**Topics:** inverse trig functions

**Student Learning Outcomes:**

1. Students will be able to evaluate the inverse trigonometric functions for sine, cosine, and tangent.
2. Students will be able to solve trigonometric equations using inverse trigonometric functions.
3. Students will be able to find the composition of trigonometric and inverse trigonometric functions.

---

Recall: A one-to-one function has an inverse function.

# 1 Evaluate the Inverse Sine Function

Let's draw a graph of sine and find it's inverse function:

**The Inverse Sine Function**

The **inverse sine function**, denoted by $\sin^{-1}$, is the inverse of the restricted sine function $y = \sin x$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. Thus,

$$y = \sin^{-1} x \quad \text{means} \quad \sin y = x,$$

where $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ and $-1 \leq x \leq 1$. We read $y = \sin^{-1} x$ as "$y$ equals the inverse sine at $x$."

Domain:

Range:

**Finding Exact Values of** $\sin^{-1}(x)$ (or $\arcsin(x)$)

1. Let $\theta = \sin^{-1} x$.
2. Rewrite $\theta = \sin^{-1} x$. as $\sin\theta = x$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.
3. Use exact values on the unit circle to find the value of $\theta$ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ that satisfies $\sin\theta = x$.

1. Find the exact values of an inverse function:

   (a) $\sin^{-1}\left(\frac{1}{2}\right) =$

   (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

   (c) $\sin^{-1}\left(-\frac{1}{2}\right) =$

## 2 Evaluate the Inverse Cosine Function

Let's draw a graph of cosine and find it's inverse function:

**The Inverse Cosine Function**

The **inverse cosine function**, denoted by $\cos^{-1}$, is the inverse of the restricted cosine function $y = \cos x$, $0 \le x \le \pi$. Thus,

$$y = \cos^{-1} x \quad \text{means} \quad \cos y = x,$$

where $0 \le y \le \pi$ and $-1 \le x \le 1$.

Domain:

Range:

**Finding Exact Values of** $\cos^{-1}(x)$ (or $\arccos(x)$)

1. Let $\theta = \cos^{-1} x$.
2. Rewrite $\theta = \cos^{-1} x$. as $\cos\theta = x$, where $0 \leq \theta \leq \pi$.
3. Use exact values on the unit circle to find the value of $\theta$ in $[0, \pi]$ that satisfies $\cos\theta = x$.

2. Find the exact values of an inverse function:

   (a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

   (b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$

   (c) $\cos^{-1}\left(\frac{1}{2}\right) =$

## 3 Evaluate the Inverse Tangent Function

Let's draw a graph of tangent and find it's inverse function:

### The Inverse Tangent Function

The **inverse tangent function**, denoted by $\tan^{-1}$, is the inverse of the restricted tangent function $y = \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Thus,

$$y = \tan^{-1} x \quad \text{means} \quad \tan y = x,$$

where $-\frac{\pi}{2} < y < \frac{\pi}{2}$ and $-\infty < x < \infty$.

Domain:

Range:

**Finding Exact Values of** $\tan^{-1}(x)$ (or $\arctan(x)$)

1. Let $\theta = \tan^{-1} x$.
2. Rewrite $\theta = \tan^{-1} x$. as $\tan\theta = x$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.
3. Use exact values on the unit circle to find the value of $\theta$ in $(-\frac{\pi}{2}, \frac{\pi}{2})$ that satisfies $\tan\theta = x$.

3. Find the exact values of an inverse function:

   (a) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) =$

   (b) $\tan^{-1}(0) =$

   (c) $\tan^{-1}(1) =$

## 4 Approximate Inverse Trigonometric Functions on a Calculator

4. Use a calculator to find the value of each expression rounded to 2 decimal places in radians and degrees.

   (a) $\sin^{-1}(.7) =$

   (b) $\cos^{-1}(-.47) =$

   (c) $\tan^{-1}(-14) =$

## 5 Inverse Properties of Trigonometric Functions

### Inverse Properties

**The Sine Function and Its Inverse**

$$\sin(\sin^{-1} x) = x \qquad \text{for every } x \text{ in the interval } [-1, 1]$$

$$\sin^{-1}(\sin x) = x \qquad \text{for every } x \text{ in the interval } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

**The Cosine Function and Its Inverse**

$$\cos(\cos^{-1} x) = x \qquad \text{for every } x \text{ in the interval } [-1, 1]$$

$$\cos^{-1}(\cos x) = x \qquad \text{for every } x \text{ in the interval } [0, \pi]$$

**The Tangent Function and Its Inverse**

$$\tan(\tan^{-1} x) = x \qquad \text{for every real number } x$$

$$\tan^{-1}(\tan x) = x \qquad \text{for every } x \text{ in the interval } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

5. Find the exact value of the expression. Do not use a calculator.

   (a) $\cos(\cos^{-1}(-0.4))$

   (b) $\sin^{-1}(\cos(\frac{\pi}{12}))$

   (c) $\sin^{-1}(\sin(\frac{3\pi}{12}))$

(d) $\tan(\tan^{-1}(11))$

(e) $\tan^{-1}(\tan(-\frac{\pi}{4}))$

(f) $\sin^{-1}(\sin(\pi))$

(g) $\sin(\sin^{-1}(\pi))$

## 6 Composing Trigonometric and Inverse Trigonometric Functions

6. Use a sketch to find the exact value of each expression.

   (a) $\cos(\sin^{-1}(\frac{24}{26}))$

(b) $\tan(\cos^{-1}(-\frac{10}{26}))$

(c) $\sec(\sin^{-1}(-\frac{1}{6}))$

**Student Learning Outcomes Check**

1. Can you evaluate the inverse trigonometric functions for sine, cosine, and tangent?
2. Can you solve trigonometric equations using inverse trigonometric functions?
3. Are you able to find the composition of trigonometric and inverse trigonometric functions?

**If any of your answers were no, please ask about these topics in class.**