Topics: basic functions, vertical translations and scaling, horizontal translations and scaling, horizontal and vertical reflections, graphing transformations

Student Learning Outcomes:

1. (In class) Students will be able to recognize basic functions.
2. Students will be able to transform graphs of functions.
3. Students will be able to graph a function based on transformations.

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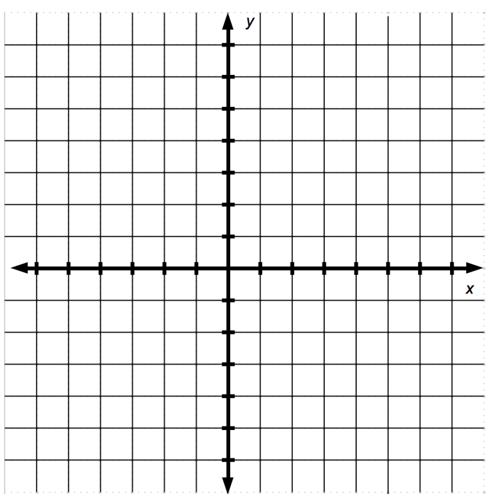
1 Vertical and Horizontal Shifts

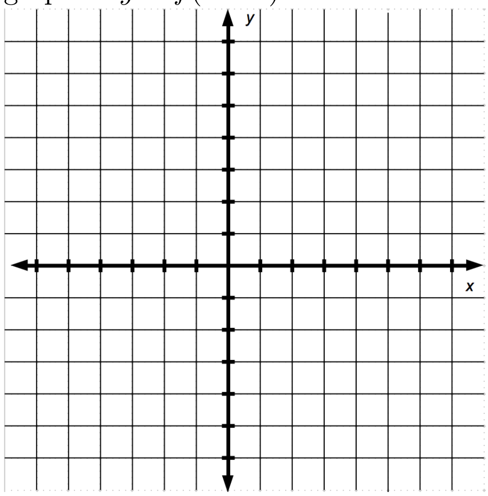
Vertical shifts. Assume that c is a positive number.
The graph of $y = f(x) + c$ is obtained from the graph of $y = f(x)$ by shifting it c units upward.
The graph of $y = f(x) - c$ is obtained from the graph of $y = f(x)$ by shifting it c units downward.

1. For this problem, let $f(x) = x^2$. Sketch the graph of the function $y = f(x) - 3$. Compare the domains and ranges of $y = f(x)$ and $y = f(x) - 3$.



Horizontal shifts. Assume that c is a positive number.
The graph of $y = f(x - c)$ is obtained from the graph of $y = f(x)$ by shifting it c units to the right.
The graph of $y = f(x + c)$ is obtained from the graph of $y = f(x)$ by shifting it c units to the left.

2. Convince yourself that the rule above is correct by using the example $f(x) = x^2$. Sketch the graph of $y = f(x - 2)$ on the axes below.



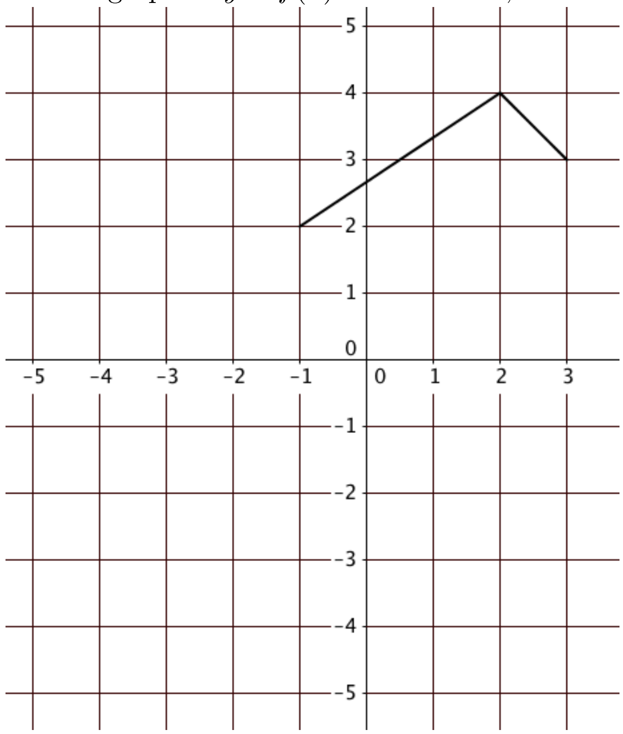
3. Compare the domains and ranges of $y = \sqrt{x}$ and $f(x) = \sqrt{x + 4}$. Think about why this makes sense and is consistent with the shifting.

4. If the point $(3, -4)$ is on the graph of $y = f(x)$, find the corresponding point on the graph of $y = f(x - 5) + 3$.

2 Reflection, Compression, and Stretching

Reflection through the x-axis. The graph of $y = -f(x)$ is obtained by reflecting the graph of $y = f(x)$ through the x-axis.

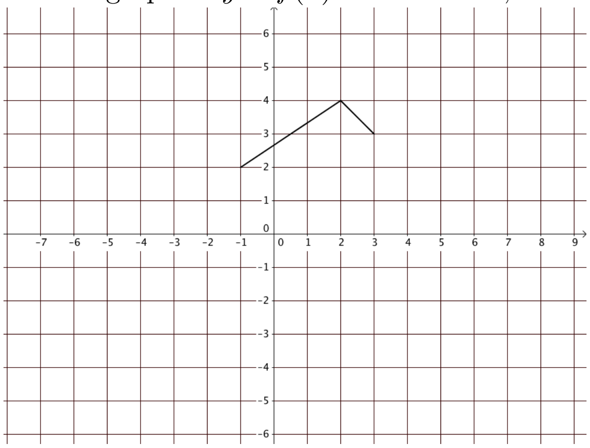
5. For the graph of $y = f(x)$ shown below, sketch the graph of $y = -f(x)$.



Vertical Compression/Stretching Assume that c is a positive number.
If $c > 1$, the graph of $y = cf(x)$ is obtained by stretching the graph of $y = f(x)$ vertically by a factor of c.
If c is between 0 and 1, the graph of $y = cf(x)$ is obtained by compressing the graph vertically by a factor of $1/c$.

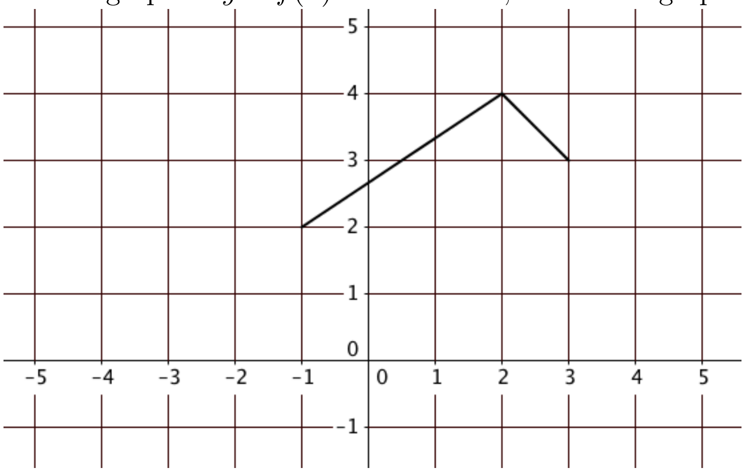
6. If the point $P(3, -1)$ is on the graph of $y = f(x)$, find the corresponding point on the graph of (a) $y = 7f(x)$ and (b) $y = \frac{1}{4}f(x)$.

7. For the graph of $y = f(x)$ shown below, sketch the graph of $y = 2f(x+3) - 1$.



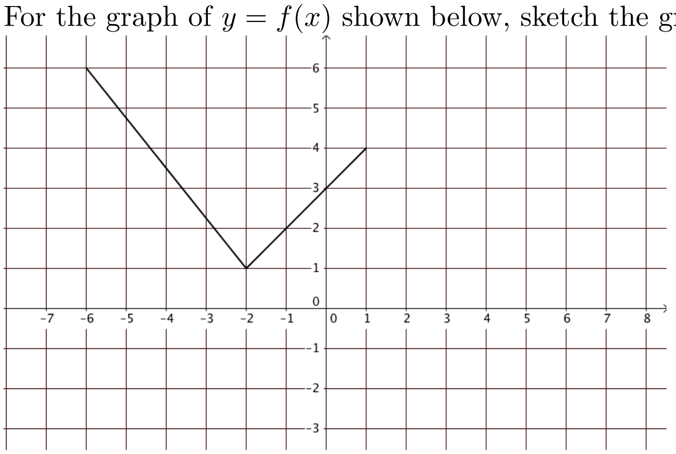
Reflection through the y-axis. The graph of $y = f(-x)$ is obtained by reflecting the graph of $y = f(x)$ through the y-axis.

8. For the graph of $y = f(x)$ shown below, sketch the graph of $y = f(-x)$.

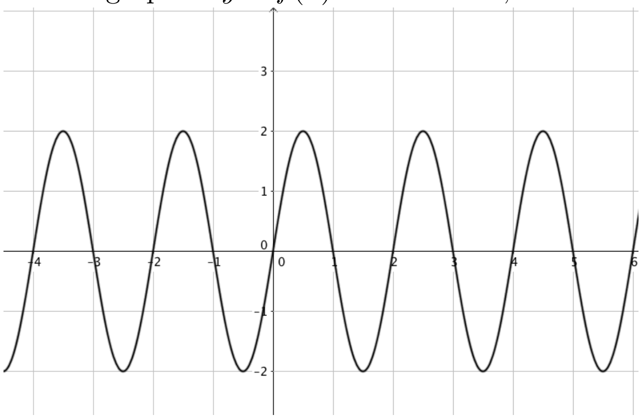


Horizontal Compression/Stretching Assume that c is a positive number.
If $c > 1$, the graph of $y = f(cx)$ is obtained by compressing the graph horizontally by a factor c.
If c is between 0 and 1, the graph of $y = f(cx)$ is obtained by stretching the graph of $y = f(x)$ horizontally by a factor of $1/c$.

9. For the graph of $y = f(x)$ shown below, sketch the graph of $y = f(2x)$.



10. For the graph of $y = f(x)$ shown below, sketch the graph of $y = |f(x)|$.



Student Learning Outcomes Check

1. Can you transform graphs of functions?
2. Do you understand the difference between a shift and a reflection? Or vertical and horizontal transformations?

If any of your answers were no, please ask about these topics in class.