Topics: exponential functions, compound interest, the number e, exponential functions with base e, growth and decay

Student Learning Outcomes:

1. Students will be able to recognize a logarithic function graphically and algebraically.
2. Students will be able to evaluate the logarithmic expressions.
3. Students will be able to apply basic logarithmic properties.

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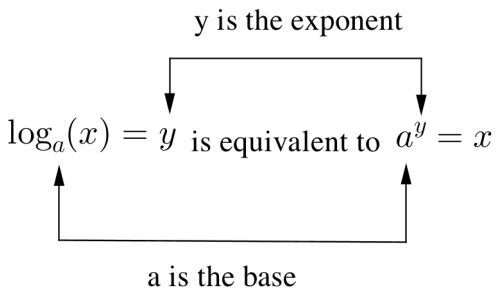
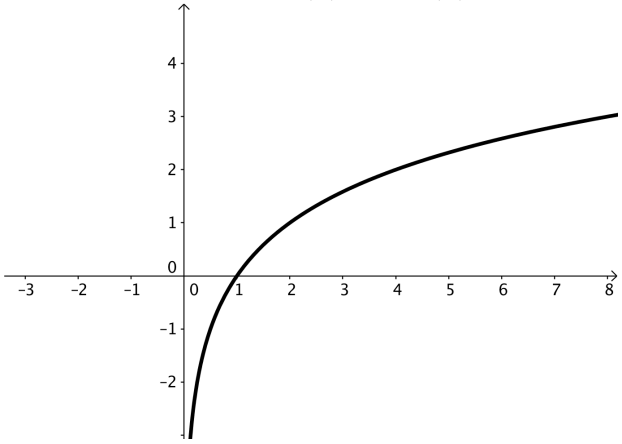
1 Logarithmic Functions

Let a be a positive real number different from 1. The *logarithm of x with base a* is defined by $y = \log_a(x)$ if and only if $a^y = x$.

Above, the left-hand equation is said to be in *logarithmic form*, while the right-hand equation is said to be in *exponential form*. The equations are *equivalent*: they have the same solutions.



Example: $f(x) = \log_2(x)$



1. Write in exponential form.
   model: $\log_a(x) = y$ $\log_2(16) = 4$ $\log_p(13) = y$

2. Write in logarithmic form.
   model: $a^y = x$ $4^{-3} = \dfrac{1}{64}$ $\pi^t = 9.4$

3. The expression $\log x$, called the *common logarithm*, is shorthand for $\log_{10}(x)$. Write in logarithmic form: $10^{2x+3} = 7$.

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5. Find the domain of the function $f(x) = \ln(9 - 6x)$.

2 Evaluating Logarithmic Expressions

6. Find the number, if possible. Rewrite in exponential form, either to solve, or to check.
$\log_2\left(\frac{1}{8}\right)$ $\log_3(27)$ $\log_4(0)$ $\log_b\left(\frac{1}{b^3}\right)$

Basic Logarithmic Properties Involving One

1. $\log_b b = 1$ because 1 is the exponent to which b must be raised to obtain b. $(b^1 = b)$
2. $\log_b 1 = 0$ because 0 is the exponent to which b must be raised to obtain 1. $(b^0 = 1)$

Inverse Properties of Logarithms

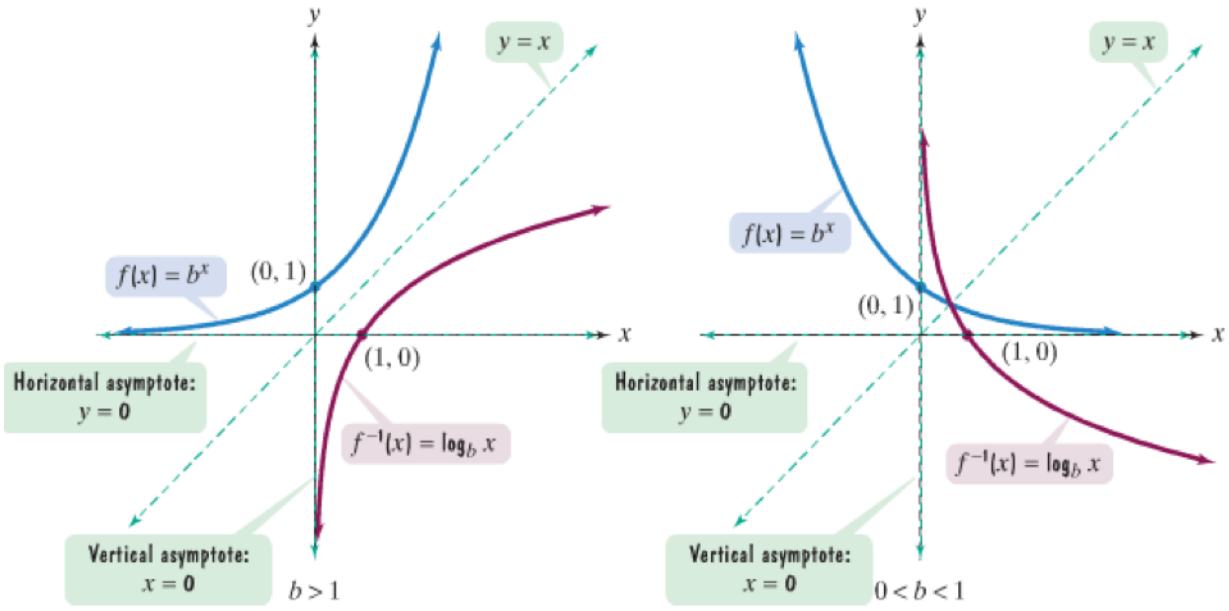
For $b > 0$ and $b \neq 1$,

$\log_b b^x = x$ The logarithm with base b of b raised to a power equals that power.

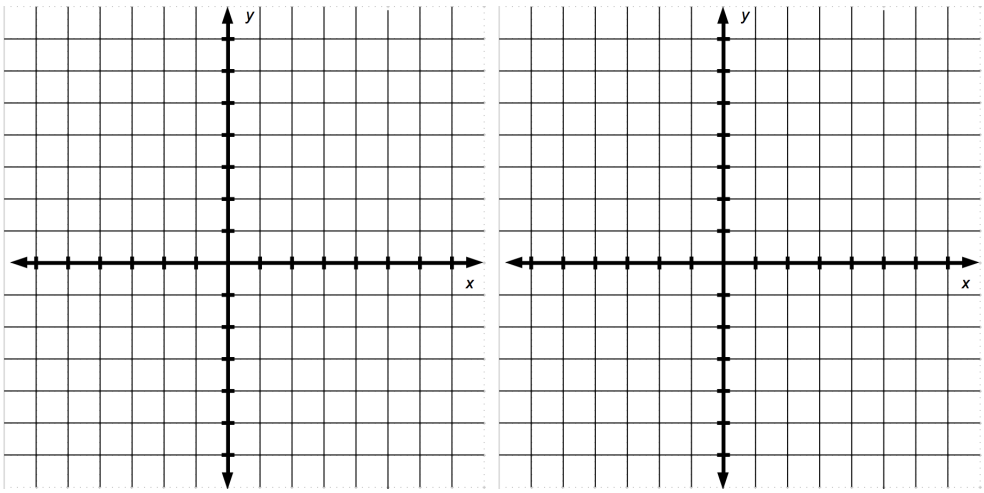
$b^{\log_b x} = x.$ b raised to the logarithm with base b of a number equals that number.

7. Evaluate the common and natural logarithms.
$\log(100,000)$ $\log(0.001)$ $\ln(e^4)$ $\ln\left(\frac{1}{e}\right)$

3 Graphing Logarithmic Functions



8. Graph $\log_2 x$ and $\log_{1/4} x$



Student Learning Outcomes Check

1. Can you recognize a logarithic function graphically and algebraically?
2. Can you evaluate the logarithmic expressions?
3. Are you able to apply basic logarithmic properties?

If any of your answers were no, please ask about these topics in class.