Topics: sum and difference formulas

Student Learning Outcomes:

- 1. Students will be able apply the sum and difference formulas for sine and cosine.
- 2. Students will be able to apply the sum and difference formulas for tangent.
- 3. Students will be able to use sum and difference formulas to verify identities.

1 Apply the Sum and Difference Formulas

Sum and Difference Identities

$$\sin(u+v) = \sin u \cos v + \cos u \sin v \qquad \qquad \sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v \qquad \qquad \cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u-v) = \frac{\tan u - \tan v}{1 - \tan u \tan v}$$

- 1. Evaluate each of the following using the above identities and the unit circle.
 - (a) $\cos 15^{\circ}$

(b) $\sin \frac{5\pi}{12}$

(c) $\sin 25^{\circ} \cos 35^{\circ} + \cos 25^{\circ} \sin 35^{\circ}$

(d) $\tan 75^{\circ}$

(e) $\sin \frac{\pi}{12}$

(f) $\cos 99^{\circ} \cos 36^{\circ} - \sin 99^{\circ} \sin 36^{\circ}$

2. Find the exact value of $\sin(\alpha + \beta)$ given that $\sin \alpha = \frac{5}{13}$ and $\cos \beta = \frac{5}{6}$ for α in Quadrant II and β in Quadrant IV.

3. Find the exact value of each of the following.

$$\sin\left(\arctan\left(-\frac{9}{4}\right) + \arccos\left(\frac{8}{17}\right)\right)$$

2 Use Sum and Difference Formulas to Verify Identitities

4. Prove that each of the following is an identity.

(a)
$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

(b)
$$\sin(x+y) - \sin(x-y) = 2\cos x \sin y$$

(c)
$$\tan(\pi - x) = -\tan x$$

Student Learning Outcomes Check

- 1. Can you apply the sum and difference formulas for sine and cosine?
- 2. Can you apply the sum and difference formulas for tangent?
- 3. Are you able to use sum and difference formulas to verify identities?

If any of your answers were no, please ask about these topics in class.