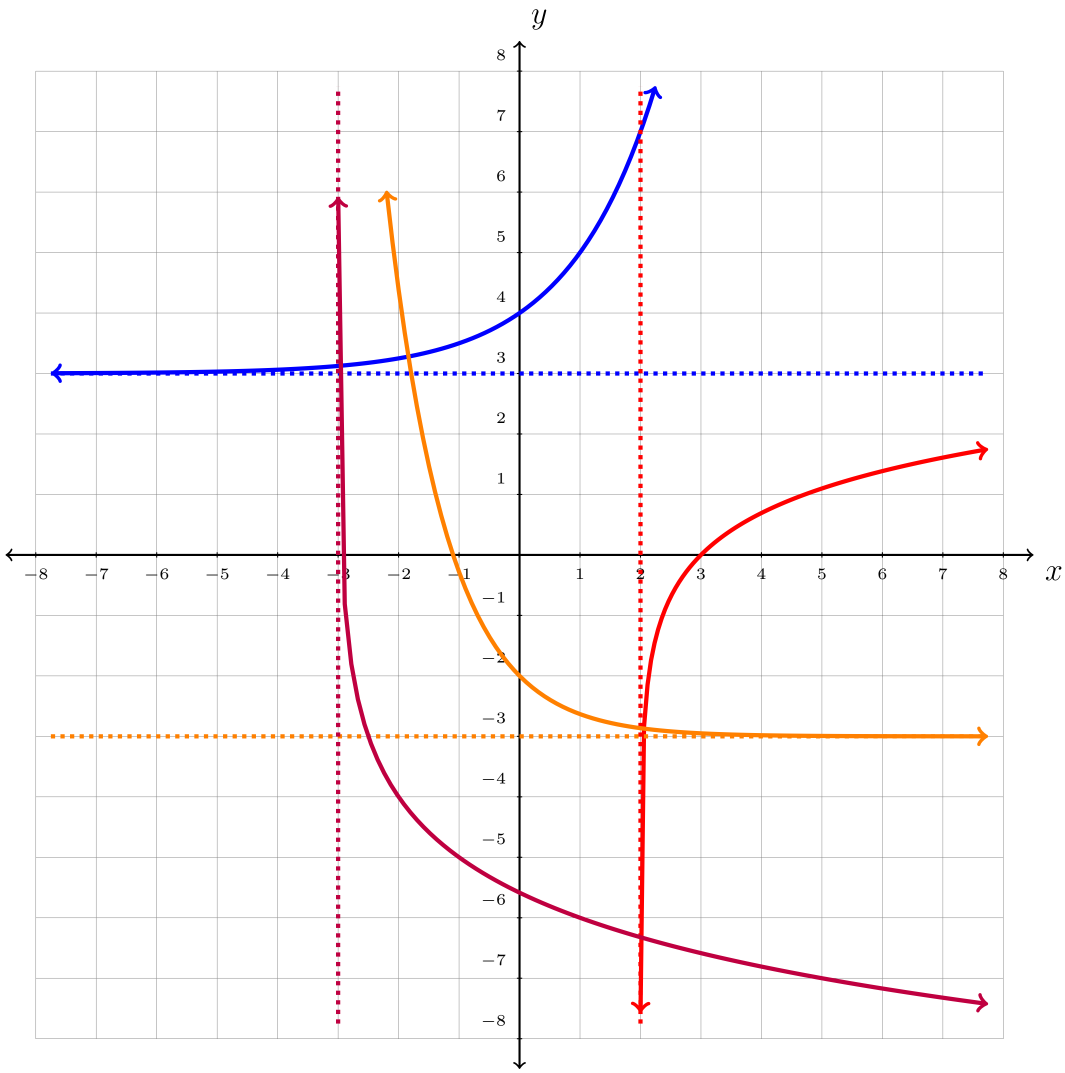
   (b) $y = \ln(x - 2)$

   (c) $y = 2^{-x} + 3$

   (d) $y = e^{-x} - 3$

   (e) $y = -\ln(x - 2)$

   (f) $y = -\log_2(x + 3) - 4$



8. Use the change-of-base formula to write $\Big(\log_2(5)\Big)\Big(\log_5(9)\Big)$ as a single logarithm.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Which functions are exponential functions?

   $f(x) = 4.2^x$ $\quad g(x) = x^{4.2}$ $\quad h(x) = 4.2x$ $\quad k(x) = (\sqrt{4.2})^x$ $\quad m(x) = (-4.2)^x$

2. Consider $y = 3^x$. Determine the domain, range, and asymptote for each of the following functions.

   (a) $f(x) = 3^x$

   (b) $g(x) = 3^x + 2$

   (c) $b(x) = 3^{x+2} - 1$

   (d) $h(x) = \left(\frac{1}{3}\right)^x$

3. Alice needs to borrow \$15,000 to buy a car. She can borrow the money at 6.4% compounded monthly for 5 years or she can borrow the money at 6.7% interest compounded continuously for 5 years. Which option is most cost effective for Alice?

4. Determine the domain, range, and asymptote for each of the following functions.

   (a) $f(x) = \log(8 - x)$

   (b) $g(x) = \log_2(x^2 - 16)$

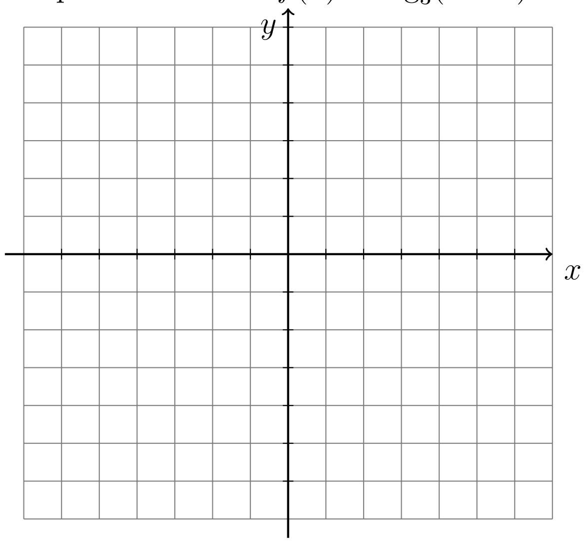
   (c) $h(x) = \ln(x^2 + 14)$

   (d) $m(x) = 3 + \log_4\left(\dfrac{1}{\sqrt{11 - x}}\right)$

5. Determine the domain of the following function and explain your answer.

$$f(x) = \ln(-6x^2)$$

6. Graph the function $f(x) = \log_3(x+2) + 4$ along with it's asymptote.



7. Find a logarithmic function of the form $f(x) = b + \log_a(x + c)$ that has the vertical asymptote $x = -14$, passes through the point $(-13, 2)$ and crosses the x-axis at $x = \frac{-685}{49}$.

8. Simplify the expression without using a calculator.

   (a) $\log_3(9)$

   (b) $\log_2(\frac{1}{16})$

   (c) $\log_{1/7}(49)$

9. Simplify the expression without using a calculator.

   (a) $\log_4(4^{11})$

   (b) $5^{\log_5(x+y)}$

   (c) $\log_\pi(1)$

10. Write the logarithm as a sum or difference of logarithms and simplify as much as possible. (Expand the logarithmic expression.)

    (a) $\log_7\left(\frac{1}{7}mn^2\right)$

    (b) $\log_5\left(\frac{p^5}{mn}\right)$

    (c) $\log\left(\frac{10}{\sqrt{a^2+b^2}}\right)$

    (d) $\ln\left(\sqrt[5]{\frac{e^2}{c^2+5}}\right)$

11. Write the logarithmic expression as a single logarithm with coefficient 1, and simplify as much as possible. (Condense the logarithmic expression.)

    (a) $\ln(y) + \ln(4)$

    (b) $\log_3(693) - \log_3(33) - \log_3(7)$

    (c) $3[\ln(x) - \ln(x+3) - \ln(x-3)]$

    (d) $15\log(c) - \dfrac{1}{4}\log(d) - \dfrac{3}{4}\log(k)$

Watch the Pre-Class videos for Section 3.5A (parts 1-4) and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Solve the following equations. Then check your answers.

   (a) $4^{x+2} = 64$

   (b) $\ln(2x - 3) = \ln(11)$

2. Solve the following equation. Then check your answer.

$$5 + e^{x+1} = 20$$

3. Solve the following equation.

$$3^x = 4^{2x-5}$$

Instructions: Work together in groups of 3 or 4 to complete the following problems.

(1) **Equivalence Property of Exponential Expressions:** If b, x, and y are real numbers where $b > 0, b \neq 1$. Then

$$b^x = b^y \text{ implies that } x = y.$$

(2) **Equivalence Property of Logarithmic Expressions:** If b, x, and y are positive real numbers with

$$\log_b(x) = \log_b(y) \text{ implies that } x = y.$$

(3) **Solving Exponential Equations by Using Logarithms:** Isolate the exponential expression on one side of the equation and take a logarithm of the same base on both sides.

1. We have learned the 3 techniques above to solve exponential and logarithmic equations. For each equation, determine the best technique to use to solve the equation. **Do not solve the equations.**

   (a) $3^x = 81$

   (b) $\log(x^2 + 6x) = \log(7)$

   (c) $11^{3x+1} = \left(\frac{1}{11}\right)^{x-5}$

   (d) $6^x = 87$

   (e) $10^{3+4x} - 8100 = 120,000$

   (f) $\log_4(3x + 11) = \log_4(3 - x)$

   (g) $1024 = 19^x + 4$

   (h) $5^{2x+2} = 625$

2. Solve the following equations using the appropriate techniques. Then check your answers.

   (a) $2^t = 32$

   (b) $\sqrt[3]{5} = 5^x$

(c) $7^{2p-3} = \left(\dfrac{1}{49}\right)^{p+1}$

(d) $100^{3m-5} = 1000^{3-m}$

(e) $2^x = 70$

(f) $801 = 23^y + 6$

(g) $80 = 320e^{-0.5t}$

(h) $\log_3(12 - x) = \log_3(x + 6)$

(i) $\ln(w^2 + 7w) = \ln(18)$

Note: Sometimes, we need to use other techniques and properties to solve exponential equations. For example, when there are multiple exponential functions, it can be helpful to take ln or log of both sides.

3. Solve the equations. Then check your answers.

   (a) $3^{6x+5} = 5^{2x}$

   (b) $2^{1-6x} = 7^{3x+4}$

Note: You may also notice that an exponential equation has the same form as a quadratic equation. So you may need to substitute to solve the problem like a quadratic equation.

4. Solve the equations. Then check your answers.

   (a) $e^{2x} - 9e^x - 22 = 0$

   (b) $e^{2x} - 6e^x - 16 = 0$

Once you have finished this worksheet, go back and solve the equations from #1.

Watch the Pre-Class videos for Section 3.5B and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Solve the following equations. Then check your answers. Leave your answers as symbolic expressions (no decimals).

   (a) $5\log_6(7w+1) = 10$

   (b) $2\log_8(3y-5) + 20 = 24$

2. If \$10,000 is invested in an account earning 5.5% interest compounded continuously, determine how long it will take for the money to triple. Round your final answer to the nearest year.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Solve the equation $4^{-x+12} = 19$ in three different ways. Note that the form your answer takes will be different depending on the method, but that all answers should agree.

   (a) by converting to logarithmic form

   (b) by taking $\log_4$ of both sides

   (c) by taking ln of both sides

2. Solve the equation $\log_5(2x + 19) = 8$ in two different ways:

   (a) by converting to exponential form

   (b) by "exponentiating" both sides with base 5

3. For the equation $5^{2x-7} = 4^{9x+12}$, why don't we want to convert to logarithmic form? Solve the equation in two different ways:

(a) by taking the ln of both sides

(b) by taking $\log_4$ of both sides

The point: You can use *any* log to solve an exponential equation. Some are just more convenient than others. A good candidate is the common log, the natural log, or a log using a "base" in the equation.

4. Solve the logarithmic equation. Then check your answers.

   (a) $6\log_5(4p-3)-2=16$

   (b) $\log(q-6)=3.5$

(c) $\log_3(y) + \log_3(y+6) = 3$

(d) $\log(x) + \log(x-7) = \log(x-15)$

(e) $\log_3(n-5) + \log_3(n+3) = 2$

5. If a couple has \$80,000 in a retirement account, how long will it take the money to grow to \$1,000,000 if it grows by 6% compounded continuously? Round to the nearest year.

6. A \$2500 bond grows to \$3729.56 in 10 years under continuous compounding. Find the average interest rate. Round to the nearest whole percent.

7. An \$8000 investment grows to \$9289.50 at 3% interest compounded quarterly. For how long was the money invested? Round to the nearest year.

8. \$20,000 is invested at 3.5% interest compounded monthly. How long will it take for the investment to triple? Round to the nearest tenth of a year.

9. Use the formula $\text{pH} = -\log(\text{H}^+)$ to determine the value of H^+ for the following liquids given their pH values.

   (a) Seawater pH: 8.5

   (b) Acid rain pH: 2.3

Watch the Pre-Class videos for Section 3.6 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Suppose that \$50,000 from a retirement account is invested in a large cap stock fund. After 20 years, the value is \$194,809.67.

   (a) Use the model $A = Pe^{rt}$ to determine the average rate of return under continuous compounding. (Do not simplify or round your answer.)

   (b) Assuming interest continues to accumulate at this average rate, how long will it take the investment to reach \$250,000? Round your final answer to the nearest tenth of a year.

2. A sample from a mummified bull was taken from a pyramid in Dashur, Egypt. The sample shows that 78% of the carbon-14 still remains. How old is the sample? Round to the nearest year. Use the model $Q(t) = Q_0 e^{-0.000121t}$ for radiocarbon dating.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Determine which are exponential **decay** functions.

$$f(t) = 5^t \qquad g(t) = 5^{-t} \qquad h(t) = \left(\frac{1}{5}\right)^t \qquad p(t) = \left(\frac{1}{5}\right)^{-t}$$

NOTE 1: Half-life is the time it takes for 50% (or half) of a substance to decay.
NOTE 2: Leave all of your answers symbolic. (Do not put your final answer in the calculator.)

2. Carlos has taken an initial dose of a prescription medication. The relationship between elapsed time t, in hours, since he took the first dose, and the amount of medication, $M(t)$, in milligrams (mg), in his bloodstream is modeled by the following function.

$$M(t) = 20e^{-0.8t}$$

(a) How much medication is in Carlos' bloodstream after 3 hours?

(b) In how many hours will Carlos have 1 mg of medication remaining in his bloodstream?

3. You invest at 3% per annum, compounded continuously. Determine the time t required for your investment to triple.

4. Money is invested at an interest rate of r (where r is a decimal) and is compounded continuously. Express the time required for the money to triple, as a **function of r.**

5. The amount of a radioactive compound in a sample decays exponentially ($P = P_0e^{kt}$). The sample initially contains 50g of the compound, and after three years contains 40g. How long will it take until there is 30g of material?

   (a) Determine the two points (t, P) given in the question.

   (b) Use the two points in the given equation to determine the decay constant k.

   (c) How long will it take until there is 30g of material?

   (d) What is the half-life of the compound?

6. The human population grew exponentially from 1.6 billion people in the year 1900
to 6 billion in the year 2000.

   (a) If the population continues to grow at this rate, what will be the population in 2100?
   To simplify calculations, I recommend using the year 1900 at year $t = 0$.
   Round your answer to the nearest tenth of a billion.

   (b) WOW! That is a lot of people!
   Suppose the population growth slows to follow the **logistic model**

$$P = \frac{12}{1 + 22.3e^{-0.031t}}$$

   where P is measured in billions of people and t in years since 1900.
   In this model with reduced growth, what will be the population in 2100?
   Round your answer to the nearest tenth of a billion.

   (c) Following the exponential growth, the population will continue to grow indefinitely. However the logistic model levels off to a certain maximum population.
   What is the maximum (long-term) population of the logistic model?

7. If a certain bacteria population triples in 5 days, determine the time t (in days) that it takes the population to quadruple.

8. 78% of Carbon-14 remains after 2053 years.

   (a) Determine the decay constant for Carbon-14.

   (b) Determine the half-life of Carbon-14. (Determine how long it takes for half of the Carbon-14 to remain.)

   (c) Determine the age of a piece of wood that has 42% of its Carbon-14 remaining.

9. A pendulum swings back and forth over the ground. Its height above the ground oscillates with an *amplitude that decays exponentially.*

   Originally, that amplitude is 2.3 cm. After 2 hours of swinging, the amplitude is 1.9 cm.

   Determine how long it will take for the amplitude to be 1% of its original value.

10. A patient has 80 milligrams of a drug administered at 9AM. At noon, there is 20 mg of the drug in his bloodstream. If the amount of drug in the patient's blood decays exponentially, how much of the drug do we expect to be in his bloodstream at 5PM?

11. Radioactive iodine is used in thyroid testing. Its half-life is 8 days. The amount of iodine remaining after t days is $A(t) = A_0 b^{-t}$, where A_0 is the initial amount. Determine b.

12. Suppose that you have an exponential decay function of the form $P = P_0 e^{kt}$, and you know that the points (4, 6000) and (10,2700) are on the graph.

    (a) Determine the decay constant k.

    (b) Determine the P_0.

Chapter 3

Angle Measurement

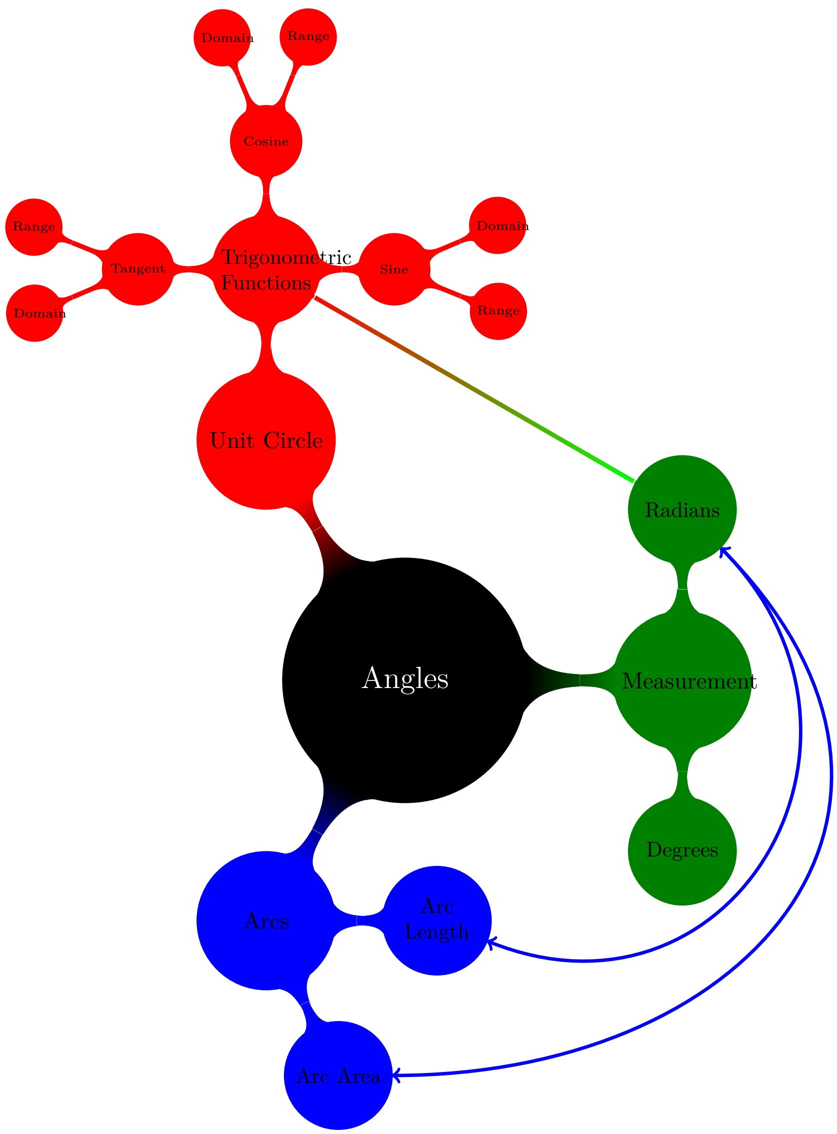


Figure 3.1: Topics for the section on angles.

Watch the Pre-Class videos for Section 4.1 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Convert the following angles measured in degrees to radian measure. Your answers must be written as fractions and not rounded decimals.

   (a) $\theta = 60^\circ$

   (b) $\beta = 110^\circ$

2. Find one positive angle and one negative angle that is coterminal to $\theta = \dfrac{3}{2}\pi$.

3. Answer the following questions about a circle that has radius 7 and an angle θ that subtends an arc of length 15.

   (a) Determine θ in radians and degrees.

   (b) Determine the area of the circular sector with central angle θ found in part (a).

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Find a positive angle less than $360°$ that is coterminal with the given angle.

   (a) $\theta = 400°$

   (b) $\alpha = -160°$

2. Find a positive angle less than 2π that is coterminal with the given angle.

   (a) $\beta = -\dfrac{\pi}{15}$

   (b) $\phi = \dfrac{34\pi}{9}$

3. Find the radian measure of the central angle θ of a circle of radius $r = 8$ meters that intercepts an arc of length $s = 14$ meters.

4. The minute hand of a clock is 3 inches long. How far does the tip of the minute hand move in 45 minutes?

5. The minute hand of a clock moves from 12:10 to 12:30.

   (a) How many degrees does it move during this time?

   (b) How many radians does it move during this time?

   (c) If the minute hand is 10 inches in length, determine the exact distance the tip of the minute hand travels during this time.

6. Find the exact area of the following sectors given the radius of the circle r and the subtended angle θ. Then round the result to the nearest tenth of a unit.

   (a) $r = 6$ m; $\theta = \frac{5\pi}{3}$

   (b) $r = 1.2$ ft; $\theta = \frac{\pi}{6}$

   (c) $r = 3$ cm; $\theta = 120°$

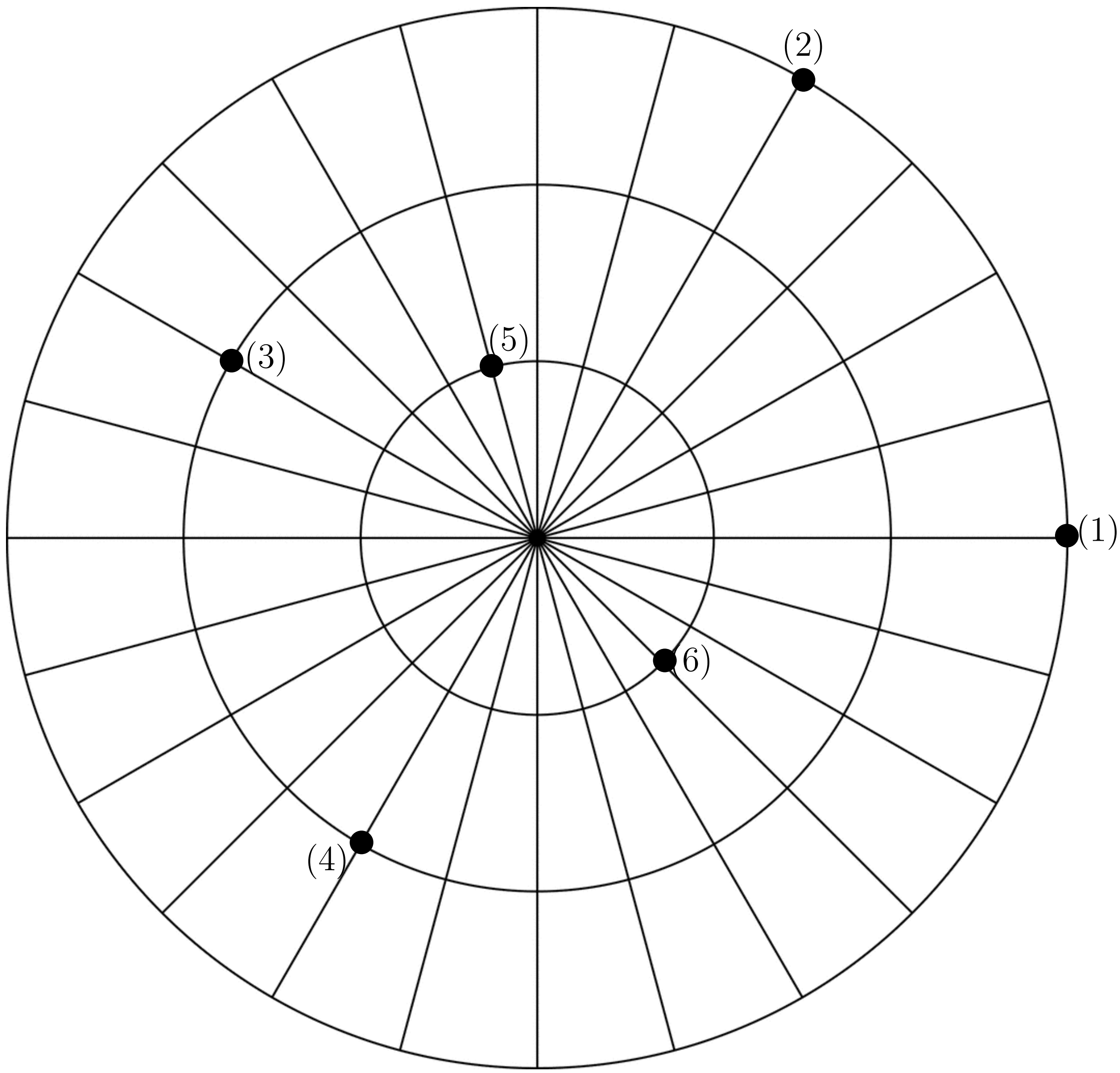
7. You are one member of a group of 8 friends who are going out for pizza. A small pizza has a 6" radius, while a large pizza has 9" radius. Answer the following questions.

   (a) How much pizza will each of you eat if you order two small pizzas? Will you get more of less pizza if you order one large pizza? (Assume everyone eats the same amount and all the pizza is eaten.)

   (b) How many inches of crust will each person eat if you order two smalls? If you order one large will you get more crust?

   (c) Suppose you and your friends want to order one pizza and that you each want to eat 50 square inches worth of pizza. What should the radius of the pizza be?

8. Suppose that the three circles drawn below have radii of length 1, 2, and 3. For each pair of points given below, find the shortest path connecting the points.



(a) From (1) to (2).
(b) From (3) to (4).
(c) From (5) to (6).
(d) From (3) to (5).
(e) From (2) to (4), avoiding the center

Watch the Pre-Class videos for Section 4.2 A and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Suppose that the real number t corresponds to the point $P\left(-\frac{\sqrt{13}}{4}, \frac{\sqrt{3}}{4}\right)$ on the unit circle. Evaluate the six trigonometric functions at t.

   (a) $\sin(t) =$

   (b) $\cos(t) =$

   (c) $\tan(t) =$

   (d) $\csc(t) =$

   (e) $\sec(t) =$

   (f) $\cot(t) =$

2. Use the coordinates on the unit circle to find the value of each trig function at the indicated real number.

   (a) $\sin\left(\frac{4\pi}{3}\right) =$

   (b) $\csc\left(\frac{4\pi}{3}\right) =$

3. Evaluate the trig functions at the indicated real number.

   (a) $\cos\left(-\frac{\pi}{6}\right) =$

   (b) $\tan\left(-\frac{\pi}{6}\right) =$

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Suppose the real number t corresponds to the point $P\left(\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}\right)$ on the unit circle. (The ray at angle t intersects the unit circle at P.) Evaluate the six trigonometric functions of t.

   (a) $\sin(t) =$ (d) $\csc(t) =$

   (b) $\cos(t) =$ (e) $\sec(t) =$

   (c) $\tan(t) =$ (f) $\cot(t) =$

2. Use (x, y) coordinates in the unit circle to find the value of each trig function at the indicated real number $t = \frac{5\pi}{3}$.

   (a) $\sin(\frac{5\pi}{3}) =$ (d) $\csc(\frac{5\pi}{3}) =$

   (b) $\cos(\frac{5\pi}{3}) =$ (e) $\sec(\frac{5\pi}{3}) =$

   (c) $\tan(\frac{5\pi}{3}) =$ (f) $\cot(\frac{5\pi}{3}) =$

3. Use (x, y) coordinates in the unit circle to find the value of each trig function at the indicated real number $t = -\frac{5\pi}{4}$.

   (a) $\sin(-\frac{5\pi}{4}) =$

   (b) $\cos(-\frac{5\pi}{4}) =$

   (c) $\tan(-\frac{5\pi}{4}) =$

   (d) $\csc(-\frac{5\pi}{4}) =$

   (e) $\sec(-\frac{5\pi}{4}) =$

   (f) $\cot(-\frac{5\pi}{4}) =$

4. Evaluate the trig function.

   (a) $\sin(\frac{3\pi}{4}) =$

   (b) $\tan(\frac{4\pi}{3}) =$

   (c) $\sec(\frac{5\pi}{6}) =$

   (d) $\cos(\frac{19\pi}{6}) =$

   (e) $\sec(-\frac{2\pi}{3}) =$

   (f) $\cot(-\frac{\pi}{4}) =$

5. Given $\sin(t) = \frac{3}{7}$ and $\cos(t) = \frac{2\sqrt{10}}{7}$, use reciprocal and quotient identities to find the values of the other trigonometric functions of t.

6. (a) Use the unit circle to evaluate $\cos\left(\dfrac{3\pi}{2}\right)$.

   (b) Evaluate $\tan\left(\dfrac{3\pi}{2}\right)$.

   (c) Are there any other trigonometric functions that are undefined at $t = \dfrac{3\pi}{2}$?

   (d) Determine another value for t where $\tan(t)$ and $\sec(t)$ are undefined.

7. For each trigonometric function, determine for which angles the function is undefined.

8. Use the unit circle to determine the following.

   (a) Determine two values of t for which $\csc(t)$ is undefined.

   (b) Determine two values of t for which $\cos(t) = -\frac{\sqrt{2}}{2}$.

   (c) Determine two values of t for which $\tan(t) = 1$.

   (d) Determine two values of t for which $\cot(t) = -1$.

   (e) Determine two values of t for which $\csc(t) = -2$.

   (f) Determine two values of t for which $\tan(t) = -\sqrt{3}$.

Watch the Pre-Class videos for Section 4.2B and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Evaluate the functions if possible. Your answers must be exact and not decimal approximations.

   (a) $\sin(0) =$

   (b) $\cot(\pi) =$

   (c) $\tan(3\pi) =$

   (d) $\sec(\pi) =$

   (e) $\csc(0) =$

   (f) $\cos(\pi) =$

2. Given $\cos(t) = \dfrac{7}{25}$ for $\dfrac{3\pi}{2} < t < 2\pi$. Use an appropriate Pythagorean Identity to find the value of $\sin(t)$.

3. Circle all properties that apply to $\csc(t)$.

   (a) The function is even.

   (b) The function is odd.

   (c) The period is 2π.

   (d) The period is π.

   (e) The domain is all real numbers.

   (f) The domain is all real numbers excluding odd multiples of $\frac{\pi}{2}$.

   (g) The domain is all real numbers excluding multiples of π.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. $P(x, y)$ on the unit circle corresponding to the real number t is $(5/6, -\sqrt{11}/6)$.

   (a) Make a diagram of the unit circle with P on it.

   (b) Determine the values of the six trig functions for t.

   $\sin(t) =$ $\cos(t) =$ $\tan(t) =$

   $\csc(t) =$ $\sec(t) =$ $\cot(t) =$

   (c) Determine $\cos(-t)$ and $\sin(-t)$.

   (d) Determine $\sin(t + 2\pi)$ and $\cos(t + 2\pi)$.

2. $P(x, y) = (4/5, 3/5)$ corresponds to a real number t.

   (a) Make a diagram of the unit circle with P on it.

   (b) Determine $\cos(t)$ and $\sin(t)$.

   (c) Determine $\cos(t + 4\pi)$ and $\sin(t - 6\pi)$.

   (d) Determine $\cos(-t)$ and $\sin(-t)$.

   (e) Determine $\cos(-t - 4\pi)$ and $\sin(-t + 100\pi)$.

3. Given $\cot(t) = \frac{45}{28}$ for $\pi < t < \frac{3\pi}{2}$. Use an appropriate Pythagorean identity to find the value of $\csc(t)$

4. Write $\tan(t)$ in terms of $\sec(t)$ for

   (a) t in Quadrant 2.

   (b) t in Quadrant 4.

5. Use the periodic properties of the trigonometric functions to simplify each expression to a **single** function of t.

   (a) $\sin(t + 2\pi) \cdot \cot(t + \pi)$

   (b) $\sin(t + 2\pi) \cdot \sec(t + 2\pi)$

6. Use the even-odd and periodic properties of the trigonometric functions to simplify.

   (a) $\csc(t) - 4\csc(-t)$

   (b) $-2\sin(3t + 2\pi) - 3\sin(-3t)$

7. Simplify using properties of trigonometric functions.

$$\sin^2(t + 2\pi) + \cos^2(t) + \tan^2(t + \pi)$$

8. Identify values t on the interval $[0, 2\pi]$ that make the given function undefined (if any).

   (a) $y = \sin(t)$

   (b) $y = \cot(t)$

   (c) $y = \cos(t)$

   (d) $y = \tan(t)$

   (e) $y = \csc(t)$

   (f) $y = \sec(t)$

9. Write down all trig functions for which each property applies.

   (a) The function is even.

   (b) The function is odd.

   (c) The period is 2π.

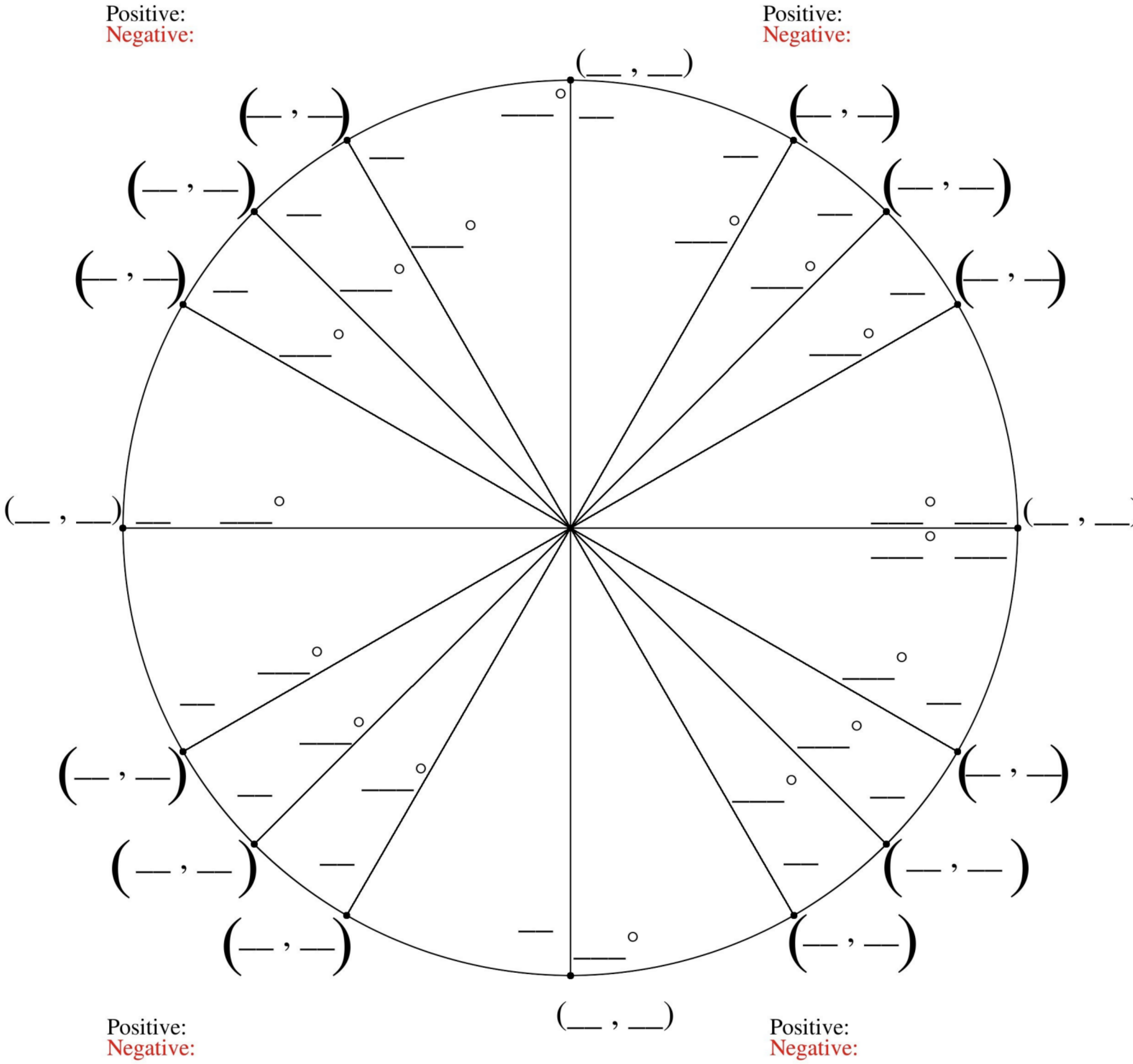
   (d) The period is π.

   (e) The domain is all real numbers.

   (f) The domain is all real numbers excluding odd multiples of $\frac{\pi}{2}$.

   (g) The domain is all real numbers excluding multiples of π.

10. If you plan on using the unit circle instead of special triangles and the chart for angles on the x and y axes, start memorizing the angles of the unit circle in radians as well as the points along the unit circle.



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Negative:

Positive:
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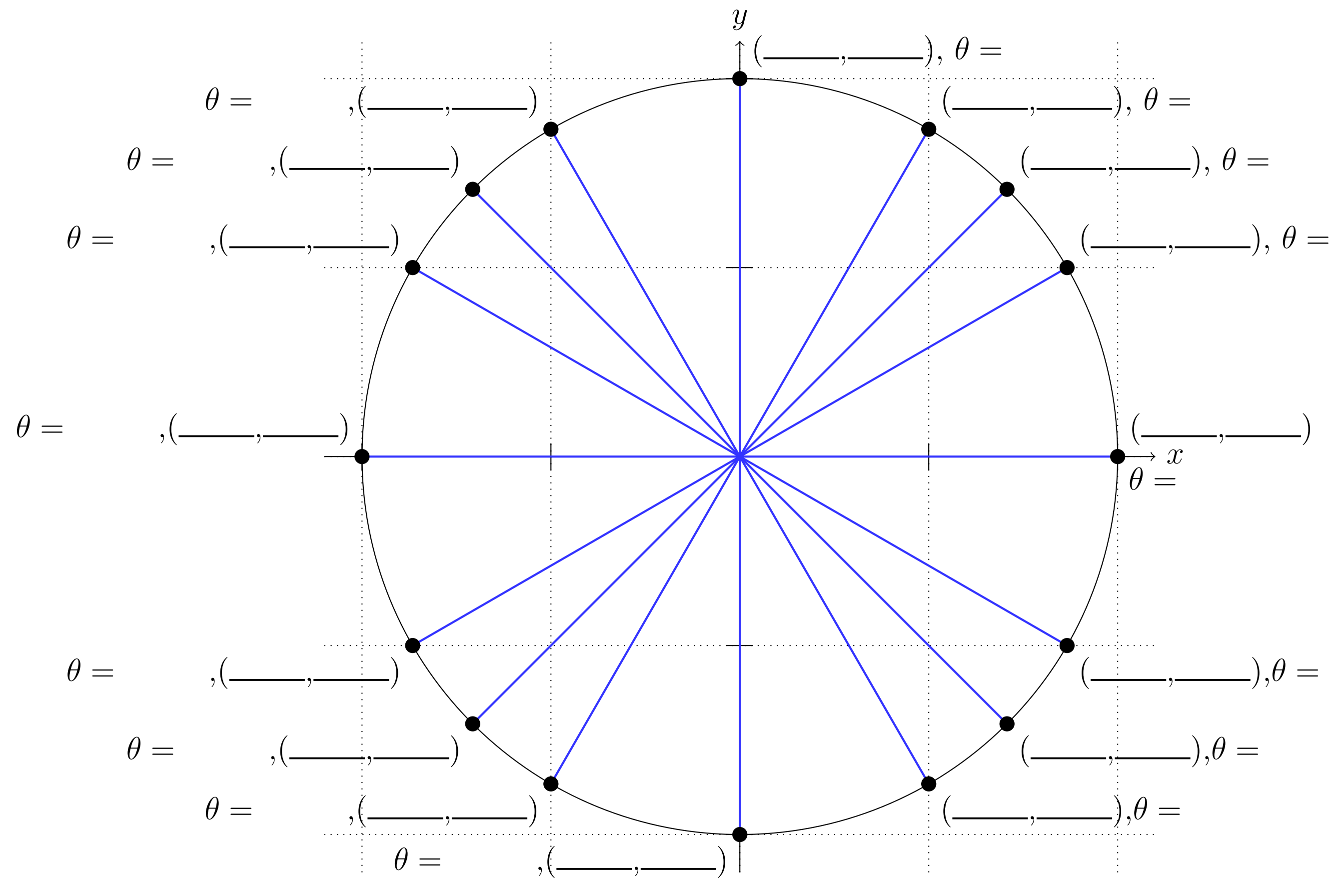

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Chapter 4

Trigonometric Functions

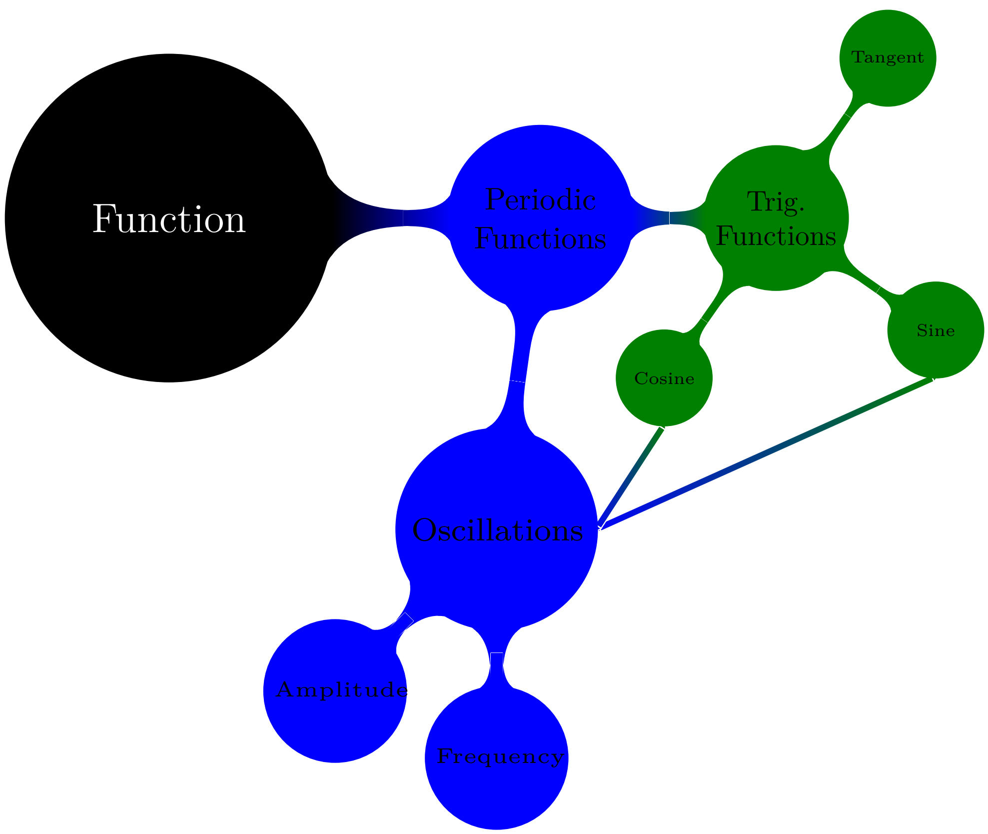
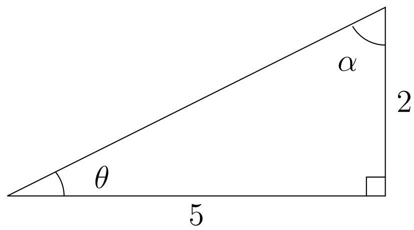


Figure 4.1: Topics for the section on trigonometric functions.

Watch the Pre-Class videos for Section 4.3 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Find the exact values of the six trigonometric functions θ and α. Use the following triangle.



(a) $\sin(\theta) =$

(g) $\sin(\alpha) =$

(b) $\cos(\theta) =$

(h) $\cos(\alpha) =$

(c) $\tan(\theta) =$

(i) $\tan(\alpha) =$

(d) $\csc(\theta) =$

(j) $\csc(\alpha) =$

(e) $\sec(\theta) =$

(k) $\sec(\alpha) =$

(f) $\cot(\theta) =$

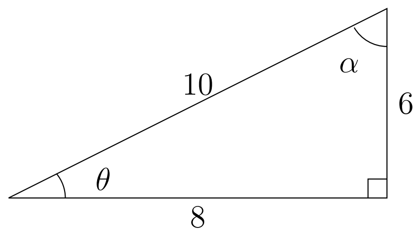
(l) $\cot(\alpha) =$

2. An observer at the top of a 462 ft mountain cliff measures the angle of depression from the top of the cliff to a point on the ground to be $5°$. What is the distance from the base (bottom) of the mountain to the point on the ground? Round to the nearest foot.

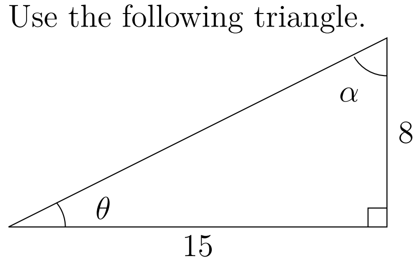
Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Find the exact values of the six trigonometric functions of θ and α

   (a) Use the following triangle.

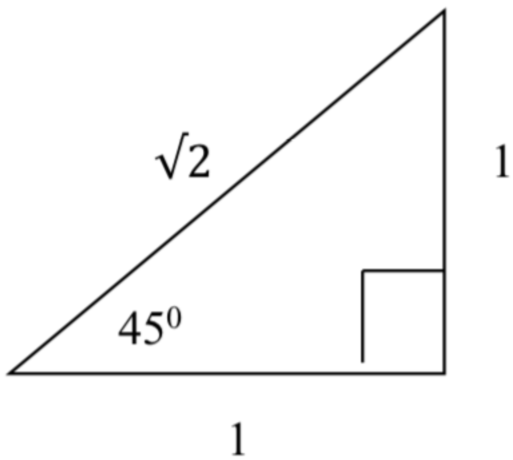


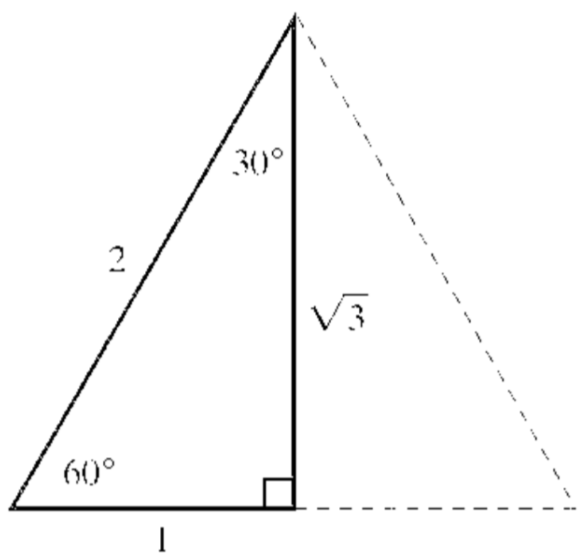
   (b) Use the following triangle.



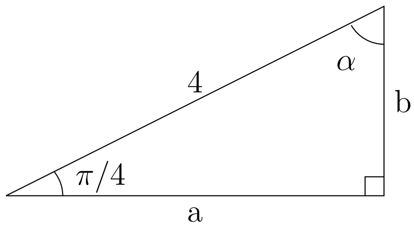
2. Use the isosceles right triangle and the 30/60/90 triangle to complete the table.

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
$30^\circ = \frac{\pi}{6}$						
$45^\circ = \frac{\pi}{4}$						
$60^\circ = \frac{\pi}{3}$						





3. For each problem below determine the values of the missing quantities. All angles are in radians, and your answers for angles should be in radians. (The triangles are not drawn to scale.)

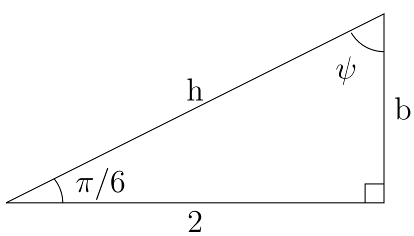


(a)

$a =$

$b =$

$\alpha =$



(b)

$b =$

$h =$

$\psi =$

4. If a 15 ft ladder is leaning against a wall at an angle of 62° with the ground, how high up the will will the ladder reach? Round to the nearest tenth of a foot.

5. A 30 ft boat ramp makes a 7° angle with the water. What is the height of the ramp above the water at the ramp's highest point? Round to the nearest tenth of a foot.

6. At a tree farm, palm trees are harvested once they reach a height of 20 feet. Suppose a farm worker determine's that the distance along the ground from her position to the base of a palm tree is 22 feet. She then uses an instrument called a clinometer held at her eye level of 6 feet to measure the angle of elevation tot he top of the tree as 30.2°. Is the tree tall enough to harvest?

Watch the Pre-Class videos for Section 4.4 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Let $P(-2,-5)$ be a point on the terminal side of θ. Find each of the six trig functions of θ.

   (a) $\sin(\theta) =$

   (b) $\cos(\theta) =$

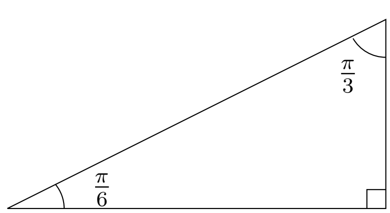
   (c) $\tan(\theta) =$

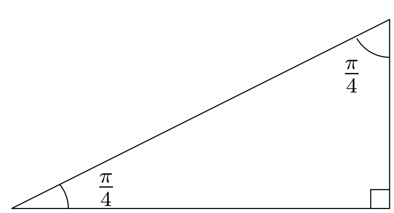
   (d) $\csc(\theta) =$

   (e) $\sec(\theta) =$

   (f) $\cot(\theta) =$

2. Label the side lengths of the given reference triangles.





3. Let $\theta = \dfrac{8\pi}{3}$.

   (a) Determine the reference angle θ_R for $\theta = \dfrac{8\pi}{3}$.

   (b) Determine $\sin(\frac{8\pi}{3})$.

   (c) Determine $\cos(\frac{8\pi}{3})$.

Instructions: Work together in groups of 3 or 4 to complete the following problems. **NOTE: Do not use your unit circle to answer the following questions. Only use reference triangles.**

1. Let $P(-7, \frac{\sqrt{3}}{4})$ be a point on the terminal side of θ. Find each of the six trig functions of θ.

   (a) $\sin(\theta) =$

   (b) $\cos(\theta) =$

   (c) $\tan(\theta) =$

   (d) $\csc(\theta) =$

   (e) $\sec(\theta) =$

   (f) $\cot(\theta) =$

2. (a) If $\theta = 2\pi/3$ is in standard position, what quadrant is θ in?

   (b) Determine the reference angle for θ.

   (c) Determine exact values of the following.

      i. $\sin(2\pi/3)$

      ii. $\cos(2\pi/3)$

      iii. $\tan(2\pi/3)$

3. Determine the following.

   (a) $\sin(7\pi/2)$

   (b) $\tan(7\pi/2)$

   (c) $\sec(7\pi/2)$

4. (a) If θ is an angle in standard position, determine the quadrant corresponding to $\theta = 16\pi/3$. Then determine the reference angle for $\theta = 16\pi/3$.

   (b) Determine the exact values of $\sin(\theta)$, $\sec(\theta)$, and $\cot(\theta)$ for $\theta = 16\pi/3$.

5. (a) If θ is an angle in standard position, determine the quadrant corresponding to $\theta = -210°$. Then determine the reference angle for $\theta = -210°$.

   (b) Determine the exact values of $\cos(\theta)$, $\csc(\theta)$, and $\tan(\theta)$ for $\theta = -210°$.

6. Determine the exact value of each of the following.

   (a) $\sin(7\pi/6)$

   (b) $\sin(11\pi/6)$

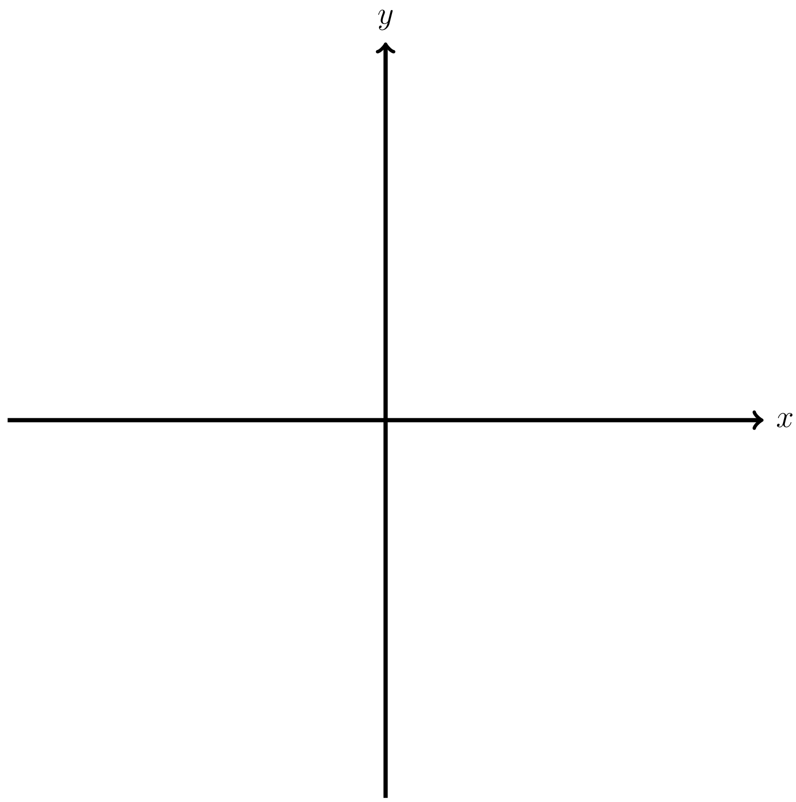
   (c) $\sin(5\pi/6)$

7. Suppose θ is an angle in the third quadrant with reference angle θ_R satisfying $\cos(\theta_R) = 5/13$ and $\sin(\theta_R) = 12/13$. Determine the exact values of $\cos(\theta)$ and $\csc(\theta)$.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Consider the angle $\theta = \dfrac{7\pi}{4}$.

   (a) Draw a picture labeling $\theta = \dfrac{7\pi}{4}$ and it's corresponding reference angle.



   (b) Determine the reference angle.

   (c) Determine $\tan\left(\dfrac{7\pi}{4}\right)$.

2. An angle θ is in the fourth quadrant, and $\cos\theta = 0.25$. Determine the tangent of the angle.

3. Determine all the angles, θ between 0 and 3π such that $\cos(\theta) = -\frac{\sqrt{3}}{2}$. Give exact answers in radians.

4. The terminal side of an angle θ in standard position goes through the point $(-6, -5)$. Find an exact answer for the following.

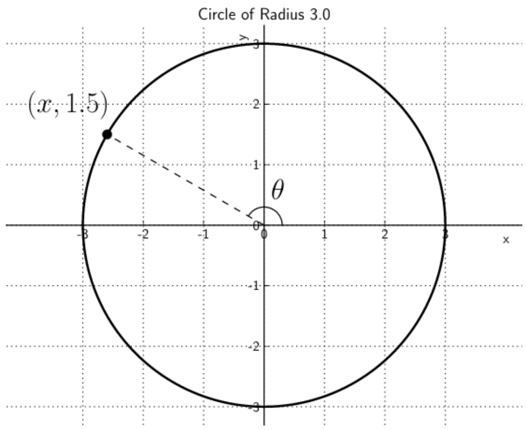
   (a) $\sin(\theta)$

   (b) $\cos(\theta)$

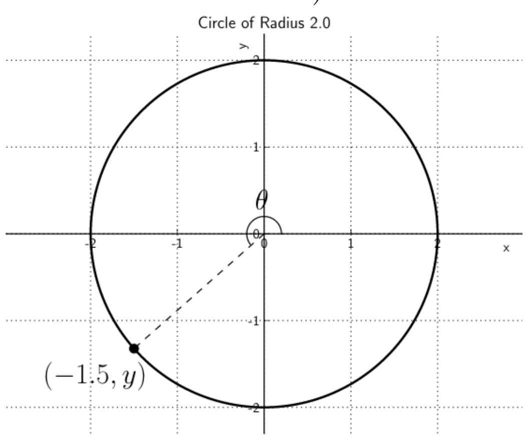
   (c) $\cot(\theta)$

5. For each question below, a diagram of a point on a circle is given. Answer each question about the angle formed by the line through the point, the origin, and the positive x-axis.

   (a) Determine the cosine of the angle θ. (Your answer should be a number and not have an x in it.)



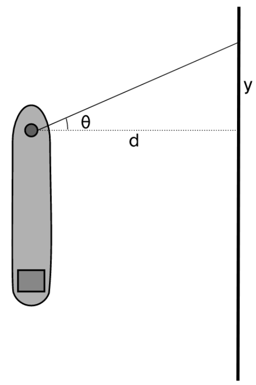
   (b) Determine the tangent of the angle θ. (Your answer should be a number and not have an x in it.)



6. Given the following, find the exact value of $\sin(\theta)$.

$$\cos(\theta) = -\frac{1}{3} \quad \text{and} \quad \tan(\theta) \quad \text{is negative}$$

7. A ship is anchored a distance of $d = 150$ from a beach. A spotlight on the bow of the ship can rotate, and the angle is measured from a line perpendicular to the beach that goes through the base of the spotlight. Determine the position, y, along the beach that the spotlight will illuminate the given angle, θ. (Your answer should be a function of θ with $-\pi/2 < \theta < \pi/2$.)



8. Two observers are standing a distance of 100m apart. They both spot an eagle and watch it closely. The moment it passes between them, the first observer measures an angle of elevation from the ground of 45°, and the second observer measures an angle of elevation from the ground of 35°. How high in the air was the eagle when it passed between the two observers?

9. A frog rides a unicycle that has a wheel with a diameter of 1.5 inches. If the frog travels a distance of 320 inches, what is the angle that the wheel turned? Give an exact answer.

10. A pizza shop owner feels inspired after taking a precalculus class at his local college. He decides that he wants to sell his 16 inch (diameter) pizzas at a price of \$0.09 per square inch.

    (a) Find the price of a slice of pizza as a function of θ.

    $$\text{Price} = \text{Area} \times (\text{Price Per Square Inch})$$

    (b) He decides that one slices of pizza must cost \$2. Find the angle formed by one slice of pizza. Give your answer **rounded to the nearest degree**.

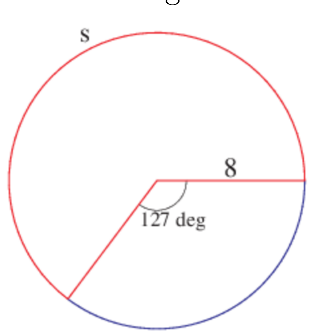
    (c) About how many slices can he get out of each pizza?

11. From a point 15 meters above ground level, a surveyor measures the angle of depression of an object on the ground at 68°. Approximate the distance from the object to the point on the ground directly beneath the surveyor. (Round to the nearest hundredth.)

12. A surveyor standing 57 meters from the base of a building measures the angle to the top of the building and finds it to be 36°. The surveyor then measures the angle to the top of the radio tower on the building and finds it is 50°. How tall is the radio tower. (round to the nearest hundredth).

13. Find the radian and degree measures of the central angle θ subtended by the arc of length $s = 11$ cm on a circle of radius $r = 5$ cm. Then find the area of the sector determined by θ. Give an exact answer.

14. Find the length of the arc s shown in the figure below. Give an exact answer.



Watch the Pre-Class videos for Section 4.5 A and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Let $f(x) = 2\sin(3x - \frac{\pi}{2})$.

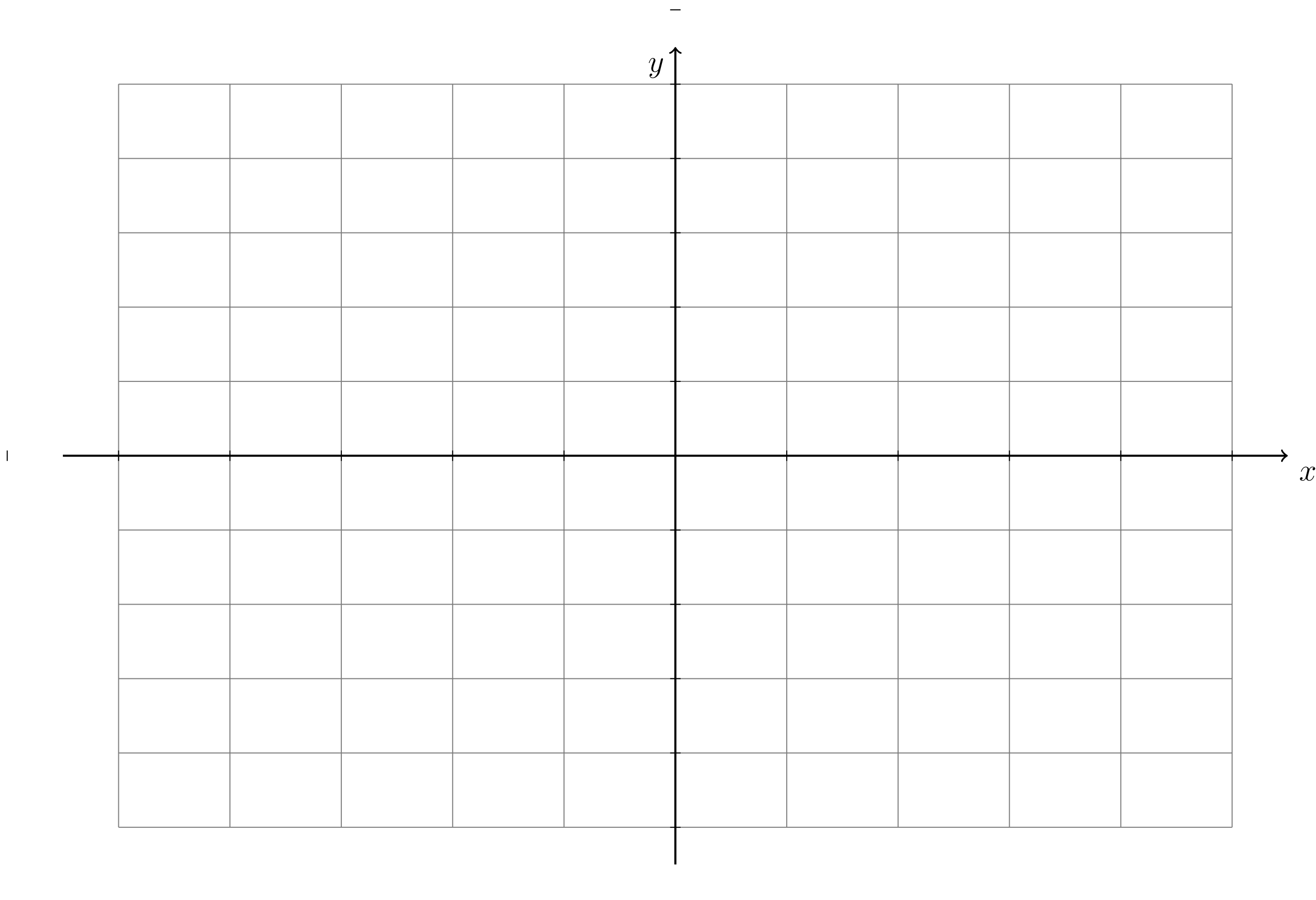
   (a) Determine the period, amplitude, and phase shift of $f(x) = 2\sin(3x - \frac{\pi}{2})$.

   (b) Find an interval containing exactly one cycle (period).

   (c) Determine the x-values of the five key points in the cycle above.

   $x_1 =$ $x_2 =$ $x_3 =$ $x_4 =$ $x_5 =$

2. (2 points) Graph $f(x) = 2\sin(3x - \frac{\pi}{2})$.



3. Determine the period, amplitude, and phase shift of $f(x) = -7\cos(\pi x - 4)$

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Determine the amplitude and period of the function.
   (a) $y = 7\sin(2x)$

   (b) $y = \frac{1}{7}\sin(2\pi x)$

   (c) $y = -7\cos\left(-\frac{2}{3}x\right)$

2. Identify the phase shift and indicate whether the shift is to the left or to the right.
   (a) $\cos(x - \frac{\pi}{3})$

   (b) $\cos(2x + \frac{\pi}{3})$

   (c) $\sin(2\pi x - \frac{\pi}{8})$

3. Let $f(x) = 2\cos(x + \pi) - 1$.

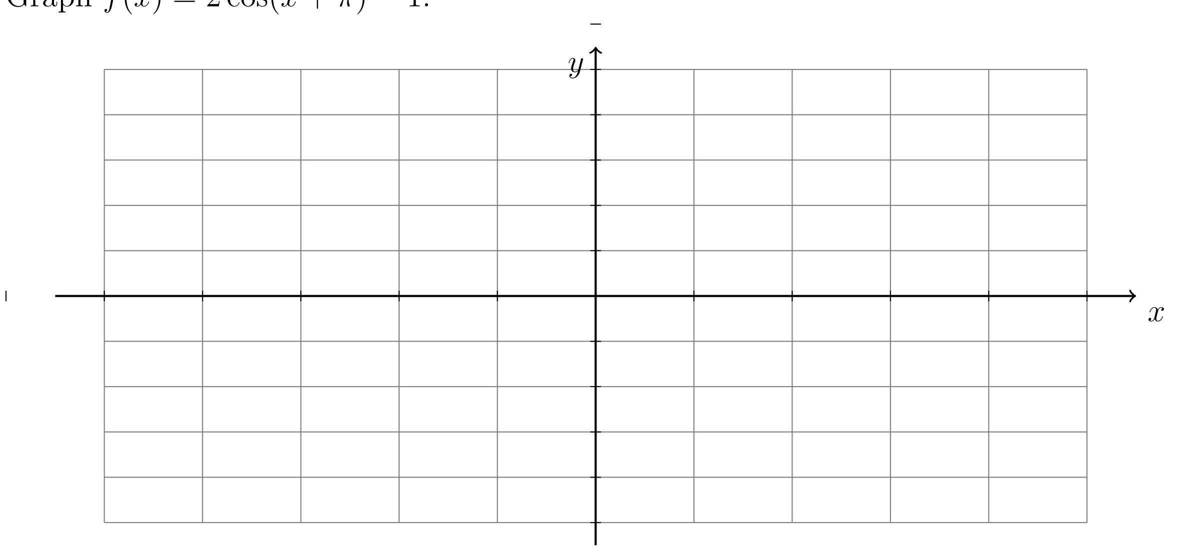
   (a) Determine the period, amplitude, and phase shift of $f(x) = 2\cos(x+\pi)-1$.

   (b) Find an interval containing exactly one cycle (period).

   (c) Determine the x-values of the five key points in the cycle above.

   $x_1 =$ $\qquad$ $x_2 =$ $\qquad$ $x_3 =$ $\qquad$ $x_4 =$ $\qquad$ $x_5 =$

   (d) Graph $f(x) = 2\cos(x + \pi) - 1$.



4. Let $f(x) = -5\sin\left(\frac{1}{3}x + \frac{\pi}{6}\right)$.

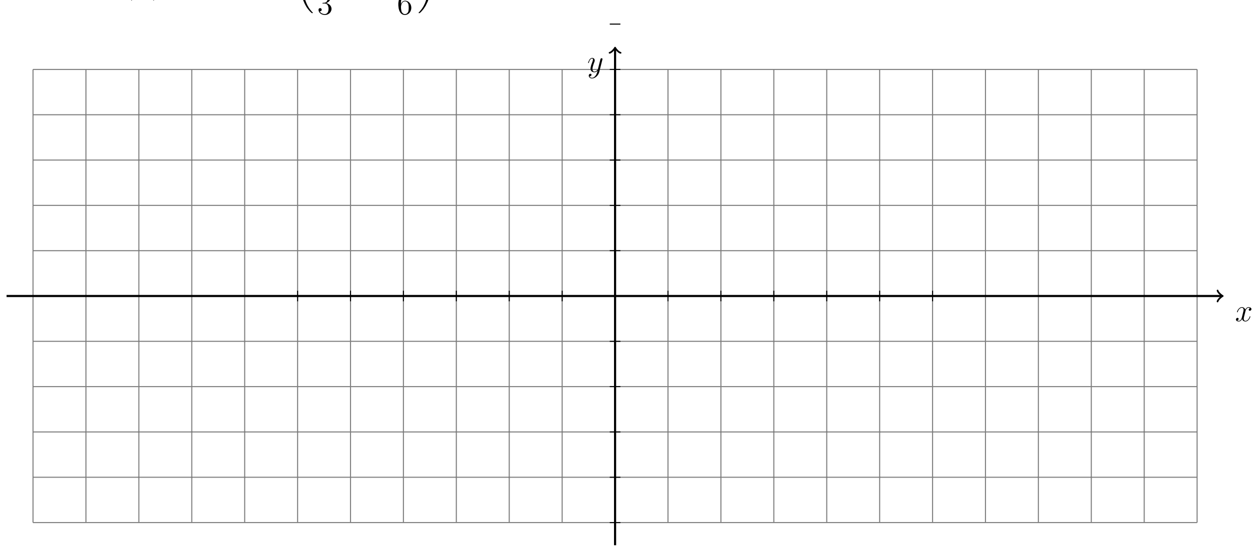
   (a) Determine the period, amplitude, and phase shift of $f(x) = -5\sin\left(\frac{1}{3}x + \frac{\pi}{6}\right)$.

   (b) Find an interval containing exactly one cycle (period).

   (c) Determine the x-values of the five key points in the cycle above.

   $x_1 =$ $x_2 =$ $x_3 =$ $x_4 =$ $x_5 =$

   (d) Graph $f(x) = -5\sin\left(\frac{1}{3}x + \frac{\pi}{6}\right)$.



5. The function $y = a\sin(x) + d$ has range $[-10, 28]$. Assuming that a is positive, determine the values for a and d.

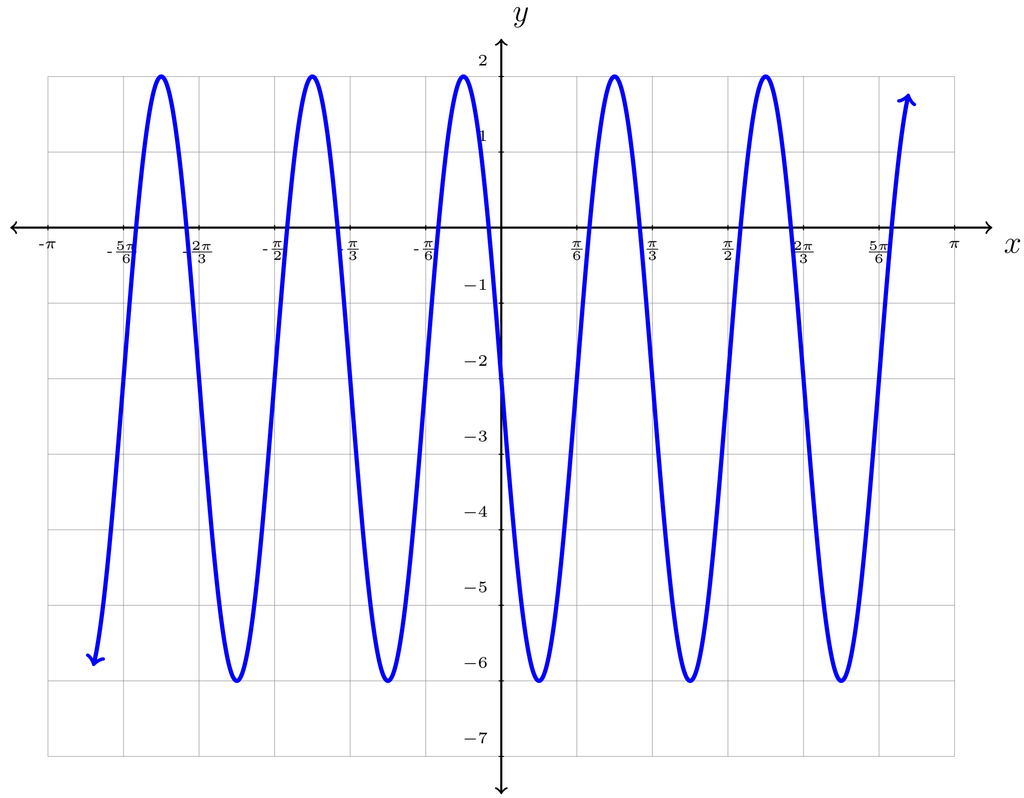
6. The function $y = a\cos(x) + d$ has range $[-26, 10]$. Assuming that a is positive, determine the values for a and d.

7. Let $f(x) = -7\sin(6x)$.

   (a) Determine the coordinates (x, y) of the first maximum turning point on the graph $f(x)$ in the interval $(0, 2\pi)$.

   (b) Determine the coordinates (x, y) of the first minimum turning point on the graph $f(x)$ in the interval $(0, 2\pi)$.

Watch the Pre-Class videos for Section 4.5 B and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.



1. The graph above is a **sine** graph that has been transformed.

   (a) Determine the amplitude, period, phase shift, and vertical shift of function above.

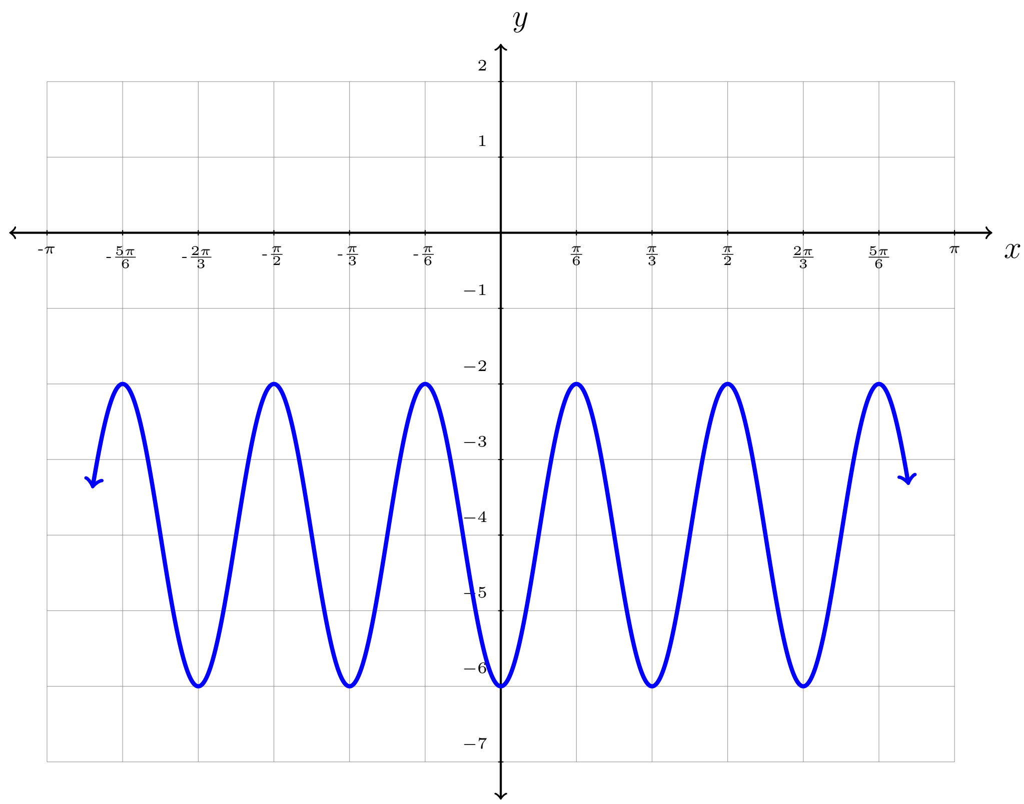
   Amplitude: **Period**: .

   Phase Shift: **Vertical Shift**: .

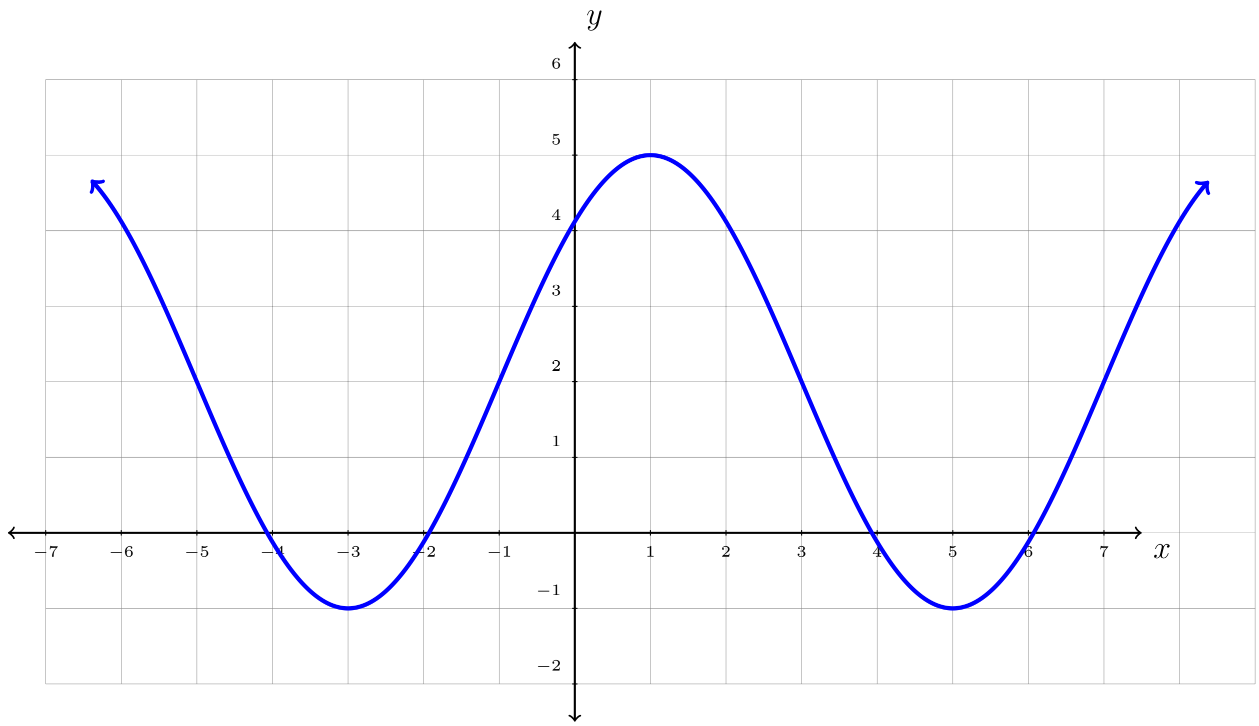
   (b) Determine a formula for $f(x) = A\sin(Bx + C) + D$ for the graph above.

Instructions: Work together in groups of 3 or 4 to complete the following problems.

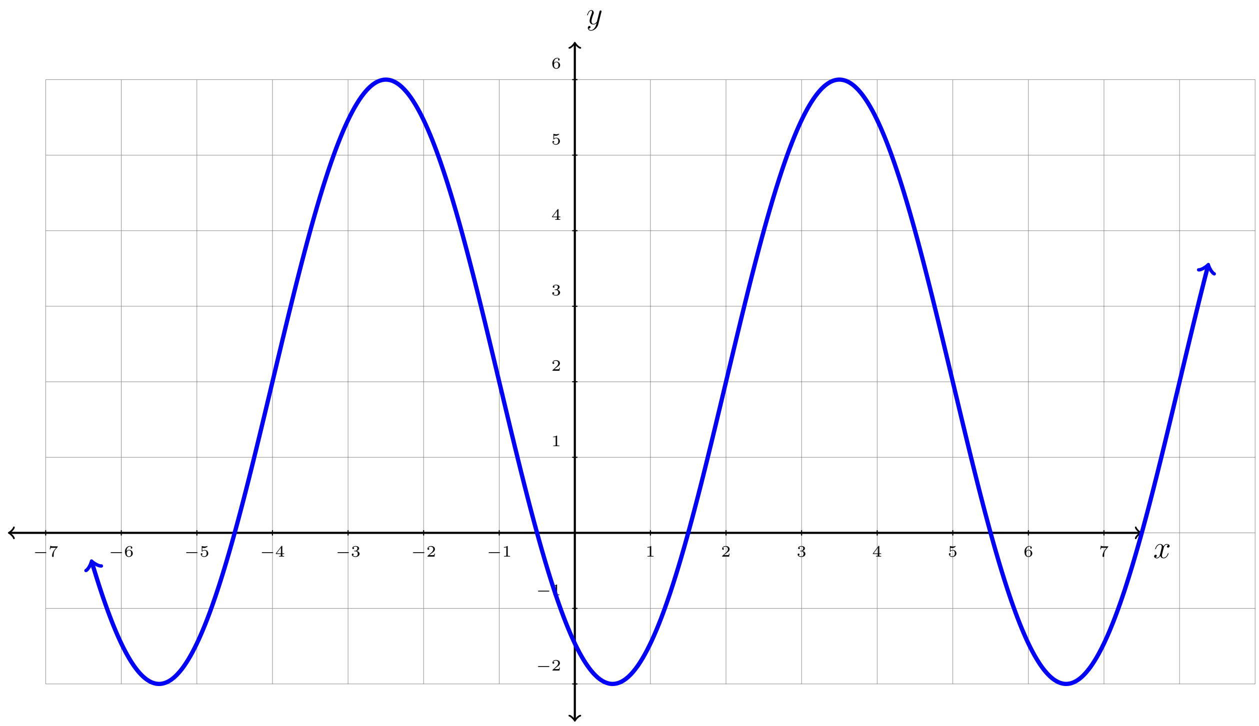
1. Write an equation of the form $A\cos(Bx + C) + D$ for the given graph where $A > 0$ and $B > 0$.



2. Write an equation of the form $A\sin(Bx + C) + D$ for the given graph where $A > 0$ and $B > 0$.



3. Write equations of the form $A\sin(Bx+C)+D$ **AND** $A\cos(Bx+C)+D$ and for the given graph where $A>0$ and $B>0$.



4. The water level relative to the top of a boat dock varies with the tides. One particular day, low tide occurs at midnight and the water level is 7ft below the dock. The first high tide of the day occurs at approximately 6:00 AM, and the water level is 3ft below the dock. The next low tide occurs at noon and the water level is again 7ft below the dock.
   Assuming that this pattern continues indefinitely and behaves like a cosine wave, write a function of the form $w(t) = A\cos(Bt + C) + D$. The value $w(t)$ is the water level (in ft) relative to the top of the dock, t hours after midnight.

5. The function $f(x) = A\sin(Bx) + D$ has a period of 13π. If the graph of $f(x)$ oscillates between 2 and 20, determine the numeric values for A, B, and D. (You may assume that A, B, and D are positive.)

6. Write the range of the function in interval notation.

   (a) $y = \sin(x)$

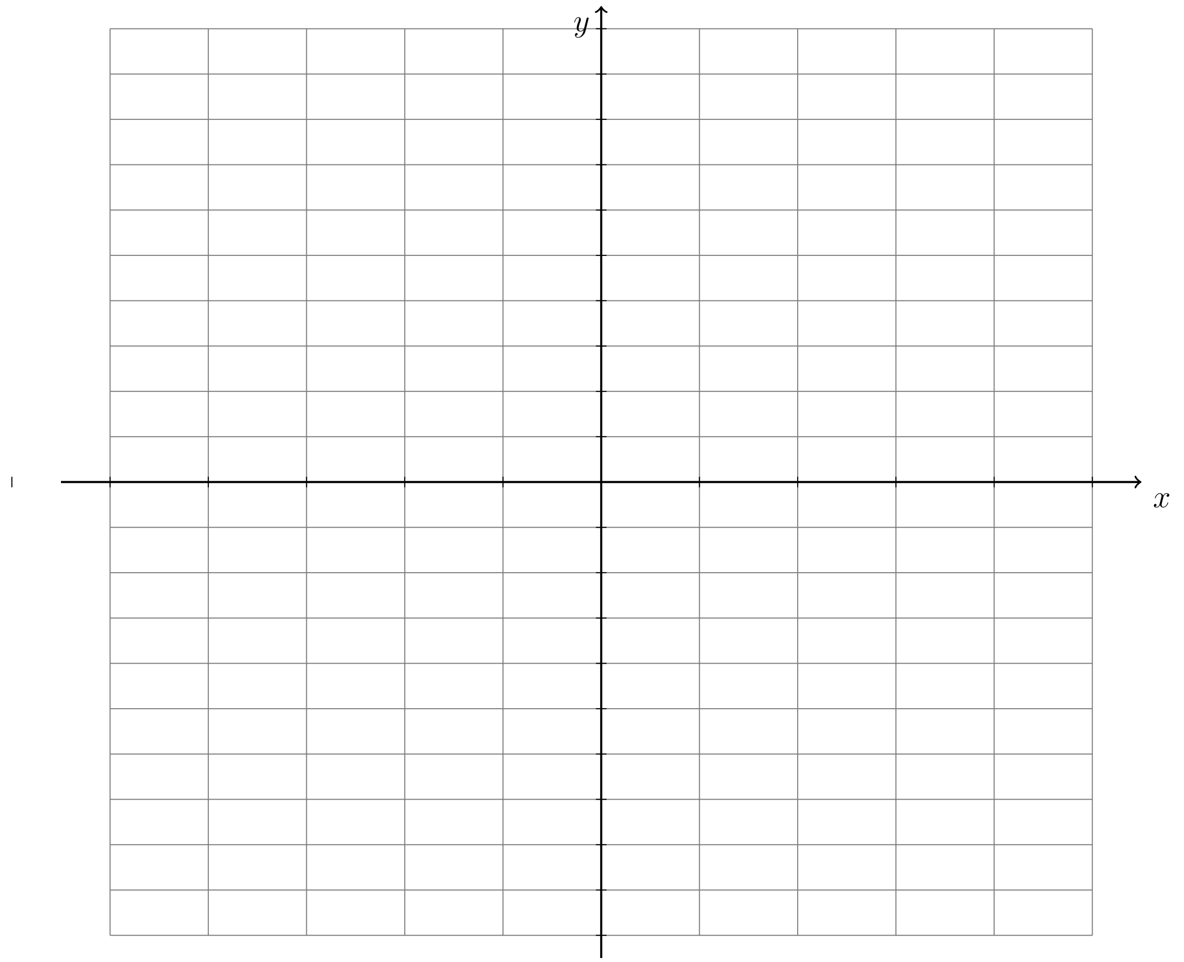
   (b) $y = \cos(x)$

   (c) $y = 8\cos(2x - \pi) + 4$

   (d) $y = -3\cos(x + \frac{\pi}{3}) - 5$

   (e) $y = -6\sin(3x - \frac{\pi}{2}) - 2$

7. Graph $f(x) = -6\sin(3x - \frac{\pi}{2}) - 2$.



Watch the Pre-Class videos for Section 4.7 A and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Find the exact value of the following inverse functions.

   (a) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

   (b) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

   (c) $\tan^{-1}(-1)$

2. Find the exact value of the expression. Do not use a calculator.

   (a) $\sin(\sin^{-1}(1))$

   (b) $\sin^{-1}\left(\sin\left(\frac{7\pi}{6}\right)\right)$

3. Find the exact value of the expression. Do not use a calculator.

   (a) $\cos(\cos^{-1}(1))$

   (b) $\cos^{-1}(\cos(1))$

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Use the unit circle/reference triangles to determine the value of each of the following.
   (a) $\arcsin\left(\frac{\sqrt{2}}{2}\right)$
   (b) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
   (c) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
   (d) $\arcsin\left(-\frac{1}{2}\right)$
2. Use the unit circle/reference triangles to determine the value of each of the following.
   (a) $\arccos\left(\frac{\sqrt{2}}{2}\right)$
   (b) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
   (c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
   (d) $\arccos\left(-\frac{1}{2}\right)$
3. Use the unit circle/reference triangles to determine the value of each of the following.
   (a) $\arctan(1)$
   (b) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
   (c) $\tan^{-1}\left(\sqrt{3}\right)$
   (d) $\arctan(-1)$

4. Find the exact value of the expression. Do not use a calculator.

   (a) $\arcsin\left(\sin\frac{\pi}{3}\right) =$

   (b) $\arcsin\left(\sin\frac{5\pi}{4}\right) =$

   (c) $\arccos\left(\cos\frac{11\pi}{6}\right) =$

   (d) $\cos\left(\arccos 0.56\right) =$

   (e) $\tan\left(\arctan 1754\right) =$

   (f) $\arctan\left(\tan\frac{23}{814}\right) =$

5. Use a calculator to approximate the degree measure (to 1 decimal place) or radian measure (to 4 decimal places) of the angle θ subject to the given conditions. (**Hint:** Use reference angles!)

   (a) $\cos(\theta) = -\dfrac{8}{11}$ and $180^\circ \le \theta \le 270^\circ$

   (b) $\tan(\theta) = -\dfrac{9}{7}$ and $\frac{\pi}{2} < \theta < \pi$

   (c) $\sin(\theta) = \dfrac{12}{19}$ and $90^\circ < \theta < 180^\circ$

6. A student measures the length of the shadow of the Washington Monument to be 620 ft. If the Washington Monument is 555 ft tall, approximate the angle of elevation of the Sun to the nearest tenth of a degree.

7. A balloon advertising an open house is stabilized by two cables of lengths 20 ft and 40 ft tethered to the ground. If the perpendicular distance from the balloon to the ground is
$10\sqrt{3}$ ft, what is the degree measure of the angle each cable makes with the ground? Round to the nearest tenth of a **degree** if necessary.

Watch the Pre-Class videos for Section 4.7 B and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Determine the exact value of the following. Show all work.

$$\sin\left(\tan^{-1}\left(\frac{12}{5}\right)\right)$$

2. Determine the exact value of the following. Show all work.

$$\cos\left(\sin^{-1}\left(-\frac{2}{11}\right)\right)$$

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Find the exact values of the following.

   (a) $\cos\left(\tan^{-1}(\frac{\sqrt{3}}{3})\right)$

   (b) $\tan\left(\sin^{-1}(-\frac{2}{3})\right)$

   (c) $\sin\left(\cos^{-1}(\frac{3}{4})\right)$

   (d) $\sec\left(\tan^{-1}(\frac{4}{3})\right)$

2. Write the expression as an **algebraic** expression. (There should be no trig functions in your answers.)

   (a) $\sin\left(\cos^{-1}\left(\frac{\sqrt{x^2-25}}{x}\right)\right)$ for $x > 5$

   (b) $\tan\left(\cos^{-1}\left(\frac{3}{x}\right)\right)$ for $x > 3$

   (c) $\sin\left(\tan^{-1}\left(x\right)\right)$ for $x > 0$

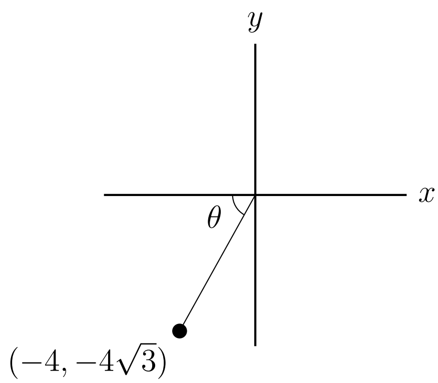
3. A group of campers hike down a steep path. One member of the group has an altimeter on his watch to measure altitude. If the path is 1250 yd and the amount of altitude lost is 480 yd, what is the angle of incline? Round to the nearest tenth of a **degree**.

4. A video camera located at ground level follows the liftoff of an Atlas V Rocket from the Kennedy Space Center. Suppose that the camera is 1000 m from the launch pad.

   (a) Write the angle of elevation θ from the camera to the rocket as a function of the rocket's height, h.

   (b) Use a calculator to find θ to the nearest tenth of a degree when the rocket's height is 400 m, 1500 m, and 3000 m.

5. Determine the value of θ associated with the coordinate in the figure below. (Numerical answers should be to within 2 decimal digits.)



6. Determine the **exact** values of each of the expressions below. (Do not use a value from a calculator but derive the true value.)

   (a) $\cos\left(\frac{\pi}{2} + \arcsin\left(\frac{3}{5}\right)\right)$

   (b) $\cos\left(\pi + \arcsin\left(\frac{3}{5}\right)\right)$

   (c) $\cos\left(2\pi - \arcsin\left(\frac{3}{5}\right)\right)$

Watch the Pre-Class videos for Section 5.1 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Simplify the expression. Write the final form with no fractions or products.

$$\cos(x)\tan(x)\csc(x)$$

2. Verify the trigonometric identities and **write the name of the fundamental identity used at each step**. Remember to only manipulate one side. The other side should stay the same.

   (a) $\sin(-x) + \csc(x) = \cot(x)\cos(x)$

(b) $\dfrac{\sin^2(x)+1}{\cos^2(x)}+1 = 2\sec^2(x)$

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Verify each identity.

   (a) $\cot(-x)\sin(-x) = \cos(x)$

   (b) $\dfrac{\csc(x) - \sec(x)}{\csc(x) + \sec(x)} = \dfrac{\cot(x) - 1}{\cot(x) + 1}$

(c) $\dfrac{\tan(x)+\tan(y)}{\tan(x)\tan(y)-1} = \dfrac{\sin(x)\cos(y)+\cos(x)\sin(y)}{\sin(x)\sin(y)-\cos(x)\cos(y)}$

2.

$$\tan(-x)\cos(x) = -\sin(x)$$

Choose the sequence of steps below that verifies the identity.

(a) $\tan(-x)\cos(x) = -\tan(x)\cdot\cos(x) = -\dfrac{\cos(x)}{\sin(x)}\cdot\cos(x) = -\sin(x)$

(b) $\tan(-x)\cos(x) = -\tan(x)\cdot\cos(x) = -\dfrac{\sin(x)}{\cos(x)}\cdot\cos(x) = -\sin(x)$

(c) $\tan(-x)\cos(x) = \tan(x)\cdot -\cos(x) = \dfrac{\cos(x)}{\sin(x)}\cdot -\cos(x) = -\sin(x)$

(d) $\tan(-x)\cos(x) = -\tan(x)\cdot -\cos(x) = -\dfrac{\sin(x)}{\cos(x)}\cdot -\cos(x) = -\sin(x)$

3. Choose the correct expression that completes the identity.

$$\frac{\cos^2(x)-\sin^2(x)}{1-\tan^2(x)} =$$

(a) -1

(b) 1

(c) $\cos^2(x)$

(d) $\sin^2(x)$

4. Choose the correct expression that completes the identity.

$$\sin^4(x) - \cos^4(x) =$$

   (a) $1 - 2\cos^2(x)$
   (b) $1 + 2\cos^2(x)$
   (c) $1 - 2\sin^2(x)$
   (d) $1 + 2\sin^2(x)$

5. Verify the identity.

   (a) $\Big(6\cos(\theta) - \sin(\theta)\Big)^2 + \Big(\cos(\theta) + 6\sin(\theta)\Big)^2 = 37.$

   (b) $\dfrac{(\sin(x) + \cos(x))^2}{1 + 2\sin(x)\cos(x)} = 1$

6. Rewrite the expression in terms of the given function.

   (a) $\dfrac{\tan(x) + \cot(x)}{\sec(x)}$ in terms of $\csc(x)$

   (b) $\dfrac{\tan(x)}{-1 + \sec(x)} - \dfrac{\sec(x)}{\tan(x)}$ in terms of $\tan(x)$

7. Verify the identity.

   (a) $\tan^4(t) - \sec^4(t) = -2\tan^2(t) - 1$

   (b) $\csc(x) + \tan^2(x)\csc(x) = \dfrac{1}{\sin(x)\cos^2(x)}$

Watch the Pre-Class videos for Section 5.2 and answer the following questions. Remember that in your written work you are graded on the correctness of your supporting work and not just your final answer. Always give an exact answer unless you are explicitly told to round; calculator approximations will not receive full credit.

1. Evaluate the following using a sum or difference identity and the unit circle.

$$\cos(165^\circ)$$

2. Find the exact value of $\cos(\alpha - \beta)$ given that $\sin\alpha = -\frac{4}{5}$ and $\cos\beta = -\frac{5}{8}$ for α in Quadrant III and β in Quadrant II.

3. Find the exact value of the following.

$$\cos\left(\arcsin\left(-\frac{12}{37}\right)+\arctan\left(\frac{5}{12}\right)\right)$$

Instructions: Work together in groups of 3 or 4 to complete the following problems.

1. Write each expression as the sine, cosine, or tangent of an angle. Then find the value the expression.

   (a) $\sin(10^\circ)\cos(80^\circ) + \cos(10^\circ)\sin(80^\circ)$

   (b) $\sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{6}\right)$

   (c) $\cos(71^\circ)\cos(19^\circ) - \sin(71^\circ)\sin(19^\circ)$

   (d) $\cos\left(\frac{5\pi}{12}\right)\cos\left(\frac{\pi}{12}\right) + \sin\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{12}\right)$

(e) $\dfrac{\tan(25°) + \tan(20°)}{1 - \tan(25°)\tan(20°)}$

(f) $\dfrac{\tan\left(\frac{4\pi}{5}\right) - \tan\left(\frac{11\pi}{20}\right)}{1 + \tan\left(\frac{4\pi}{5}\right)\tan\left(\frac{11\pi}{20}\right)}$

2. Find the exact value of each expression.

   (a) $\sin(105°)$

   (b) $\sin(15°)$

(c) $\cos\left(\frac{7\pi}{12}\right)$

(d) $\sin\left(\frac{\pi}{12}\right)$

(e) $\tan\left(\frac{\pi}{12}\right)$

3. Find the exact value of $\sin(\alpha + \beta)$, $\cos(\alpha + \beta)$, and $\tan(\alpha + \beta)$ under the given conditions.

   (a) $\sin(\alpha) = \frac{24}{25}$, α lies in quadrant I, and $\sin(\beta) = \frac{4}{5}$, β lies in quadrant II.

   (b) $\sin(\alpha) = \frac{7}{25}$, $0 < \alpha < \frac{\pi}{2}$, and $\cos(\beta) = \frac{15}{17}$, $0 < \beta < \frac{\pi}{2}$

4. Use the given information to find the exact value of $\cos(\alpha - \beta)$:

   - $\sin(\alpha) = \frac{3}{5}$, α lies in quadrant II, and
   - $\cos(\beta) = \frac{2}{5}$, β lies in quadrant I.

5. Use the given information to find the exact value of $\tan(\alpha + \beta)$:

   - $\tan(\alpha) = \frac{1}{3}$, α lies in quadrant III, and
   - $\cos(\beta) = \frac{1}{5}$, β lies in quadrant IV.

6. Verify the identities. What does each identity tell you about the graphs of sine, cosine, and tangent? Can you interpret each identity using the unit circle?

   (a) $\sin(\theta + 2\pi) = \sin(\theta)$

   (b) $\cos(\theta + 2\pi) = \cos(\theta)$

   (c) $\tan(\theta + \pi) = \tan(\theta)$

   (d) $\sin(\theta + \pi) = -\sin(\theta)$

   (e) $\cos(\theta + \pi) = -\cos(\theta)$

   (f) $\sin\left(\theta + \frac{\pi}{2}\right) = \cos(\theta)$

   (g) $\cos\left(\theta + \frac{\pi}{2}\right) = -\sin(\theta)$

Chapter 5

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