Topics: unit circle, trigonometric functions, trigonometric identities, periodic

Student Learning Outcomes:

1. Students will be able to evaluate trigonometric functions using the unit circle.
2. Students will be able to determine domains of trigonometric functions.
3. Students will be able to use trigonometric identities.

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1 Pythagorean Identities

$$\sin^2(t) + \cos^2(t) = 1 \qquad \tan^2(t) + 1 = \sec^2(t) \qquad 1 + \cot^2(t) = \csc^2(t)$$

1. Given that $\tan(t) = \frac{12}{5}$ for $\pi < t < \frac{3\pi}{2}$, use an appropriate identity to find the value of $\sec(t)$.

2. Given that $\csc(t) = \frac{5}{4}$ for $\frac{\pi}{2} < t < \pi$, use an appropriate identity to find the value of $\cot(t)$.

3. Given a real number t, express $\sin(t)$ in terms of $\cos(t)$.

2 Even, Odd, and Periodic Properties

Even and Odd Trigonometric Functions

The cosine and secant functions are *even*.

$$\cos(-t) = \cos t \qquad \sec(-t) = \sec t$$

The sine, cosecant, tangent, and cotangent functions are *odd*.

$$\sin(-t) = -\sin t \qquad \csc(-t) = -\csc t$$
$$\tan(-t) = -\tan t \qquad \cot(-t) = -\cot t$$

4. Use the unit circle to find the value of $\cos\left(\frac{2\pi}{3}\right)$ and odd trig functions to find the value of $\cos\left(-\frac{2\pi}{3}\right)$.

Periodic Properties of the Sine and Cosine Functions

$$\sin(t + 2\pi) = \sin t \quad \text{and} \quad \cos(t + 2\pi) = \cos t$$

The sine and cosine functions are periodic functions and have period 2π.

Periodic Properties of the Tangent and Cotangent Functions

$$\tan(t + \pi) = \tan t \quad \text{and} \quad \cot(t + \pi) = \cot t$$

The tangent and cotangent functions are periodic functions and have period π.

Repetitive Behavior of the Sine, Cosine, and Tangent Functions

For any integer n and real number t,

$$\sin(t + 2\pi n) = \sin t, \quad \cos(t + 2\pi n) = \cos t, \quad \text{and} \quad \tan(t + \pi n) = \tan t.$$

5. Given $\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$, determine the value of $\sin\left(\frac{49\pi}{12}\right)$.

6. Use properties of trigonometric functions to simplify $\tan(-3t) - \tan(-3t+\pi)$.

3 Approximate Trigonometric Functions on a Calculator

7. Use a calculator to approximate the function values. Round to 4 decimal places.

   (a) $\cos\left(\frac{2\pi}{7}\right)$

   (b) $\csc(0.92)$

Student Learning Outcomes Check

1. Can you evaluate trigonometric functions using the unit circle?
2. Can you determine domains of trigonometric functions?
3. Are you able to use trigonometric identities?

If any of your answers were no, please ask about these topics in class.