# Classroom Activities
# Math 1113 - Precalculus

University of Georgia
Department of Mathematics

June 19, 2018

# Contents

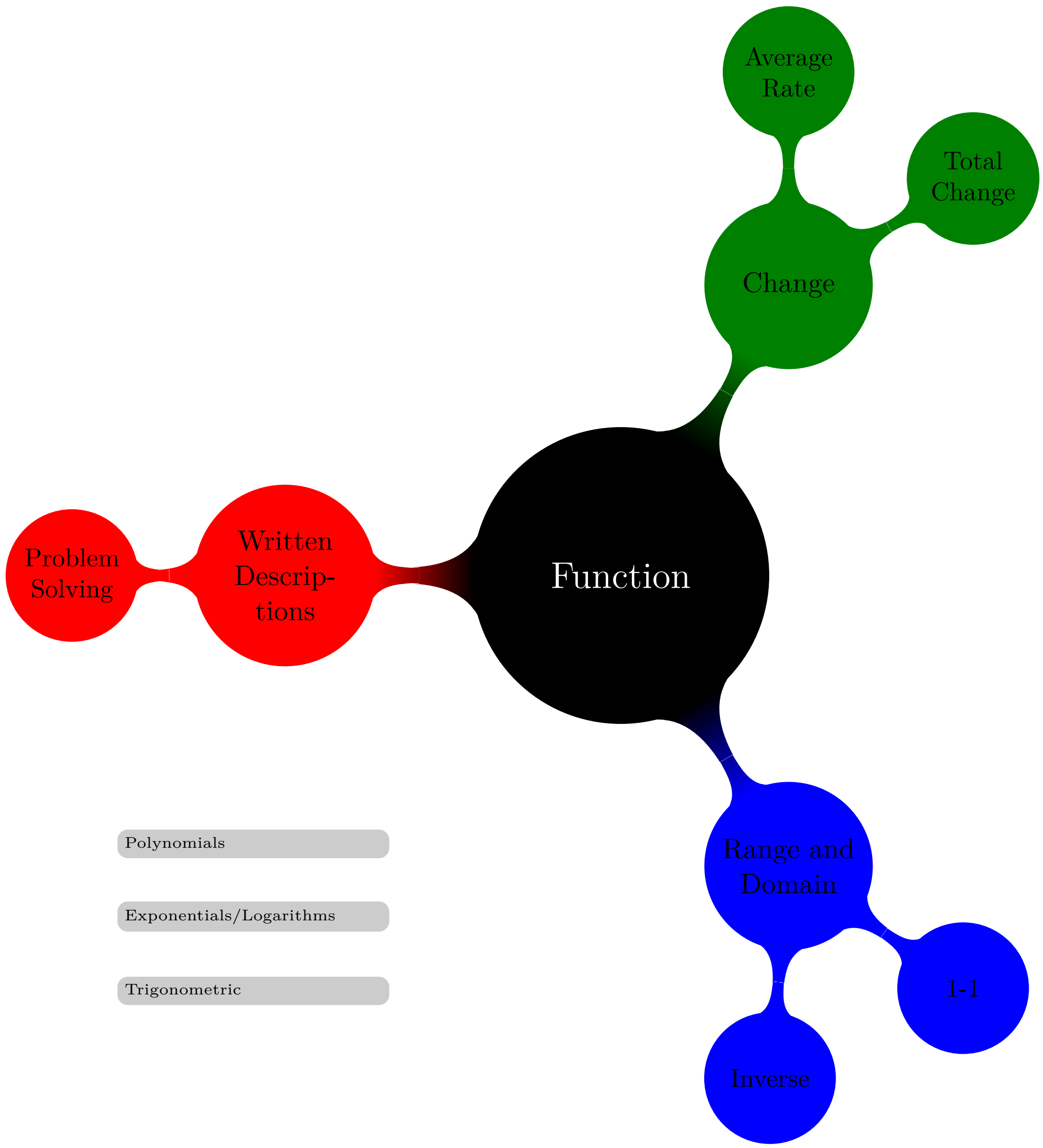



Figure 1: Broad overview of the topics for the full course.

# Chapter 1

# Functions and Preliminaries

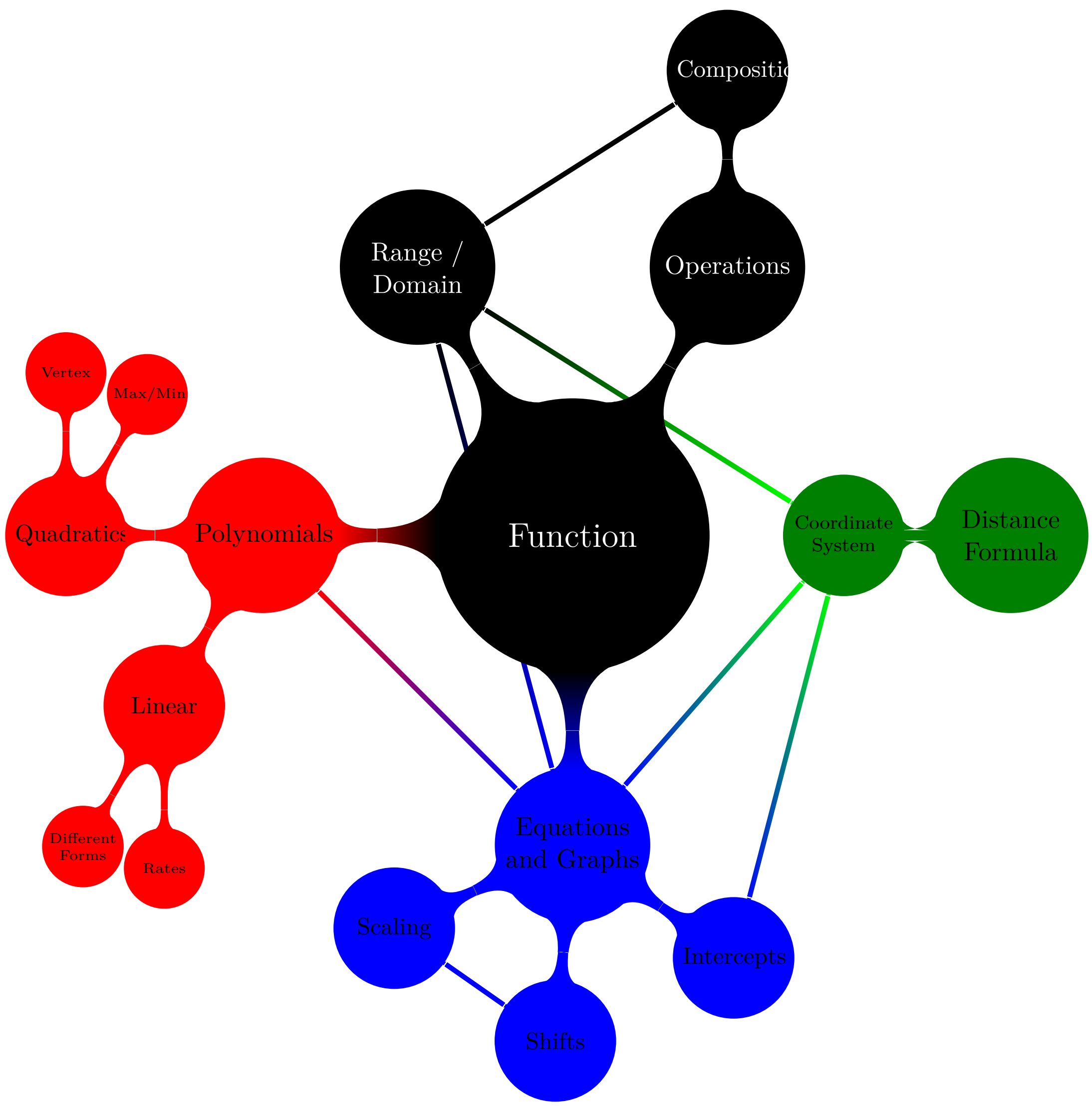



Figure 1.1: Topics for the first section of the course.

1. Make a sketch of a number line with zero at the center. Mark the locations of -2, -2.5, 1.1, and 2.3 on your number line. The relative distances between the points should be consistent.

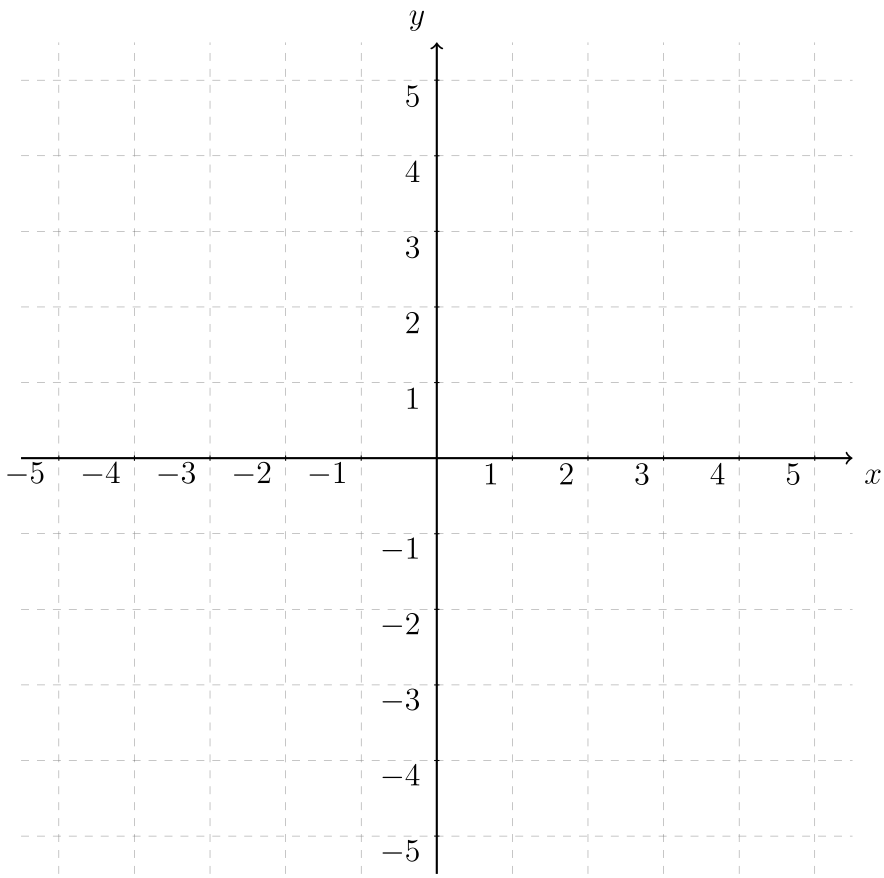
*Label the number line by putting an "$x$" to the right and an arrow indicating the positive direction.*

2. Make a sketch of a number line with zero at the center. Mark the locations of -2 and 2.15 on your number line. What is the distance between the two points? (The relative distances between the points should be consistent.)

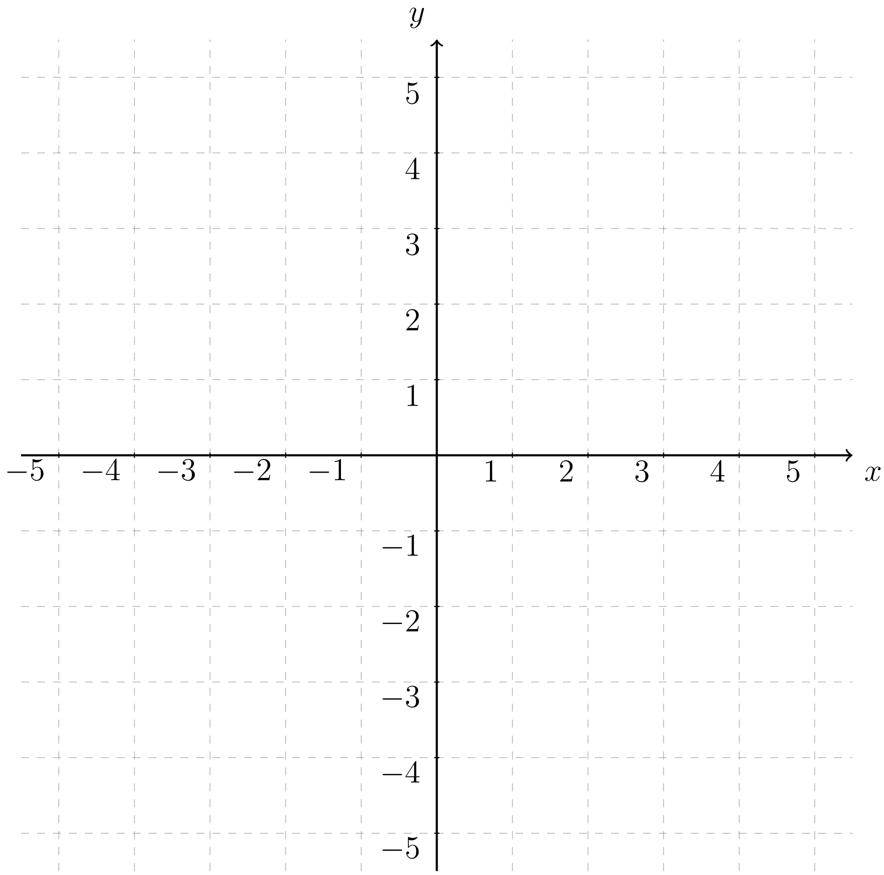
3. Make a sketch of a number line with zero at the center. Mark the locations of -1.54 and 2.07 on your number line. What is the distance between the two points? (The relative distances between the points should be consistent.)

1. A set of points is given below as a table. Each point is given on a line. The left side of the table has the $x$-coordinate of each point, and the right side has the $y$ coordinate. Plot each point on the axes given below. Next to each point indicate which quadrant the point is located.

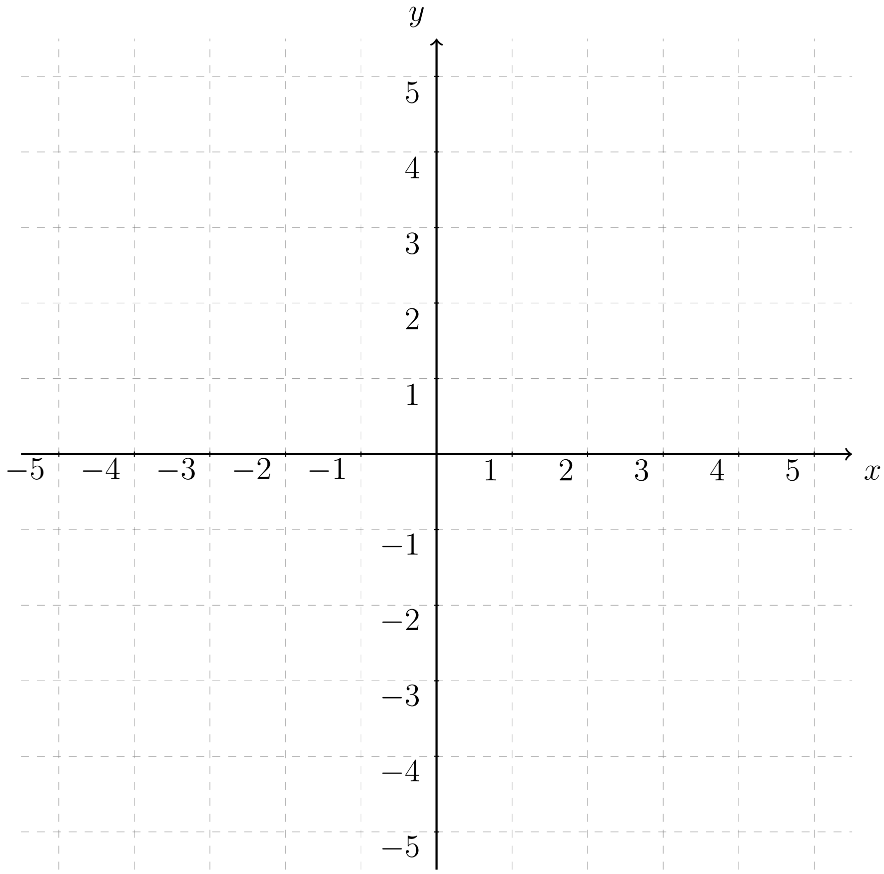
| $x$ | $y$ | Quadrant |
|---|---|---|
| 2.0 | 1.5 | |
| 2.2 | -4.5 | |
| -3.4 | 2.8 | |
| -2.8 | -1.3 | |
| -4.5 | 2.5 | |



2. Mark the points $P_1(-2.1, -4.4)$ and $P_2(4.5, 1.2)$ on the coordinate plane below. Determine the distance between the two points. Include a sketch of a right triangle whose hypotenuse represents the distance between the two points.



3. Mark the point $P_3(1.3, -2.4)$ on the coordinate plane below. Determine the points on the $x$-axis that are a distance of 3 units from $P_3$.




   (a) Place points on the graphs near to where you think the points **may** be located.

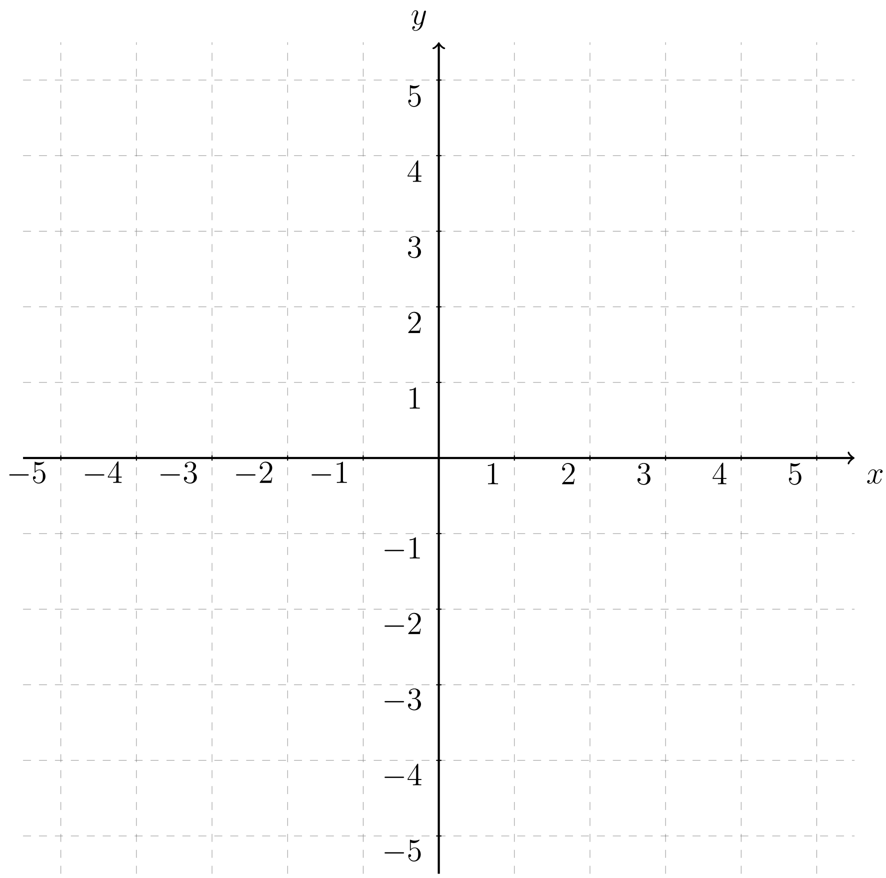
   (b) Label one of the points, $(x, y)$.

   (c) Write out the general distance formula.

   (d) What do you know about the point? Can you simplify the formula?

   (e) Solve for the unknown variable.

4. Mark the point $P_4(1, -2)$ on the coordinate plane below. Mark **all** of the points that are a distance of 2 units from $P_4$.




What kind of figure did you draw?

5. Suppose a point, $P(x, y)$ is a distance of 2 units from the point $P_4(1, -2)$.
   (a) Use the distance formula to express the distance relationship between $P$ and $P_4$.

   (b) Square both sides of the previous equation.

6. Suppose a point, $P(x, y)$ is a distance of $R$ units from the point $P_4(1, -2)$.
   (a) Use the distance formula to express the distance relationship between $P$ and $P_4$.

   (b) Square both sides of the previous equation.

   (c) How does this formula relate to the figure you drew in question 4.

1. Briefly state two ideas from today's class.
   -
   -
2. For each equation below determine the values of $x$ that satisfy the equation. Determine the exact answer and also determine an approximations to the answer using two decimal places.
   (a) $2x^2 + 5x - 3 = 0$

   (b) $5x - 1 = 8x + 7$

   (c) $3x - 1 = 2x^2 + 2x + 6$

   (d) $x^3 = 2$
3. Draw a coordinate axis, and properly label the axes. Use the axes to make a sketch of the graph of the relationship $y + x = 2$.
4. Draw a coordinate axis, and properly label the axes. Use the axes to make a sketch of the graph of the relationship $y^2 + x = 2$.
5. Make a sketch of a number line with zero at the center. Indicate the set of numbers that satisfy $x^2 > 2$.
6. Make a sketch of a number line with zero at the center. Indicate the set of numbers that satisfy $x > 2.2$ and $x < 5.4$.
7. Make a sketch of a number line with zero at the center. Indicate the set of numbers that satisfy $|x| > 1.5$.

1. Expand each of the functions below by FOILing the expression. The first one is done as an example. Recall what it means to FOIL an expression.

$$(a + b) \cdot (c + d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d$$

(a) $(x-3)^2$

$$\begin{aligned}(x-3)^2 &= (x-3)\cdot(x-3),\\ &= x\cdot x - 3\cdot x - 3\cdot x + (-3)\cdot(-3),\\ &= x^2 - 3x - 3x + 9,\\ &= x^2 - 6x + 9.\end{aligned}$$

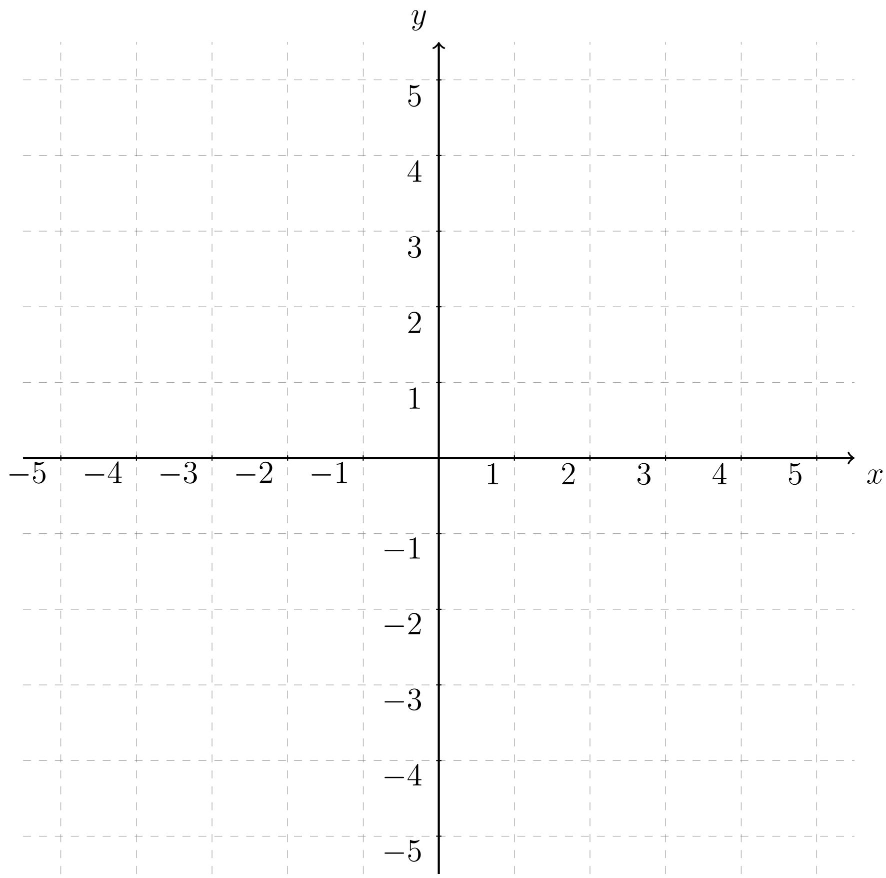
(b) $(x+4)^2$

(c) $(y-5)^2$

(d) $(y+a)^2$ where $a$ is a constant.

2. if $(x-a)^2 = x^2 + 10x + 25$ what is the value of $a$?

1. Sketch the set of all points that are a distance of two from the point $Q(1, -3)$. What kind of figure do the points represent?



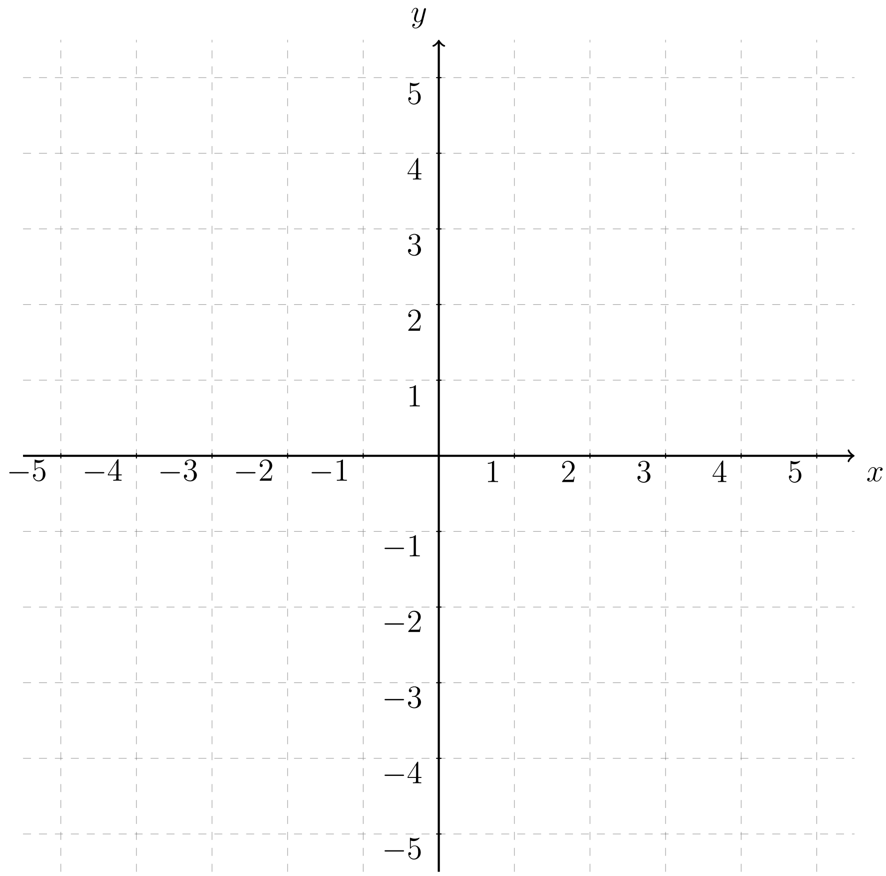
2. Suppose a point, $P(x, y)$ is a distance of $R$ units from the point $C(x_0, y_0)$.

*The values, $x_0$ and $y_0$, are constants. The notation is awkward, but this is a convention that we have to adapt to.*

   (a) Use the distance formula to express the distance relationship between $P$ and $C$.

   (b) Square both sides of the previous equation.

3. Make a sketch of the circle with a radius of two centered at the point $P(-2, 3)$. Determine a formula for the circle.



4. Sketch a graph of the relationship given by

$$x^2 + 2x + y^2 - 8y = 8.$$

Determine the center and the radius of the circle. Make a sketch of the circle. (Include the axes and label the axes.)

In mathematics the idea of proportionality has a specific definition. The idea is that when two things are proportional then any changes in one yield a similar change in the other. For example, suppose we have two quantities. The first we call $x$, and the other we call $y$. If $y$ is proportional to $x$, then if we double $x$ then $y$ will double. Likewise, if we triple $x$ then $y$ will triple.
We express this mathematically by noting that if $y$ is proportional to $x$ then the ratio of $y$ to $x$ must be a constant,

$$\frac{y}{x} = \text{constant.}$$

If we multiply both sides by $x$ then

$$y = x \cdot \text{constant.}$$

As an example, it is estimated that the length of a person's femur is proportional to the person's total height. This implies that

$$\frac{\text{height}}{\text{femur length}} = \text{constant.}$$

In a paper by Obialor *et al*[1], it is estimated that in a specific area in Nigeria the mean height of women is 161.90 cm and the mean femur length of women is 40.82 cm. If a woman's femur has a length of 42.00 cm, what is her expected height?
First, we have to estimate the value of the constant. Assuming that the means are consistent then

$$\frac{161.90}{40.82} = \text{constant.}$$

Now we look at the expression for the unidentified woman,

$$\frac{\text{height}}{42.00} = \frac{161.90}{40.82}.$$

Solving for the height we get

$$\text{height} = 42.00 \cdot \frac{161.90}{40.82}\text{cm.}$$

[1] Ambrose Obialor, Churchill Ihentuge and Frank Akpuaka, **Determination of Height Using Femur Length in Adult Population of Oguta Local Government Area of Imo State Nigeria**, The FASEB Journal, April 2015, vol. 29 no. 1 Supplement LB19

5. Windows are constructed, and their width is proportional to their height. One window is measured, and its width is 100cm, and its height is 200cm.

   (a) Another window has a width of 75cm. What is its height?

   (b) Make a sketch of the relationship of the height of a window given its width. First, express the relationship as a mathematical formula. Second, sketch the graph of the relationship. Briefly discuss the relationship. How does the height change as the width changes?

*Annotate your plot and label your axes!*

6. The surface area of a sparrow's wing is proportional to the square of the length of its wing. A sparrow is measured, and it has a wing length of 9cm and an area of 45cm$^2$.

   (a) Another sparrow is measured, and the length of its wing is 8cm. What is the area of its wing?

   (b) Make a sketch of the relationship of the area of a sparrow's wing given the length. Briefly discuss the relationship. How does the area change as the length changes?

*Annotate your plot and label your axes!*

1. Briefly state two ideas from today's class.
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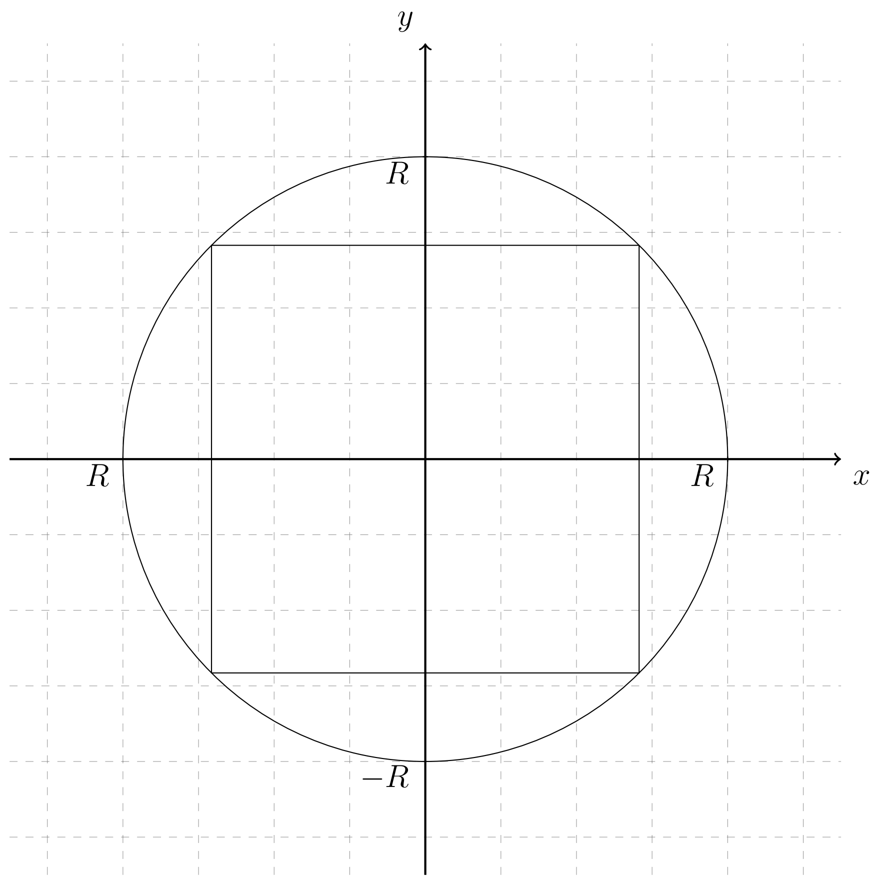
2. A square is circumscribed within a circle of radius $R$ so it just touches the circle on each of the four corners of the square.




   (a) Sketch the radius of the circle at one of the points where the square touches the circle.

   (b) Find a convenient right triangle in the new diagram using the radius that you drew, and use the triangle to determine the length of one of the sides of the square. (You may have to double the length of the triangle to get the length of the side.)

   (c) Determine the area of the square.

3. A triangle with three equal sides is circumscribed within a circle of radius $R$ so it just touches the circle on each of the three corners of the triangle.

   (a) Make a sketch of the situation. (Label your axes and label the length of the sides of the triangle.)

(b) Sketch the radius of the circle at one of the points where the triangle touches the circle.

(c) Find a convenient right triangle in the new diagram, and use the triangle to determine the length of one of the sides of the triangle. (You may have to double the length of the triangle to get the length of the side.)

1. A biologist grows four different colonies of bacteria. The number of bacteria in the colonies is estimated to be 10,000, 20,000, 30,000, and 40,000. The mass for each colony is measured and is estimated to be $2.61 \times 10^{-6}$, $5.20 \times 10^{-6}$, $7.85 \times 10^{-6}$ and $1.043 \times 10^{-5}$ grams respectively.
   Organize the information above into a table so that the mass can be more easily determined given the number of bacteria in the colony. Also, graph each point as a coordinate where the number of bacteria is on the horizontal axis, and the mass is on the vertical axis.

*Label your axes and properly annotate your plot.*

2. What is the change in mass when the number of bacteria increases by 10,000?

3. Make rough estimate for a relationship that will provide a prediction for the mass of a colony given the number of bacteria within it.

1. A balloon has a tether that is attached to the ground, and the tether can be extended or retracted as the balloon is raised or lowered. One end of the tether is attached to the ground 20m away from a point directly below the balloon, and the balloon moves straight up and down. If the length of the tether is $x$ meters what is the altitude of the balloon?

   (a) Sketch a diagram of the physical situation. Label the known and unknown quantities. Make up names for the quantities that you think you **may** need to use later.

   *Assume that the balloon only moves up and down with no lateral motion.*

   (b) Determine the important relationships between the known and unknown quantities.

   (c) How can you use this relationship to determine the variable of interest?

   (d) Determine the height of the balloon given the length of the tether.

   (e) Determine the domain and range of the function that gives the height given the length of the tether.

2. A park has two distinct areas separated by a river, and each area has its own population of mice. The population East of the river is estimated to have 10,000 individuals at the beginning of the year, and each week it grows by a constant 200 individuals. The population West of the river is estimated to have 8,000 individuals at the beginning of the year, and each week it grows by a constant 250 individuals.

   (a) Make a rough sketch of the number of mice in the two populations on the same graph. The horizontal axis should be the time from the beginning of the year in weeks.

*Label your axes and properly annotate your plot.*

   (b) Describe what is happening to the two populations. Is there a time when the two populations are equal? If so when is it?

   (c) Determine a formula for the total number of mice in the park at any week after the beginning of the year.

3. The mass of a sparrow is proportional to the cube of the length of its wing.

   (a) A sparrow is measured, and it has a wing length of 9cm and a mass of 30 grams. Determine the mass of a sparrow whose wing length is 10cm.

   (b) It is estimated that the mass of a sparrow is 27 grams. Determine an estimate of its wing length.

   (c) Determine the general function that returns the mass of a sparrow given the length of its wing.

4. A common task is to convert units. For each statement below determine the function that returns a quantity in the second unit given a quantity in the first units.

   (a) One kilometer is approximately 0.62 miles.

   (b) One meter is 100 centimeters.

   (c) One US dollar is approximately 1.35 Canadian dollars.

   (d) How do these questions relate to the idea of proportionality?

1. Briefly state two ideas from today's class.
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   - 
2. The surface area of a sphere of radius $r$ is $4\pi r^2$, and the volume is $\frac{4}{3}\pi r^3$. Determine the equation for the surface area of a sphere given its volume.
3. In the Star Trek television series a ship's velocity is given in terms of its warp factor, $w$. According to wikipedia[2], the actual speed is the warp factor cubed multiplied by the speed of light which is approximately $3.0 \times 10^8$ m/s.
   (a) Determine the speed of a ship that is moving at warp factor 0.2.
   (b) Determine the speed of a ship that is moving at warp factor 2.5.
   (c) Determine the speed of a ship that is moving at warp factor 3.0.
   (d) A ship is moving at warp factor 3.1. What warp factor would be required to double the ship's speed?
   (e) A ship is moving at warp factor 4.1. What warp factor would be required to double the ship's speed?
   (f) What is the general formula to determine the new warp factor required to double the speed given the current warp factor.
4. You watch a video from your favourite conspiracy theorist. He says that scientists are supressing evidence about prehistoric sparrows. He says that giant sparrows once existed whose wing length was 10 meters. Use the results from exercises 3a and 3b to determine if this makes sense. Based on your result write out the comment that you will post in the comments section in response to the video.

[2] https://en.wikipedia.org/wiki/Warp_drive accessed June 2016

1. A tortoise and a hare move in a straight line, and the both start at $x = 0$. The tortoise's position is given by

$$x_T = \frac{1}{2}t,$$

where $t$ is in minutes and $x$ is in meters. The hare's position is given by

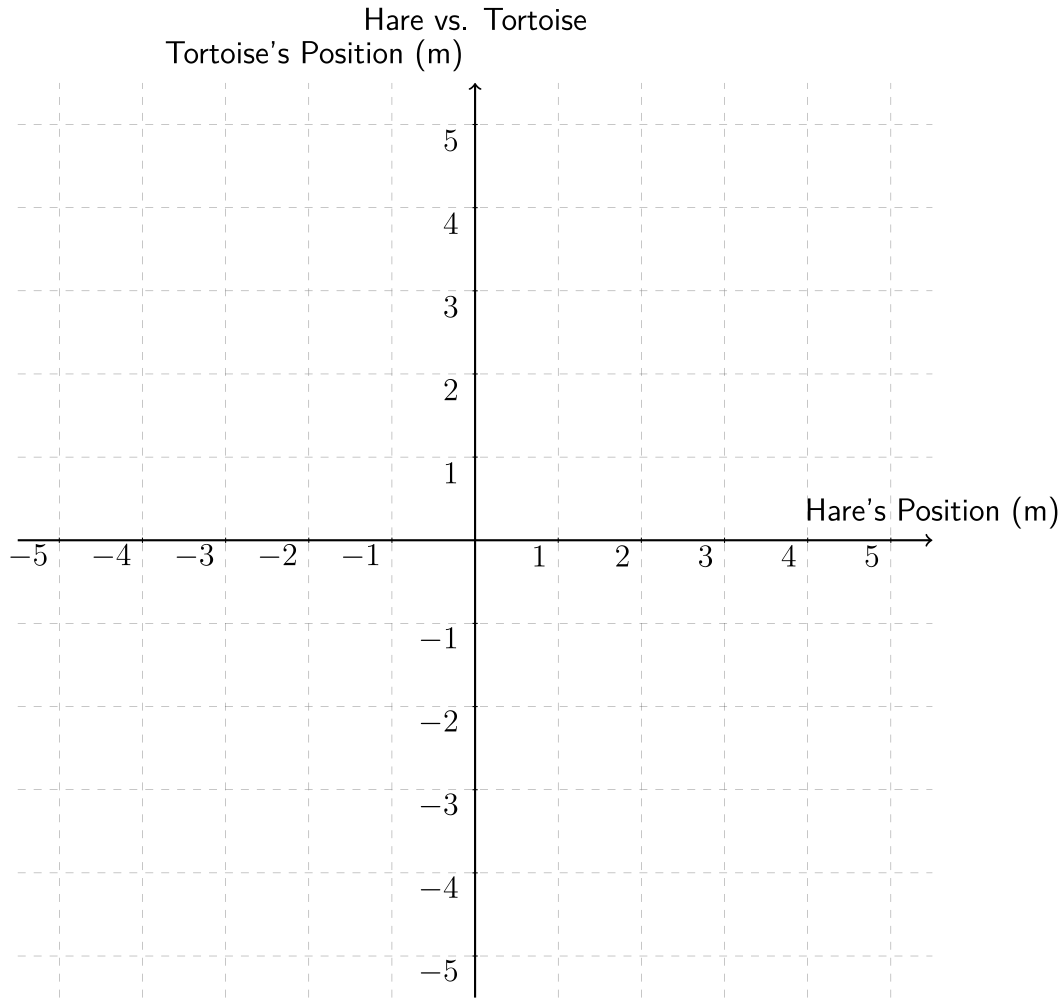
$$x_H = 2t,$$

where $t$ is in minutes and $x$ is in meters.

(a) Determine the positions of the tortoise at $t = 0$, $t = 1$, and $t = 2$.

(b) Determine the positions of the hare at $t = 0$, $t = 1$, and $t = 2$.

(c) For each time, plot the coordinate of the relative positions on the set of axes below. Use the tortoise's position for the $x$-coordinate, and use the hare's position for the $y$-coordinate. For example, if the tortoise's position is 1m, and the hare's position is 4m, then the coordinate would be $P(1,4)$.



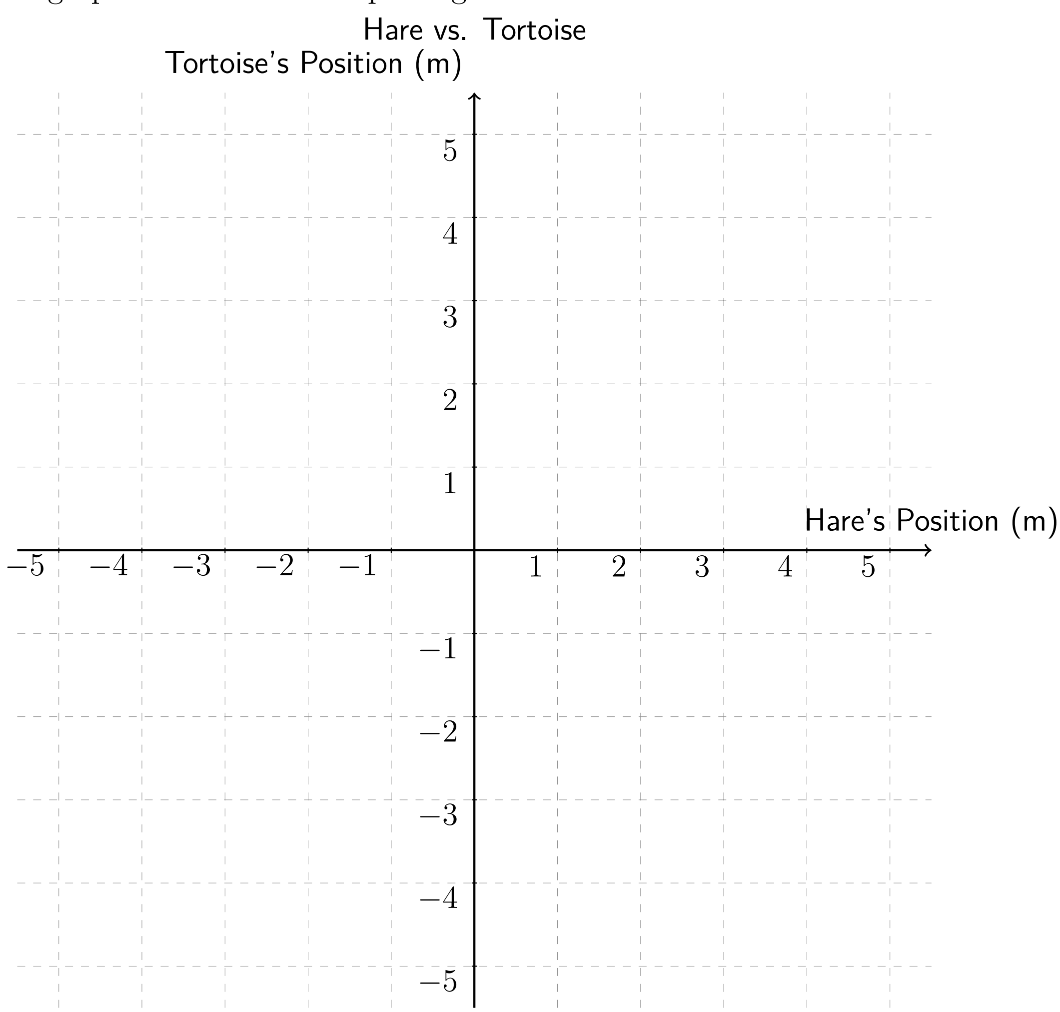
2. A tortoise and a hare move in a straight line, and the both start at $x = 0$. The tortoise's position is given by

$$x_T = \frac{1}{2}t, \tag{1.1}$$

where $t$ is in minutes and $x$ is in meters. The hare's position is given by

$$x_H = 2t, \tag{1.2}$$

where $t$ is in minutes and $x$ is in meters.
Determine the relationship between the hare's and the tortoise's position. That is, given the hare's position determine the tortoise's position. Make a sketch of the graph of the relationship using the axes below.




What is the tortoise's position when the hare's position is 15 meters? (Mark the associated coordinate on the plot above.)
In equation 1.1 and 1.2 how many variables are there? How many equations? Which variables are changing?

1. In each case below determine the formulas for the lines that satisfy the given requirements. In each case make a rough sketch of the line.

   (a) Goes through the point $P(-2, 5)$ and has a slope of -3.

   (b) Goes through the points $P_1(-3, -4)$ and $P_2(4, 1)$.

2. Two test plots are used to study the spread of an invasive plant. In the first test plot the conditions are dryer than in the second test plot. In the first test plot the invasive plant begins with a coverage of 10 square meters, and each day the area covered by the plant increases by 2 square meters. In the second test plot the invasive plant begins with a coverage of 15 square meters, and each day the area increases by 1 square meters.

   (a) Will there be a time when the area covered by the invasive test plant will be the same in the two test plots. Explain your reasoning.

   (b) Determine the area covered by the invasive plant in each test plot for a given time.

   (c) Determine the time that the area covered will be the same.

3. A group of researchers studies birds near a park. The birds tend to use cigarette butts in their nests, and it is believed to help reduce the number of parasitic insects. It is estimated that the number of cigarette butts used for nesting materials varies linearly with the distance from the nest to a nearby open air theater. A nest that is a distance of 30 meters appears to have 10 cigarette butts, and a nest that is a distance of 40 meters appears to have 8 cigarette butts.

   (a) Determine the relationship that will predict the number of cigarette butts in a nest given its distance from the theater. Use the relationship to predict the number of cigarette butts in a nest 50 meters from the theater. Also, make a sketch of the relationship.

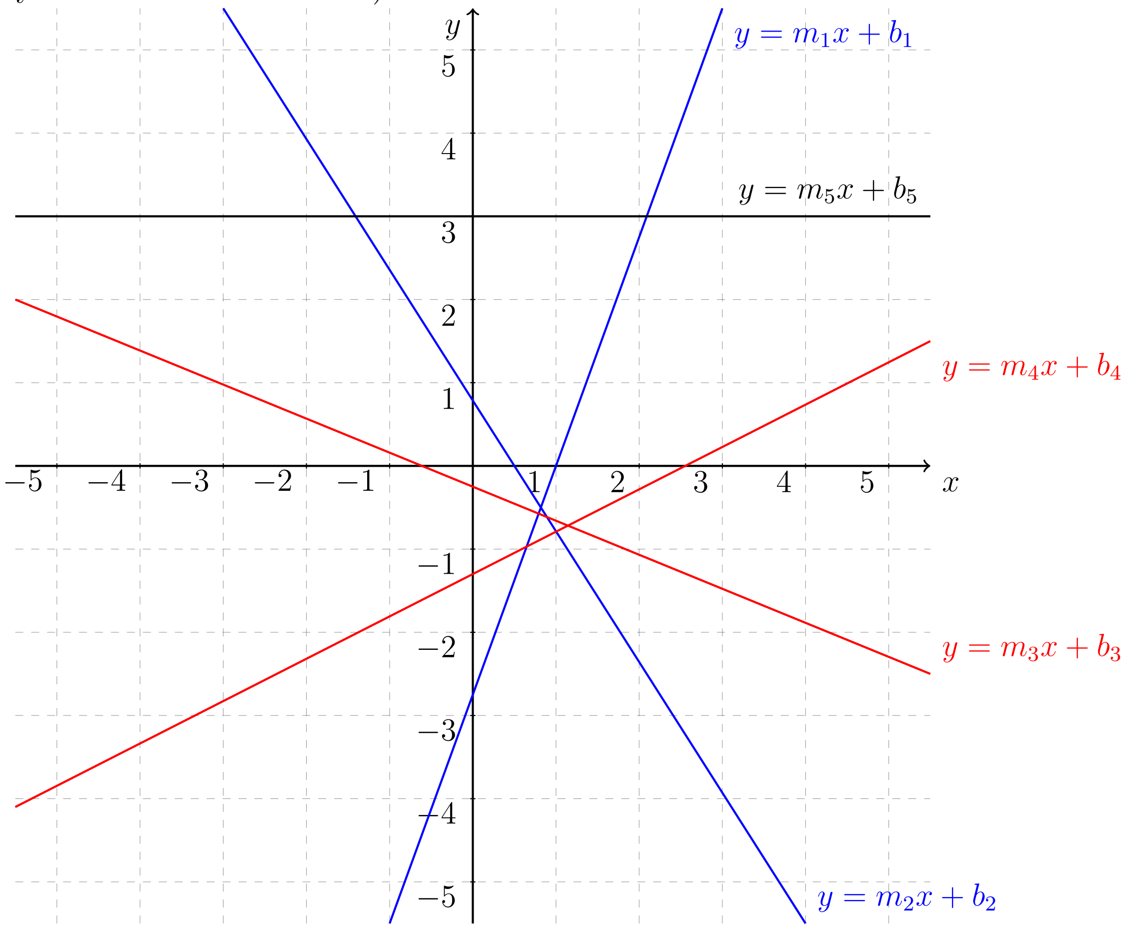
   *Be sure to label your axes and annotate your plot.*

   (b) What is the domain for the relationship?

   (c) A nest is found that has 4 cigarette butts. What is the prediction for the distance the nest is from the theater.

   (d) If the conjecture for the reason why birds use cigarette butts in their nests is true what would you expect is the general relationship between the fledgling success rate for birds and the location of their nests?

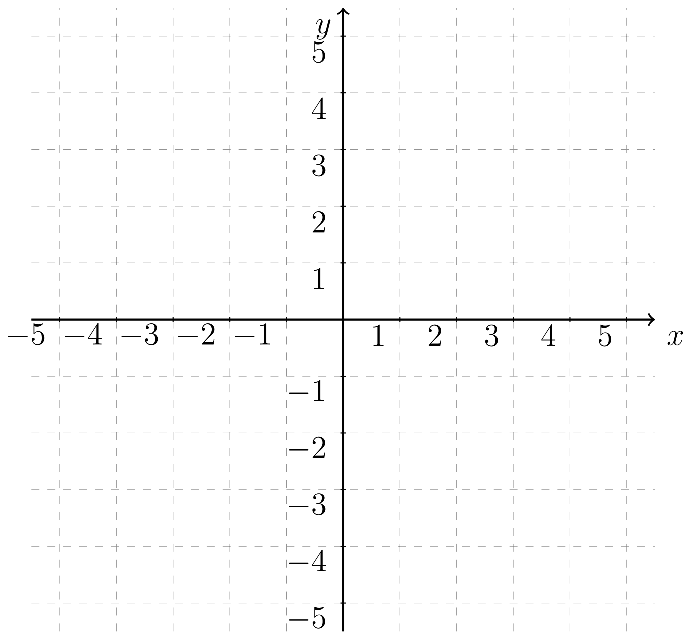
4. For the lines in the following plot sort the slopes for the lines in increasing order. (Write out the slopes, $m_1$, $m_2$, etc., in order from lowest to highest and do not try to estimate their values.)



5. The function, Larry($x$), is defined to be

$$\text{Larry}(x) = \frac{1}{2}(x-1)^2 + 2.$$

(a) Make a sketch of Larry.



(b) Determine the average rate of change of Larry from $x = -2$ to $x = 3$. Add a sketch of the secant line for these points on your sketch above.

(c) Determine a value of $x_0$ where the average rate of change from $x = 2$ to $x = x_0$ is zero. Add the a sketch of the resulting secant line for these points on your sketch above.

(d) Is there any value of $x = a$ where you cannot find another point so that the resulting average rate of change is zero? Explain your reasoning.

1. Briefly state two ideas from today's class.
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   -
2. The growth rate for a population is the change in the number of individuals per unit time. The per-capita growth rate is the growth rate divided by the total number of individuals in the population. Suppose that the per-capita growth rate for a particular species is approximated as a linear function. It is estimated that when the population is near zero the per-capita growth rate is highest due to a lack of competition and approaches 0.5 (the time units are hours). When the population approaches 1,000 the per-capita growth rate is estimated to be zero the death rate and birth rate are balanced, and the total growth rate is zero.
   (a) What are the units for the per capita growth rate?
   (b) Is the slope of the per-capita growth rate positive or negative? Explain why your answer makes sense given the physical situation.
   (c) Determine the relationship that gives the per-capita growth rate as a function of the population, $p$.
   (d) Make a sketch of the graph of the per-capita growth rate. (Make sure to annotate your graph and label your axes.)
   (e) What happens to the per-capita growth rate as the population increases? Why might this happen?
   (f) Determine the values where the per-capita growth rate is negative. Why would the per-capita growth rate be negative?

1. A chemical reaction has a single reactant that breaks down, and the resulting reaction produces two different products. It is estimated that for each gram of the reactant that $\frac{1}{3}$g of the first product is produced and $\frac{1}{6}$g of the original reactant remains. Everything else that remains is the second product.
   (a) Write down all of the relevant information that is given in the statement above.

   (b) If you start with four grams of reactant how many grams of the reactant will you get in the end?

   (c) If you start with four grams of reactant how much of each product do you get? (Keep in mind that mass is conserved so you have to have the same total mass that you started with.)

   (d) Determine the number of grams of the reactants will result when you start with $x$ grams of reactant.

   (e) Determine the number of grams of the products will result when you start with $x$ grams of reactant.

1. The time required to bake a ceramic depends on the mass of the ceramic. It is estimated to be a linear relationship between the time and mass. **The goal is to determine the time required to bake a ceramic sample given its mass.**

   (a) Define your variables for the quantities described above. Then make a rough sketch of the relationships described above that will given an idea of the general relationship between the variables.

   *Label your axes and properly annotate your sketch.*

   (b) Should there be a positive or negative slope for the relationship? Briefly justify your answer.

   (c) With respect to the cost for each ceramic, would you prefer a larger or smaller slope for the relationship? Briefly justify your answer.

   (d) A set of samples will be baked, and it is estimated that the baking time for a ceramic whose mass is 2,000g is five hours. It is also estimated that the baking time for a ceramic whose mass is 3,000g is five and a half hours. Determine the baking time given the mass.

   (e) A sample ceramic will be tested, and its mass is 4,500g. How long would you expect it to take to bake the ceramic?

2. Alice was born on the same day as her father, Bob. This year Bob's age is three times Alice's age. In fifteen years, Bob will be twice his daughter's age. What are their ages this year?

   (a) Define the variables that you will be using for Alice's and Bob's ages.

   (b) Write the algebraic expression that indicates that Bob's age is three times Alice's age.

   (c) If Alice's age is now $A$ what will her age be in fifteen years?

   (d) If Bob's age is now $B$ what will his age be in fifteen years?

   (e) Use the two previous expressions, and write out the algebraic expression that indicates that Bob's future age will be twice Alice's future age.

   (f) Use the two expressions from parts 2b and 2e to draw a sketch of the two linear relationships. Assume that Bob's age is a function of Alice's age, and the horizontal axis will Alice's age. Will the system have a solution that makes sense?

   (g) Use the two expressions from parts 2b and 2e to determine Alice's and Bob's age.

   (h) Next to each of the previous steps, write a note in the left margin. Your note should indicate the relationship of the step and what part of the original statement it corresponds to.

3. Trucks are unloaded at a warehouse, and during the summer it is estimated that it takes longer to unload a truck if the weather is warmer. When the temperature is 22C it is estimated that it takes sixty minutes to unload a truck. For each increase of one degree Celsius it is estimated to take four additional minutes to unload a truck.

   (a) Define the variables that you will use.

   (b) How long will it take to unload a truck if the temperature is 23C?

   (c) How long will it take to unload a truck if the temperature is 24C?

   (d) How long will it take to unload a truck given a temperature of T degrees Celsius?

4. In the previous problem it was assumed that the truck was being unloaded during the summer. In the winter the relationship between unloading time and the temperature is different. When the temperature is 5C it is estimated that it takes fifty minutes to unload a truck, and for each decrease of one degree Celsius it is estimated to take three additional minutes to unload the truck.

   (a) Define the variables that you will use.

   (b) Determine how long it will take to unload the truck for any temperature less than 5C.

   (c) At what temperature should you switch from using the formula on the previous problem to using the formula above?

   (d) Write out the formula to determine the time required to unload the truck given **any** temperature in Celsius.

   (e) What is the shortest time to unload a time and what is the best temperature to unload a truck? (Explain your reasoning.)

1. Briefly state two ideas from today's class.
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   -
2. Determine the formula for a line that goes through the points $P(2, 50)$ and $Q(5, 80)$. What is the height of the graph of the line when $x = 3.5$?
3. The number of items sold at a given time of day is a linear function of the number of hours the office has been open. After the office has been open for two hours it is estimated that fifty items have been sold. After the office has been open for five hours it is estimated that eighty items have been sold. What is the estimate for the number of items sold after the office has been open for three and a half hours?
4. What is the relationship between the previous two questions?

1. Two populations of different species of bacteria interact. The number of bacteria (in tens of thousands) in the first population is given by

$$B(t) \quad = \quad 10 + t^2,$$

where $t$ is the time in days since the beginning of the year. The number of bacteria (in tens of thousands) in the second population is given by

$$C(t) \quad = \quad 10 + (t-2)^2,$$

where $t$ is the time in days since the beginning of the year.

(a) Make a sketch of the two functions below.

*Label your axes and properly annotate your plot.*

(b) For what values of $t$ does it make sense to use these functions?

(c) A researcher decides to alter the situation and adds 50,000 bacteria to the first population given by $B(t)$. Determine a formula for the altered population. Make a sketch of the original and altered populations below.

1. The height, in meters, of a certain tree changes by the relationship

$$h(t) = \sqrt{\frac{t}{3}},$$

where $t$ is the time in years from when the seed was germinated.

(a) Make a sketch of the height of a tree as a function of time.

*Label your axes and properly annotate your plot.*

(b) Two seeds are planted, and the first seed germinates immediately. The second seed germinates one year after the first is germinated, and then begins to grow.

(i) Make a sketch of the graphs of the heights of the two trees.

(ii) Determine the formulas for the height of the two trees with respect to the time that they were planted.

(c) A new strain of the tree is developed that grows to the same height in half the time.

(i) Make a sketch of the graph of the height of the new strain and the previous strain.

(ii) Determine the formula for the height of the new strain with respect to the time that it is planted.

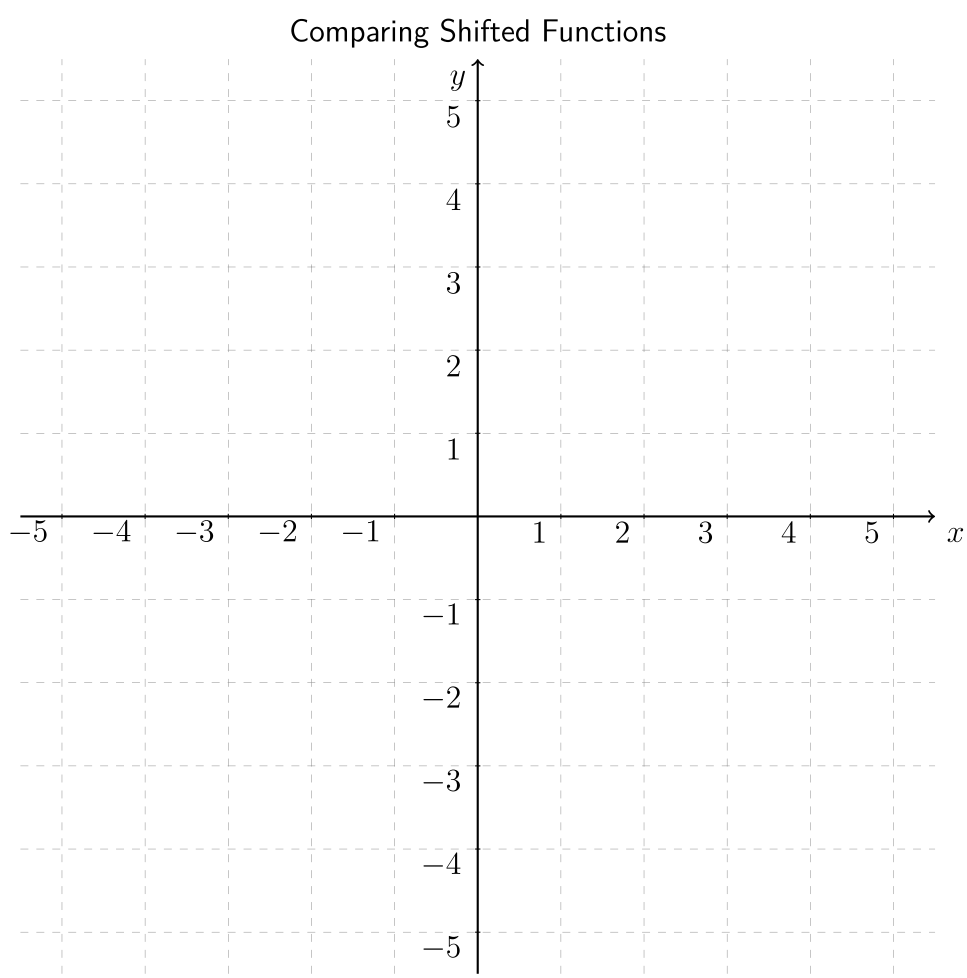
2. A function is defined as

$$f(x) = |x|.$$

(a) Make a sketch of the function on the axes below.

(b) Make a sketch of the following new functions on the same graph as well. (Clearly annotation your plot.)

$$\begin{aligned} g(x) &= f(3x), \\ h(x) &= f(x) + 2, \\ p(x) &= f(x+2), \\ q(x) &= 3f(x), \\ r(x) &= -f(x) - 2. \end{aligned}$$

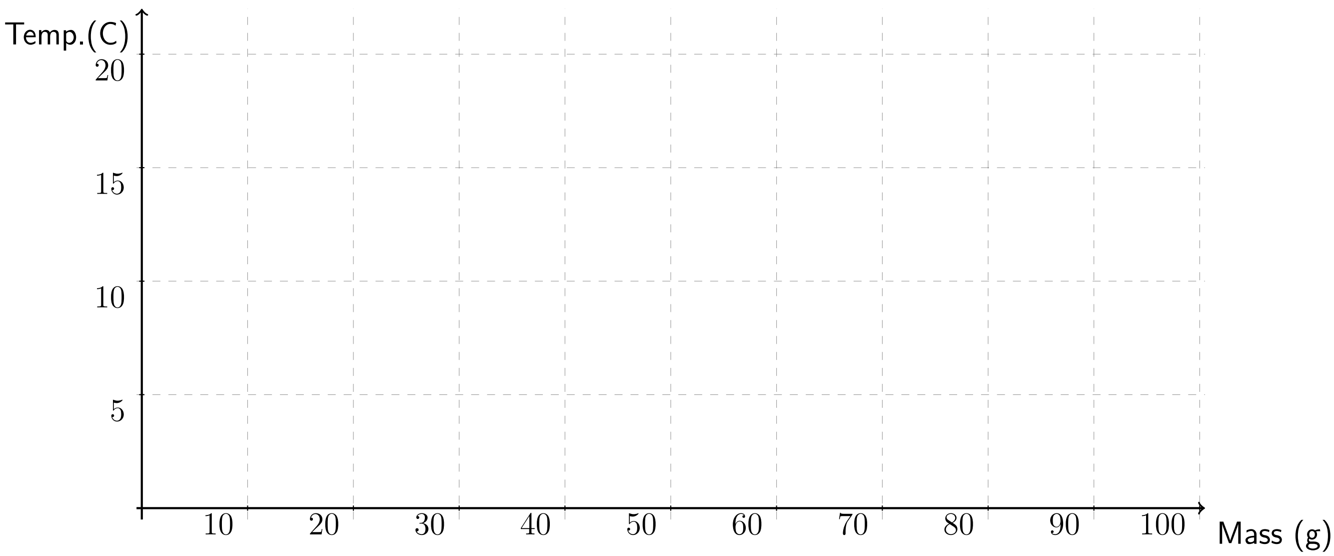


3. The temperature of a snake can change based on its activity level. Suppose that the temperature of a snake is taken at a fixed time after it consumes a rodent, and the snake's temperature depends on the mass of the rodent, denoted $m$,

$$\text{Temperature(m)} = 12 + 0.04m,$$

where the temperature is in Celsius, and $m$ is in grams.

(a) Make a sketch of the relationship on the axes below.




(b) To convert Celsius to Kelvin, you add 273.15K. What will happen to the graph if the temperature is converted to Kelvin? Can you draw the new plot on the existing axes?

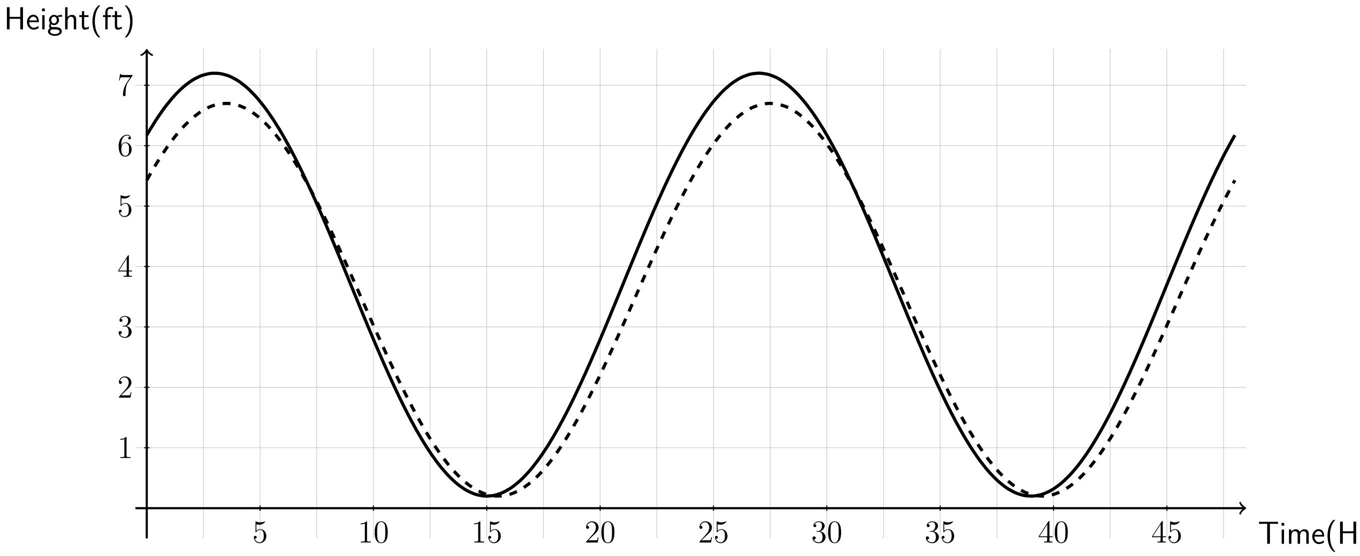
(c) To convert Celsius (C) to Fahrenheit (F), you use the function

$$\text{Fahrenheit}(C) = \frac{9}{5}C + 32.$$

If you convert the temperature to Fahrenheit how will the graph change?

(d) If you convert grams to kilograms how will the graph change? (1kg=1,000g)

4. The water levels for the ocean near the Tybee lighthouse and the St. Simon's lighthouse are shown in the plot below. The solid line is the water level at Tybee, and the dotted line is at St. Simon's. According to the tidal charts for a given day it is estimated that the low tide at Tybee is at 3pm, and the low tide at St. Simon's is at 3:24pm. Determine the expression that will give the water level at St. Simon's in terms of the water level at Tybee.



1. Briefly state two ideas from today's class.
   - 
   - 
2. An enzyme in a solution decays, and the concentration (mg/liter) as a function of time in hours is

$$C(t) = \frac{3.5}{5.0+t}.$$

   (a) Make a sketch of the concentration as a function of time. Assume that the time is positive. Annotate your plot and label your axes.

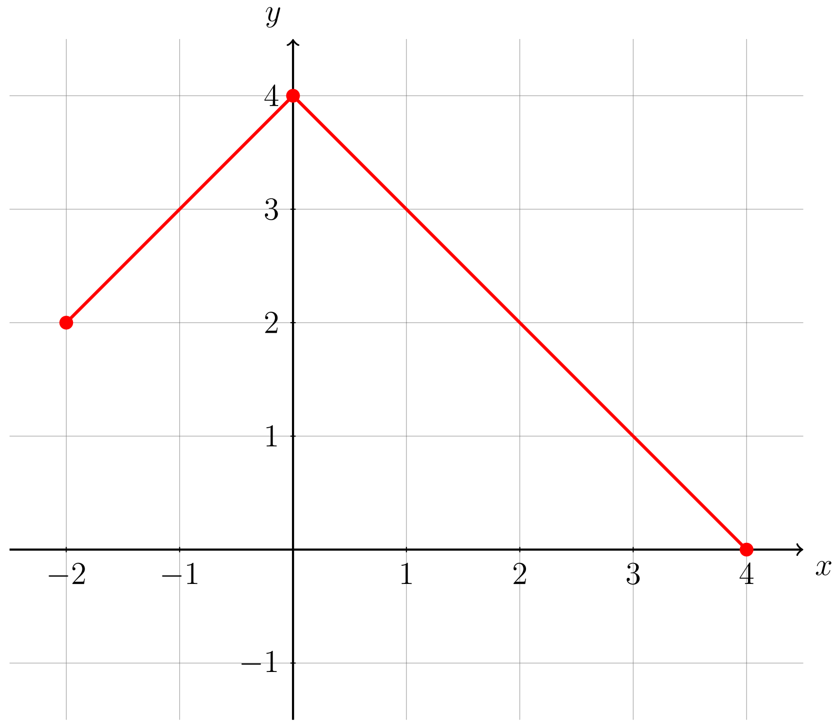
   (b) How long will it take for the enzyme to be reduced to half its original concentration?

   (c) Another enzyme is present, and its concentration is linked to the first. Its concentration is half the first enzyme's concentration 30 minutes in the past. Determine the formula for the second enzyme's concentration. (This is referred to as a delay relationship.)

   (d) How long will it take for the second enzyme to be reduced to half its original concentration?

   (e) Make a sketch of the concentration of both enzymes as a function of time. Assume that the time is positive. Annotate your plot and label your axes.

1. The graph of a function, $g$, is shown shown below:




   (a) Determine the domain and range of the function.

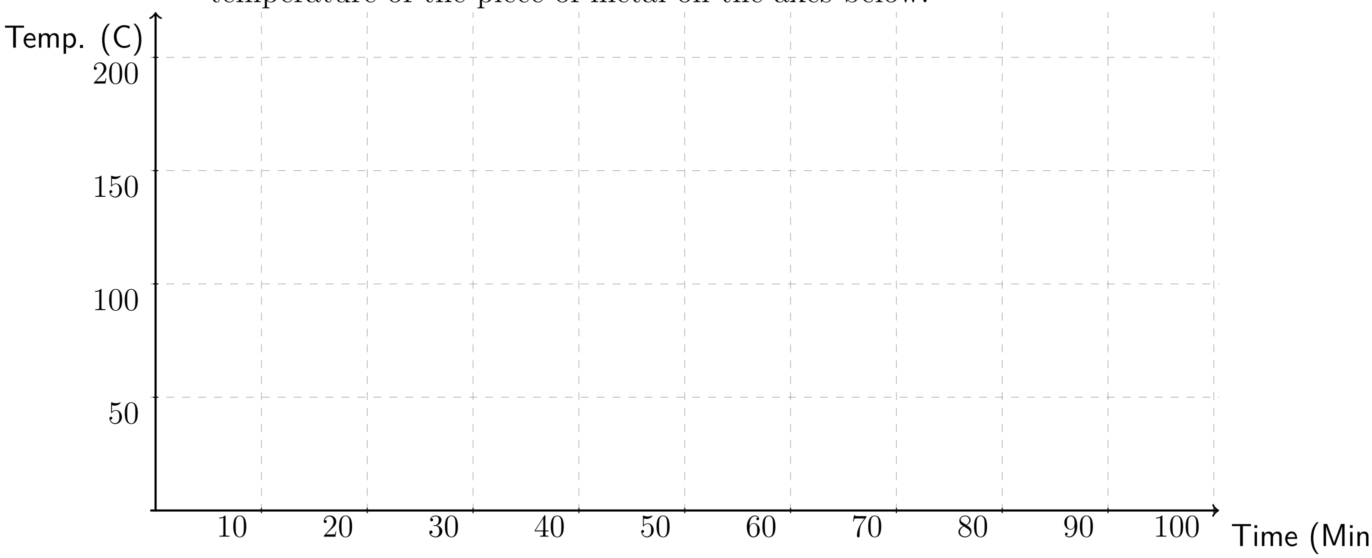
   (b) Determine the formula for the function if $x \geq -2$ and $x < 0$.

   (c) Determine the formula for the function if $x > 0$ and $x \leq 4$.

   (d) For what values of $x$ is the function increasing?

   (e) For what values of $x$ is the function decreasing?

1. The temperature of a piece of metal is initially 20C. It is placed in a hot furnace for twenty-five minutes, and its temperature is raised to 200C. It is then removed from the oven and placed in cold water and cooled to room temperature.. After five minutes it is placed back into the furnace, and the process is repeated twice for a total number of three cycles. Make a sketch of the graph of the temperature of the piece of metal on the axes below.

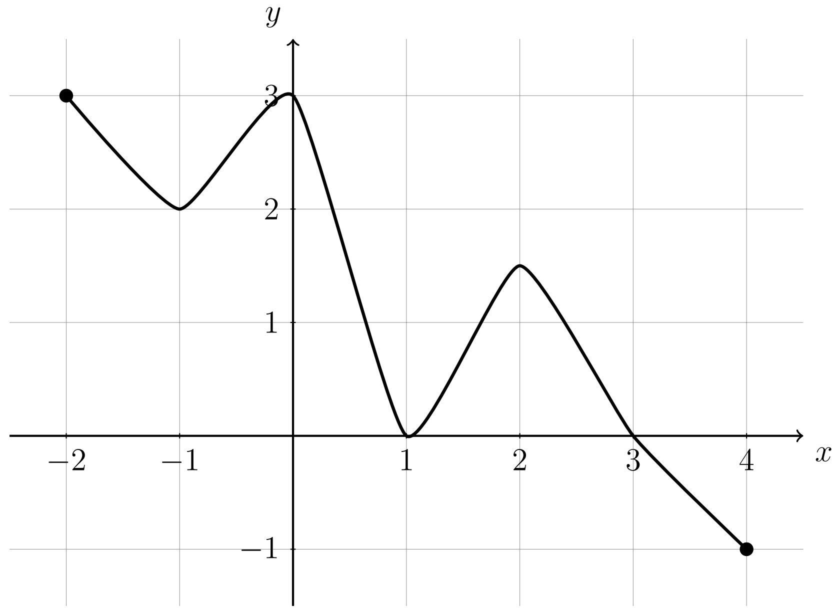



   (a) What is the range and domain of the function?

   (b) Determine the values of the time where the function is increasing.

   (c) Determine the values of the time where the function is decreasing.

2. The questions below refer to the function whose graph is given below.




(a) Determine the domain and range of the function.

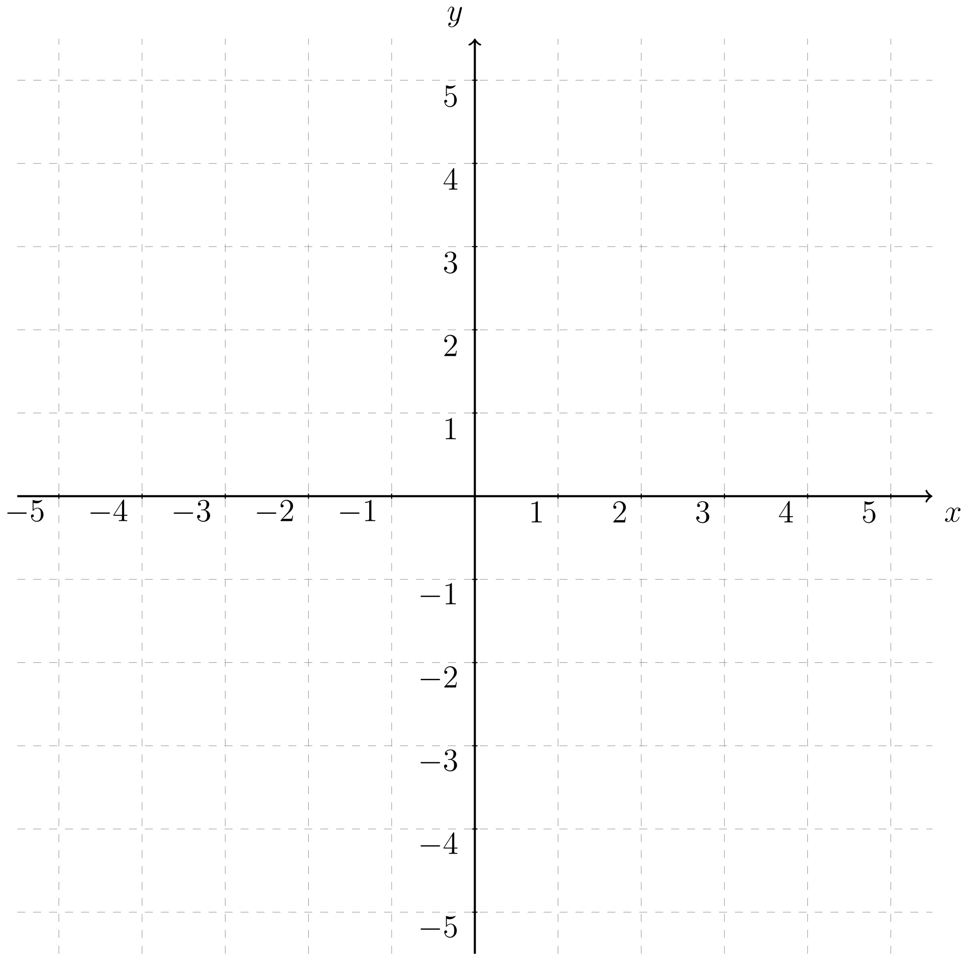
(b) Determine the values of $x$ where the function is increasing.

(c) Determine the values of $x$ where the function is decreasing.

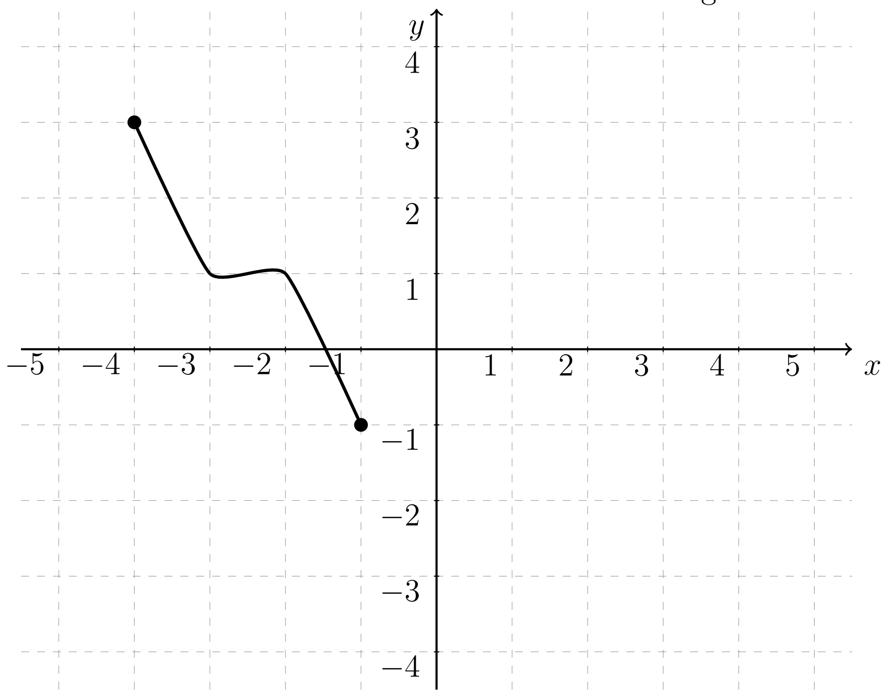
3. Make sketch of the graph of the function

$$\text{Harold}(x) = \begin{cases} \frac{1}{2}(x+2) & -2 \leq x < 0, \\ 4 - x^2 & 0 < x < 2. \end{cases}$$

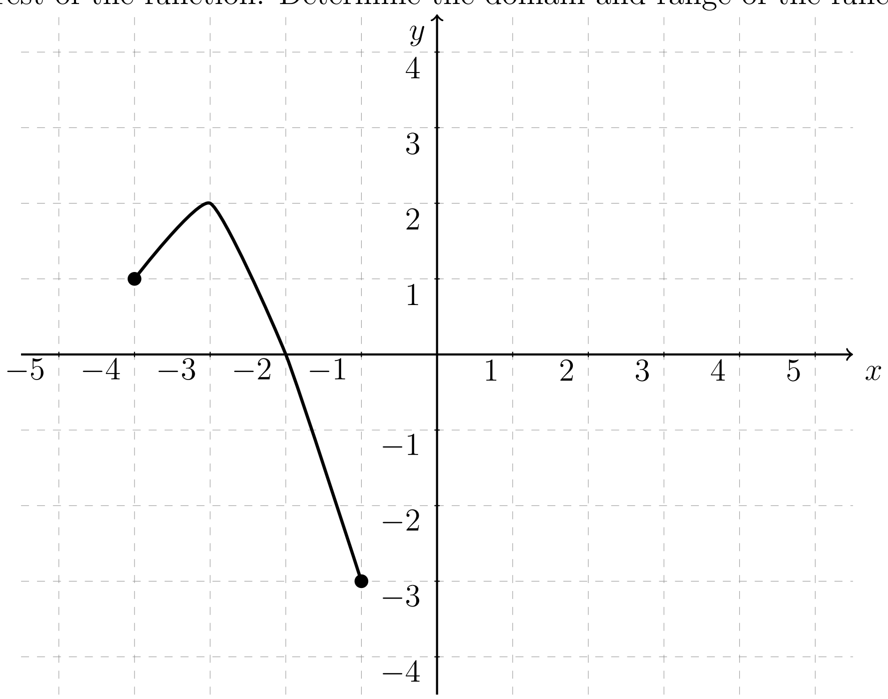
Determine the domain and range of the function. Determine where the function is increasing. Also determine where it is decreasing.



4. Part of the graph of a function is given below. The function is even. Sketch the rest of the function. Determine the domain and range of the function.




5. Part of the graph of a function is given below. The function is odd. Sketch the rest of the function. Determine the domain and range of the function.



6. A small company builds solar panels. The amount of electricity produced by a panel is proportional to the intensity of sun light. When the sunlight is bright, 100,000 lux, the system produces 4,000 watts. On one day of operation there is a good deal of cloud cover, and the amount of sunlight varies linearly from 6am to noon from 0 lux to 50,000 lux. After noon it varies linearly to 0 lux at 6pm. On the second day the cycle repeats, but the maximum amount of light is 100,000 lux. Determine the amount of power produced by the panel at any time during the two days. (Include night time!)

   (a) Read the question again. Organize the information given to you. Underline the important parts, and determine the underlying assumption. (Hint: what happens when the light intensity is zero.)

   (b) Make a rough sketch of what the relationship of interest might look like. (Label your axes.)

   (c) Define the variables that you will use.

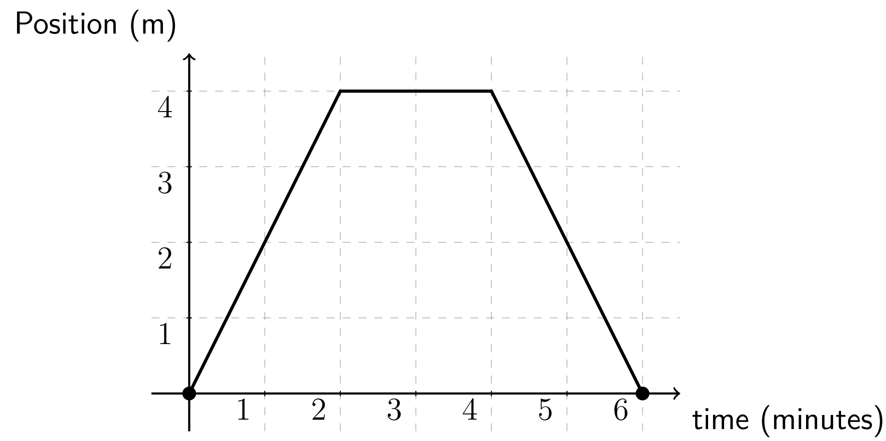
   (d) List the relevant relationships.

   (e) How will you solve this problem?

   (f) Determine the relationship.

   (g) Go back and check your work. Is your answer consistent with your prediction? Does it make sense?

1. Briefly state two ideas from today's class.
   -
   -
2. A train moves along a track. From $t = 0$ to $t = 12$ hours it moves from its initial position at a constant velocity to a new location twenty-five miles away. From $t = 12$ to $t = 24$ hours the train moves with a constant velocity to its start position. Determine a formula for the train's position. Use proper notation for piecewise defined functions.
3. A car moves along a straight track. The graph of the car's position is shown in the plot below. Write out a brief story describing the car's position but also how its position is changing in time.



1. A restaurant would like to test a new menu item. They estimate that the cost for producing $x$ servings in market A is 12$ per serving. They estimate that the cost for producing $y$ servings in market B is $15 per serving. They will allocate a total of $36,000 for the test. Write out an expression that relates $x$, $y$, and the total cost of the test.

2. The number of mosquitoes per acre in an area is estimated to be 600 times the area of open water measured in acres. The area of open water in a location is declining over time and is $A(t) = 50 - \frac{1}{3}t$, where $t$ is the number of years since January 1 of the current year. Determine the number of mosquitoes per acre in given the time, make $t$.

3. The delay time required for a neuron to recharge is estimate to be a function of the calcium concentration,

$$\text{Recharge}([\text{Ca}]) \quad = \quad 0.05 - [\text{Ca}]^2.$$

The concentration of calcium in an experiment is changed over time and is estimated to be

$$[\text{Ca}](t) \quad = \quad 0.01 + \frac{1}{1+t}.$$

Determine the formula used to estimate the recharge delay as a function of time, $t$. *(Do not simplify the expression.)*

1. A function is defined to be

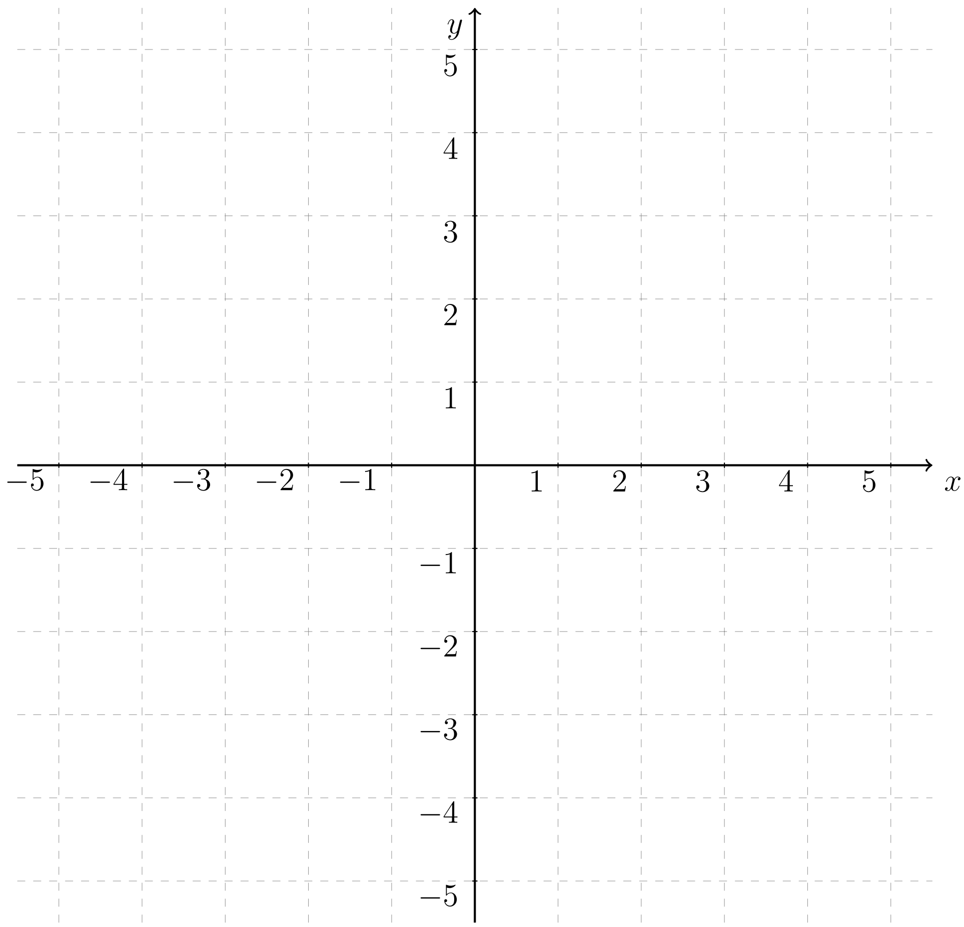
$$f(x) \quad = \quad x^2.$$

Determine the value of $a$ and $b$ so that the function

$$g(x) \quad = \quad f(x-a)+b$$

is the original function that is shifted up two units and left three units. Plot the graphs of $f(x)$ and $g(x)$ on the coordinate plane below.

Comparing Shifted Functions



2. Two functions are given in the tables below.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(x)$ | a | m | k | a | h |

| $x$ | a | c | h | j | m |
|---|---|---|---|---|---|
| $g(x)$ | $\natural$ | $\Diamond$ | $\Box$ | $\heartsuit$ | $\Diamond$ |

   (a) Determine the range and domain of $f$.

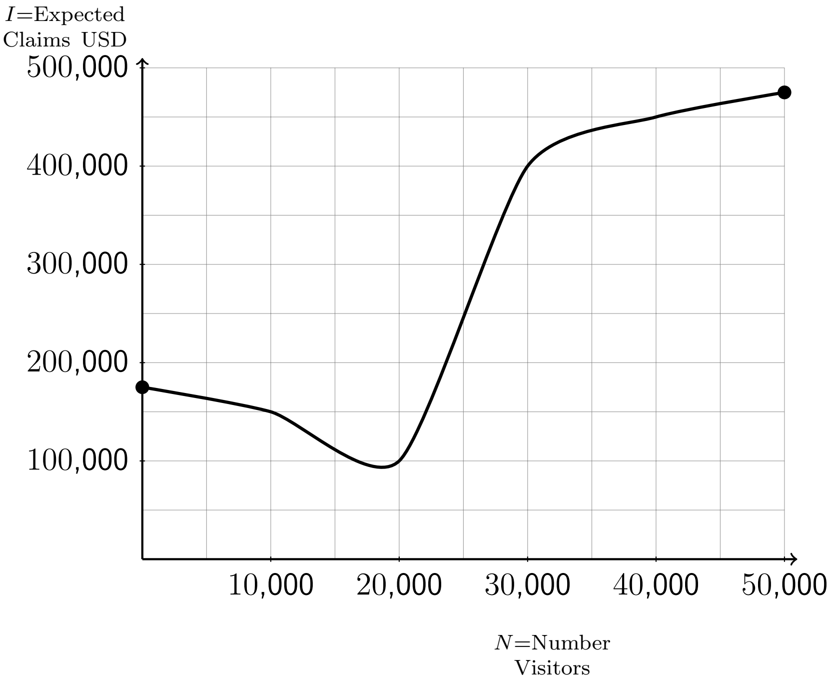
   (b) Determine the range and domain of $g$.

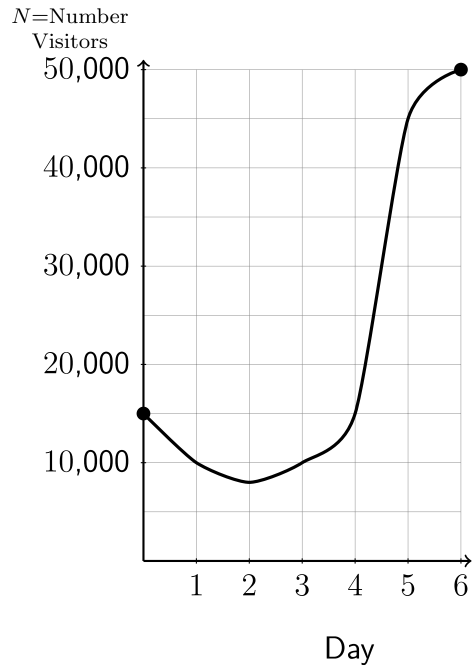
   (c) Determine the values of each of the following expressions:

$$\begin{aligned} f(2) &= \\ f(4) &= \\ g(f(1)) &= \\ g(f(2)) &= \\ g(f(0)) + g(f(3)) &= \end{aligned}$$

   (d) If $f(x)$ =h what is the value of $x$?

3. The expected amount for insurance claims, $I$, for a small, college town depends on the number, $N$, of outside visitors to the town, and the graph of the relationship is shown in the plot below, on the left. The number of visitors to the town changes during a week, and the number of visitors based on the day of the week is shown in the graph on the right. (Zero corresponds to Sunday.)




(a) What is the expected amount for insurance claims on Monday?

(b) As the day of the week increases from Sunday to Monday what is the change in the expected amount for insurance claims?

*A negative number indicates a decrease, and a positive number indicates an increase.*

(c) As the day of the week increases from Wednesday to Thursday what is the change in the expected insurance claims?

(d) Explain what meaning the expression $I(N(d))$ has where $d$ is the day of the week.

(e) How can the function $I(N(d))$ increase from Sunday to Monday when both functions are decreasing?

4. The owner of a shop will be selling two similar items and hopes that customers will choose to buy either one or the other of the items. The cost to the owner depends on how many are purchased,

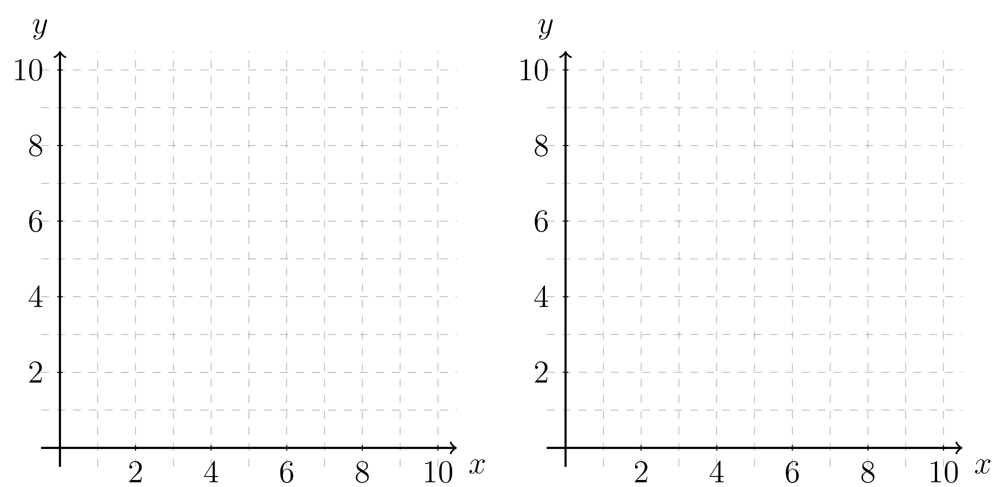
$$
\begin{aligned}
C_1(x) &= 100 - (x-10)^2, \\
C_2(y) &= 100 - (y-10)^2,
\end{aligned}
$$

where $x$ is the number of the first item purchased, and $y$ is the number of the second item purchased. The owner plans on allocating **at least** 100$ for purchasing the items. The owner has **at most** 5 $\mathrm{m}^2$ of total space for the two sets of items, and each individual item takes up the same amount of space, 0.5 $\mathrm{m}^2$.

(a) Determine the relationship between $x$, $y$, and the total costs.

(b) Determine the relationship between $x$, $y$, and the total space required.

(c) Make a sketch of the graphs of the two relationships. Use the left axes for the costs, and use the right axes for the space.




(d) If the owner chooses to purchase 4 of the first item, determine the possible numbers of the second item that could be purchased. (It is a range of values.)

(e) If the owner chooses to purchase 8 of the first item, determine the the possible numbers of the second item that could be purchased. (It is a range of values.)

1. Briefly state two ideas from today's class.
   - 
   - 
2. Two functions are given in the tables below.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(x)$ | a | m | k | a | h |

| $x$ | a | c | h | j | m |
|---|---|---|---|---|---|
| $g(x)$ | $\natural$ | $\Diamond$ | $\Box$ | $\heartsuit$ | $\Diamond$ |

   (a) If $g(f(x)) = \natural$ what are the possible values of $x$? Is this reverse procedure a function?

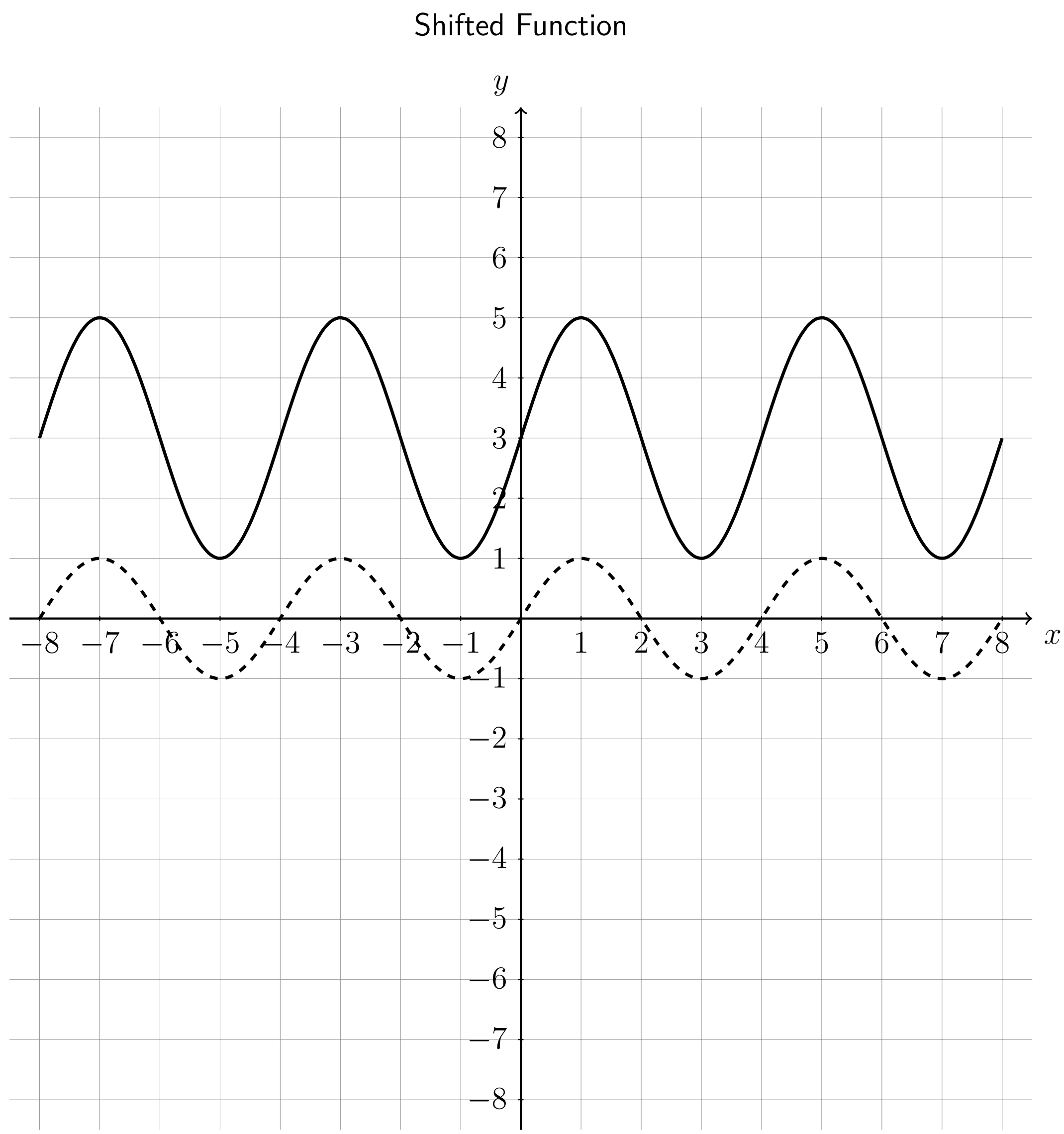
   (b) If $g(x) = \Diamond$ what are the possible values of $x$? Is this reverse procedure a function?

   (c) Express the function $g(f(x))$ as a table.

   (d) Determine the range and domain of $g(f(x))$.

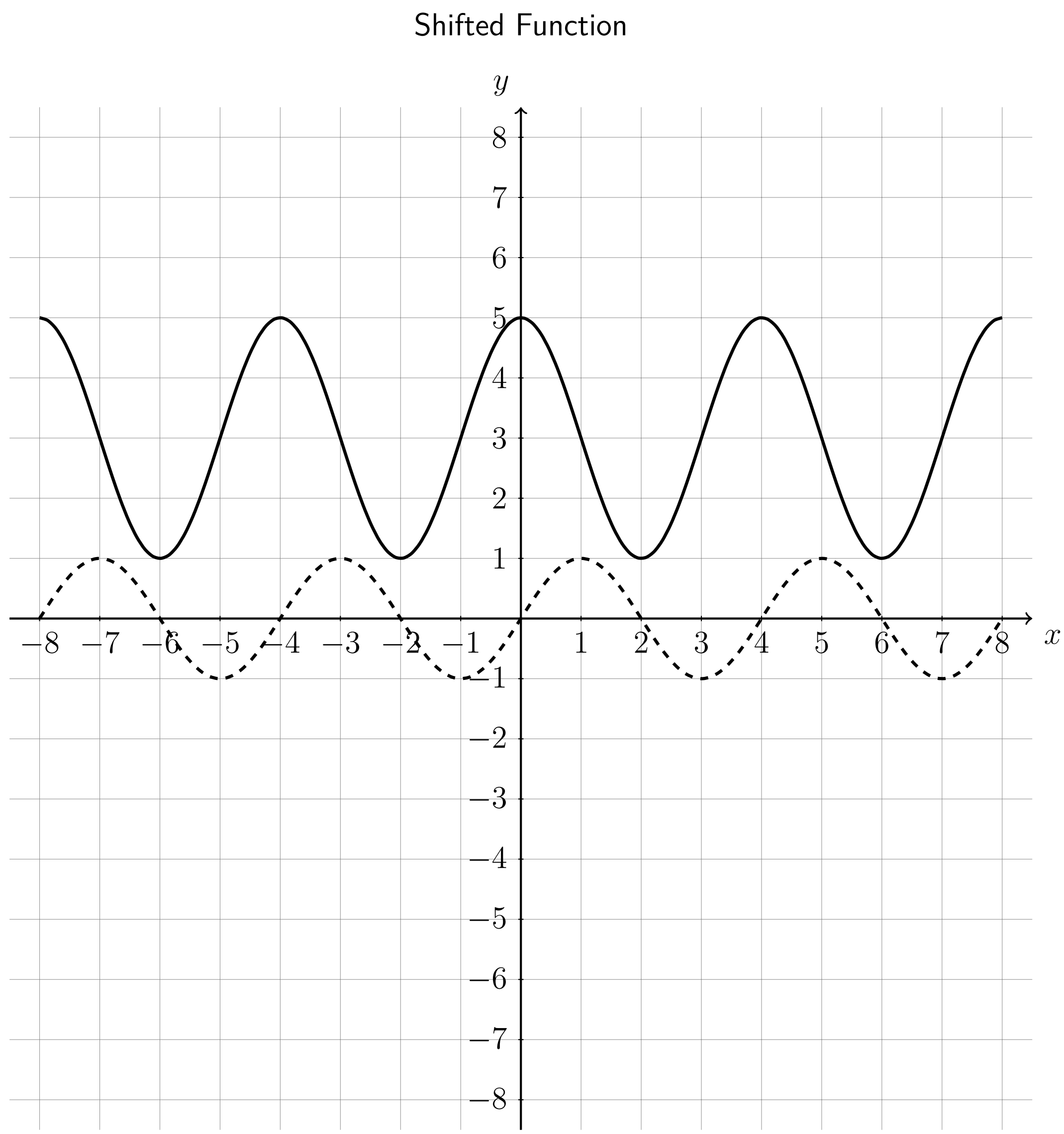
3. Two functions are shown in the figure below. The function plotted with the dotted line is $f(x)$, and the function plotted with the solid line is $g(x)$. Express $g(x)$ in terms of $f(x)$,

$$g(x) \quad =$$



4. Two functions are shown in the figure below. The function plotted with the dotted line is $f(x)$, and the function plotted with the solid line is $g(x)$. Express $g(x)$ in terms of $f(x)$.

$$g(x) \quad =$$




Add a sketch of the graph of the function $h(x) = 3f(x+2) - 5$ to the plot above.
Can you find a different formula whose graph looks exactly the same as $g(x)$?

1. A function is defined to be

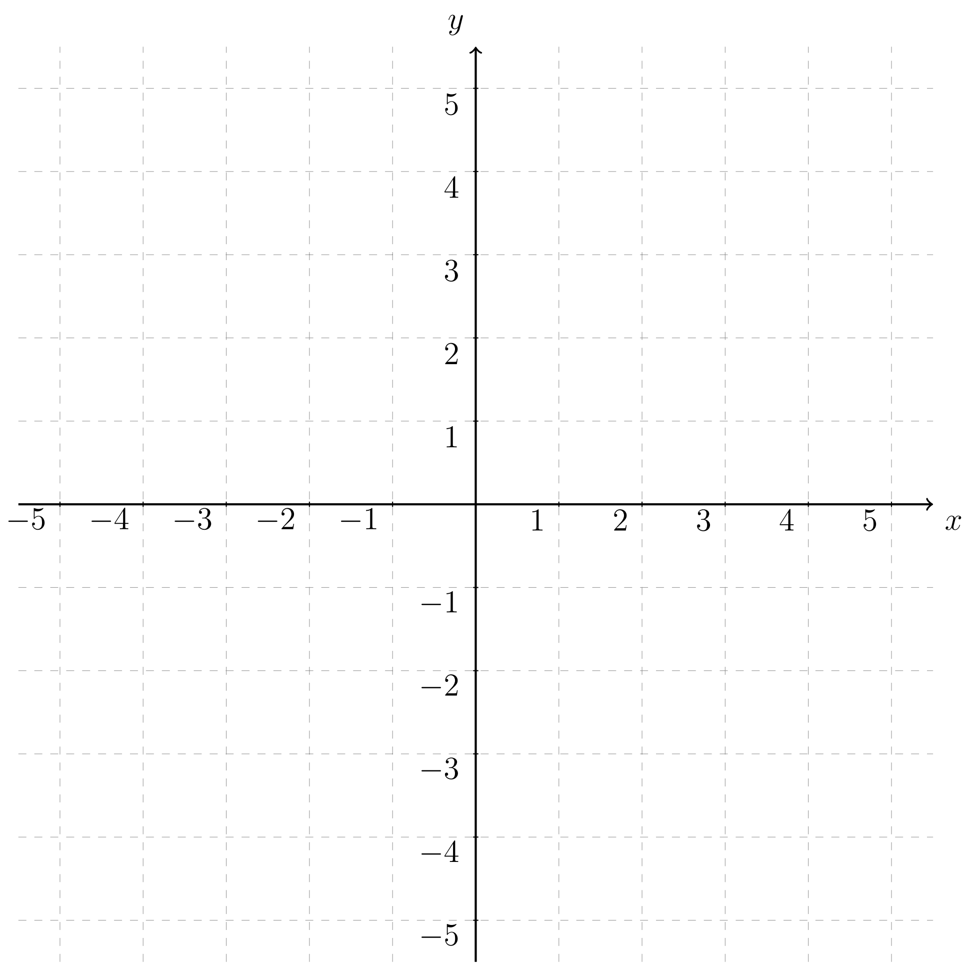
$$f(x) = x^2.$$

   (a) Make a sketch of the function on the axes below.

   (b) Make a sketch of the following new functions on the graph as well with clear annotations:

$$\begin{aligned} g(x) &= f(3x), \\ h(x) &= f(x)+2, \\ p(x) &= f(x+2), \\ q(x) &= -f(x). \end{aligned}$$

Comparing Shifted Functions



1. A traffic engineer observes the movement of traffic on a busy street. She estimates that when the density of cars is low the rate of flow is also low, and as the density increases the rate of flow increases. However, once the density of cars reaches a certain point the traffic slows, and the rate of flow begins to decrease.
   Some tests are run, and it is estimated that when the traffic density is 0 cars/meter that the rate of flow is 0 cars/minute. When the density is high, roughly 0.1 cars/meter or more, there is gridlock, and the rate of flow is 0 cars/minute. Finally, when there is 0.05 cars per meter the rate of flow is at a maximum of 20 cars per minute.

   (a) Assuming that the rate of flow is 0 if the density is greater than 0.1 make a rough sketch of the rate of flow as a function of the density of cars.

   *Label your axes and properly annotate your graph.*

   (b) Assuming that the rate of flow is a quadratic function of the traffic density when the density is between 0 and 0.1 cars/meter inclusive, determine the formula for the traffic density.

2. Maximize the product of two positive numbers whose sum is five.

   (a) The product of two numbers can be thought of as the area of a rectangle. Make a rough sketch of a rectangle.

*The product of two numbers is the area of a rectangle!*

   (b) Within the sketch above, identify and label the variables that you will use.

   (c) Determine the formula for the function to be maximized.

*This is called the "Cost Function." It should have two variables.*

   (d) Determine any other relevant relationships between your variables that must be true in all circumstances.

*This is called the "Constraint." It should have two variables.*

   (e) How will you solve this problem?

   (f) Determine the values of the two numbers.

*Check your work and make sure that you maximized the profit and did not minimize it.*

3. A developer has a square plot of land that is 100 meters by 100 meters, and the land is next to a river. The land will be divided into two parts. One part will be formed by cutting out a rectangle in one corner of the large plot, and its width will be along the river. The height of the rectangle will be ten meters shorter than its width. It is estimated that this new rectangular plot of land will sell for \$2.00 per square meter plus 100\$/meter times the width. The remaining land will be sold for \$4.00 per square meter. Determine the dimensions of the rectangular plot that will result in the highest profit.

   (a) Make a sketch of the land with a corner designated for one of the plots.

   *Label important aspects of the sketch.*

   (b) Within the sketch above, identify and label the variables that you will use.

   (c) Determine the total selling price for the land in terms of both variables.

   *This is called the "Cost Function." It should have at least two variables in it.*

(d) Determine any other relevant relationships between your variables that must be true in all circumstances.

*This is called the "Constraint." It should have two variables in it.*

(e) How will you solve this problem?

(f) Determine the dimensions of the rectangular plot that will result in the highest profit.

*Check your work and make sure that you maximized the profit and did not minimize it.*

1. Briefly state two ideas from today's class.
   - 
   - 
2. Determine the vertex of the following parabolas.
   (a) $y = -x^2 + 10x - 5$

   (b) $y = 2x^2 - 10x + 18$

   (c) $y = 3x^2 + 4 + 1$

   (d) $y = 5x^2 + 2$

   (e) $y = 5x^2 + x + 2$
3. Two super hero capes will be sewn as part of a demonstration. One cape will be in the shape of a square. The other cape will be in the shape of a triangle, and the height is the same length as its base. They will be displayed together, and the sum of the two lengths must be 6 feet. What are the dimensions of the capes that will minimize the total area of the capes?
   (a) What are the steps you will take to solve this problem?

   (b) Follow your steps and determine the dimensions of the two capes that will meet the given criteria.
4. Determine the numbers whose sum is 30 and their product is as large as possible. (Show your work and justify your answers mathematically.)
5. Two types of bacteria will be used to produce a medicinal compound. Bacteria A will be given $x$ hours, and it produces 3g of the compound per hour. Bacteria B will be given $y$ hours, and it produces 4g of the compound per hour. The total cost for the type A bacteria is $x^2$ dollars, and the total cost for the type B bacteria is $2y^2$ dollars. A total of 40g of product will be required. How many hours should be allocated to each bacteria to minimize the total cost
6. You have been asked to help design a channel taking water from a lake into a reservoir. The channel must have a rectangular cross section with a fixed depth and width throughout its entire length. The bigger the cross section, the more the channel will cost. In fact, the total cost of building the channel will be \$30,000 dollars per meter of depth and \$10,000 per meter of width of its cross section, and the government has budgeted a total of \$100,000 to construct the channel. The government wants you to use this money to build a channel which will maximize the rate at which water flows through the channel (which is proportional to the cross sectional area of the channel). What is the depth and width that will accomplish this?
7. Two chicken coops will be constructed using 120 feet of fencing. The coops will be in the shape of a rectangle with fencing across the middle dividing the

rectangle into two equal rectangles. What dimensions will give the largest total area for the two coops?

1. The height, in meters, of a certain tree is given by the relationship

$$h(t) = \frac{50 \cdot t}{25 + t},$$

where $t$ is the time in years from when the seed was germinated.

(a) Make a sketch of the height of a tree as a function of time.

(b) A tree is measured, and it is estimated that its height is 5 meters. How long ago did its seed germinate?

(c) A tree is measured, and it is estimated that its height is 10 meters. How long ago did its seed germinate?

(d) Determine the function that takes the height of the tree and then determines its time since germination.

1. Two functions are defined in the following tables. Determine the values of each expression below. If a value does not exist write "NA."

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(x)$ | 2 | 6 | 5 | 1 | 4 |

| $x$ | 1 | 2 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $g(x)$ | 3 | 3 | 2 | 9 | 4 |

   (a) $f(g(4))$

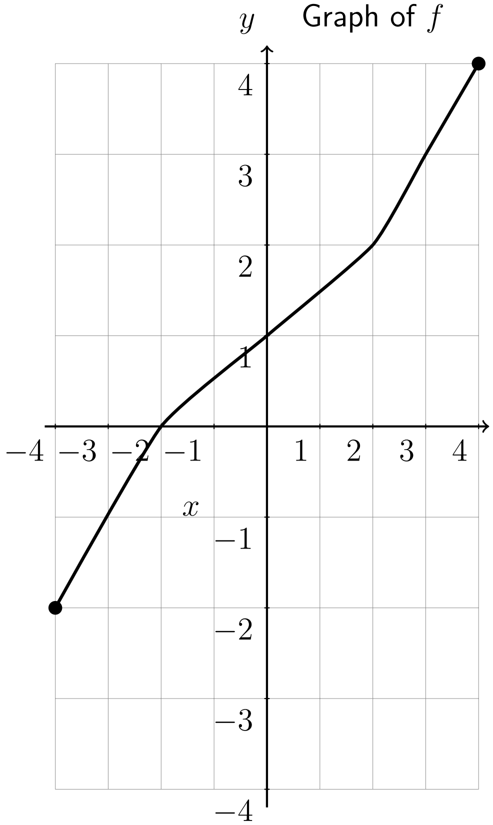
   (b) $g(f(4))$

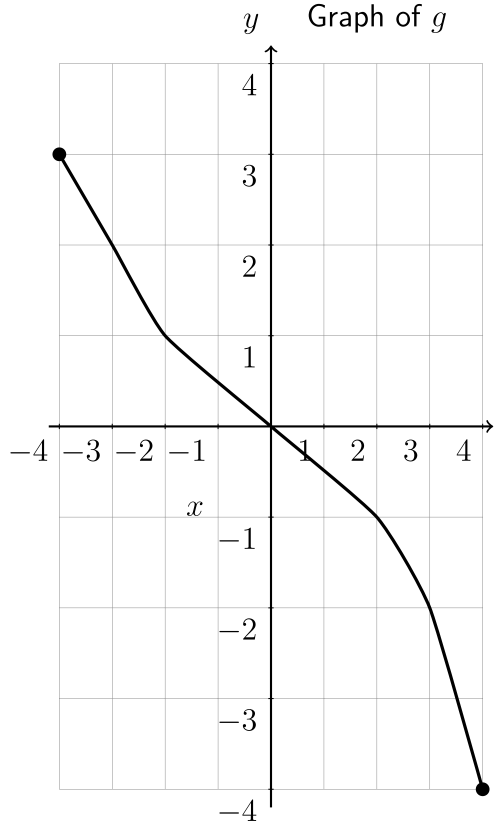
   (c) $f^{-1}(g(4))$

   (d) $f(g^{-1}(4))$

   (e) $f^{-1}(g^{-1}(3))$

2. Given the graphs of the two functions below determine the values of the expressions below the graph.




(a) $f(g(3)) =$

(b) $f^{-1}(0) =$

(c) $f^{-1}(-2) =$

(d) $g^{-1}(f(0)) =$

3. The height, in meters, of a certain tree changes by the relationship

$$h(t) = \frac{50 \cdot t}{25 + t},$$

where $t$ is the time in years from when the seed was germinated.

(a) Determine the range and the domain of $h(t)$. (For the domain also consider the physical description as well as the definition of the function.)

(b) Determine the inverse of $h(t)$. How can it be interpreted?

(c) Determine the range and the domain of the inverse of $h(t)$.

4. For each function below determine if it is one-to-one. For each function that is one-to-one determine the inverse of the function. For each function that is not one-to-one determine a subset of the domain for which the function is one-to-one on that subset.

   (a) $f(x) = 3x + 1$

   (b) $f(x) = \frac{1}{1+x}$

   (c) $f(x) = x^2$

   (d) $f(x) = \sqrt{3x + 1}$

5. The impact of sex determination in Pine Snakes was explored in a paper in **The American Naturalist**.[3] It was found that if the temperature of the Pine Snake's eggs was kept constant then the sex ratio in the brood can be approximated using a linear function of the temperature. The data suggests the following estimate:

$$\text{Sex Ratio(temp.)} \approx 0.68 \cdot \text{temp.} - 0.95.$$

(The sex ratio is calculated by taking the number of male snakes and dividing by the number of female snakes in the brood.)

(a) What is the largest possible domain that can be used for this function? Does this result make physical sense?

(b) What is the meaning for the inverse function? What is the domain and range of the inverse?

(c) Determine the inverse function.

(d) Sketch a graph of the function and its inverse function on the following page. Label your axes and annotate your plots.

[3] *Effects of Incubation Temperature on Sex Ratios in Pine Snakes: Differential Vulnerability of Males and Females*, Joanna Burger and R. T. Zappalorti, **The American Naturalist** Vol. 132, No. 4 (Oct., 1988), pp. 492-505.

1. Briefly state two ideas from today's class.
   - 
   - 
2. We examine the function

$$f(x) = x^2.$$

   (a) Determine if the function is 1-1.

   (b) Determine a restriction on the domain of $f$ so that the function is 1-1 on the restricted domain.

   (c) Determine the inverse on the restricted domain.

3. We examine the function

$$f(x) = x^2 - 4x.$$

   (a) Determine if the function is 1-1.

   (b) Determine a restriction on the domain of $f$ so that the function is 1-1 on the restricted domain.

   (c) Determine the inverse on the restricted domain.

4. Finish question 5.

# Chapter 2

# Exponential and Logarithmic Functions

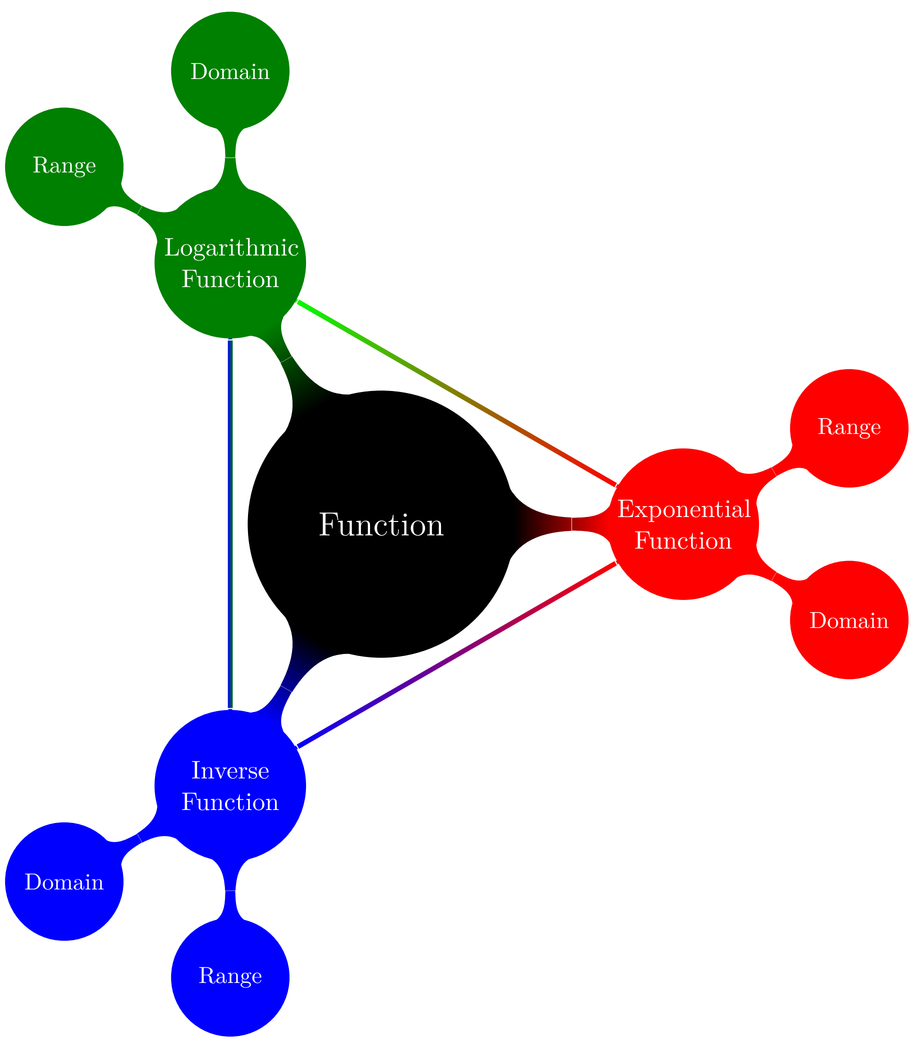



Figure 2.1: Topics for the second section of the course.

1. Carbon-15 has a half life of about 2.5 seconds. If an object has 2 grams of carbon-15 within it now, then in 2.5 seconds it will only have 1 gram due to the decay of carbon-15. After an additional 2.5 seconds there will only be about $\frac{1}{2}$ gram within the object.
   Suppose that an object has $8.0 \times 10^{-6}$ grams of carbon-15, and it is placed in a sealed container. Determine how much carbon-15 is contained in the object at the following times:

   (a) After 2.5 seconds.

   (b) After an additional 2.5 seconds for a total of 5.0 seconds.

   (c) After an additional 2.5 seconds for a total of 7.5 seconds.

   (d) After an additional 2.5 seconds for a total of 10.0 seconds.

1. A species of bacteria is able to divide every three hours, and every three hours each individual bacteria splits into two new individuals. Suppose that a colony starts with 10,000 individuals. For each time below determine the number of bacteria in the colony and also determine an expression for the total time in terms of the three hour time span. (For example, 6 hours = $2 \times 3$ hours.)

   (a) At $t = 3$ hours. (Do not simplify the expression.)

   *Your answer should be expressed as the product of two numbers.*

   (b) At $t = 6$ hours.

   *Your results should be kept in terms of products of terms and do not simplify. Look for a pattern.*

   (c) At $t = 9$ hours.

   (d) At $t = n \times 3$ hours where $n$ is an integer greater than or equal to zero.

   (e) How many bacteria were in the colony 3 hours before the start of the experiment?

2. A species of bacteria is able to divide every five hours. That is every five hours each bacteria splits into two new individuals. **After** each division, only 75% of the bacteria survive. A colony starts with 10,000 individuals. For each time below determine the number of bacteria and also determine an expression for the total time in terms of the three hour time span. (For example, 6 hours $= 2 \times 3$ hours.)

*Keep the fractions and do not simplify. Look for a pattern.*

   (a) At $t = 5$ hours.

   (b) At $t = 10$ hours.

   (c) At $t = 15$ hours.

   (d) At $t = n \times 5$ hours where $n$ is an integer greater than zero.

   (e) How many bacteria were in the colony 5 hours before the start of the experiment?

3. A bank offers a savings account in which the interest is compounded 1.5% annually, and the interest is accrued at the end of each month. If a person places \$1,000 in an account how much money is in the account after $n$ months? From the resulting expression determine the money in the account for any time, $t$, where $t$ is measured in years. *Determine the amount of money in the account after the first, second, and third months. Do not simplify your results, and try to determine the pattern.*

*Try to mimic the steps in the previous question.*

*Break each operation you make into separate steps and clearly show your work.*

4. Simplify each expression below.

   (a) $\frac{3^5 \cdot 3^2}{3^4}$

   (b) $\frac{2^8}{2^5}$

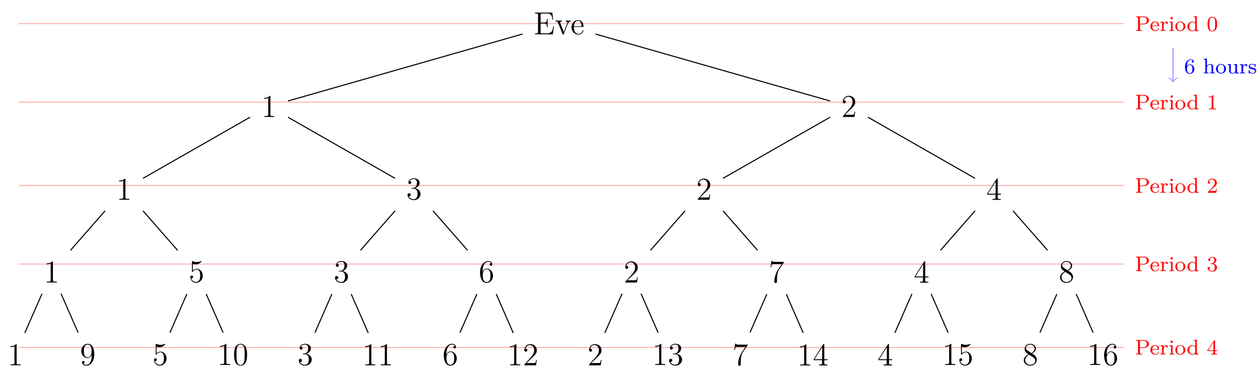
   (c) $\frac{4^2 \cdot 2^2}{4^3}$

   (d) $5^9 \cdot 5^2 \cdot 5^{-7}$

   (e) $\left(\frac{1}{2}\right)^5 \cdot 2^9 \cdot 2^{-3}$

1. Briefly state two ideas from today's class.
   -
   -
2. A bank offers 1.5% annual interest compounded weekly (assume 52 weeks in a year). You will deposit \$5,000 into the account. How much money will be in the account at any time?
3. A bank offers 1.5% annual interest compounded monthly. You will deposit some money into an account and wish to have \$25,000 after two years. How much money should you deposit?
4. A compound is created that decays over time. It takes four years until half of the compound decays in a sample. You wish to store the compound for 5 years. How much should you store so that there will be 4 kg of material at the end of the time period?
5. A colony of bacteria starts with 10,000 individuals. Each individual divides into two new individuals every 4 hours. After each division only 80% of the total number of bacteria survive to divide later. Determine the number of bacteria present at any time.

Exponential functions are used whenever some quantity has a proportional increase over fixed time spans. An example is a bacteria population that increases by 100% every six hours. That means that every six hours the population doubles. In the diagram below, a single bacteria starts in a sample. After the first time period, six hours, there are two bacteria. After another six hours, a total of 12 hours, there are four bacteria. In each six hour time period that follows the population doubles.




Exponential functions satisfy the algebraic properties given below. In each example it is assumed that $a$, $b$, and $c$ are constants, and $a > 0$.

$$a^b \cdot a^c = a^{b+c} \tag{2.1}$$

$$\frac{a^b}{a^c} = a^{b-c} \tag{2.2}$$

$$\left(a^b\right)^c = a^{b \cdot c} \tag{2.3}$$

Also, $e$ is a constant number, and we define the number $e$ to be

$$e \approx 2.718.$$

It is common to use the number $e$ as the base for exponentials. The number $e$ plays the same role as the constant $a$ in the equations above:

$$e^b \cdot e^c = e^{b+c} \tag{2.4}$$

$$\frac{e^b}{e^c} = e^{b-c} \tag{2.5}$$

$$\left(e^b\right)^c = e^{b \cdot c} \tag{2.6}$$

1. Determine an approximation for the value of each expression below. Use a calculator, and your approximation should be to the nearest 0.01.

   (a) $2^3$

   (b) $2.5^3$

   (c) $2.7^3$

   (d) $2.718^3$

   (e) $2.7183^3$

1. A bank manager is considering the impact of different terms for an account that offers compounded interest. She assumes that the interest rate is a constant annual one percent rate and then checks to see what happens for different lengths of time between compounding. Assume that one dollar is initially deposited.

   (a) Determine the amount of money in the account after one hundred years, if the interest is compounded yearly.

   (b) Determine the amount of money in the account after 100 years, if the interest is compounded once every six months.

   (c) Determine the amount of money in the account after 100 years, if the interest is compounded once a month.

   (d) Determine the amount of money in the account after 100 years, if the interest is compounded once a day.

   (e) Determine the amount of money in the account after 100 years, if the interest is compounded twice a day.

   (f) What is happening to the balance as the number of terms increases?

2. Generalize the value found on the previous page.

   (a) Determine a formula for the balance for 1% annual interest after 100 years if the interest is compounded $n$ times per year.

   (b) Make a substitution, $u = 100n$, in the previous expression. Write out the expression for the balance as a function of $u$.

*You can solve the substitution for* $n$ *if you are not sure how to do proceed.*

   (c) Determine the values of the balance for the following values of $u$.

| $u$ | balance |
|---|---|
| 1 | |
| 2 | |
| 12 | |
| 365 | |
| 1000 | |

   (d) What is the value approaching as $u$ gets large? This is a number that occurs in many situations, and we do not want to write it out every time we use it, so we use the symbol $e$ as a form of short hand notation.

$$e \approx$$

3. When interest is compounded continuously, the balance is determined using the function

$$\text{Balance}(t) = Pe^{rt},$$

where $P$ is the initial balance, $r$ is the annual interest rate, and $t$ is the time in years.

(a) Sketch a plot of the balance over time if 1$ is deposited with a rate of 1%.

*Label your axes and annotate your plot.*

(b) What happens to the balance as time increases?

(c) What would happen to the graph if you make $r$ bigger? What if $r$ is smaller?

4. As radioactive isotopes decay, the amount of isotope in a sample decreases. If the decay rate of an isotope is $r$ then the amount of an isotope in a sample is expressed using the function

$$\text{Amount}(t) \quad = \quad Ae^{-rt},$$

where $t$ is measured in years.

(a) What is the physical interpretation of the constant $A$?

(b) If a sample of a radioactive substance initially contains 3 grams, and the radioactive decay is 0.00004, sketch a plot of the amount of the substance in the sample over time.

*Label your axes and annotate your plot. Your plot does not have to be perfect. Just try to get the general shape.*

(c) What happens to the amount of the radioactive substance in the sample as time increases?

(d) What would happen to the graph if you make $r$ bigger? What if $r$ is smaller?

5. Simplify each of the following expressions.

   (a) $\left(e^{4.22}\right)^3$

   (b) $e^{3.2} \cdot e^{1.8}$

   (c) $e^{9.33} \cdot e^{t}$

   (d) $\left(e^{4.19} \cdot e^{2t}\right)^2$

   (e) $\left(e^{1.9}\right)^t$

   (f) $(e^r)^t = \left(\dfrac{1}{2}\right)^t$

6. We look at one important property of the logarithm. In particular we want to examine ways to write the expression $e^a \cdot e^b$. For each question below, solve the relationships as requested.

   *Write the result as* $e^a \cdot e^b = e^{\#}$

   (a) Use the properties of the exponential to represent the product $e^a \cdot e^b$ as a single exponential.

   $$e^a \cdot e^b \quad = \tag{2.7}$$

   (b) If $e^a = x$ substitute this value for $e^a$ into the left hand side of equation 2.7.

   (c) If $e^b = y$ substitute this value for $e^b$ into the left hand side of equation 2.7.

   (d) Take the logarithm of both sides of your current form of the expression and simplify the expression.

   (e) Solve $e^a = x$ for $a$ and solve $e^b = y$ for $b$ in the previous expression and substitute the result into the expression.

   (f) What does this imply about the expression

   $$\ln(x \cdot y) \quad = \qquad\qquad\qquad .$$

7. We now examine another property with respect to raising a number to a power. In particular we look at the logarithm of $x^r$ where $r$ is a constant.

   (a) We can use the algebraic rule $x^r = (x)^r$. Substitute the identity $x = e^{\ln(x)}$ to rewrite the right side of the expression:

   $$x^r = \tag{2.8}$$

   (b) Expand and simplify the exponent in the right hand side of the expression.

   (c) Take the logarithm of both sides of the expression. Do not simplify the left hand side of the expression, but simplify the right hand side of the expression.

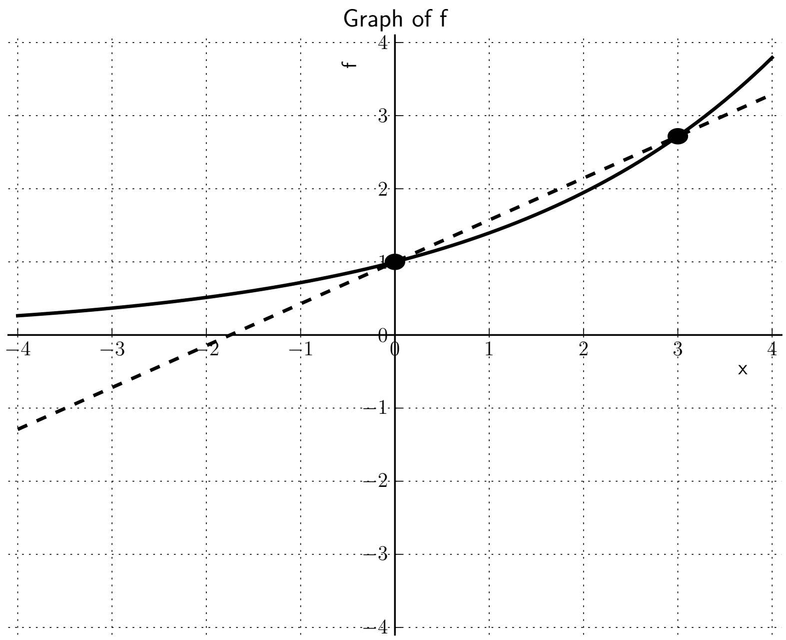
   (d) What does this imply about the expression

   $$\ln\left(x^r\right) = \qquad .$$

1. Briefly state two ideas from today's class.
   - 
   - 

2. The average rate of change for a function from $x = a$ to $x = b$ is defined to be

$$\text{Avg. Rate of Change} \quad = \quad \frac{f(b) - f(a)}{b - a}.$$




Visually, it can be thought of as the slope of the line that goes through the graph of the function at two points, $(a, f(a))$ and $(b, f(b))$.

(a) The population of a colony of bacteria is two times its previous population every one hour. Determine the average rate of change from $a = 0$ to the times given in the table below.

| $b$ | 1 | $\frac{1}{2}$ | $\frac{1}{2^2}$ | $\frac{1}{2^3}$ | $\frac{1}{2^4}$ | $\frac{1}{2^5}$ |
|---|---|---|---|---|---|---|
| Number Bacteria | | | | | | |
| Avg Rate Change | | | | | | |

What number is the average rate of change approaching as $b$ gets close to zero?

(b) The population of a colony of bacteria is three times its previous population every one hour. Determine the average rate of change from $a = 0$ to the times given in the table below.

| $b$ | 1 | $\frac{1}{2}$ | $\frac{1}{2^2}$ | $\frac{1}{2^3}$ | $\frac{1}{2^4}$ | $\frac{1}{2^5}$ |
|---|---|---|---|---|---|---|
| Number Bacteria | | | | | | |
| Avg Rate Change | | | | | | |

What number is the average rate of change approaching as $b$ gets close to zero?

(c) The population of a colony of bacteria is 2.5 its previous population every one hour. Determine the average rate of change from $a = 0$ to the times given in the table below.

| $b$ | 1 | $\frac{1}{2}$ | $\frac{1}{2^2}$ | $\frac{1}{2^3}$ | $\frac{1}{2^4}$ | $\frac{1}{2^5}$ |
|---|---|---|---|---|---|---|
| Number Bacteria | | | | | | |
| Avg Rate Change | | | | | | |

What number is the average rate of change approaching as $b$ gets close to zero?

(d) The population of a colony of bacteria is 2.7 its previous population every one hour. Determine the average rate of change from $a = 0$ to the times given in the table below.

| $b$ | 1 | $\frac{1}{2}$ | $\frac{1}{2^2}$ | $\frac{1}{2^3}$ | $\frac{1}{2^4}$ | $\frac{1}{2^5}$ |
|---|---|---|---|---|---|---|
| Number Bacteria | | | | | | |
| Avg Rate Change | | | | | | |

What number is the average rate of change approaching as $b$ gets close to zero?

(e) Determine a value so that if a population multiplies its population by that number every hour the average rate of change approaches 1 from $a = 0$ and $b$ gets close to zero.

1. A computer virus is constructed that will infect two new computer systems each day and then wipe the disk drive of its current computer clean. The virus is installed on one computer on 1 January.
   (a) How many new computers will it infect on 2 January?

   (b) How many new computers will it infect on 3 January?

   (c) How many new computers will it infect on 4 January?

2. A computer virus is constructed that will infect two new computer systems each day and then wipe the disk drive of its current computer clean. The virus is installed on a computer but it is not clear when it was first installed.
   (a) It is estimated that 32 computers were infected on a given day. How many days beforehand was it installed on the first computer?

   (b) It is estimated that 64 computers were infected on a given day. How many days beforehand was it installed on the first computer?

   (c) It is estimated that 512 computers were infected on a given day. How many days beforehand was it installed on the first computer?

3. Make a sketch of the number line. Mark zero on your number line. Indicate the locations of 1, 1.5, -0.5, and 9.5. Try to make the points consistent with the relative distances between the points. Also, label the number line.

1. The approximate distances in kilometers from the earth to various destinations is given in the table below. Use this information to answer each of the questions below.

| Destination | Distance |
|---|---|
| The moon | $3.84 \times 10^5$ km |
| Mars | $5.46 \times 10^7$ km |
| Saturn | $1.28 \times 10^9$ km |
| Proxima Centauri (Closest star) | $4.01 \times 10^{13}$ km |
| Center of the Milky Way | $2.83 \times 10^{17}$ km |

   (a) Make a sketch of a number line with the distances for each destination marked on the number line. Try to keep the relevant distances between points consistent.

   (b) For each distance in the table above what is the approximate power of ten for the distance? For example, if a distance is $1 \times 10^4$ km it is a power of 4. Sketch a number line and indicate the powers of ten on the number line. Try to keep the relevant distances between points consistent.

The approximate average lengths of various items are given in the table below. Use this information to answer each of the questions below.

| Item | Size |
|---|---|
| Person | 1.75 m |
| Finger | $9.2 \times 10^{-2}$ m |
| DNA | $5.0 \times 10^{-2}$ m |
| Hair (Width) | $9.0 \times 10^{-5}$ m |
| Bacteria | $0.8 \times 10^{-7}$ m |

(a) Make a sketch of a number line with the lengths for each item marked on the number line.

*Try to keep the relevant distances between points consistent.*

(b) For each item in the table above what is the approximate power of ten for the length? For example, if a length is $1 \times 10^{-4}$ m it is a power of -4. Sketch a number line and indicate the powers of ten on the number line.

*Try to keep the relevant distances between points consistent.*

2. In one cycle a type of mayfly lays eggs and on average three females survive to lay more eggs. One of the mayfly are introduced to a stream for the first time and lays eggs.

   (a) Draw a tree diagram of the individuals from each cycle that survive starting with the original mother. Include three cycles in your diagram. The node for the first female mayfly should be at the top center of the space, and then the generations that follow should be below the previous generations in the diagram.

   (b) On the right side of the tree diagram indicate the total number of female mayflies that survive each cycle.

   (c) On the left side of the tree diagram indicate the coresponding cycle with the first cycle labeled as cycle 0 (zero), and the second cycle is 1 (one).

   (d) It is estimated that there are 2187 female mayfly in the river. How many cycles have there been?

   (e) It is estimated that there are 19683 female mayfly in the river. How many cycles have there been?

   (f) If the number of female mayfly, $M$, is given by $M = 3^n$ where $n$ is the number of cycles, solve the equation for $n$. (What is your new equation called in relation to the original function?)

3. Mosquitoes lay between 50 to 200 eggs each cycle. Assume that for a given species in a particular area roughly 34 eggs hatch and survive to be female adults and lay eggs.

   (a) Assume that in the spring there is one surviving female mosquito starting with cycle 0. Make a **table** to indicate how many mosquitoes there will be from cycle 0 to cycle 4.

   (b) Using your table, if it is estimated that there are 39304 female mosquitoes what cycle in the season is it?

   (c) Using your table, if it is estimated that there are 1,336,336 female mosquitoes what cycle in the season is it?

   (d) If the number of female mosquitoes, $f$, is given by $f = 34^n$ where $n$ is the number of cycles, solve the equation for $n$. (What is this equation called in relation to the original function?)

4. Determine the value(s) of $x$ that satisfy the equation

$$\log_5(x) + \log_5(x+1) \;=\; 2.$$

   (a) Raise both sides of the equation to the power of 5.

   (b) Take advantage of the property that $5^{a+b} = 5^a \cdot 5^b$ to rewrite the left hand side as the product of two values.

   (c) Use the property of the inverse to rewrite each of the terms in a simpler form.

   (d) Simplify the right hand side and solve for $x$.

1. Briefly state two ideas from today's class.
   -
   -
2. Determine the value of $x$ in each expression below. Any approximations should be to at least two decimal places.
   (a) $\log_5(2x+1) = 9$
   (b) $\log_7(x-1) + \log_7(x+1) = 2$
   (c) $3^{\log_3(x+1)} = 4$
   (d) $\log_8\left(8^{5-2x}\right) = 9$
   (e) $\log_{10}(2x+1) = \log_3(4.1)$
3. Use the natural logarithm to determine the value of $x$ in each expression below. Any approximations should be to at least two decimal places.
   (a) $3.5^{x+1} = 2^{x-1}$
   (b) $10 = e^{-2x}$
   (c) $1 = 2^{3x} \cdot 4^{8x-1}$
   (d) $5 = e^{3x-1}$
   (e) $e^{9x} = 2^{9x}$

1. Determine an approximation for the numerical value of each number below. Also determine the natural logarithm of each of the following values. Express the numbers to two decimal places.

   (a) $e$

   (b) $e^2$

   (c) $e^3 \cdot e^{-4}$

   (d) $e^{-2} \cdot e^{-3}$

1. Determine the natural logarithm of each value below, and use the properties of logarithms to express the value as a sum or differences of logarithms.

   (a) $a \cdot b \cdot c$

   (b) $a \cdot b^2 \cdot c \cdot d^3$

   (c) $\frac{(x-1)\cdot(x+3)}{(x-2)}$

   (d) $\frac{(x-4)\cdot(x+2)^3\cdot(x+4)}{(x-9)^2}$

2. A computer virus is constructed that will infect two new computer systems each day and then wipe the disk drive of its current computer clean. The virus is installed on one computer on day 0 (zero).

   (a) Determine the formula that gives the number of computers infected on the number of days since the first computer is infected.

   $$\text{Number(t)} \quad =$$

   (b) Make a rough sketch of the number of new computers infected as a function of time.

   *Label your axes and annotate your plot.*

   (c) Solve the equation above for $t$. That is determine a formula for $t$ given the number of newly infected computers.

   (d) It is determined that 524288 new computers were infected on a given day. How long ago was the first virus installed?

   (e) It is determined that 67108864 new computers were infected on a given day. How long ago was the first virus installed?

3. Radon 222 has a half life of 3.8 days. It is estimated that there is 0.2g of radon 222 in a basement. Assume that no more radon enters the basement after a treatment is applied, and the area is not ventilated.

   (a) Determine a formula for the amount of radon in the basement at a given time, in days, from when the treatment is applied.

   $$\text{Amount(t)} \quad =$$

   (b) Make a rough sketch of the amount of Radon 222 in the basement as a function of time.

   *Label your axes and annotate your plot.*

   (c) Solve the original equation for $t$. That is determine a formula for $t$ given the amount of radon in the basement.

   (d) How long will it take before the amount of radon is down to one tenth the original amount?

4. A patient has a growth, and a treatment is applied that is estimated to reduce the mass of the growth by ten percent each week. At the start of the treatment the mass of the growth is estimated to be two grams.

   (a) Determine a formula for the mass of the growth at a given time, in days, from when the treatment is applied.

   $$\text{Size(t)} \quad =$$

*Label your axes and annotate your plot.*

   (b) Make a rough sketch of the mass of the growth as a function of time.

(c) Solve the equation for $t$. That is determine a formula for $t$ given the size of the growth.

(d) How long will it take before the growth is reduced to one tenth of its original mass?

1. Briefly state two ideas from today's class.
   - 
   - 
2. Determine the value of $x$ in each expression below:
   (a) $e^{2x} - 2e^x - 3 = 0.$

   (b) $e^{2x} - 4e^x + 4 = 0.$

   (c) $e^{2x} - e^x + 6 = 0.$

   (d) $e^{2x} - e^x - 12 = 0.$

   (e) $e^{2x} - e^x + 4 = 0.$
3. Complete question 4.
4. A bank offers 1.5% annual interest compounded weekly (assume 52 weeks in a year). How long will it take for the balance to double?
5. A bank offers 1.5% annual interest compounded monthly. How long will it take for the balance to double?
6. A compound is created that decays over time. It takes four years until half of the compound decays in a sample. How long will it take for 80% of the compound to decay?
7. A species of plant produces two hundred seeds each year, but on average only 5% of the seeds germinate and grow in to plants that produce seeds. A survey is done in a large area and it is estimated that the area contains fifty plants. How many plants are expected to be present after 3 years? Determine a formula to estimate the number of plants at any year, $t$, in the future.

Logarithmic functions satisfy the algebraic properties given below. In each example it is assumed that $a$, $b$, and $r$ are constants. Both $a$ and $b$ must be positive numbers.

$$\ln(a \cdot b) = \ln(a) + \ln(b) \tag{2.9}$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b) \tag{2.10}$$

$$\ln\left(a^r\right) = r\ln(a) \tag{2.11}$$

$$\log_{10}(a \cdot b) = \log_{10}(a) + \log_{10}(b) \tag{2.12}$$

$$\log_{10}\left(\frac{a}{b}\right) = \log_{10}(a) - \log_{10}(b) \tag{2.13}$$

$$\log_{10}\left(a^r\right) = r\log_{10}(a) \tag{2.14}$$

1. Solve for the variable $x$ in each expression below. Provide exact answers in each case.

   (a) $\ln(x-1) = 19$

   (b) $e^{3x+1} = 5$

   (c) $\log_4(x+1) = 2$

   (d) $3^{-x+1} = 4$

1. Determine the value of $x$ or $t$ as appropriate in each of the following expressions. Justify the steps that you make. Take one step at a time, and write out the complete step in each case.

   (a) $10 = 20\left(1 - e^{-3t}\right)$

   (b) $\frac{1}{10} = \frac{1}{\sqrt{12\pi}} e^{-x^2/6}$

   (c) $xe^{-x} + e^{-x} = 0$

(d) (Skip this equation and come back if you have time.)
$e^t + 2 - e^{-t} = 0$

(e) (Skip this equation and come back if you have time.)
$e^{-x^2} = 3 \cdot 4^{2x-1}$

2. The goal is to determine the value of $x$ that satisfies the equation

$$\log_4(x) = 3 + \log_8(x).$$

   (a) First focus on the left hand side of the equation.

      (i) Define a new variable, $u$, by setting $u$ equal to the left hand side of the equation.

      (ii) Exponentiate both sides of the equation using a clever choice for the base so that there will not be any logarithms in the equation. Simplify the result.

      (iii) Take the natural logarithm of both sides and solve for $u$.

   (b) Now focus on the right hand side of the equation.

      (i) Define a new variable, $v$, by setting $v$ equal to the logarithm in the right hand side of the equation.

      (ii) Exponentiate both sides of the equation using a clever choice for the base so that there will not be any logarithms in the equation. Simplify the result.

      (iii) Take the natural logarithm of both sides and solve for $v$.

(c) Substitute your value for $u$ into the left hand side and the value for $v$ into the right hand side.

(d) Solve the new equation for $x$.

3. A population of bacteria doubles every four hours. Determine a function that gives the number of individuals in the population for a given time.

   (a) Assume that the population is given by $P(t)$. Express the given information in function form.

   (b) The assumption is that the population is modeled as an exponential function. Determine the general form of the function. (Why would an exponential function be used in this instance?)

   (c) Substitute your previous expression into the first expression above. You should get a new equation using your general exponential formula.

   (d) Simplify the expression. Which variable can you solve for? Identify the variable and solve for it.

   (e) What is the general form for the function that models the population?

4. Carbon 14 has a half life of 5,730 years. How long will it take for a sample to decay to two thirds of its original value?

   (a) Express the given information in function form. (Assume the amount is given by $C(t)$.)

   (b) The assumption is that the amount is modeled as an exponential function. Determine the general form of the function. (Why would an exponential function be used in this instance?)

   (c) Substitute your previous expression into the first expression above. You should get a new equation using your general exponential formula.

   (d) Simplify the expression. Which variable can you solve for? Identify the variable and solve for it.

   (e) What is the general form for the function that models the amount of material?

1. Briefly state two ideas from today's class.
   - 
   - 
2. Determine the value of $x$ in each expression below.
   (a) $3^x = 4^{x+1}$

   (b) $3^x = 9 \cdot 4^{x+1}$

   (c) $2^x = 9^{5x-1} \cdot 4^{x+1}$

   (d) $14^{3x+4} = 10^{9x/2} \cdot 20^{6x-1}$

   (e) $3^{x^2} = 3 \cdot 8^x$

1. Solve for the variable $x$ in each expression below. Take one step at a time, and write out every step as a separate expression.

   (a) $e^{4x-1} = 8$

   (b) $\ln(2x+1) = -4$

   (c) $3^{2x+1} = e^{4x}$

   (d) $8^{3x+2} = 6e^{4x}$

1. The number of animals in a population of a small mammals follows a logistic function,

$$P(t) = \frac{10,000}{1 + e^{-\frac{1}{2}t}},$$

where $t$ is the number of years since the initial observation.

(a) How many animals are in the population initially?

(b) How many animals will the population approach after a very long time?

(c) How long will it take for the number of animals to reach 75% of its long term value?

(d) How long will it take for the number of animals to reach 80% of its long term value?

2. A spill occurs and a chemical is introduced into a lake. The amount of chemical decays over time.

   (a) In this case an exponential function should be used. Why?

   (b) Should the rate, $r$, used in the exponential function be positive or negative? (Briefly explain how you arrive at your conclusion.)

   (c) Write the general form for the equation for the amount of the chemical at any time given in months.

   (d) At some time after the spill occurs it is estimated that there are 4,000 kg of the chemical in the lake. A month later it is estimated that there is 3,500 kg in the lake. What is the half life of the chemical?

      (i) Write out the equations that result from the given information.

      (ii) Use the two equations to solve for the value of the decay rate.

      (iii) Determine the half life of the chemical.

3. A population of bacteria doubles every four hours. How long does it take for the population to triple?

   (a) Is this decay or growth? Make a (very) rough sketch of the graph of the function you expect to get.

   *Label the axes and annotate your plot.*

   (b) Write out the information that is given to you.

   (c) Write out a general form of a function that will model this situation.

   (d) Use the given information and the function above to construct a system of equations.

(e) Solve the system of equations for the constants necessary to solve the problem.

(f) How long will it take for the population to triple?

4. Carbon 14 has a half life of 5,730 years. How long will it take for a sample to decay to two thirds of its original value?

*Use the same steps used in the previous question.*

1. Briefly state two ideas from today's class.
   -
   -
2. The number of animals in a population is approximated using a logistic function,

$$P(t) = \frac{80,000}{1 + 10e^{-\frac{1}{10}t}}.$$

   (a) Make a sketch of the function for $t$ going from 0 to 50.

   (b) How long will it take for the population to double with respect to its original population?

   (c) How many animals does the population approach in the long run? (i.e. what happens to the population after a very long time?)

# Chapter 3

# Angle Measurement

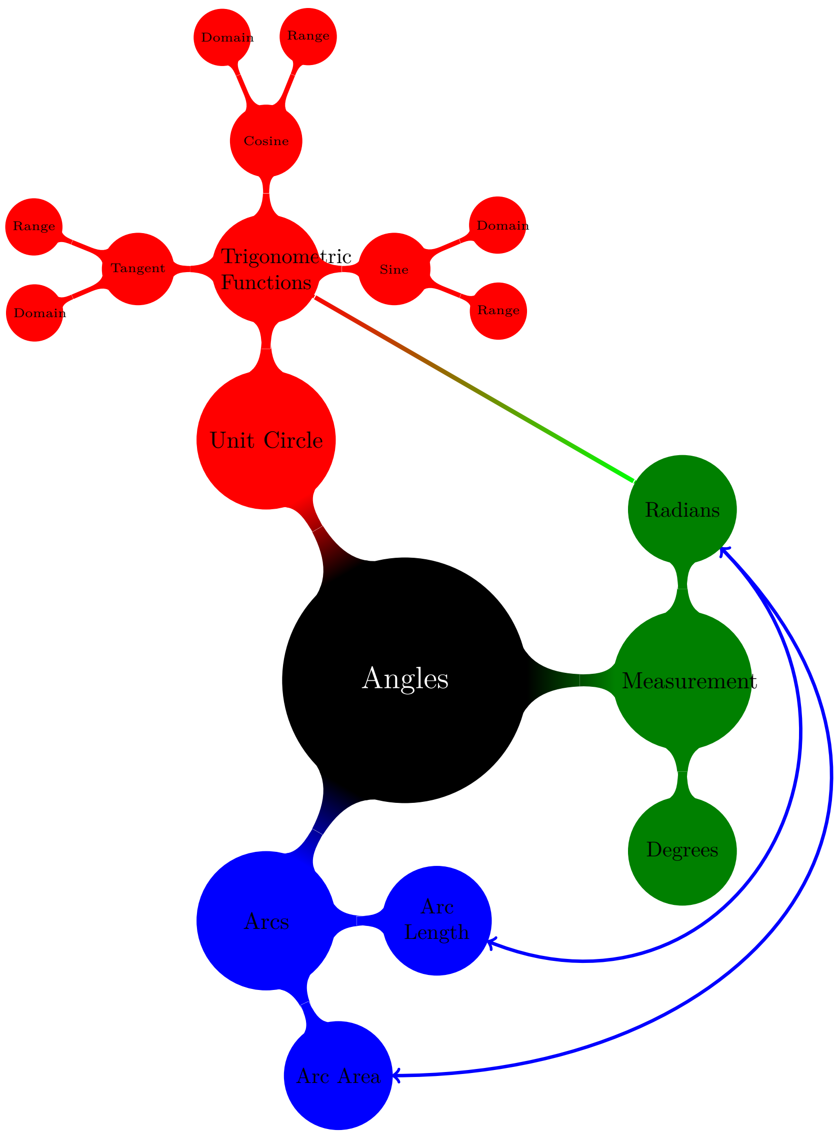
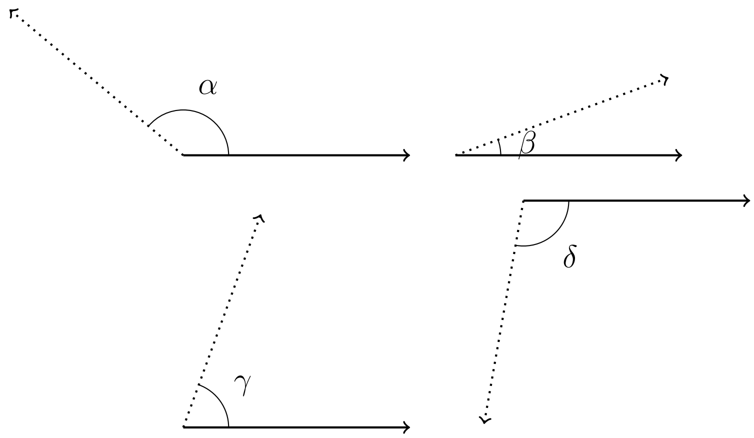



Figure 3.1: Topics for the section on angles.

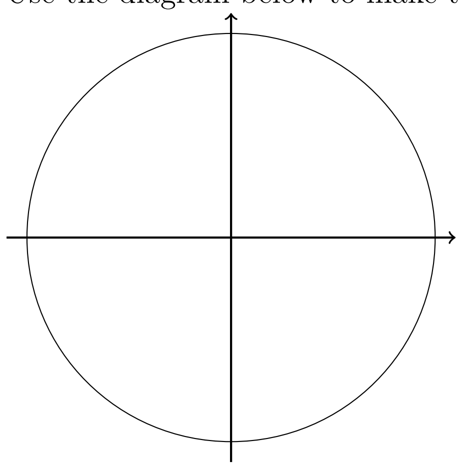
1. A circle has a radius of $r$ meters. Answer each of the following questions:
   (a) Determine the arc lengths described below.
      (i) What is the circumference of the whole circle?
      (ii) What is the arc length around half of the circle?
      (iii) What is the arc length around one-fourth of the circle?
   (b) Determine the areas of the circle described below.
      (i) What is the area of the whole circle?
      (ii) What is the area of half of the circle?
      (iii) What is the area of one-fourth of the circle?
   (c) How many degrees are there in a full circle? Where did this number come from?
   (d) Make a sketch of a circle centered at the origin. Include the $x$ and $y$-axes. Add the ray from the origin that forms a 45 degree angle with the positive $x$ axis.

*Label the axes and annotate your plot.*

1. For each of the angles below, print the angles in order from smallest to largest. In each picture the solid line is the initial side, and the dotted line is the terminal side.




2. Use the diagram below to make the points indicated in the descriptions below.



   (a) Mark and label a point, $P$, where the ray from the origin to the point form an angle of $\pi$ radians.

   (b) Mark and label a point, $Q$, where the ray from the origin to the point form an angle of $2\pi$ radians.

   (c) Mark and label a point, $R$, where the ray from the origin to the point form an angle of $\frac{\pi}{2}$ radians.

   (d) Mark and label a point, $S$, where the ray from the origin to the point form an angle of $\frac{\pi}{4}$ radians.

   (e) Mark and label a point, $T$, where the ray from the origin to the point form an angle of 0 radians.

   (f) Mark and label a point, $U$, where the ray from the origin to the point form an angle of $\frac{3\pi}{4}$ radians.

3. Make a sketch of a circle of radius $r$, and mark the radius and circumference of the circle.

   (a) What is the general relationship between the radius and the circumference?

   (b) Mark a sector on your circle above whose angle is one half of the angle needed to make one complete turn around the circle. What is the length of the sector of the circle?

   *This should be a function of $r$.*

   (c) Mark a sector on your circle above whose angle is one third of the angle needed to make one complete turn around the circle. What is the length of the sector?

   *This should be a function of $r$.*

   (d) If the angle of a sector is a fraction, $p$, of one whole turn around the circle, what is the length of the sector? (If $p = 0.5$ then it represents one half of a full turn around the circle.)

   *This should be a function of $r$ and $p$.*

4. From the previous problem you should have a general formula that relates the length of the sector with radius $r$ given the fraction, $p$, that its angle is of one complete turn around a circle.

   (a) Rewrite your expression, and label the distance along the sector as $s$.

   (b) Divide both sides of your formula by the radius, and you should have an expression for

$$\frac{s}{r} \quad =$$

   (c) The value on the right side of your expression is the definition of radian measure for an angle. In one sentence explain the meaning of the value on the ride side of the expression.

5. A hare is placed on a track that is a circle with radius 10m. The hare moves around the track in the counter-clockwise direction.
   (a) Make a sketch of the track below.

   (b) Determine the angle that the hare moves around after moving 1m.

   *All angle measures should be in radians.*

   (c) Determine the angle that the hare moves around after moving 10m.

   (d) Mark and annotate the sector formed after the hare moves 10m. What is the angle of the sector, and what is the area of the sector?

   (e) Mark and annotate the sector formed after the hare moves 2m. What is the angle of the sector, and what is the area of the sector?

   (f) Mark and annotate the sector formed after the hare moves 15m. What is the angle of the sector, and what is the area of the sector?

6. A turtle and a hare are placed at the same start point, and they move around a circle of radius 10m. The hare moves around the circle counter-clockwise at 1m per minute. The turtle moves around the circle counter-clockwise at 0.1m per minute.
   (a) After one hour where is the hare on the circle?

   (b) After one hour what angle has the hare moved around the circle?

   (c) After one hour where is the turtle on the circle?

   (d) After one hour what angle has the turtle moved around the circle?

   (e) How long will it take until the hare and the turtle are at the start point at the same time?

1. Briefly state two ideas from today's class.
    - 
    - 
2. The following questions refer to the measure of an angle in radians.
    (a) How many radians are there in one complete turn around a circle?

    (b) How many radians are there in one half of one complete turn around a circle?

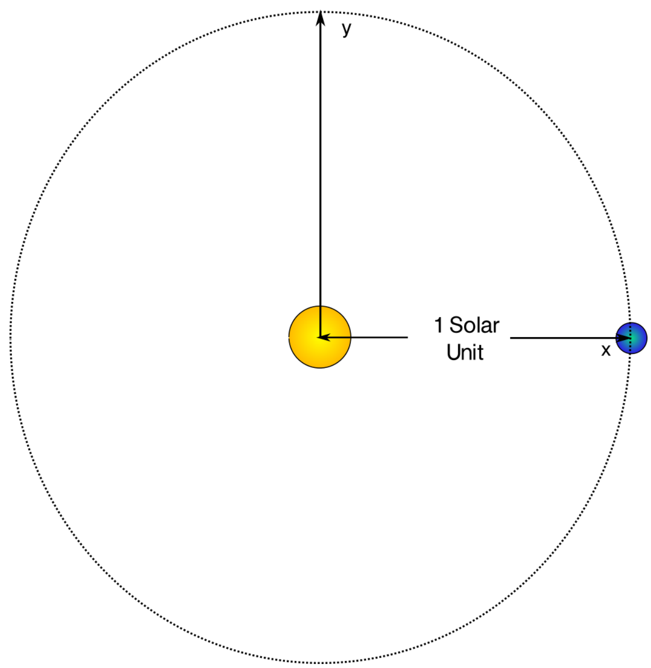
    (c) How many radians are there in one fourth of one complete turn around a circle?

    (d) How many radians are there in one third of one complete turn around a circle?

    (e) If an angle is measured as being 45 degrees, how many radians is it?

    (f) If an angle is measured as being 120 degrees, how many radians is it?

1. When viewed above the north pole of the Sun, the earth appears to move around the sun in a counter-clockwise direction. The path can be roughly approximated as a circle, and its coordinate at any time is $(x, y)$. It takes one year to make one revolution, and assume that the distance from the center of the sun to the earth is one solar unit.




(a) What distance does the earth traverse in one year?

(b) What distance does the earth traverse in two years?

(c) What distance does the earth traverse in ten years?

(d) What is the largest value that the $x$-coordinate can be? What is the smallest value that the $x$-coordinate can be? What is the largest value that the $y$-coordinate can be? What is the smallest value that the $y$-coordinate can be?

1. An angle is in the second quadrant, and the sine of the angle is 0.2. Determine the cosine and tangent of the angle.

   (a) Make a sketch of the unit circle and place an angle somewhere in the second quadrant. Mark the point on the edge of the circle, $(x, y)$.

   (b) Will the cosine be positive or negative? (Explain how you came to that conclusion.) Do the same for the tangent.

   (c) Determine the cosine of the angle using the Pythagorean relationship between the sine and the cosine.

   (d) Determine the tangent of the angle using the definition of the tangent.

2. A turtle moves around the edge of a circle of radius 1 meter.

   (a) After the turtle moves a distance of 2m what is its position? What quadrant is the point in?

*Make a sketch of the unit circle first and estimate its location.*

   (b) After the turtle moves a distance of 4m what is its position? What quadrant is the point in?

*Make a sketch of the unit circle first and estimate its location.*

   (c) After the turtle moves a distance of 6m what is its position? What quadrant is the point in?

*Make a sketch of the unit circle first and estimate its location.*

3. A turtle moves around the edge of a circle of radius 1 meter.
   (a) After it moves a distance of $\frac{\pi}{2}$ meters where is it on the circle?

   *Make a rough sketch of the circle and indicate its position.*

   (b) After it moves a distance of $\pi$ meters where is it on the circle?

   (c) After it moves a distance of $\frac{3\pi}{2}$ meters where is it on the circle?

   *Make a rough sketch of the circle and indicate its position.*

   (d) After it moves a distance of $2\pi$ meters where is it on the circle?

   (e) After it moves a distance of $5\pi$ meters where is it on the circle?

   *Make a rough sketch of the circle and indicate its position.*

4. A turtle moves around the edge of a circle of radius 1 meter.

*Make a rough sketch of the circle and indicate its position.*

   (a) After it moves a distance of $\frac{\pi}{4}$ meters where is it on the circle?

   (b) As the turtle moves from a distance of 0 meters to $\frac{\pi}{2}$ meters what is happening to the value of its $x$ coordinate. What about its $y$ coordinate?

   (c) As the turtle moves from a distance of $\frac{\pi}{2}$ meters to $\pi$ meters what is happening to the value of its $x$ coordinate. What about its $y$ coordinate?

   (d) As the turtle moves from a distance of $\pi$ meters to $\frac{3\pi}{2}$ meters what is happening to the value of its $x$ coordinate. What about its $y$ coordinate?

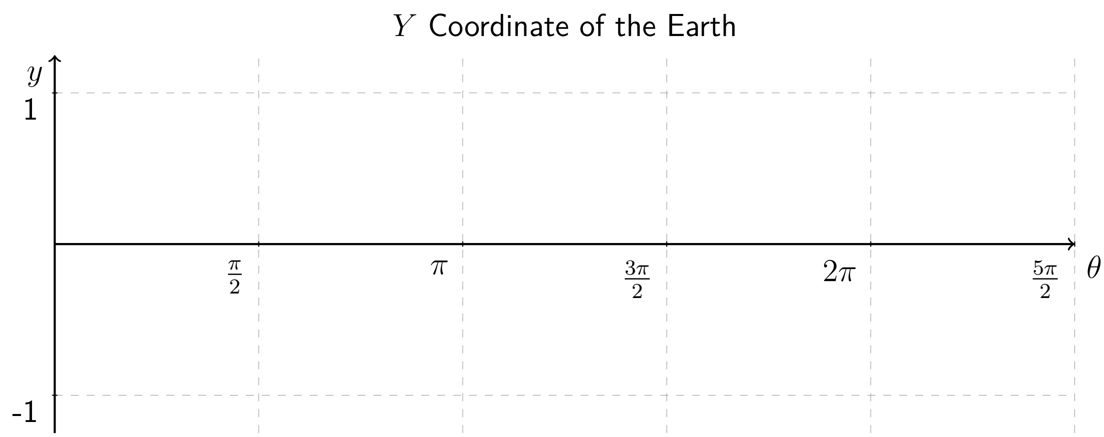
   (e) As the turtle moves from a distance of $\frac{3\pi}{2}$ meters to $2\pi$ meters what is happening to the value of its $x$ coordinate. What about its $y$ coordinate?

5. When viewed above the north pole of the Sun, the earth appears to move around the sun in a counter-clockwise direction. The path can be roughly approximated as a circle. It takes one year to make one revolution, and assume that the distance from the center of the sun to the earth is one solar unit. (See the image in the preclass activity.)

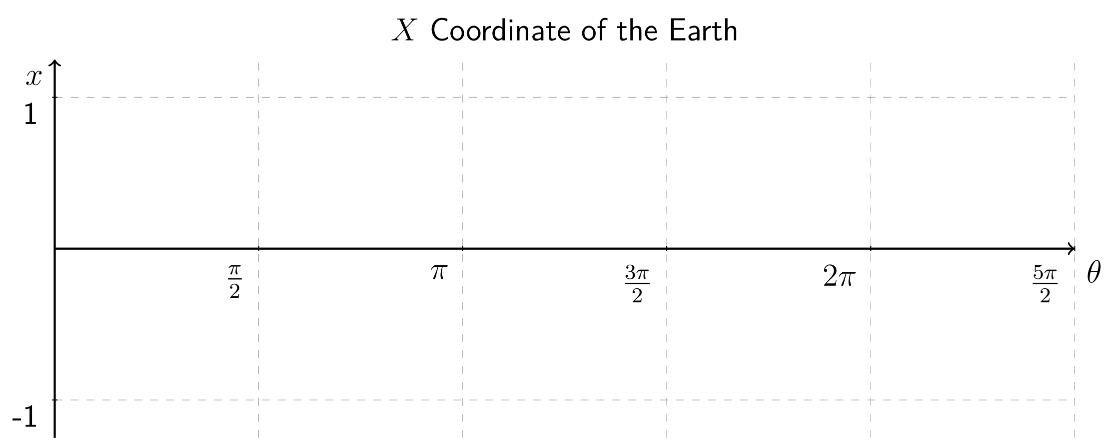
   (a) What angle (in radians) does the earth make from the $x$-axis after 3 months?

   (b) What angle (in radians) does the earth make from the $x$-axis after 6 months?

   (c) Make a rough sketch of the earth's $y$ position as a function of the angle.




   (d) Make a rough sketch of the earth's $x$ position as a function of the angle.



6. At the annual Plainfield $500\pi$ race a tractor will make 250 laps around a circular track. The track has a radius of 1 km. A single tractor will make a trial run by going around the track at 1 km per hour. (It is not a very fast race.) The car starts on the point furthest East and is initially moving to the North.

   (a) How far will the tractor travel in all? Determine the distance traveled as a function of time, and then determine the angle, $\theta$, at any time.

   (b) What are the possible values of the angle, $\theta$?

   (c) Assuming that the origin is the center of the track, sketch a plot of the tractor's $y$ position as a function of time for the first two laps.

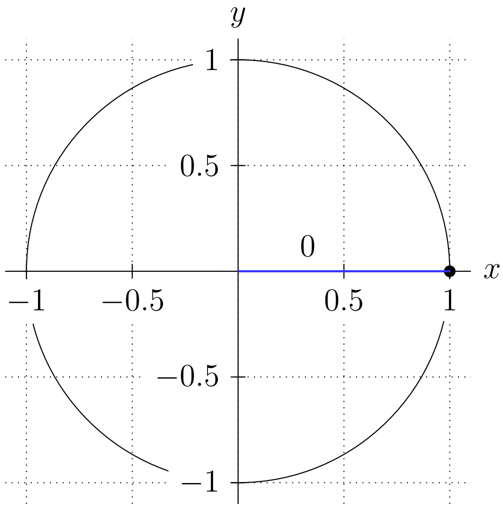
   *Label the axes and annotate the intercepts.*

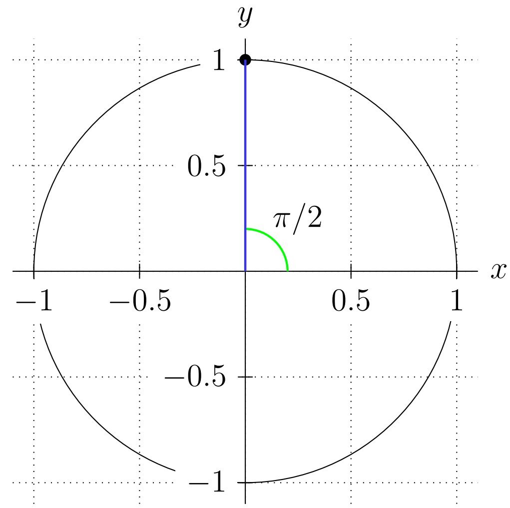
   (d) Assuming that the origin is the center of the track, sketch a plot of the tractor's $x$ position as a function of time for the first two laps.

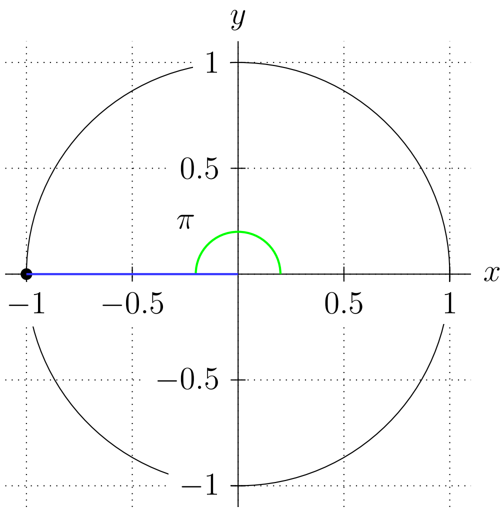
   *Label the axes and annotate the intercepts.*

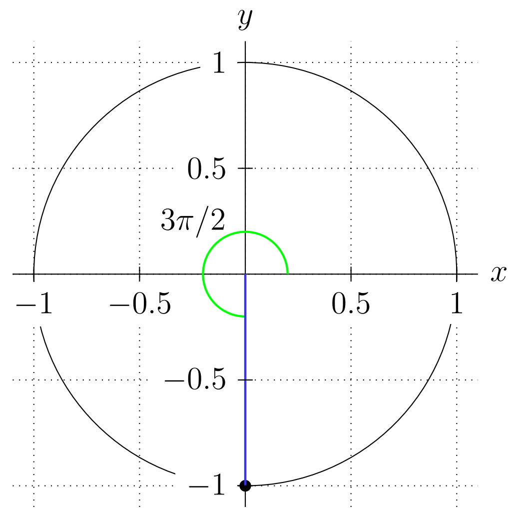
1. Briefly state two ideas from today's class.
   -
   -
2. Make a sketch of the sine function. Annotate the graph to show the amplitude and period of the function.
3. Make a sketch of the cosine function. Annotate the graph to show the amplitude and period of the function.
4. Make a sketch of the function $f(x) = \sin(\pi x)$. Annotate the graph to show the amplitude and period of the function.
5. Make a sketch of the function $f(x) = \cos(\pi x)$. Annotate the graph to show the amplitude and period of the function.
6. Make a sketch of the function $f(x) = \sin(x) + 2$. Annotate the graph to show the amplitude and period of the function.
7. Make a sketch of the function $f(x) = \cos(x) + 2$. Annotate the graph to show the amplitude and period of the function.
8. Make a sketch of the function $f(x) = 3\sin(x) + 1$. Annotate the graph to show the amplitude and period of the function.
9. Make a sketch of the function $f(x) = 3\cos(x) + 1$. Annotate the graph to show the amplitude and period of the function.

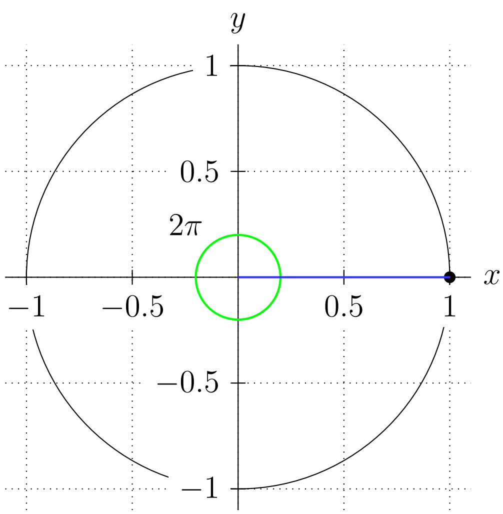
Angles that are a multiple of $\frac{\pi}{2}$ are aligned with the $x$ and $y$ axis. For each plot below determine and label the sine and cosine of the angle based on the $x$ and $y$ position of the point on the unit circle.



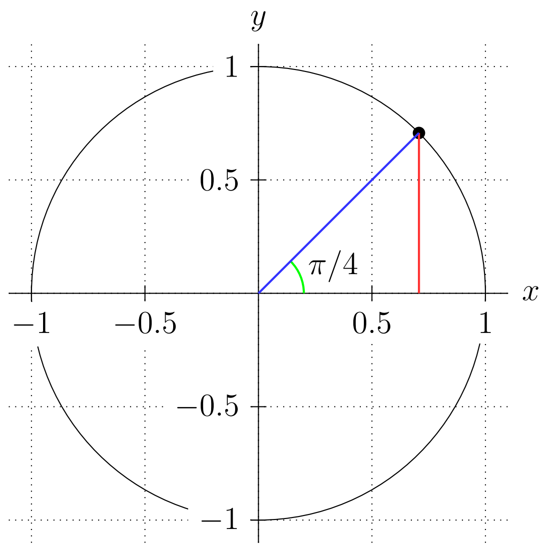



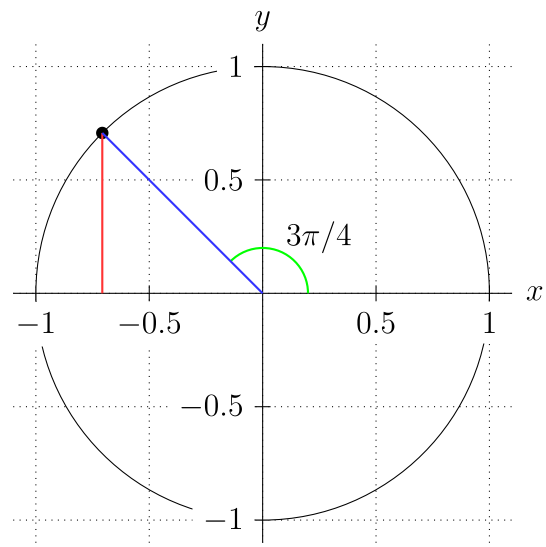



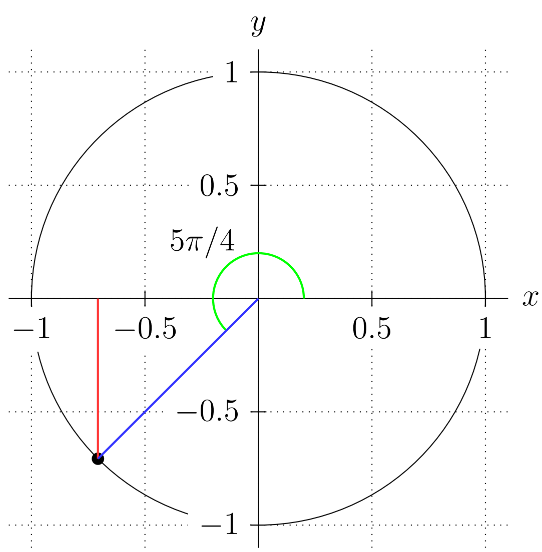



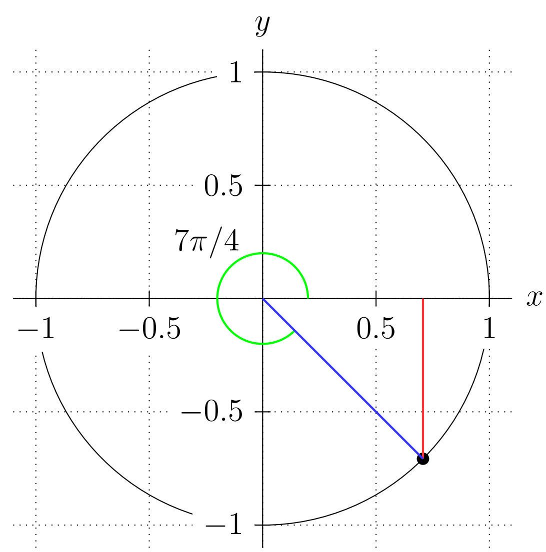



Angles whose reference angles are $\frac{\pi}{4}$ are aligned with the diagonal lines from the origin. In each plot below label and define the reference angle. Also, determine and label the sine and cosine of the angle based on the $x$ and $y$ position of the point on the unit circle.

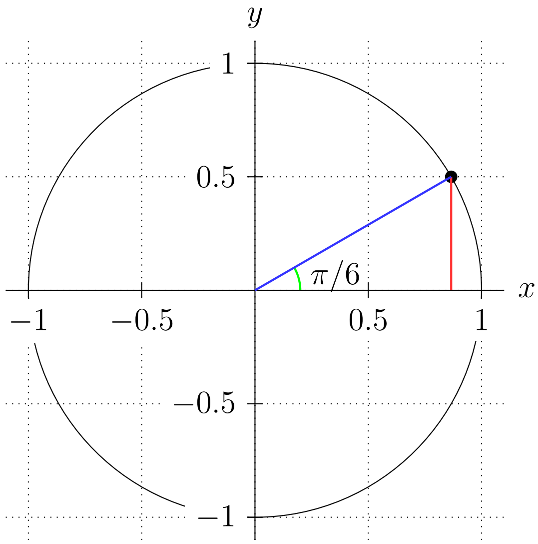


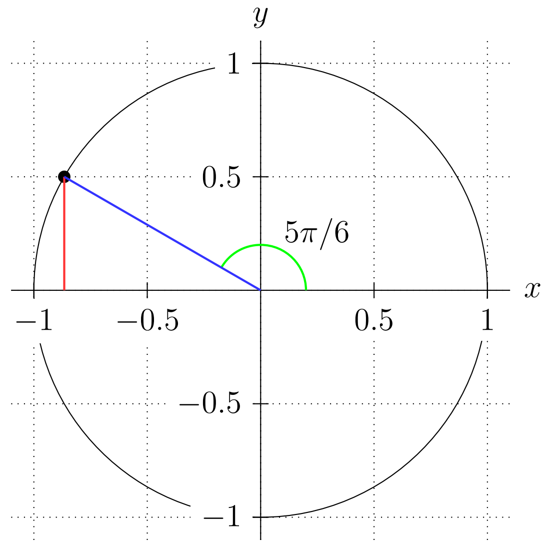



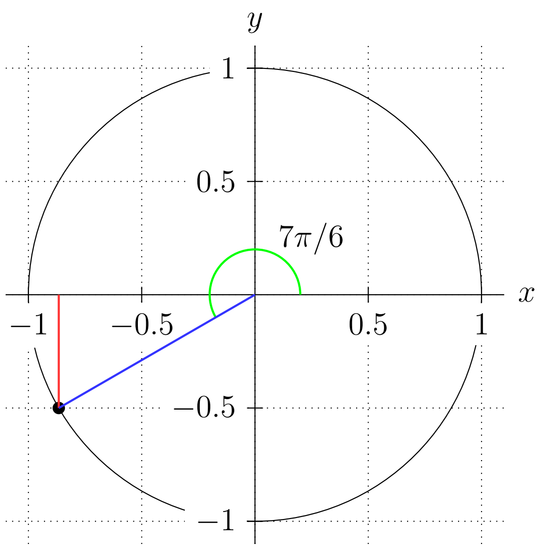



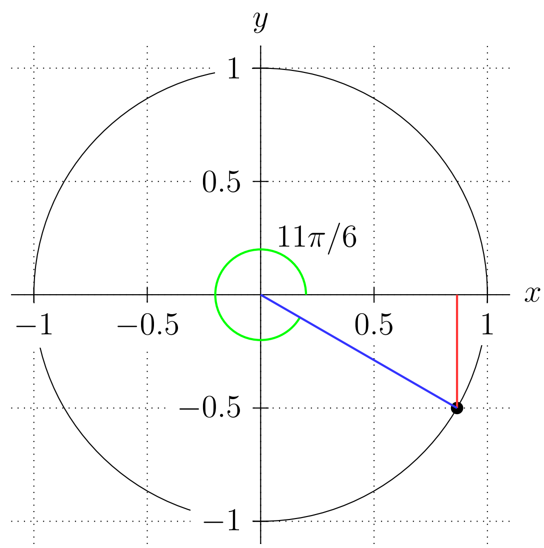



Angles whose reference angles are $\frac{\pi}{6}$ have $y$ values that are $\pm\frac{1}{2}$. In each plot below label and define the reference angle. Also, determine and label the sine and cosine of the angle based on the $x$ and $y$ position of the point on the unit circle.

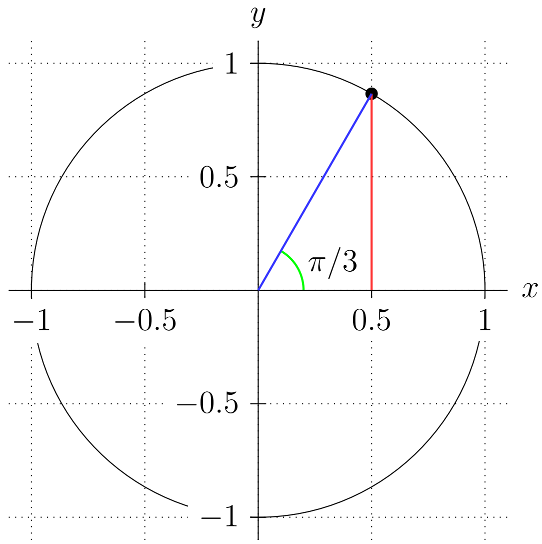


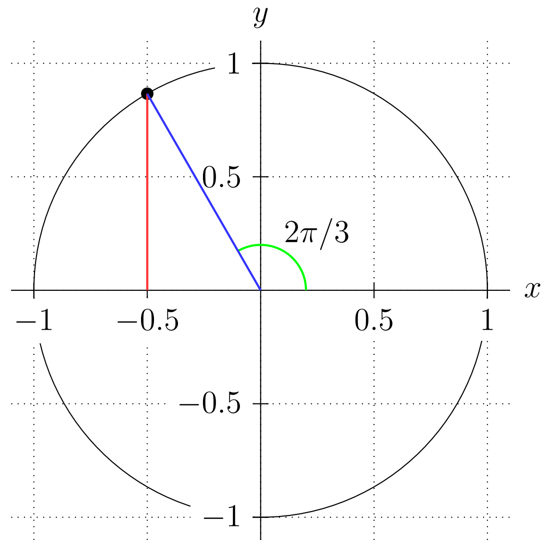



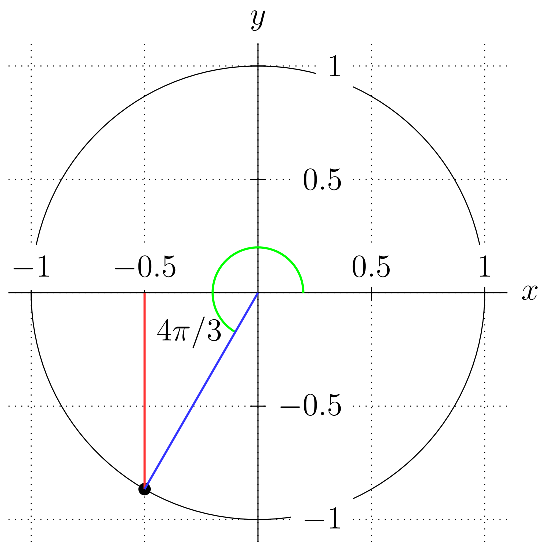



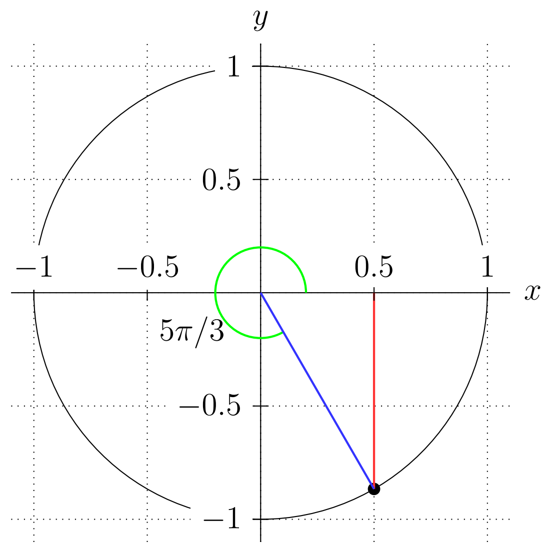



Angles whose reference angles are $\frac{\pi}{3}$ have $x$ values that are $\pm\frac{1}{2}$. In each plot below label and define the reference angle. Also, determine and label the sine and cosine of the angle based on the $x$ and $y$ position of the point on the unit circle.



# Chapter 4

# Trigonometric Functions

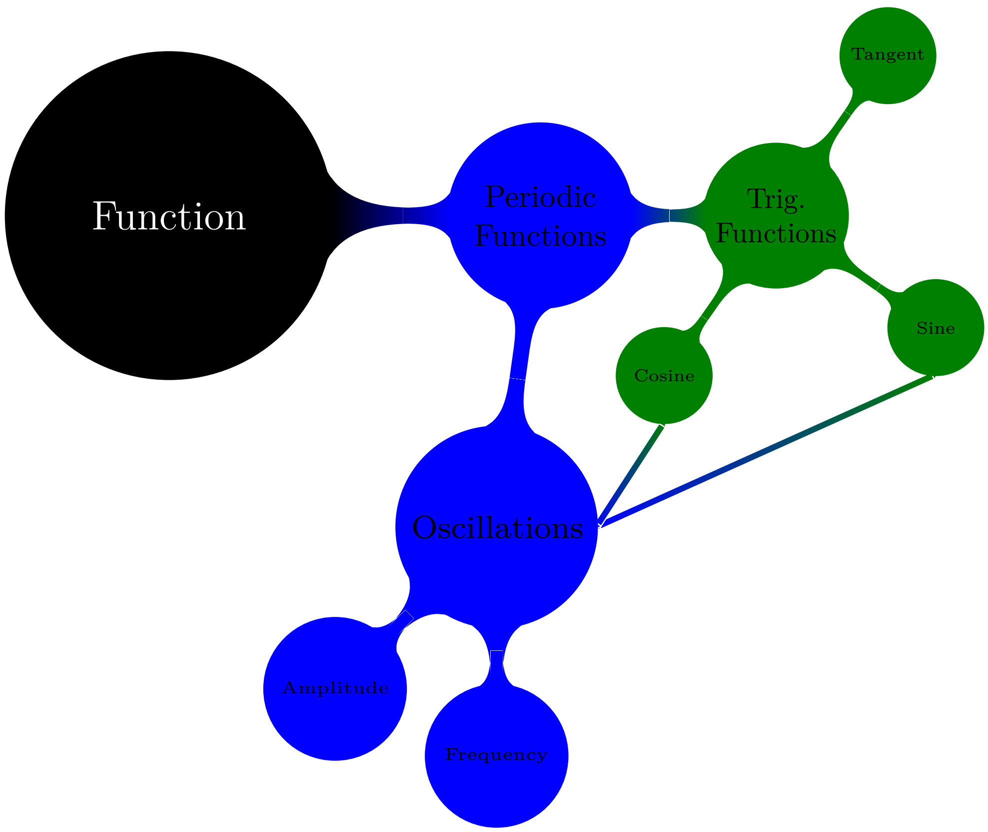
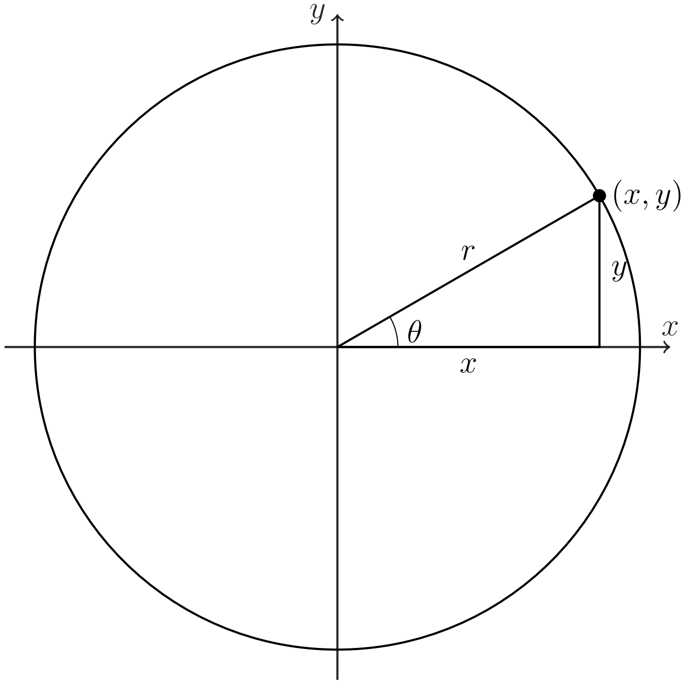



Figure 4.1: Topics for the section on trigonometric functions.

1. A circle of radius $r$ is centered at the origin, and a ray originates from the origin at a given angle, $\theta$. The ray passes through the circle at the coordinate $(x, y)$. A function of $\theta$ can be defined in terms of the coordinate.




   (a) A function, sine, is defined to be

   $$\sin(\theta) = \frac{y}{r}.$$

   Determine the formula for the value of $y$ given $r$ and $\theta$ in terms of the sine function.

   *Solve the equation above for $y$.*

   (b) A function, cosine, is defined to be

   $$\cos(\theta) = \frac{x}{r}.$$

   Determine the formula for the value of $x$ given $r$ and $\theta$ in terms of the cosine function.

   *Solve the equation above for $x$.*

1. Answer each of the following questions where the given point is $P(2,4)$.
   (a) Make a sketch of the coordinate plane and include the point $P(2,4)$. Draw the ray from the origin to the point.

   *Label your axes and annotate your plot.*

   (b) Add a circle to your sketch whose center is the origin and goes through the point. Label the angle $\theta$ as the angle between the ray and the $x$-axis.

   (c) What is the radius of the circle? (Add a label to your plot for the radius.)

   (d) Determine the values of the sine and cosine for the angle.

$$\sin(\theta) =$$

$$\cos(\theta) =$$

2. Different points are given below. For each point assume it is on the edge of a circle whose center is at the origin. Determine the radius of the circle as well as the value of the sine and cosine of the angle associated with each point on its circle.

   (a) $P(1, 0)$

   (b) $P(0, 1)$

   (c) $P(-1, 0)$

   (d) $P(0, -1)$

   (e) $P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

3. In each item below some information is given about an angle. Use the information to determine the sine, cosine, and tangent of the angle. In each case determine if the sine, cosine, and tangent will increase or decrease if the angle is increased by a small amount.

   (a) The angle is in the first quadrant and $\cos(\theta) = 0.3$.

   (b) The angle is in the third quadrant and $\sin(\theta) = -0.25$.

   (c) The angle is in the second quadrant and $\cos(\theta) = -0.8$.

4. An airplane is observed, and it is flying directly away from an observer in level flight. The observer measures the angle of elevation for the plane at two different times. You wish to determine the height of the plane.

   (a) The plane is moving away from the observer. Will the angle increase or decrease?

   (b) At the first time the angle of elevation is $20°$. At the second time the angle of elevation is $13°$. It is estimated that the plane flew a distance of 3 miles between observations. Draw a rough sketch of the situation.

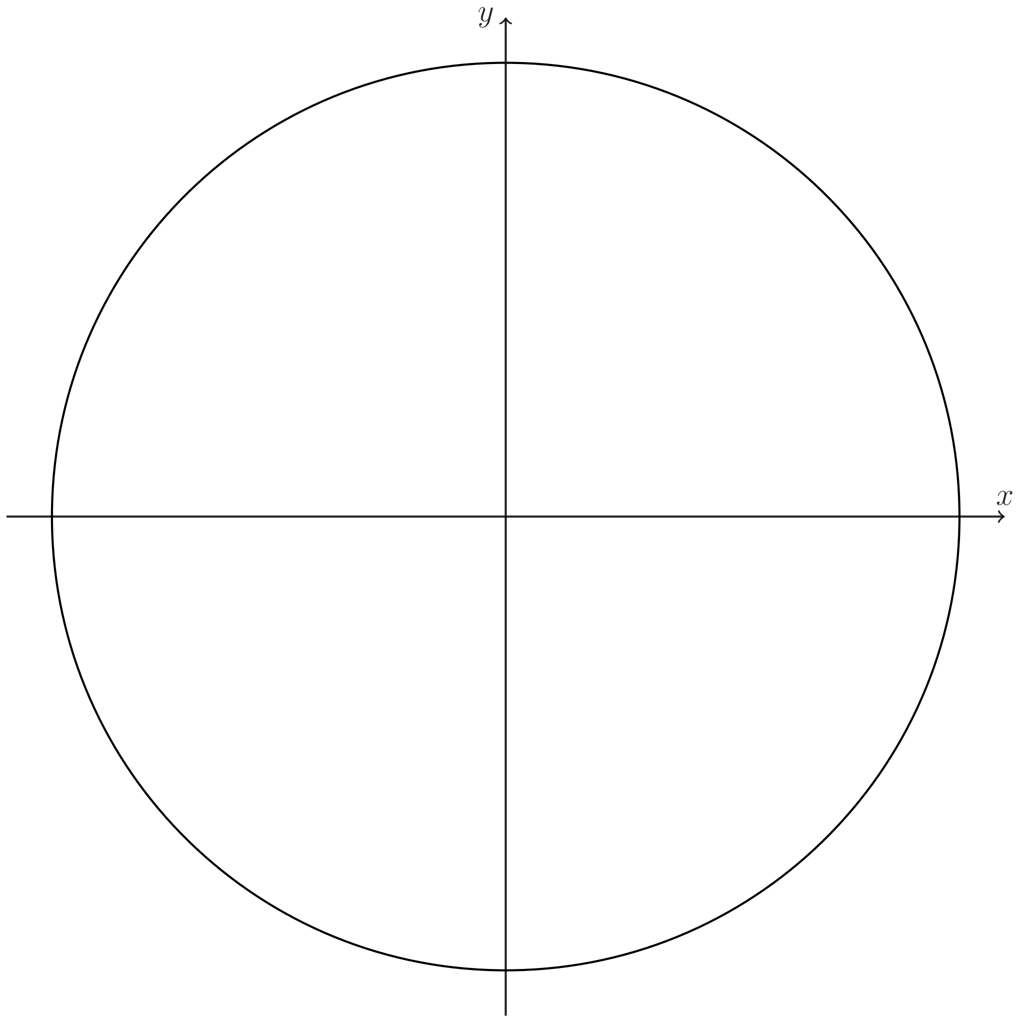
*You should have two triangles.*

   (c) Determine the number of unknowns and determine how many equations you need.

   (d) Determine a relevant formula each of the two triangles plus any other equations you may need.

   (e) Solve the system of equations to determine the height of the plane.

1. Briefly state two ideas from today's class.
   - 
   - 
2. The circle below is centered at the origin and has a radius of one.




   (a) Mark the locations on the circle whose associated angles are 0, $\pi/4$, $\pi/2$, $3\pi/4$, $\pi$, $5\pi/4$, $3\pi/2$, and $7\pi/4$. Determine the coordinates for the points. (Label the points and annotate your plot.)

   (b) Determine the $(x, y)$ coordinates for each angle.

   (c) Determine the cosine and sine of each angle.

3. An airplane is observed, and it is flying directly away from an observer in level flight. The observer measures the angle of elevation for the plane at three different times. You wish to determine the height of the plane.

   (a) The plane is moving away from the observer. Will the angle increase or decrease?

(b) At the first time the angle of elevation is $20°$. At the second time the angle of elevation is $13°$. At the third time the angle of elevation is $10°$. It is estimated that the plane is flying at a constant speed. The time between each measurement is 20 seconds, and the distance the plane flies between each measurement is $20 \cdot v$ where $v$ is measured in meters per second. Draw a rough sketch of the situation.

*You should have three triangles.*

(c) Determine the number of unknowns and determine how many equations you need. other equations you may need.

(d) Solve the system of equations to determine the height of the plane.

1. Determine the sine and cosine of the angle $\frac{\pi}{4}$.

2. Determine the sine and cosine of the angle $\frac{3\pi}{4}$.

3. Determine the sine and cosine of the angle $\frac{5\pi}{4}$.

4. Determine the sine and cosine of the angle $\frac{7\pi}{4}$.

5. Make a sketch of the unit circle and include the points on the unit circle whose angle coincides with the angles above. What is the relationship between the $x$ and $y$ values of the points in relationship to one another?

1. Make a sketch of the unit circle. Indicate the point whose angle coincides with the angle $\theta = \frac{11\pi}{6}$.

   (a) Determine the reference angle for $\theta$.

   (b) According to some website, $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$. Assuming this is correct determine the sine of the original angle, $\theta$.

   (c) Determine the cosine of the original angle, $\theta$.

*Do not just use your caclulator! Use reference angles and justify your answer.*

2. Make a sketch of the unit circle. Indicate the point whose angle coincides with the angle $\theta = \frac{13\pi}{12}$.
   (a) Determine the reference angle for $\theta$.

   (b) According to some website, $\cos\left(\dfrac{\pi}{12}\right) = \dfrac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$. Assuming this is correct determine the cosine of the original angle, $\theta$.

   (c) Determine the sine of the original angle, $\theta$.

*Do not just use your caclulator! Use reference angles and justify your answer.*

3. The angle $\theta$ is in the second quadrant.

   (a) How do you calculate the reference angle given $\theta$?

   *Make a sketch of the unit circle and label the angles.*

   (b) What happens to the sine of the angle if its reference angle is increased and the angle remains in the second quadrant?

   (c) What happens to the cosine of the angle if its reference angle is increased and the angle remains in the second quadrant?

*Make a sketch of the unit circle and label the angles.*

4. The angle $\theta$ is in the third quadrant.

   (a) How do you calculate the reference angle given $\theta$?

   (b) What happens to the sine of the angle if its reference angle is increased and the angle remains in the third quadrant?

   (c) What happens to the cosine of the angle if its reference angle is increased and the angle remains in the third quadrant?

1. Briefly state two ideas from today's class.
   -
   -
2. (a)

1. An ant starts at the coordinate $P(1, 0)$, and it moves counter-clockwise around a circle of radius one centered at the origin. It moves at a constant 1 meter per minute.

   (a) Sketch a plot of the ant's path.

   *Label your axes and annotate your plot.*

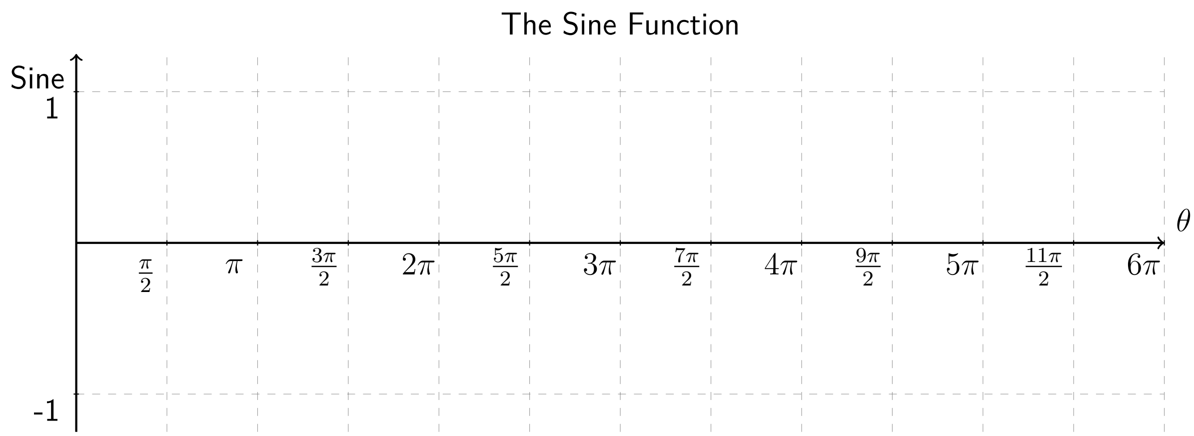
   (b) Sketch a plot of the ant's $x$-coordinate as a function of time.

   *Label your axes and annotate your plot.*

   (c) Sketch a plot of the ant's $y$-coordinate as a function of time.

   *Label your axes and annotate your plot.*

1. (a) Make a sketch of the sine function.



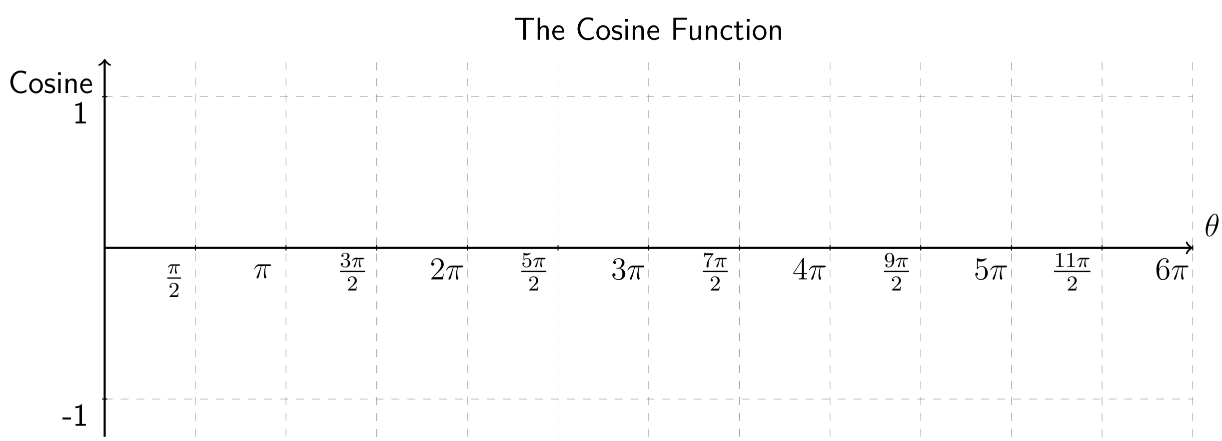

(b) After what values of $\theta$ does the sine function start to repeat itself?

(c) For what value of $a$ is

$$\sin(\theta) \quad = \quad \sin(\theta - a)?$$

(d) Is the sine function an invertible function?

(e) Make a sketch of the cosine function.




(f) For what values of $\theta$ does the cosine function start to repeat itself?

(g) For what value of $a$ is

$$\cos(\theta) \quad = \quad \cos(\theta - a)?$$

(h) Is the cosine function an invertible function?

2. What is the relationship between the sine and cosine functions? For each relationship below determine the value and use the unit circle to explain why these relationships should be expected.

   (a) Determine a value of $a$ where

   $$\cos(\theta) = \sin(\theta + a).$$

   (b) Determine a value of $a$ where

   $$\cos(\theta) = -\sin(\theta + a).$$

3. Determine values of $a$, and $b$ so that the function

$$f(\theta) = a\sin(\theta) + b$$

oscillates between 2 and 6.

4. Determine values of $a$, and $b$ so that the function

$$g(\theta) = a\cos(\theta) + b$$

oscillates between -5 and -3.

5. Determine values of $m$, and $c$ so that the function

$$f(\theta) \quad = \quad a\sin(mt + c) + b$$

has a minimum when $t = 1$ and has a period of 3.

6. Determine values of $m$, and $c$ so that the function

$$g(\theta) \quad = \quad a\cos(mt + c) + b$$

has a minimum when $t = 3$ and has a period of 2.

7. Determine values of $a$, $b$, $m$, and $c$ so that the function

$$f(\theta) = a\sin(\theta) + b$$

oscillates between 2 and 6 and has a minimum when $t = 1$ and has a period of 3.

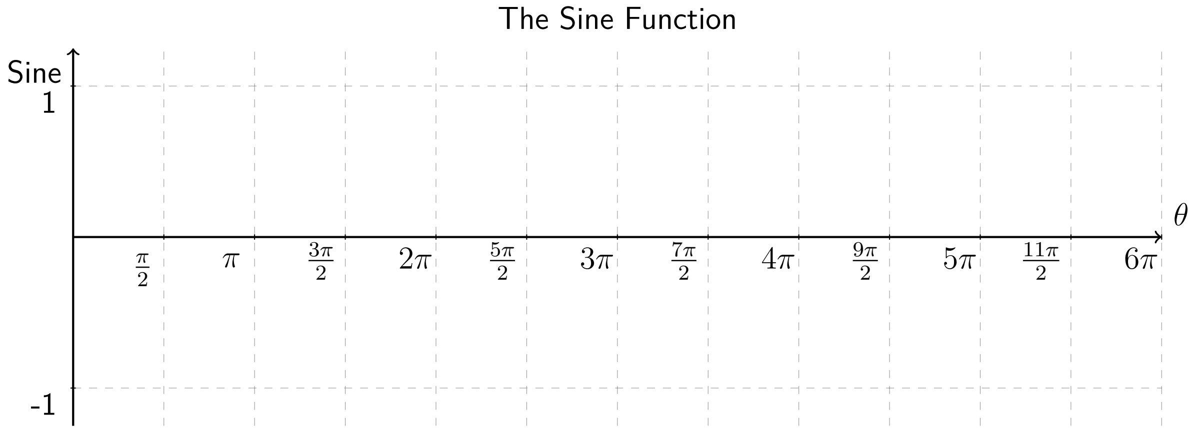
8. Determine values of $a$, $b$, $m$, and $c$ so that the function

$$f(\theta) = a\cos(\theta) + b$$

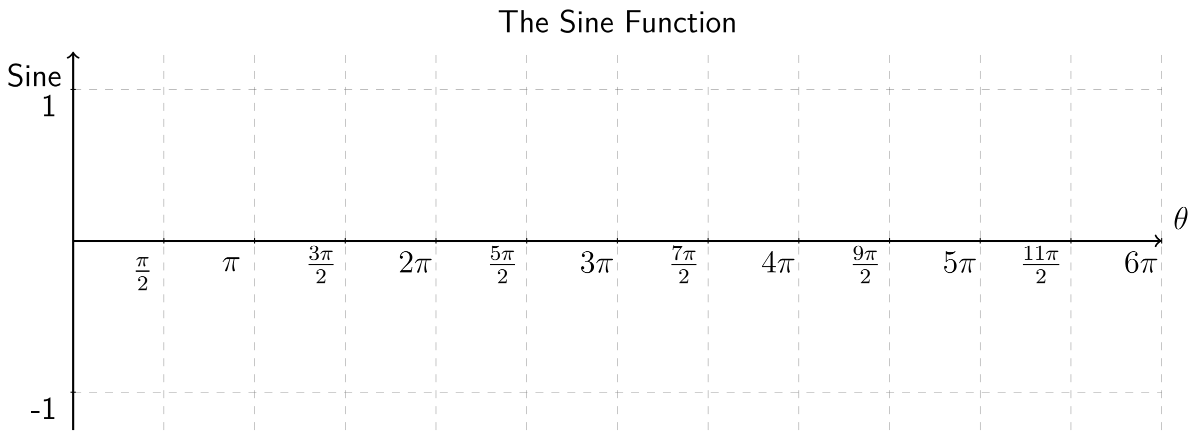
oscillates between -5 and -3 and has a minimum when $t = 3$ and has a period of 2.

1. Briefly state two ideas from today's class.
   - 
   - 
2. (a)

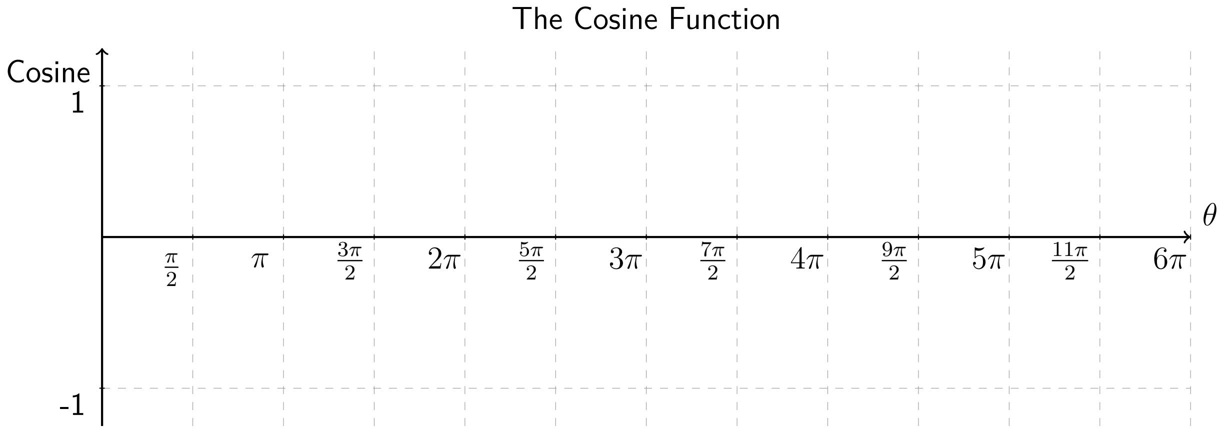
1. Make a sketch of the sine function.



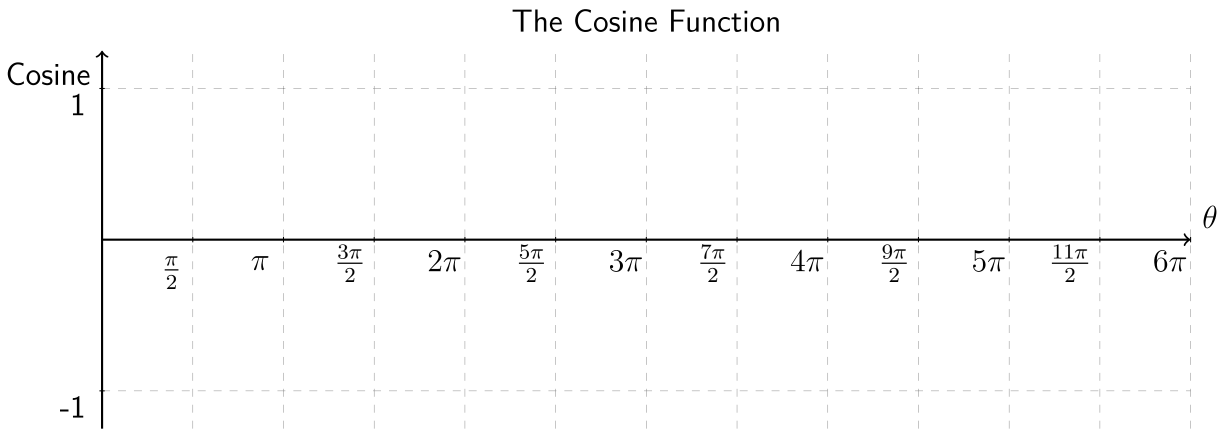

2. Make a sketch of the sine function shifted left $\frac{\pi}{2}$ units.



3. Make a sketch of the cosine function.

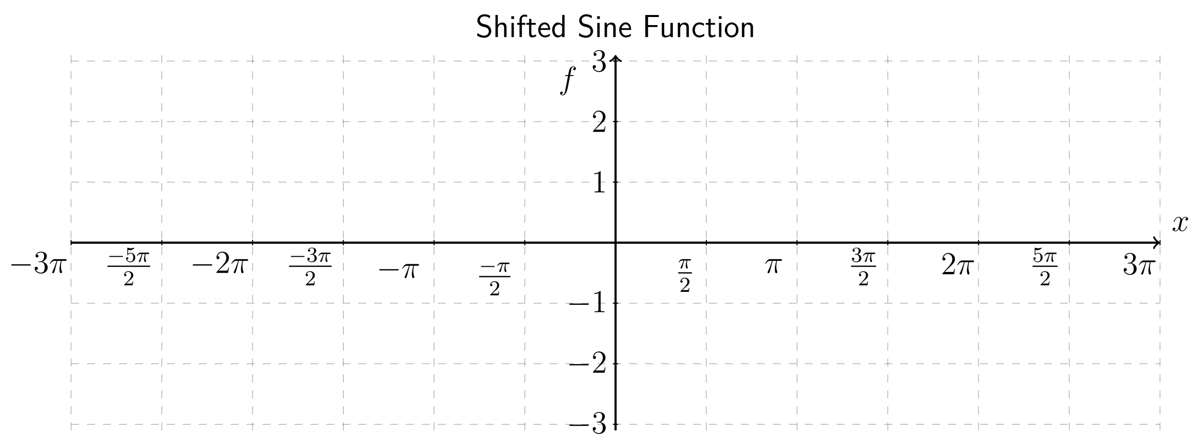



4. Make a sketch of the cosine function shifted left $\frac{\pi}{2}$ units.



1. A sine function is shifted and scaled as described below. Make a sketch of its graph first, and then determine the formula for the new function.

   (a) Make a sketch of a sine function that is shifted left $\frac{\pi}{2}$ units, oscillates between 2 and -2, and has a period of $2\pi$.

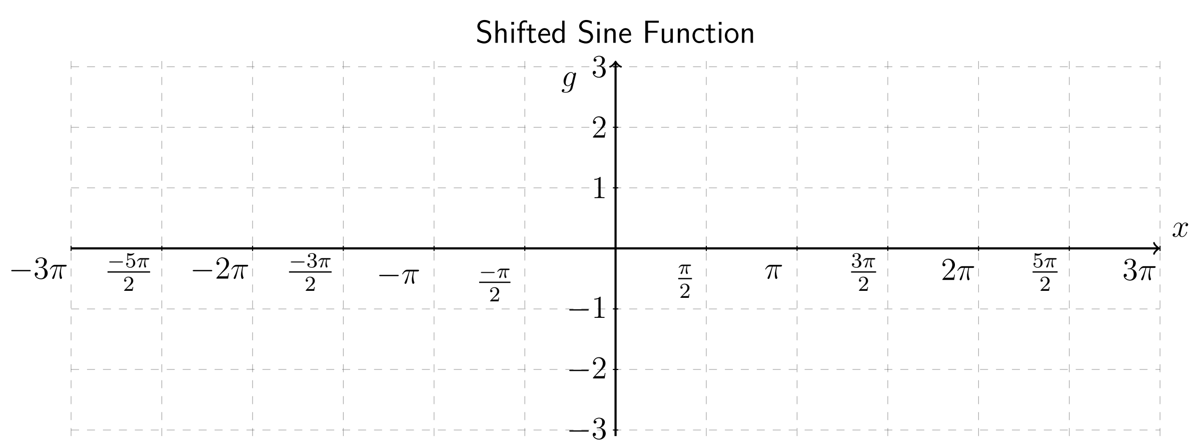



   (b) Determine the formula for the new function

$$f(x) \quad =$$

2. A sine function is shifted and scaled as described below. Make a sketch of its graph first, and then determine the formula for the new function.

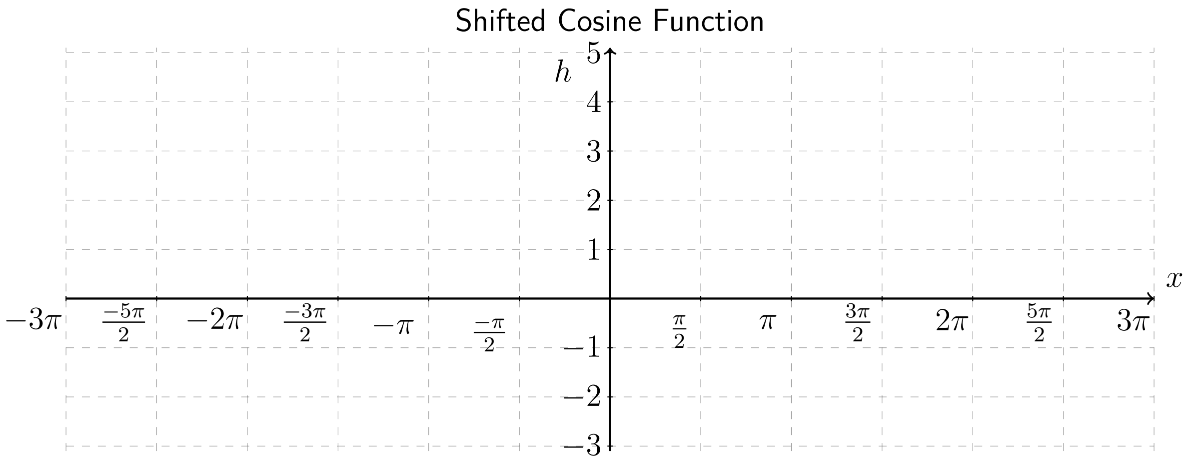
   (a) Make a sketch of a sine function that is shifted left $\frac{\pi}{2}$ units, oscillates between 3 and -1, and has a period of $2\pi$.




   (b) Determine the formula for the new function

$$g(x) \quad =$$

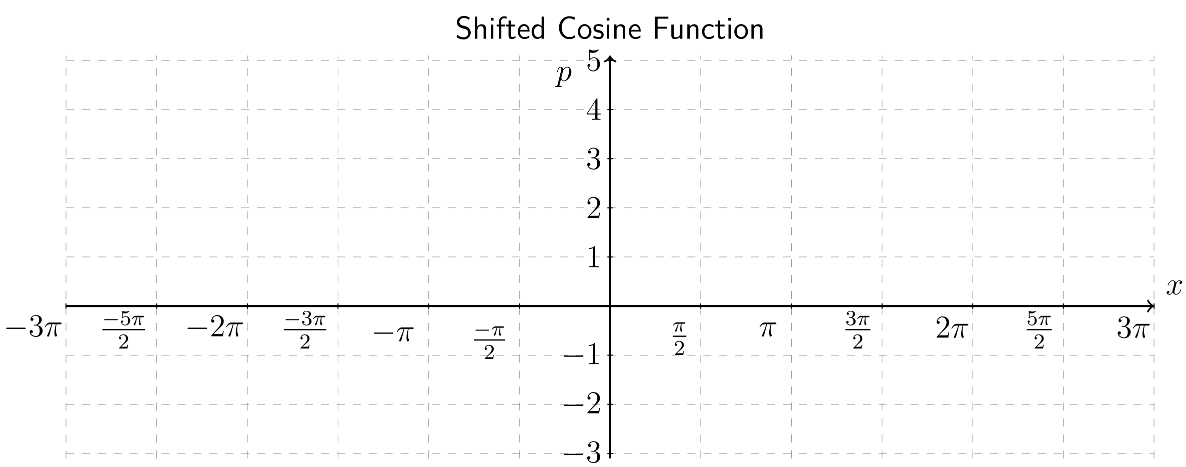
3. A cosine function is shifted and scaled as described below. Make a sketch of its graph first, and then determine the formula for the new function.
   (a) Make a sketch of a cosine function that is shifted right $\frac{\pi}{2}$ units, oscillates between 3 and -3, and has a period of $2\pi$.




   (b) Determine the formula for the new function

$$h(x) \quad =$$

4. A cosine function is shifted and scaled as described below. Make a sketch of its graph first, and then determine the formula for the new function.
   (a) Make a sketch of a cosine function that is shifted right $\frac{\pi}{2}$ units, oscillates between 5 and -1, and has a period of $2\pi$.

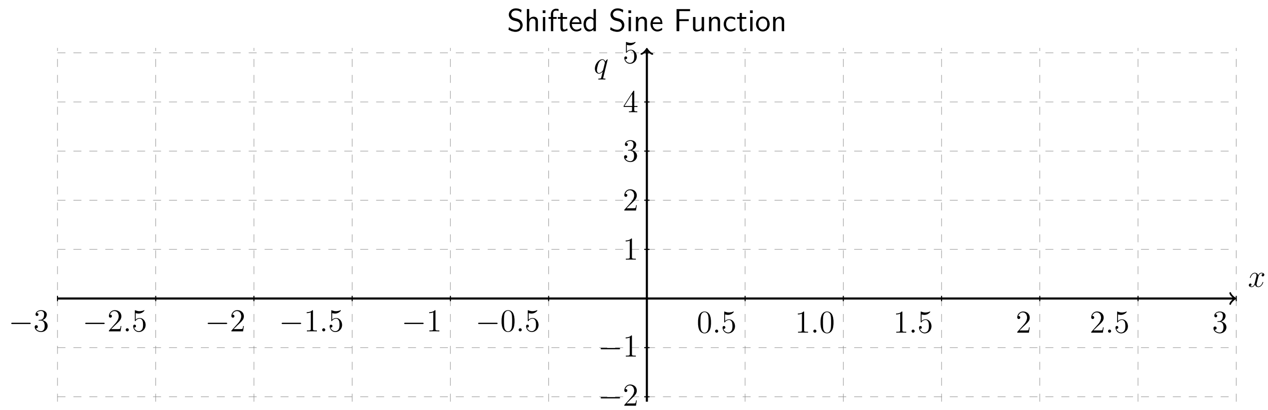



   (b) Determine the formula for the new function

$$p(x) \quad =$$

5. A sine function is scaled as described below. Make a sketch of its graph first, and then determine the formula for the new function.

   (a) Make a sketch of a sine function that has a period of 1 and an amplitude of 1.

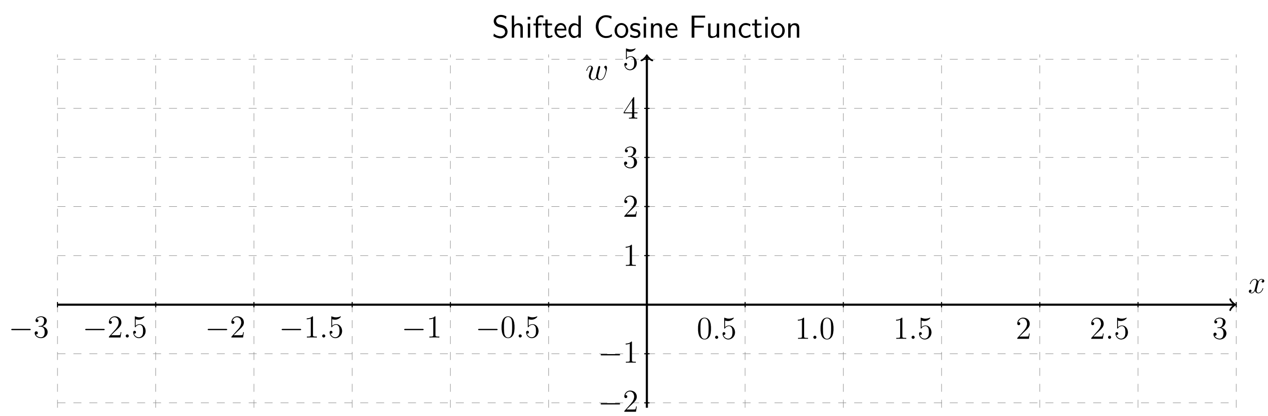



   (b) Determine the formula for the new function

$$q(x) \quad =$$

6. A cosine function is shifted and scaled as described below. Make a sketch of its graph first, and then determine the formula for the new function.

   (a) Make a sketch of a cosine function that is shifted right 2 units, oscillates between 5 and -1, and has a period of 1.

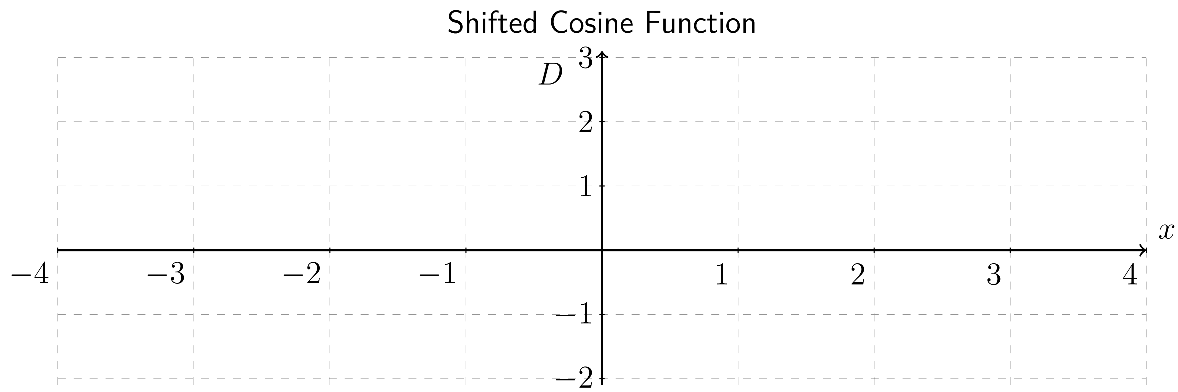



   (b) Determine the formula for the new function

$$w(x) \quad =$$

7. A cosine function is shifted and scaled as described below. Make a sketch of its graph first, and then determine the formula for the new function.

   (a) Make a sketch of a cosine function that is shifted right 1 units, oscillates between 2 and -1, and has a period of 2.

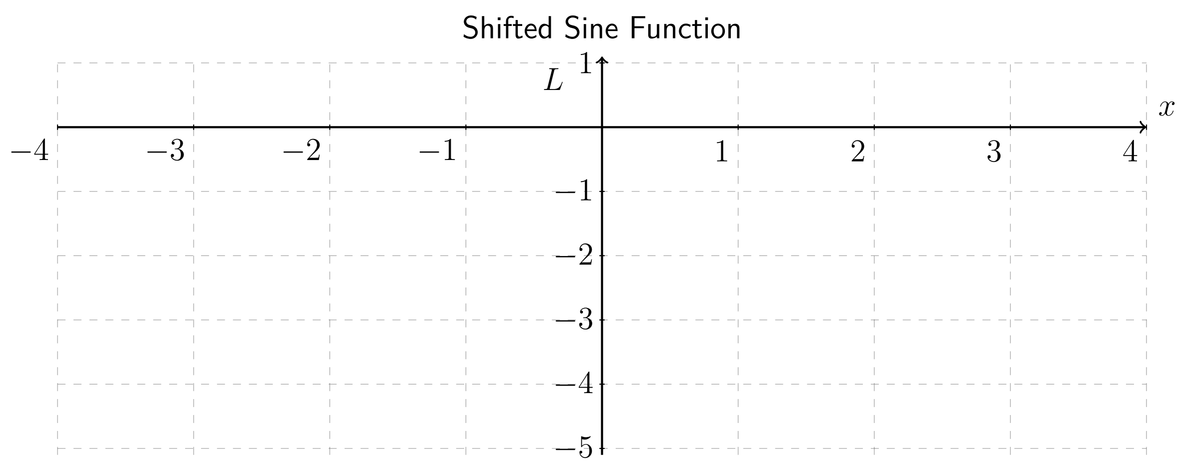



   (b) Determine the formula for the new function

$$D(x) \quad =$$

8. A sine function is shifted and scaled as described below. Make a sketch of its graph first, and then determine the formula for the new function.

   (a) Make a sketch of a sine function that is shifted left 2 units, oscillates between -1 and -4, and has a period of $\frac{2}{3}$.




   (b) Determine the formula for the new function

$$L(x) \quad =$$

1. Briefly state two ideas from today's class.
   - 
   - 
2. (a)

1. A triangle, ABC, has lengths $a = 5$, $b = 4$, and the angle directly across from $c$ is $\gamma = \frac{\pi}{6}$. (The triangle is **not** a right triangle.)
   (a) Draw a picture of the triangle.

   (b) Label the sides and the angle in your picture.

   (c) Determine the area of the triangle. (Hint: draw a vertical line that represents the height in your triangle and use the appropriate trigonometric functions to determine the height.)

1. A surveyor sets up a transit 2m above the surface of the ground. The transit is 80m away from the base of a building. The transit is pointing at the top of the building, and its angle of elevation is 35 degrees. How tall is the building?
   (a) Make a sketch of the situation. (It may take a couple tries!)

   (b) Indicate and label all of the information that is given and indicate any variables that are not known in your diagram above.

   (c) Identify the relationships between the variables.

   (d) How do you plan on solving the problem?

   (e) Determine the height of the building.

2. Chris Hadfield is in the International Space Station and is 360km above the surface of the earth. He looks down toward the center of the earth and then to the horizon of the earth. He measures an angle of 71 degrees between the two directions. What is the radius of the earth?

   (a) Make a sketch of the situation. (It may take a couple tries!)

   (b) Indicate and label all of the information that is given and indicate any variables that are not known in your diagram above.

   (c) Identify the relationships between the variables.

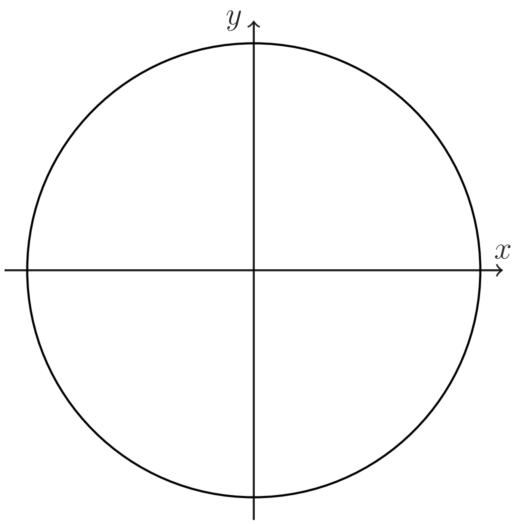
   (d) How do you plan on solving the problem?

   (e) Determine the radius of the earth.

1. Briefly state two ideas from today's class.
   -
   -
2. Captain Horatio McCallister is standing on the bridge of his ship. He spies a buoy through his telescope. He estimates that the straight line distance to the buoy is 180m, and the angle of depression is 4 degrees. How far must he sail to get to the buoy?
3. A regular polygon has seven equal sides. The distance from each vertex to the center of the polygon is 0.5m. What is the area of the polygon?
4. A regular polygon with 8 sides is inscribed within a circle of radius 2m. What is the area between the circle and the polygon?
5. A circle of radius 2m is inscribed within a regular polygon with six sides. What is the area between the polygon and the circle?
6. The base of a rectangular box has dimensions 20cm by 15cm, and the height is $h$ cm. The angle between the bottom of the box and the diagonal is 21 degrees. What is the height of the box?

1. Use the unit circle and the definition of sine and cosine to provide a justification for the identity

$$\sin^2(\theta) + \cos^2(\theta) \quad = \quad 1.$$




*Choose a point on the circle, draw the associated triangle, and then use the appropriate definitions.*

1. Is the equation

$$(\sec(\theta) - \tan(\theta)) \cdot (\csc(\theta) + 1) \quad = \quad \cot(\theta)$$

true for all $\theta$? (Fully justify your answer!)

2. Is the equation

$$\frac{1+\csc(3\beta)}{\sec(3\beta)} - \cot(3\beta) \quad = \quad \cos(3\beta)$$

true for all $\beta$? (Fully justify your answer!)

3. Is the equation

$$\cos^2(4\alpha) \quad = \quad 1 - \sin^2(4\alpha)$$

true for all $\alpha$? (Fully justify your answer!)

4. Is the equation

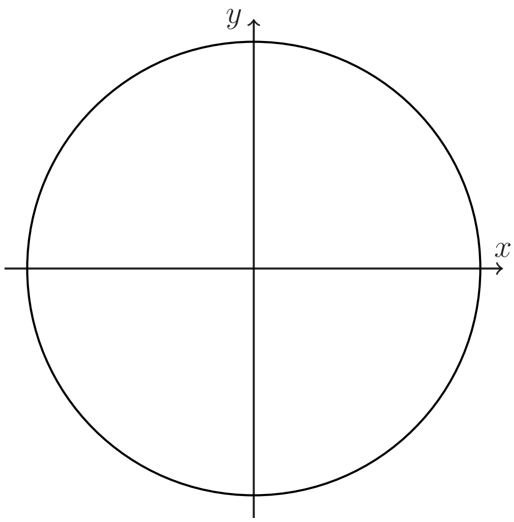
$$\tan^2(\theta) + 1 \quad = \quad \sec^2(\theta)$$

true for all $\theta$? (Fully justify your answer!)

1. Briefly state two ideas from today's class.
   - 
   - 
2. (a)

1. Determine all values of $\theta$ that satisfy

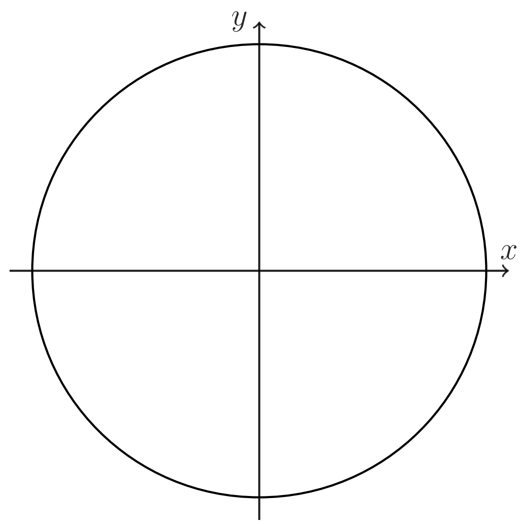
$$\sin(\theta) \quad = \quad \frac{\sqrt{2}}{2}.$$




Use the plot of the unit circle above, and sketch the angles that satisfy the equation above. Use the plot to determine the values of the angles in radians.

1. Determine all values of $\theta$ that satisfy

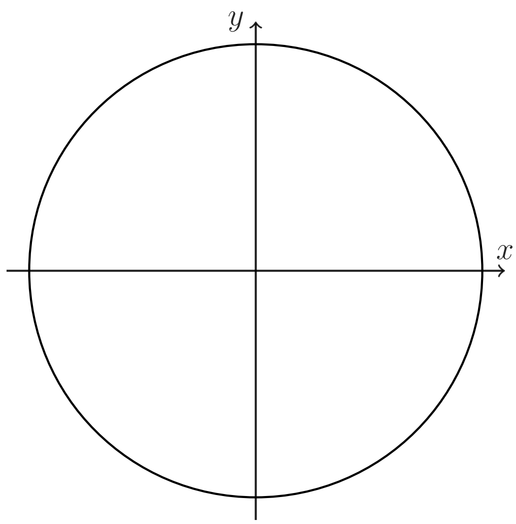
$$\cos(\theta) \quad = \quad -\frac{\sqrt{2}}{2}.$$




Use the plot of the unit circle above, and sketch the angles that satisfy the equation above. Use the plot to determine the values of the angles in radians.

2. Determine all values of $x$ that satisfy

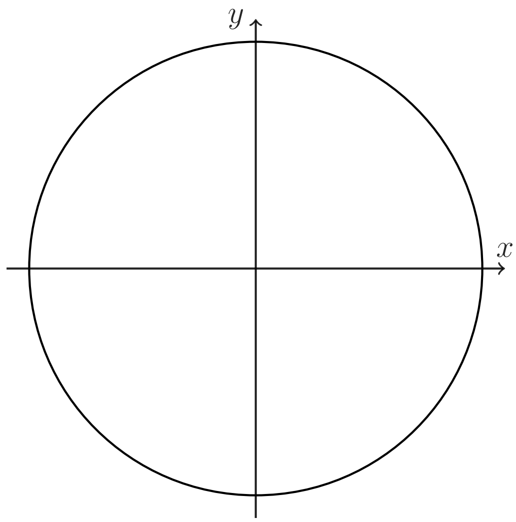
$$\cos(2x+1) \quad = \quad -\frac{\sqrt{2}}{2}.$$




Use the plot of the unit circle above, and sketch the angles that satisfy the equation above. Use the plot to determine the values of the angles in radians.

3. Determine all values of $x$ that satisfy

$$\tan(x^2+1) = \frac{\sqrt{3}}{3}.$$




Use the plot of the unit circle above, and sketch the angles that satisfy the equation above. Use the plot to determine the values of the angles in radians.

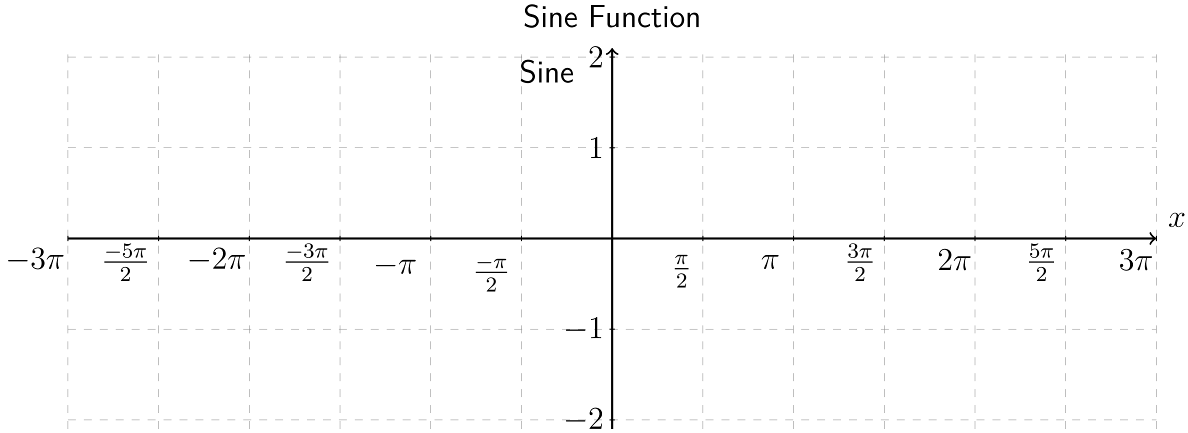
4. The voltage within a circuit element is given by

$$V(t) \quad = \quad 100 - 50\cos\left(\frac{3\pi}{120}t + 3\pi\right).$$

What is the minimum voltage and when does it occur? (Make a sketch of the unit circle!)

1. Briefly state two ideas from today's class.
   - 
   - 
2. (a)

1. Use the axes below to make a sketch of the graph of $\sin(x)$.




(a) Is the sine function one-to-one? (Justify your answer!)

(b) What is the domain for the sine function?

(c) Define a small part of the domain for which the sine function is one-to-one. (There is not a unique answer - just find one small part.) Make a sketch of the sine function restricted to your domain and explain why it is now one-to-one.

1. Use your calculator to evaluate the following expressions. Explain why it gives your results.
   (a) $\arcsin\left(\sin\left(\frac{\pi}{4}\right)\right)$

   (b) $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right)$

2. For each equation below determine all possible values of $x$ that satisfy the equations.

   (a) $\sin(3x - 1) = \dfrac{\sqrt{3}}{2}$

   (b) $\ln(\cos(x + 1) + 2) = 0.6$

3. The commander of a radar station just received word that a super secret spy plane will fly directly over it. The plane is expected to have an elevation of two miles, and the distance between the station and a point on the ground directly below the plane is roughly ninety miles away from the station's antenna.

   (a) What should the angle of elevation of the antenna be to point directly at the plane.

   (b) Due to a mix up it is estimated that the orders are thirty minutes late. If the plane is traveling level flight at a constant speed of five-hundred miles per hour determine the angle of elevation.

   (c) Determine the necessary angle of elevation if the orders were actually thirty-five minutes late.

4. Determine the value of

$$\tan\left(\arcsin\left(\frac{1}{3}\right)\right)$$

(a) Let $\theta$ be the angle that satisfies $\sin(\theta) = \frac{1}{3}$ How does this definition relate to the expression above? Rewrite the expression in terms of the angle $\theta$.

(b) Make a sketch of a right triangle. Mark the appropriate angle and sides given that $\sin(\theta) = \frac{1}{3}$.

(c) Determine the value of the expression above based on your diagram.

5. Determine the value of

$$\tan\left(\operatorname{arcsec}\left(x\right)\right)$$

for any value of $x$ without using any trigonometric functions.

(a) Make a sketch of a right triangle. Mark the appropriate angle and sides.

(b) Determine the value based on your diagram.

1. Briefly state two ideas from today's class.
    - 
    - 
2. (a)

# Chapter 5

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