Topics: exponential functions, compound interest, the number e, exponential functions with base e, growth and decay

Student Learning Outcomes:

1. Students will be able to recognize an exponential function graphically and algebraically.
2. Students will be able to evaluate the exponential function base e.
3. Students will be able to use exponential functions in compound interest and growth/decay problems.

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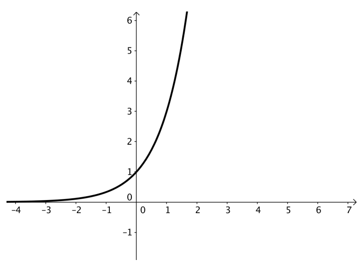
1 Exponential Functions

Exponential functions $y = a^x$ (always assume $a > 0$)

exponential growth: $a > 1$

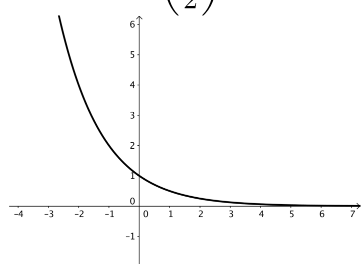
Ex. $f(x) = 3^x$



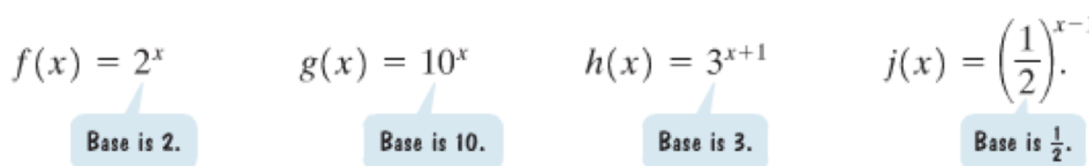
always increasing
one-to-one
has an asymptote of $y = 0$
y-intercept is $(0, 1)$, no x-intercept

exponential decay: $0 < a < 1$

Ex. $g(x) = \left(\frac{1}{2}\right)^x$



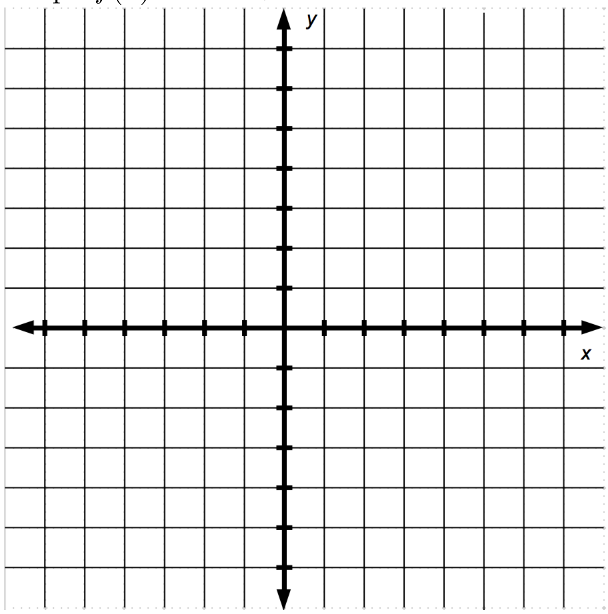
always decreasing
one-to-one
has an asymptote of $y = 0$
y-intercept is $(0, 1)$, no x-intercept



1. Is $f(x) = 1^x$ an exponential function?

2. Is $f(x) = (-4)^x$ an exponential function?

3. Graph $f(x) = 3^{x-2} + 4$.



4. Determine the domain and range of $y = 5^{x-3} + 4$.

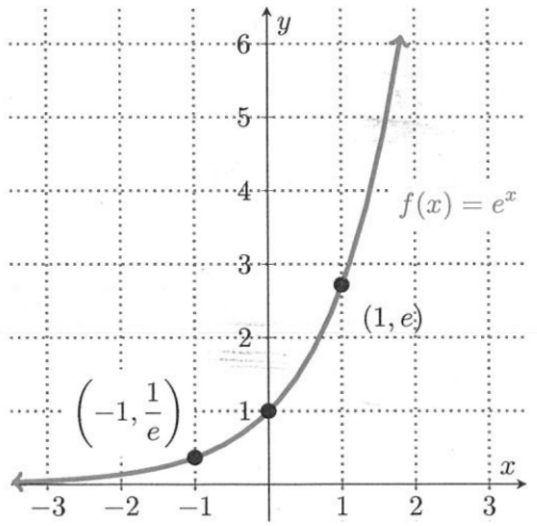
2 Exponential Function Base e

Just like the number $\pi \approx 3.14159$ is important to the study of circles and angle measures, the number $e \approx 2.71828$ is important to problems involving exponential exponential functions (and their inverses).

Where does the number e come from? Consider the expression

$$\left(1 + \frac{1}{n}\right)^n$$

If you plug in larger and larger values for n, this expression gets closer and closer to e. Try it out!



3 Compound Interest

Compound Interest Formula (Annually, Monthly, Quarterly, Daily, Etc.) $A = P\left(1 + \frac{r}{n}\right)^{nt}$
In this formula, P is the principal, r is the annual interest rate in decimal form, n is the number of interest periods per year, t is the number of years P is invested, and A is the amount after t years.

5. Suppose \$ 2000 is invested at a rate of 3% compounded monthly. Find the *principal after 18 months.* (Round your answer to the nearest cent.)

Continuously compounded interest formula $A = Pe^{rt}$
In this formula, P is the principal, r is the annual interest rate in decimal form, t is the number of years P is invested, and A is the amount after t years.

6. If \$1500 is deposited in a savings account that pays interest at a rate of .1% compounded continuously, find the balance after 7 years.

4 Exponential Functions in Applications

Increasing and decreasing exponential functions can be used in a variety of real world applications. For example:

- Population growth can often be modeled by an exponential function.
- The growth of an investment under compound interest increases exponentially.
- The mass of a radioactive substance decreases exponentially with time.

A substance that undergoes radioactive decay is said to be radioactive. The **half-life** of a radioactive substance is the amount of time it takes for one-half of the original amount of the substance to change into something else.

7. The half-life of radium 226 is 1620 years. In a sample originally having 1 gram of radium 226, the amount $A(t)$ in grams of radium 226 present after t years is given by $A(t) = (\frac{1}{2})^{t/1620}$ where t is the time in years after the start of the experiment. How much radium will be present after 3240 years?

Student Learning Outcomes Check

1. Can you recognize an exponential function graphically and algebraically?
2. Can you evaluate the exponential function base e?
3. Are you able to use exponential functions in compound interest and growth/decay problems?

If any of your answers were no, please ask about these topics in class.