Topics: acute angles, right triangle trigonometry, cofunction identities

Student Learning Outcomes:

1. Students will be able to evaluate trigonometric functions of acute angles.
2. Students will be able to use trigonometric identities.
3. Students will be able to use trigonometric functions in applications.

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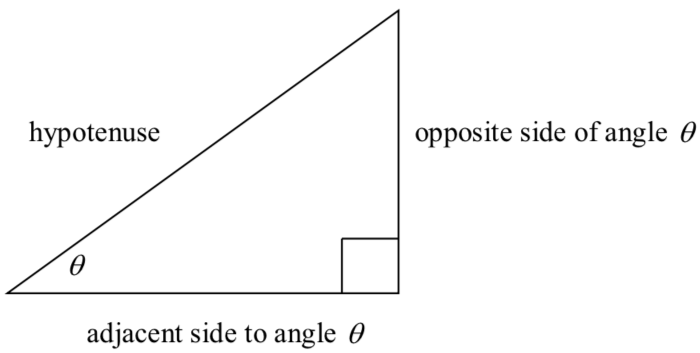
1 Trigonometric Functions of Acute Angles

An angle is **acute** if it measures less than 90°.
A **right angle** measures exactly 90°.
The sum of angles of a triangle is 180°.

If we know two sides of a right triangle, we can use the **Pythagorean Theorem** $a^2 + b^2 = c^2$ to find the 3rd side.

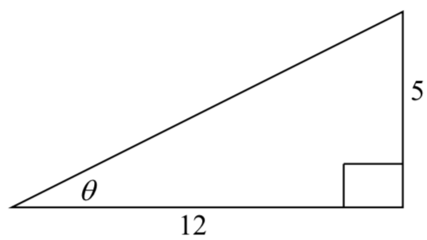


In the **right** triangle above, the ***six trigonometric functions*** of an angle θ are defined as follows:

$$\sin\theta = \frac{opp.}{hyp.} \qquad \cos\theta = \frac{adj.}{hyp.} \qquad \tan\theta = \frac{opp.}{adj.}$$

$$\csc\theta = \frac{hyp.}{opp.} \qquad \sec\theta = \frac{hyp.}{adj.} \qquad \cot\theta = \frac{adj.}{opp.}$$

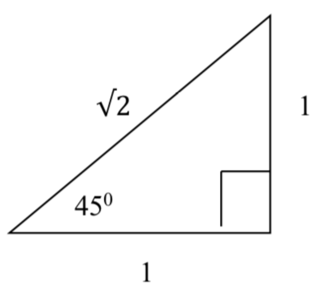
1. Find the six trig functions of θ in the triangle below.



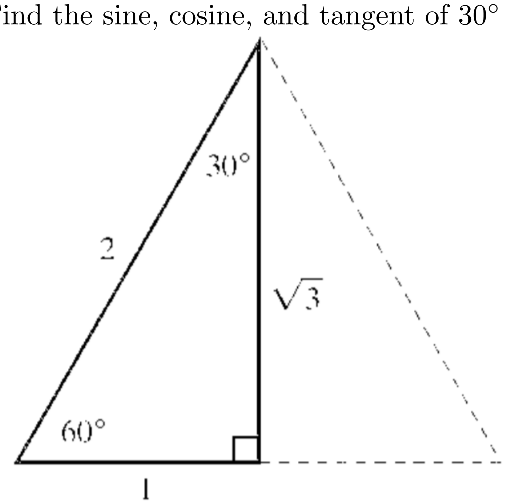
$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$ $\cot\theta =$

2. Find the sine, cosine, and tangent of $45°$ using the triangle below.



3. Find the sine, cosine, and tangent of $30°$ and $60°$ using the triangle below.



2 Fundamental Trigonometric Identities

Reciprocal Identities:

$$\sin(\theta) = \frac{1}{\csc(\theta)} \qquad \cos(\theta) = \frac{1}{\sec(\theta)} \qquad \tan(\theta) = \frac{1}{\cot(\theta)}$$

Let's find three more:

Quotient Identities:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

4. Given $\sin(\theta) = \frac{2}{3}$ and $\cos(\theta) = \frac{\sqrt{5}}{3}$, find the value of each of the four remaining functions.

3 Using Trigonometric Functions in Applications

5. A forester, 300 feet from the base of a redwood tree, observes that the angle between the ground and the top of the tree is $45°$. Determine the height of the tree.

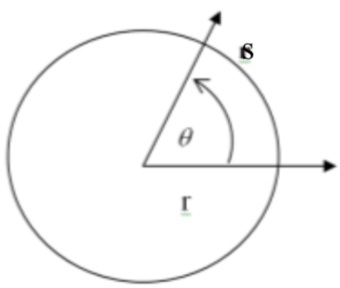
6. A pilot flying an airplane at an altitude of 1 mile sights a point at the end of a runway. The angle of depression is $3°$. What is the distance d from the plane to the point on the runway? Round to the nearest tenth of a mile.

4 Trigonometric Functions of Real Numbers

A **unit circle** is a circle of radius 1 with center at the origin. The equation of a unit circle is:

$$x^2 + y^2 = 1$$

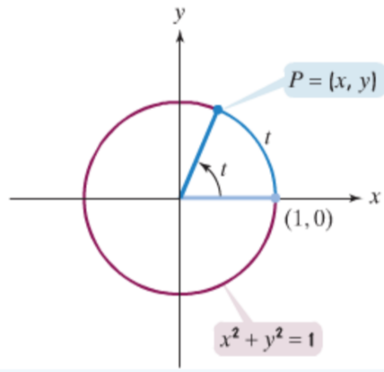
Recall:

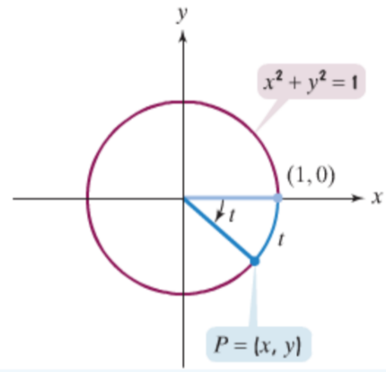


The length of a circular arc:

$$s = r\theta$$

In a unit circle, the radian measure of the central angle is equal to the length of the subtended arc. Both are given by the same real number t.





Definitions of the Trigonometric Functions in Terms of a Unit Circle

If t is a real number and $P = (x, y)$ is a point on the unit circle that corresponds to t, then

$$\sin t = y \qquad \csc t = \frac{1}{y},\ y \neq 0$$

$$\cos t = x \qquad \sec t = \frac{1}{x},\ x \neq 0$$

$$\tan t = \frac{y}{x},\ x \neq 0 \qquad \cot t = \frac{x}{y},\ y \neq 0.$$

7. Suppose that the real number t corresponds to the point $P(-\frac{2}{3}, -\frac{\sqrt{5}}{3})$ on the unit circle. Evaluate the six trigonometric functions of t.

   (a) $\sin(t) =$

   (b) $\cos(t) =$

   (c) $\tan(t) =$

   (d) $\csc(t) =$

   (e) $\sec(t) =$

   (f) $\cot(t) =$

8. Use the (x, y) coordinates in the unit circle to find the value of each trig function at the indicated real number t.

   (a) $\cos\left(\frac{3\pi}{2}\right) =$

   (b) $\tan\left(\frac{11\pi}{6}\right) =$

   (c) $\sec\left(\frac{11\pi}{6}\right) =$

9. Evaluate the six trigonometric functions of the real number t.

(a) $t = -\dfrac{5\pi}{4}$

(b) $t = \pi$

5 Domains of the Trigonometric Functions

Student Learning Outcomes Check

1. Can you evaluate trigonometric functions of acute angles?
2. Are you able to use able to use trigonometric identities?
3. Can you use trigonometric functions in applications?

If any of your answers were no, please ask about these topics in class.