Topics: polynomials, leading term test, intermediate value theorem

Student Learning Outcomes:

1. Students will be able to determine the end behavior of a polynomial function.
2. Students will be able to identify zeros and multiplicities of zeros.
3. Students will be able apply the Intermediate Value Theorem.

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1 End Behavior of Polynomial Functions

Definition of a Polynomial Function Let n be a positive, whole number and $a_n, a_{n-1}, a_{n-2}..., a_1, a_0$ be real numbers, where $a_n \neq 0$. Then a function defined by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_0$$

is called a **polynomial function of degree** n.

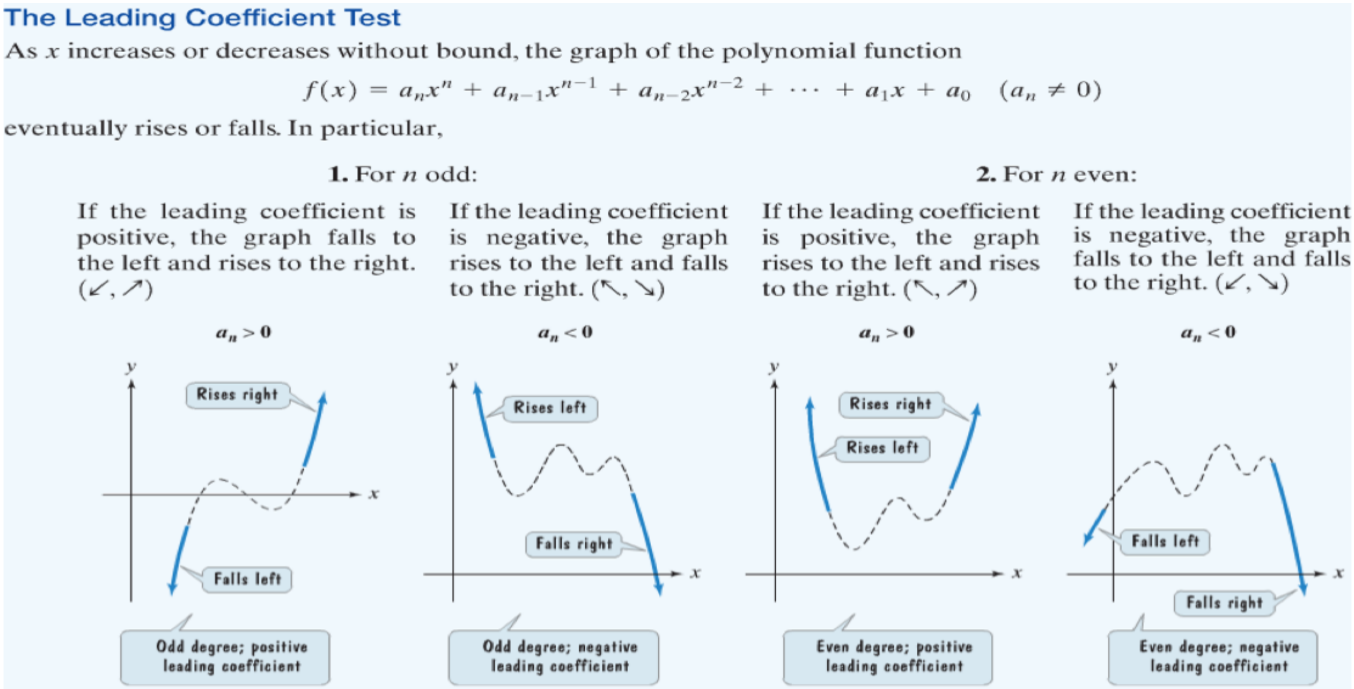
Examples:

$f(x) = 5x^2 + 8x^4$ $\quad g(x) = \dfrac{x^2+3}{x^2}$ $\quad h(x) = 4x^5 - 3x^4 + 2x^2$ $\quad k(x) = 4\sqrt{x} - \dfrac{3}{x} + x^2$

We have already studied several special cases of polynomial functions:

$$f(x) = 2 \qquad g(x) = 3x + 1 \qquad h(x) = 4x^2 + 7x - 1$$

Polynomial functions of degree 2 or higher have graphs that are smooth and continuous.


The Leading Coefficient Test

As x increases or decreases without bound, the graph of the polynomial function

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1x + a_0 \quad (a_n \neq 0)$$

eventually rises or falls. In particular,

1. For n odd:

If the leading coefficient is positive, the graph falls to the left and rises to the right. ($\swarrow, \nearrow$)

If the leading coefficient is negative, the graph rises to the left and falls to the right. ($\nwarrow, \searrow$)

2. For n even:

If the leading coefficient is positive, the graph rises to the left and rises to the right. ($\nwarrow, \nearrow$)

If the leading coefficient is negative, the graph falls to the left and falls to the right. ($\swarrow, \searrow$)

1. Use the leading term test to determine the function's end behavior.

   (a) $f(x) = -7x^5 + 2x^3 + 7x + 5$

   (b) $f(x) = \frac{1}{4}x(2x-3)^3(x+4)^2$

2 Identify Zeros and Multiplicities of Zeros

Multiplicities and x-Intercepts If f is a polynomial function, then the values of x for which $f(x) = 0$ are called the **zeros (roots or solutions)** of $f(x)$. Each real root of the polynomial equations appears as an x-intercept of the graph of the polynomial function.

If r is a zero of **even multiplicity**, then the graph **touches** the x-axis and **turns around** at r. If r is a zero of **odd multiplicity**, then the graph **crosses** the x-axis at r. Regardless of whether the multiplicity of a zero is even or odd graphs tend to flatten out near zeros with multiplicity greater than one.

2. Find the zeros for each polynomial function and give the multiplicity for each. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each zero.

   (a) $f(x) = 4(x+3)(x-7)^2$

   (b) $f(x) = x^3 - 6x^2 + 9x$

3 Apply the Intermediate Value Theorem

Intermediate Value Theorem Let f be a polynomial function. For $a < b$, if $f(a)$ and $f(b)$ have opposite signs, then f has at least one zero on the interval $[a, b]$.

3. Use the Intermediate Value Theorem to determine if $f(x) = 4x^4 - 8x^2 + 2$ has a real zero on the interval $[-1, 0]$.

4. Sketch a graph of $f(x) = x^3 - 9x$

Student Learning Outcomes Check

1. Are you able to determine the end behavior of a polynomial function?
2. Can you identify zeros and multiplicities of zeros?
3. Are you able apply the Intermediate Value Theorem?

If any of your answers were no, please ask about these topics in class.