**Topics:** basic operations on functions, composition of functions, and difference quotient

**Student Learning Outcomes:**

1. Students will be able to perform basic operations on functions.

2. Students will be able to compose functions.

3. Students will be able to evaluate a difference quotient.

# 1   Algebra on Functions

1. Find the following values for the functions $f(x) = x + 3$ and $g(x) = x^2$.

    (a) $(f + g)(2)$

    (b) $(f - g)(7)$

    (c) $(fg)(5)$
       (same as $(f \cdot g)(5)$)

    (d) $(f/g)(4)$

## 2 Function Composition

The composition of two functions, $f(x)$ and $g(x)$, is given by

$$(f \circ g)(x) = f[g(x)] \qquad \text{or} \qquad (g \circ f)(x) = g[f(x)]$$

that is, by "plugging one function into another". One must be careful, however, as the above two quantities are **not equal** in general (it is also important not to confuse this notation with multiplying two functions, hence the use of the "open circle" symbol).

2. Find the following values for the functions $f(x) = x + 3$ and $g(x) = x^2$.

   (a) $f(g(4))$

   (b) $g(f(4))$

3. Use $f(x) = \dfrac{1}{3 - x}$ and $g(x) = 3x^2 - 16$ to answer the following.

   (a) Determine $f(g(x))$ and the domain of $f(g(x))$.

   (b) Determine $(g \circ f)(x)$ and the domain of $(g \circ f)(x)$.
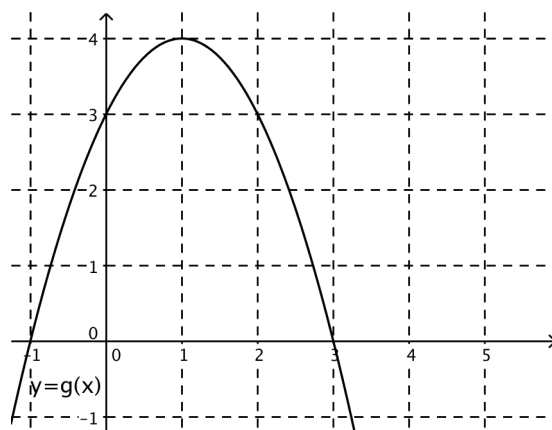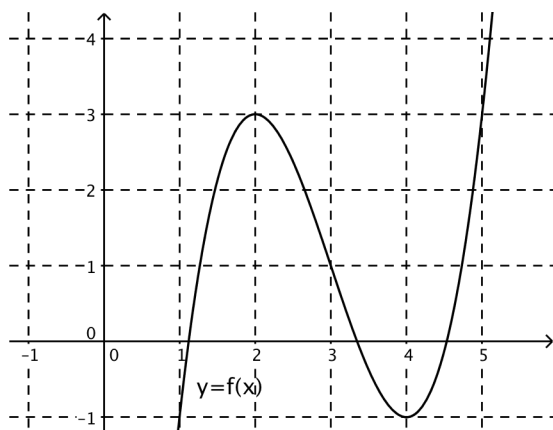
   (c) Determine $g(f(3))$.

4. **Decomposition** Find functions $f$, $g$, such that $f(g(x)) = y$ for the function $y = 6 + \sqrt{9 - x^2}$.
   (There is more than one correct answer.)

5. Use the graphs of $f$ and $g$ below to determine the following.

   (a) $f(g(2))$

   (b) $g(f(4))$

   (c) $f(g(1))$

# 3    Diference Quotient

**Recall:** The slope of a line through two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{change in y}}{\text{change in x}}$$

**Average Rate of Change**

The **average rate of change** of $y = f(x)$ with respect to $x$ over the interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{h}, h \neq 0.$$

This is the **difference quotient.** Let's draw a graph.

6. Given $f(x) = 3x - 5$.

   (a) Find $f(x + h)$.

   (b) Find the difference quotient, $\frac{f(x+h) - f(x)}{h}$.

**Student Learning Outcomes Check**

1. Can you perform basic algebra operations on functions?

2. Can you compose functions two functions?

3. Are you able to evaluate a difference quotient?

**If any of your answers were no, please ask about these topics in class.**