Topics: increasing and decreasing, reference angles

Student Learning Outcomes:

1. Students will be able to evaluate trigonometric functions of any angle.
2. Students will be able to determine reference angles.
3. Students will be able to evaluate trigonometric functions using reference angles.

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1 Trigonometric Functions of Any Angle

Recall: We have used trigonometric functions with acute angles. What if the angle is not acute?

A circle with center $(0,0)$ and radius r has the equation $x^2+y^2=r^2$. Choose a point $P(x,y)$. We can create reference triangles inside the circle in order to define our trig functions.

If the point (x,y) lies on the **terminal side** of θ, the six trig functions of θ can be defined as follows:

$$\sin(\theta)=\frac{y}{r} \qquad \cos(\theta)=\frac{x}{r} \qquad \tan(\theta)=\frac{y}{x}$$

$$\csc(\theta)=\frac{r}{y} \qquad \sec(\theta)=\frac{r}{x} \qquad \cot(\theta)=\frac{x}{y}$$

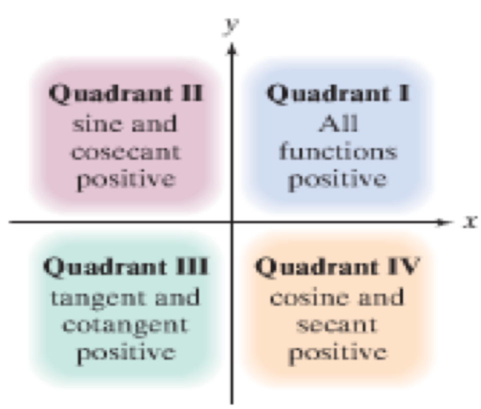
What changes when we are on the unit circle?

1. Let $P(-6, 8)$ be a point on the terminal side of θ. Find each of the six trig functions of θ.

Trigonometric Functions of Quadrantal Angles

θ	$0° = 0$	$90° = \frac{\pi}{2}$	$180° = \pi$	$270° = \frac{3\pi}{2}$
$\sin\theta$	0	1	0	-1
$\cos\theta$	1	0	-1	0
$\tan\theta$	0	undefined	0	undefined

2 The Signs of the Trigonometric Functions



2. Let θ be an angle in standard position. Name the quadrant in which θ lies.

$$\sin(\theta) < 0, \qquad \tan(\theta) < 0$$

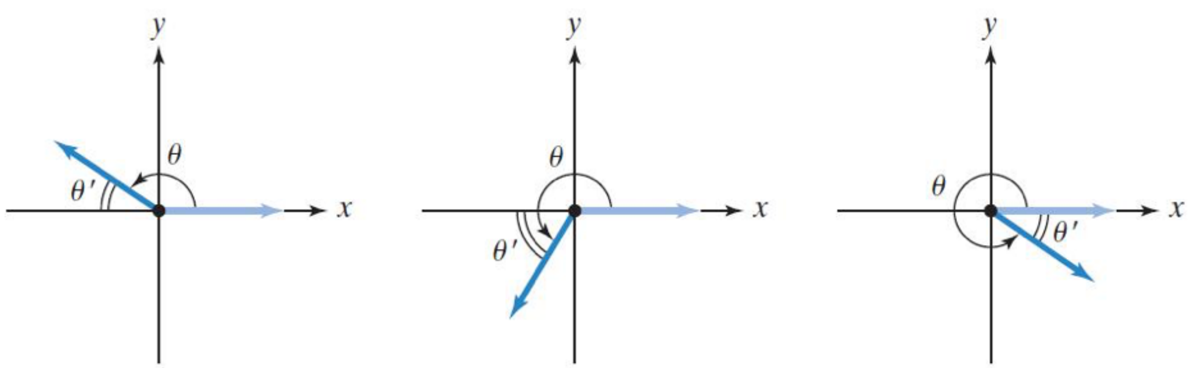
3. Find the exact value of each of the remaining trigonometric functions of θ.

$$\sin(\theta) = \frac{4}{5}, \quad \theta \text{ in Quadrant 2}$$

3 Reference Angles

Let's draw our two reference triangles as well as an $xy-$plane.

Let θ be a nonquadrantal angle in standard position. The reference angle for θ, called θ_R, is the *acute* angle θ_R that the *terminal side* of θ makes with the x-axis.



If $90° < \theta < 180°$, then $\theta' = 180° - \theta$.

If $180° < \theta < 270°$, then $\theta' = \theta - 180°$.

If $270° < \theta < 360°$, then $\theta' = 360° - \theta$.

4. Determine the reference angle θ_R for each of the following angles.

(a) $\theta = 45°$ Quadrant ________ θ_R ________

(b) $\theta = -73°$ Quadrant ________ θ_R ________

(c) $\theta = -915°$ Quadrant ________ θ_R ________

5. Determine the reference angle θ_R for each of the following angles.

(a) $\theta = \dfrac{19\pi}{6}$ Quadrant ________ θ_R ________

(b) $\theta = \dfrac{47\pi}{4}$ Quadrant ________ θ_R ________

(c) $\theta = \dfrac{-7\pi}{6}$ Quadrant ________ θ_R ________

Let θ be a nonquadrantal angle in standard position. To find the value of a trig function at θ, find its value for the *reference angle* θ_R and prefix the appropriate sign (using ASTC).

6. Determine the exact values of $\sin(2\pi/3)$ and $\cos(2\pi/3)$.

7. Determine the exact values of $\cos(210°)$, $\sec(210°)$, and $\cot(210°)$.

Student Learning Outcomes Check

1. Can you evaluate trigonometric functions of any angle?
2. Are you able to determine reference angles?
3. Can you evaluate trigonometric functions using reference angles?

If any of your answers were no, please ask about these topics in class.