Topics: acute angles, right triangle trigonometry, cofunction identities

Student Learning Outcomes:

1. Students will be able to evaluate trigonometric functions of acute angles.

2. Students will be able to use trigonometric identities.

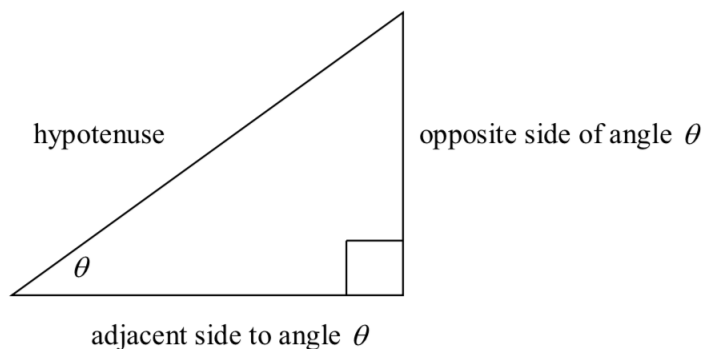3. Students will be able to use trigonometric functions in applications.

1   Trigonometric Functions of Acute Angles

An angle is **acute** if it measures less than $90°$.
A **right angle** measures exactly $90°$.
The sum of angles of a triangle is $180°$.

If we know two sides of a right triangle, we can use the **Pythagorean Theorem** $a^2 + b^2 = c^2$ to find the 3rd side.

In the **right** triangle above, the *six trigonometric functions* of an angle θ are defined as follows:

$$\sin\theta = \frac{opp.}{hyp.} \qquad \cos\theta = \frac{adj.}{hyp.} \qquad \tan\theta = \frac{opp.}{adj.}$$

$$\csc\theta = \frac{hyp.}{opp.} \qquad \sec\theta = \frac{hyp.}{adj.} \qquad \cot\theta = \frac{adj.}{opp.}$$

1. Find the six trig functions of θ in the triangle below.

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$
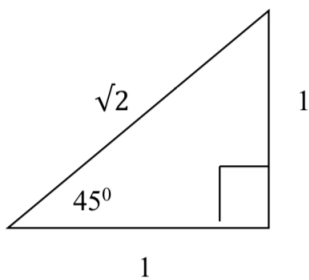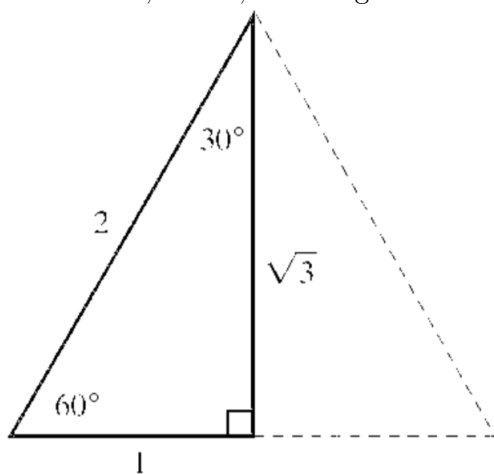
$\csc\theta =$ $\sec\theta =$ $\cot\theta =$
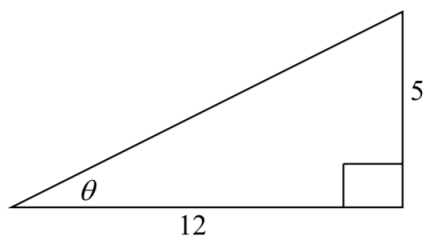
2. Find the sine, cosine, and tangent of $45°$ using the triangle below.

3. Find the sine, cosine, and tangent of $30°$ and $60°$ using the triangle below.

4. Given $\sin(\theta) = \frac{2}{3}$ and $\cos(\theta) = \frac{\sqrt{5}}{3}$, find the value of each of the four remaining functions.

2 Cofunction Identities

Cofunctions of complementary angles are equal.

$$\sin(\theta) = \cos(90° - \theta) \qquad \cos(\theta) = \sin(90° - \theta)$$

$$\tan(\theta) = \cot(90° - \theta) \qquad \cot(\theta) = \tan(90° - \theta)$$

$$\sec(\theta) = \csc(90° - \theta) \qquad \csc(\theta) = \sec(90° - \theta)$$

5. Fore each function value, find a cofunction with the same value.

(a) $\cot(15°) = 2 + \sqrt{3}$

(b) $\cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

3   Using Trigonometric Functions in Applications

6. A forester, 300 feet from the base of a redwood tree, observes that the angle between the ground and the top of the tree is 45°. Determine the height of the tree.

7. A pilot flying an airplane at an altitude of 1 mile sights a point at the end of a runway. The angle of depression is 3°. What is the distance d from the plane to the point on the runway? Round to the nearest tenth of a mile.

Student Learning Outcomes Check

1. Can you evaluate trigonometric functions of acute angles?

2. Are you able to use able to use trigonometric identities?

3. Can you use trigonometric functions in applications?

If any of your answers were no, please ask about these topics in class.