

Topics: inverse trig functions

Student Learning Outcomes:

1. Students will be able to simplify trigonometric expressions.
 2. Students will be able to verify trigonometric identities.
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1 Simplifying Trigonometric Expressions

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin x = \frac{1}{\csc x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Even and Odd Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x$$

$$\sec(-x) = \sec x$$

$$\cot(-x) = -\cot x$$

Note: It is often helpful to notice alternative forms of Pythagorean Identities, such as $\sin^2 x = 1 - \cos^2 x$ or $\cos^2 x = 1 - \sin^2 x$.

1. Simplify each of the following. Write the final form with no fractions or products.

(a) $\tan(x) \cos^2(x) \sec(x)$

(b) $\frac{\cos(\theta)}{1 + \sin(\theta)} + \tan(\theta)$

2 Verify Trigonometric Identities

Trigonometric Identities

An **identity** is an equation which it is true for all values of x for which the expressions on the left and right are defined.

Guidelines for proving Trigonometric Identities:

- (a) Work with one side of the equation (usually the more complicated side) and keep the other side in mind as your final goal.
- (b) Look for opportunities to apply the fundamental identities.
 - If the expression is a product or quotient of factors, consider the reciprocal and quotient identities.
 - If squared terms are present, look to see if the terms can be grouped in one of the forms of a Pythagorean identity.
 - If an expression involves a negative argument, consider using the even or odd function identities.
- (c) Apply basic algebraic techniques such as factoring, multiplying terms, combining like terms, and writing fractions with a common denominator.
- (d) Consider writing expressions explicitly in terms of sine and cosine.

2. Prove that each of the following equations is an identity. (Note: The **entire** proof is your answer.)

(a) $\frac{\sin(-x) \cot(-x)}{\cos x} = 1$

(b) $\frac{1}{1 - \cos(x)} - \frac{1}{1 + \cos(x)} = 2 \cot(x) \csc(x)$

$$(c) \ 1 - \frac{\sin^2 t}{1 + \cos t} = \cos t$$

$$(d) \ \frac{\cot x}{\csc x} - \frac{\csc x}{\cot x} = -\sin x \tan x$$

(e) $\frac{\sin x}{\csc x - \cot x} = 1 + \cos x$

Student Learning Outcomes Check

1. Are you able to simplify trigonometric expressions?
2. Are you able to verify trigonometric identities?

If any of your answers were no, please ask about these topics in class.