

Topics: solving exponential and logarithmic equations, exponential and logarithmic functions, defining relationships from written descriptions

Student Learning Outcomes:

1. Students will be able to solve exponential equations.
 2. Students will be able to solve logarithmic equations.
 3. Students will be able to use exponential and logarithmic equations in applications.
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1 Exponential Equations

Equivalence Property of Exponential Expressions: If b, x , and y are real numbers where $b > 0, b \neq 1$. Then

$$b^x = b^y \text{ implies that } x = y.$$

1. Solve the following exponential equations.

(a) $3^{2x-6} = 81$

(b) $25^{4-t} = \left(\frac{1}{5}\right)^{3t+1}$

Steps to Solve Exponential Equations by Using Logarithms

1. Isolate the exponential expression on one side of the equation.
2. Take a logarithm of the same base on both sides.
3. Use the power property of logarithms to "bring down" the exponent.
4. Solve the resulting equation.

2. Solve the exponential equation using logarithms.

(a) $10^{5+2x} + 820 = 49,600$

(b) $2000 = 18,000e^{-0.4t}$

3. Solve the exponential equation.

(a) $4^{2x-7} = 5^{3x+1}$

(b) $e^{2x} + 5e^x - 36 = 0$

2 Solve Logarithmic Equations

Equivalence Property of Logarithmic Expressions: If b, x , and y are positive real numbers with

$$\log_b x = \log_b y \text{ implies that } x = y.$$

4. Solve the Logarithmic Equation.

(a) $\log_2(3x - 4) = \log_2(x + 2)$

(b) $\ln(x - 4) = \ln(x + 6) - \ln(x)$

Steps to Solve Logarithmic Equations by Using Exponential Form

1. Given a logarithmic equation, isolate the logarithms on one side of the equation.
 2. Use the properties of logarithms to write the equation in the form $\log_b x = k$, where k is a constant.
 3. Write the equation in exponential form.
 4. Solve the equation from step 3.
 5. Check the potential solution(s) in the original equation.
5. Solve the logarithmic equation.

(a) $4\log_3(2t - 7) = 8$

(b) $\log(w + 47) = 2.6$

3 Exponential and Logarithmic Equations in Applications

6. A couple invests \$8000 in a bond fund. The expected yield is 4.5% and the earnings are reinvested monthly.

(a) Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to write a model representing the amount A (in \$) in the account after t years. The value r is the interest rate and n is the number of times interest is compounded per year.

(b) Determine how long it will take the initial investment to double. Round to one decimal place.

7. Suppose that the sound at a rock concert measures 124 dB (decibels).

- (a) Use the formula $L = 10 \log \left(\frac{I}{I_0} \right)$ to find the intensity of sound I (in W/m^2). The variable L represents the loudness of sound (in dB) and $I_0 = 10^{-12} \text{W}/\text{m}^2$.

- (b) If the threshold at which the sound becomes painful is $1 \text{ W}/\text{m}^2$, will the music at this concert be physically painful?

Student Learning Outcomes Check

1. Can you solve exponential equations?
2. Can you solve logarithmic equations?
3. Can you use exponential and logarithmic equations in applications?

If any of your answers were no, please ask about these topics in class.