

**Topics:** unit circle, trigonometric functions, trigonometric identities, periodic

**Student Learning Outcomes:**

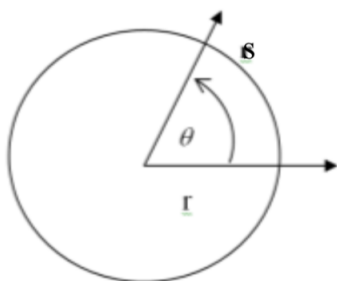
1. Students will be able to evaluate trigonometric functions using the unit circle.
2. Students will be able to determine domains of trigonometric functions.
3. Students will be able to use trigonometric identities.

## 1 Trigonometric Functions of Real Numbers

A **unit circle** is a circle of radius 1 with center at the origin. The equation of a unit circle is:

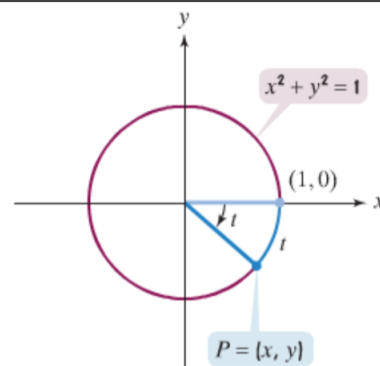
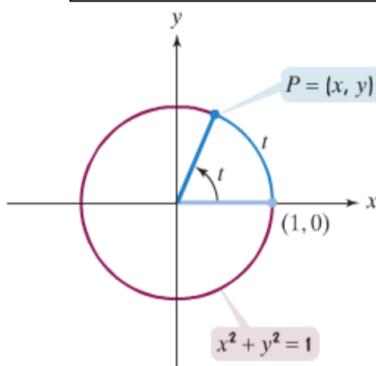
$$x^2 + y^2 = 1$$

**Recall:**



The length of a circular arc:  
 $s = r\theta$

**In a unit circle, the radian measure of the central angle is equal to the length of the subtended arc. Both are given by the same real number  $t$ .**



### Definitions of the Trigonometric Functions in Terms of a Unit Circle

If  $t$  is a real number and  $P = (x, y)$  is a point on the unit circle that corresponds to  $t$ , then

$$\sin t = y$$

$$\csc t = \frac{1}{y}, y \neq 0$$

$$\cos t = x$$

$$\sec t = \frac{1}{x}, x \neq 0$$

$$\tan t = \frac{y}{x}, x \neq 0$$

$$\cot t = \frac{x}{y}, y \neq 0.$$

1. Suppose that the real number  $t$  corresponds to the point  $P(-\frac{2}{3}, -\frac{\sqrt{5}}{3})$  on the unit circle. Evaluate the six trigonometric functions of  $t$ .

(a)  $\sin(t) =$

(b)  $\cos(t) =$

(c)  $\tan(t) =$

(d)  $\csc(t) =$

(e)  $\sec(t) =$

(f)  $\cot(t) =$

## 2 Fundamental Trigonometric Identities

**Reciprocal Identities:**

$$\sin(\theta) = \frac{1}{\csc(\theta)} \quad \cos(\theta) = \frac{1}{\sec(\theta)} \quad \tan(\theta) = \frac{1}{\cot(\theta)}$$

Let's find three more:

**Quotient Identities:**

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

2. Let  $P(\frac{15}{17}, \frac{8}{17})$  be a point on the unit circle that correspond to  $t$ . Find each of the six trig functions of  $t$ .

3. Use the  $(x, y)$  coordinates in the unit circle to find the value of each trig function at the indicated real number  $t$ .

(a)  $\cos\left(\frac{3\pi}{2}\right) =$

(b)  $\tan\left(\frac{11\pi}{6}\right) =$

(c)  $\sec\left(\frac{11\pi}{6}\right) =$

(d)  $\sin(-2\pi) =$

4. Evaluate the six trigonometric functions of the real number  $t$ .

(a)  $t = \frac{5\pi}{3}$

(b)  $t = -\frac{5\pi}{4}$

(c)  $t = \pi$

(d)  $t = \frac{5\pi}{2}$

### 3 Domains of the Trigonometric Functions

## 4 Pythagorean Identities

$$\sin^2(t) + \cos^2(t) = 1 \quad \tan^2(t) + 1 = \sec^2(t) \quad 1 + \cot^2(t) = \csc^2(t)$$

5. Given that  $\tan(t) = \frac{12}{5}$  for  $\pi < t < \frac{3\pi}{2}$ , use an appropriate identity to find the value of  $\sec(t)$ .

6. Given that  $\csc(t) = \frac{5}{4}$  for  $\frac{\pi}{2} < t < \pi$ , use an appropriate identity to find the value of  $\cot(t)$ .

7. Given a real number  $t$ , express  $\sin(t)$  in terms of  $\cos(t)$ .

## 5 Even, Odd, and Periodic Properties

### Even and Odd Trigonometric Functions

The cosine and secant functions are *even*.

$$\cos(-t) = \cos t$$

$$\sec(-t) = \sec t$$

The sine, cosecant, tangent, and cotangent functions are *odd*.

$$\sin(-t) = -\sin t$$

$$\csc(-t) = -\csc t$$

$$\tan(-t) = -\tan t$$

$$\cot(-t) = -\cot t$$

8. Use the unit circle to find the value of  $\cos\left(\frac{2\pi}{3}\right)$  and odd trig functions to find the value of  $\cos\left(-\frac{2\pi}{3}\right)$ .

### Periodic Properties of the Sine and Cosine Functions

$$\sin(t + 2\pi) = \sin t \quad \text{and} \quad \cos(t + 2\pi) = \cos t$$

The sine and cosine functions are periodic functions and have period  $2\pi$ .

### Periodic Properties of the Tangent and Cotangent Functions

$$\tan(t + \pi) = \tan t \quad \text{and} \quad \cot(t + \pi) = \cot t$$

The tangent and cotangent functions are periodic functions and have period  $\pi$ .

### Repetitive Behavior of the Sine, Cosine, and Tangent Functions

For any integer  $n$  and real number  $t$ ,

$$\sin(t + 2\pi n) = \sin t, \quad \cos(t + 2\pi n) = \cos t, \quad \text{and} \quad \tan(t + \pi n) = \tan t.$$

9. Given  $\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$ , determine the value of  $\sin\left(\frac{49\pi}{12}\right)$ .

10. Use properties of trigonometric functions to simplify  $\tan(-3t) - \tan(-3t + \pi)$ .

## 6 Approximate Trigonometric Functions on a Calculator

11. Use a calculator to approximate the function values. Round to 4 decimal places.

(a)  $\cos\left(\frac{2\pi}{7}\right)$

(b)  $\csc(0.92)$

### Student Learning Outcomes Check

1. Can you evaluate trigonometric functions using the unit circle?
2. Can you determine domains of trigonometric functions?
3. Are you able to use trigonometric identities?

**If any of your answers were no, please ask about these topics in class.**