Homework 1
Due Friday, September 13th at 11:00pm via CMS

Notes:
(1) Submit your solution in a single PDF file named HW1.pdf
(2) Put your name and NetID on the top of the first page
(3) Show how you arrived at your answers

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Problem 1.

For each of the following equations, apply the axioms and theorems of Boolean Algebra to the left hand side of the equation to derive the expression on the right. State the axiom or theorem applied in each step (apply only one in a given step).

(a) $(\underbrace{A + B'}_{Y} + \underbrace{C + D}_{X})(\underbrace{A' + B'}_{Z} + \underbrace{C + D}_{X})(B' + C + D')(B + C' + D') = (B' + C)(B + C' + D')$

(b) $X'Y + XY + YZ + XZ = Y + XZ$

a) $= (B'+C+D)(B'+C+D')(B+C'+D')$ — combining

$= (B'+C)(B+C'+D')$ — combining

$=$ RHS

b) $(X'Y+XY)+(YZ+XZ)$

$= XZ+X'Y+XY$ — consensus

$= XZ+Y$ — combining

Problem 2.

Prove the Combining theorem T10 using the axioms or theorems of Boolean Algebra. State the axiom or theorem that is used in each step (apply only one in a given step).

$XY + XY' = X$

$= X(Y+Y')$ distributive

$= X \cdot 1$ compliment

$= X$ identity

Problem 3.

Prove in a single line using De Morgan's Law that the NOR operation is not associative, that is:

$(X+(Y+Z)')' \neq ((X+Y)'+Z)'$

LHS = $(X' \cdot (Y+Z))$ RHS = $((X+Y) \cdot Z')$

$= X'Y + X'Z$ $= XZ' + YZ'$

LHS $\neq$ RHS

Problem 4.

A four-person committee consists of Alice (A), Bob (B), Carol (C), and Don (D). If, as Chair of the committee, Alice votes Yes, then the matter before the committee is approved. Otherwise, the majority vote among the remaining three members decides the outcome.

Consider representing the committee decision as digital logic, with four inputs A, B, C, and D representing the votes of the committee members (with 1 as Yes and 0 as No), and one output Decision, which is 1 when the vote is to approve, and 0 otherwise.

(a) Construct a truth table for Decision.
(b) Write the output as a canonical product (don't use the shorthand format; write out all of the terms).
(c) Express Decision in minimal sum-of-products form. If it helps, use a Karnaugh Map, but you do not have to do this to get full credit.
(d) Implement the minimal form of Decision from (c) using only NOT and NAND gates.

a)

ABCD	F
0000	0
0001	0
0010	0
0011	1
0100	0
0101	1
0110	1
0111	1
1000	1
1001	1
1010	1
1011	1
1100	1
1101	1
1110	1
1111	1

b) $F = (A+B+C+D)\cdot(A+B+C+D')\cdot(A+B+C'+D)\cdot(A+B'+C+D)$

c)

CD \ AB	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	1	1	1	1
10	0	1	1	1

$BD + AB' + AB + CD + BC$

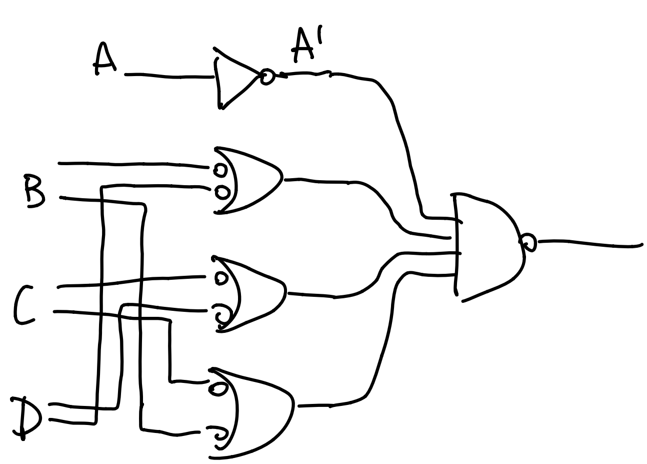
$= BD + A + CD + BC$

d)

$BD + A + CD + BC$

$= (B'+D')' + (A')' + (C'+D')' + (B'+C')'$

$= \left((B'+D')(A')(C'+D')(B'+C')\right)'$



Problem 5.

Write out the canonical sum and canonical product for each of the following functions (don't use the shorthand format; write out all of the terms):

(a) $\sum_{WXYZ}(1,3,6,9,12,13,15)$

(b) $\prod_{XYZ}(0,1,3,6,7)$

a) $\sum_{WXYZ}(1,3,6,9,12,13,15) = W'X'Y'Z + W'X'YZ + W'XYZ' + WX'Y'Z$
$+ WXY'Z' + WXY'Z + WXYZ$

b) $\prod_{XYZ}(0,1,3,6,7) = (X+Y+Z)\cdot(X+Y+Z')\cdot(X+Y'+Z')\cdot(X'+Y'+Z)\cdot$
$(X'+Y'+Z')$

Problem 6.

Use Karnaugh Maps to create minimal expressions in both sum-of-product and product-of-sum forms for the following function (*dc* = don't care).

$$F = \sum_{WXYZ} (4,5,7,12) + dc(6,14)$$

YZ \ WX	00	01	11	10
00		1	1	
01		1		
11		1		
10		d	d	

$F = \sum(4,5,7,12)$

$= W'X + XZ'$

$F = \prod(0,1,2,3,8,9,10,13,15) = X \cdot (W' + Z')$

YZ \ WX	00	01	11	10
00	0			0
01	0		0	0
11	0		0	0
10	0	d	d	0

Problem 7.

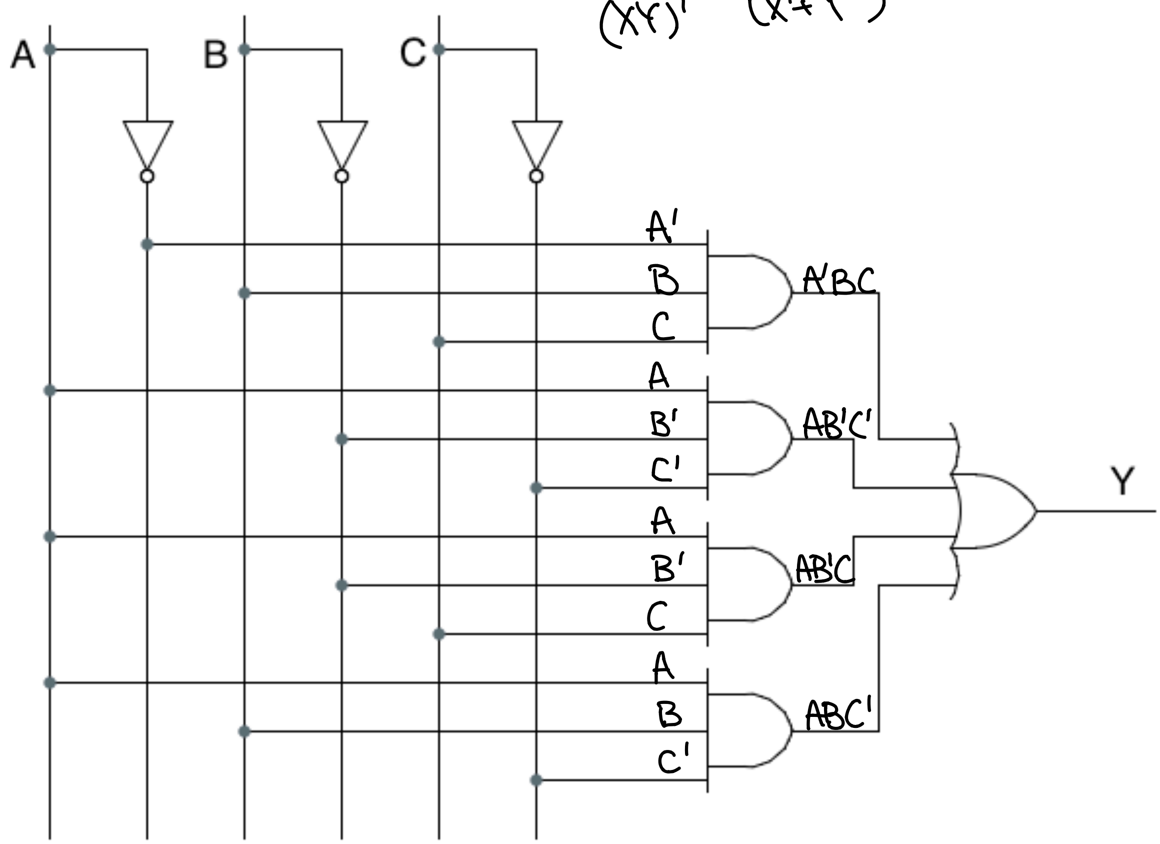
For the following circuit:

(a) Determine the expression for the output Y.

(b) Minimize the logic expression from (a).

(c) Implement (b) using only NOT and NAND gates.

$(XY)'$ $(X'+Y')$



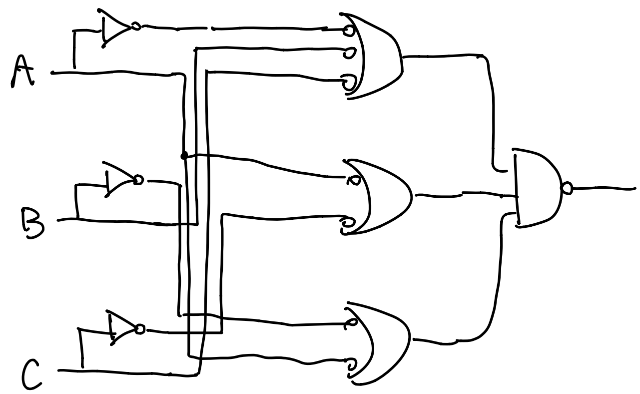
a) $Y = A'BC + AB'C' + AB'C + ABC'$

b) $Y = A'BC + AB'C' + AB'C + ABC'$

$= A'BC + AC' + AB'$

c) $Y = (A+B'+C')' + (A'+C)' + (A'+B)'$

$= \big((A+B'+C')\cdot(A'+C)\cdot(A'+B)\big)'$


A
B
C