Topics: properties of logarithms, change of base

Student Learning Outcomes:

1. Students will be able to apply the product, quotient, and power properties of logarithms.
2. Students will be able to write a logarithm in expanded form.
3. Students will be able to write a logarithm as a single logarithm.
4. Students will be able to apply the change of base formula.

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1 Properties of Logarithms

Product Property of Logarithms: Let b, x, and y be positive real numbers where $b \neq 1$. Then

$$\log_b(xy) = \log_b(x) + \log_b(y).$$

1. Write the logarithm as a sum and simplify if possible. Assume x and y represent positive real numbers.

   (a) $\log_2(8x)$

   (b) $\ln(5xy)$

   Quotient Property of Logarithms: Let b, x, and y be positive real numbers where $b \neq 1$. Then

   $$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y).$$

2. Write the logarithm as a difference of logarithms and simplify if possible. Assume x and y represent positive real numbers.

   (a) $\log_3\left(\frac{c}{d}\right)$

   (b) $\log\left(\frac{x}{100}\right)$

Power Property of Logarithms: Let b and x be positive real numbers where $b \neq 1$. Let p be any real number. Then

$$\log_b(x^p) = p \log_b(x).$$

3. Apply the power property of logarithms.

   (a) $\ln \sqrt[5]{x^2}$

   (b) $\log x^2$

2 Writing a Logarithmic Expression in Expanded Form

4. Write the expression as the sum or difference of logarithms.

   (a) $\log_2 \left(\dfrac{z^3}{xy^5} \right)$

   (b) $\log \sqrt[3]{\dfrac{(x+y)^2}{10}}$

3 Writing a Logarithmic Expression as a Single Logarithm

5. Write the expression as a single logarithm and simplify the result if possible.

   (a) $\log_2 560 - \log_2 7 - \log_2 5$

   (b) $\frac{1}{2}\ln x + \ln(x^2 - 1) - \ln(x + 1))$

4 Change of Base Formula

Change-of-Base Formula: Let a and b be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then for any positive real number x,

$$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$$

6. Use the change of base formula to approximate $\log_4 153$ by using base e.

Student Learning Outcomes Check

1. Can you apply the product, quotient, and power properties of logarithms?
2. Can you write a logarithm in expanded form?
3. Can you write a logarithm as a single logarithm?
4. Are you able to apply the change of base formula?

If any of your answers were no, please ask about these topics in class.