Topics: one-to-one functions, inverse functions, domain and range of inverse functions

Student Learning Outcomes:

1. Students will be able to determine if a function is one-to-one algebraically and graphically.
2. Students will be able to determine the inverse of a function.
3. Students will be able to determine the domain and range of an inverse function.

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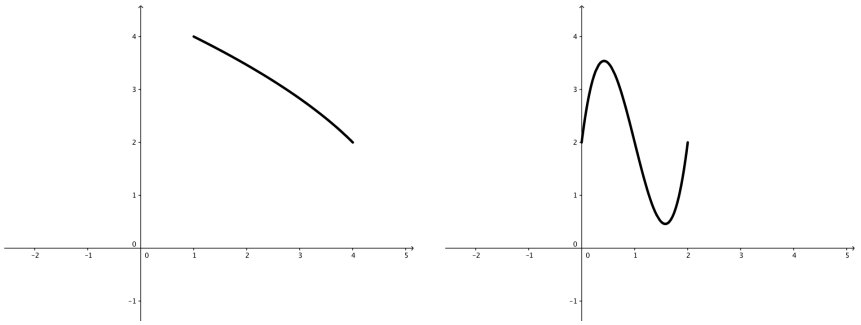
1 Identify One-to-One Functions

Horizontal Line Test. A function is one-to-one if every horizontal line intersects the graph of f at most once. Otherwise the function is not one-to-one.

1. Determine whether the graph is the graph of a one-to-one function.



2. Determine whether the following functions are one-to-one.

   (a) $f = \{(1, 4), (2, 3), (-2, 3)\}$

   (b) $f(x) = \sqrt{5x}$

   (c) $f(x) = x^2$

2 Determine Whether Two Functions are Inverses

Theorem on Inverse Functions. Let f be a one-to-one function with domain D and range R. If g is a function with domain R and range D, then g is the inverse function of f precisely when both of the following conditions hold:

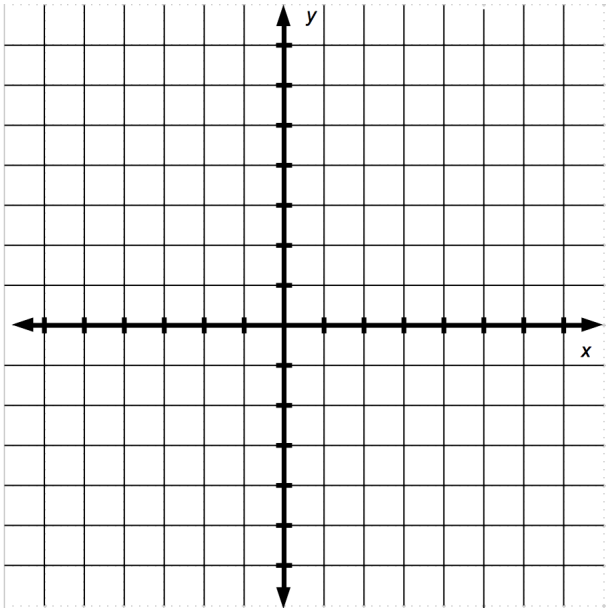
- $g(f(x)) = x$ for every x in D, and
- $f(g(y)) = y$ for every y in R.

Notation: "f inverse" is typically written as f^{-1}. It is important to note that f^{-1} does NOT mean the same thing as $\frac{1}{f}$.

3. Let $f(x) = x^2 + 4, \quad x \geq 0$, and let $g(x) = \sqrt{x-4}, \quad x \geq 4$

   (a) Use the theorem on inverse functions to determine whether f and g are inverses.

   (b) Sketch the graphs of f and g on the same coordinate axes.



3 Find the Inverse of a Function

Guidelines for Finding Inverse Functions.
Assuming that f is a one-to-one function, and that the algebra is do-able, you can find f^{-1} using the following procedure:
1. Solve the equation $y = f(x)$ for x in terms of y. You now have $x = f^{-1}(y)$.
2. Replace x with y and solve for y. This is your inverse function, replace y with $f^{-1}(x)$.
3. Check your work (if time permits): check that $f^{-1}(f(x)) = x$ whenever x is in the domain of f, and $f(f^{-1}(x)) = x$ whenever x is in the domain of f^{-1}.

4. Find the inverse function of $f(x) = 4 + 3x$.

5. Use the table for the one-to-one function $f(x)$ to compute each expression.

x	2	3	7	6	15
f(x)	-1	5	4	2	3

(a) $f^{-1}(5)$

(b) $f^{-1}(-1)$

(c) $f^{-1}(2)$

Student Learning Outcomes Check

1. Can you determine if a function is one-to-one algebraically and graphically?
2. Are you able to determine the inverse of a function?
3. Can you determine the domain and range of an inverse function?

If any of your answers were no, please ask about these topics in class.