

LARGE BAYESIAN VECTOR AUTO REGRESSIONS

MARTA BAÑBURA,^a DOMENICO GIANNONE^{b,c} AND LUCREZIA REICHLIN^{c,d*}

^a *European Central Bank, Frankfurt, Germany*

^b *ECARES, Brussels, Belgium*

^c *CEPR, London, UK*

^d *London Business School, UK*

SUMMARY

This paper shows that vector auto regression (VAR) with Bayesian shrinkage is an appropriate tool for large dynamic models. We build on the results of De Mol and co-workers (2008) and show that, when the degree of shrinkage is set in relation to the cross-sectional dimension, the forecasting performance of small monetary VARs can be improved by adding additional macroeconomic variables and sectoral information. In addition, we show that large VARs with shrinkage produce credible impulse responses and are suitable for structural analysis. Copyright © 2009 John Wiley & Sons, Ltd.

Received 14 May 2007; Revised 28 October 2008

1. INTRODUCTION

Vector auto regressions (VAR) are standard tools in macroeconomics and are widely used for structural analysis and forecasting. In contrast to structural models, for example, they do not impose restrictions on the parameters and hence provide a very general representation allowing the capture of complex data relationships. On the other hand, this high level of generality implies a large number of parameters even for systems of moderate size. This entails a risk of over-parametrization since, with the typical sample size available for macroeconomic applications, the number of unrestricted parameters that can reliably be estimated is rather limited. Consequently, VAR applications are usually based only on a small number of variables.

The size of the VARs typically used in empirical applications ranges from three to about ten variables and this potentially creates an omitted variable bias with adverse consequences both for structural analysis and for forecasting (see, for example, Christiano *et al.*, 1999; Giannone and Reichlin, 2006). For example, Christiano *et al.* (1999) point out that the positive reaction of prices in response to a monetary tightening, the so-called price puzzle, is an artefact resulting from the omission of forward-looking variables, like the commodity price index. In addition, the size limitation is problematic for applications which require the study of a larger set of variables than the key macroeconomic indicators, such as disaggregate information or international data. In the VAR literature, a popular solution to analyse relatively large datasets is to define a core set of indicators and to add one variable, or group of variables, at a time (the so-called marginal approach; see, for example, Christiano *et al.*, 1996; Kim, 2001). With this approach, however, comparison of impulse responses across models is problematic.

* Correspondence to: Lucrezia Reichlin, London Business School, Regent's Park, London NW1 4SA, UK.
E-mail: reichlin@london.edu

To circumvent these problems, recent literature has proposed ways to impose restrictions on the covariance structure so as to limit the number of parameters to estimate. For example, factor models for large cross-sections introduced by Forni *et al.* (2000) and Stock and Watson (2002b) rely on the assumption that the bulk of dynamic interrelations within a large dataset can be explained by few common factors. Those models have been successfully applied both in the context of forecasting (Bernanke and Boivin, 2003; Boivin and Ng, 2005; D'Agostino and Giannone, 2006; Forni *et al.*, 2003, 2005; Giannone *et al.*, 2004; Marcellino *et al.*, 2003; Stock and Watson, 2002a,b) and structural analysis (Stock and Watson, 2005b; Forni *et al.*, 2008; Bernanke *et al.*, 2005; Giannone *et al.*, 2004). For datasets with a panel structure an alternative approach has been to impose exclusion, exogeneity or homogeneity restrictions, as in global VARs (cf. Dees *et al.*, 2007) and panel VARs (cf. Canova and Ciccarelli, 2004), for example.

In this paper we show that by applying Bayesian shrinkage, we are able to handle large unrestricted VARs and that therefore the VAR framework can be applied to empirical problems that require the analysis of more than a handful of time series. For example, we can analyse VARs containing the wish list of any macroeconomist (see, for example, Uhlig, 2004) but it is also possible to extend the information set further and include the disaggregated, sectorial and geographical indicators. Consequently, Bayesian VAR is a valid alternative to factor models or panel VARs for the analysis of large dynamic systems.

We use priors as proposed by Doan *et al.* (1984) and Litterman (1986a). Litterman (1986a) found that applying Bayesian shrinkage in the VAR containing as few as six variables can lead to better forecast performance. This suggests that over-parametrization can be an issue already for systems of fairly modest size and that shrinkage is a potential solution to this problem. However, although Bayesian VARs with Litterman's priors are a standard tool in applied macroeconomics (Leeper *et al.*, 1996; Sims and Zha, 1998; Robertson and Tallman, 1999), the imposition of priors has not been considered sufficient to deal with larger models. For example, the marginal approach we described above has been typically used in conjunction with Bayesian shrinkage (see, for example, Maćkowiak (2006, 2007). Litterman himself, when constructing a 40-variable model for policy analysis, imposed (exact) exclusion and exogeneity restrictions in addition to shrinkage, allowing roughly ten variables per equation (see Litterman, 1986b).

Our paper shows that these restrictions are unnecessary and that shrinkage is indeed sufficient to deal with large models provided that, contrary to the common practice, we increase the tightness of the priors as we add more variables. Our study is empirical, but builds on the asymptotic analysis in De Mol *et al.* (2008), which analyses the properties of Bayesian regression as the dimension of the cross-section and the sample size go to infinity. That paper shows that, when data are characterized by strong collinearity, which is typically the case for macroeconomic time series, as we increase the cross-sectional dimension, Bayesian regression tends to capture factors that explain most of the variation of the predictors. Therefore, by setting the degree of shrinkage in relation to the model size, it is indeed possible to control for over-fitting while preserving the relevant sample information. The intuition of this result is that, if all data carry similar information (near collinearity), the relevant signal can be extracted from a large dataset despite the stronger shrinkage required to filter out the unsystematic component. In this paper we go beyond simple regression and study the VAR case.

We evaluate forecasting accuracy and perform a structural exercise on the effect of a monetary policy shock for systems of different sizes: a small VAR on employment, inflation and interest rate, a VAR with the seven variables considered by Christiano *et al.* (1999), a 20-variable VAR extending the system of Christiano *et al.* (1999) by key macro indicators, such as labor market

variables, the exchange rate or stock prices and finally a VAR with 131 variables, containing, besides macroeconomic information, also sectoral data, several financial variables and conjunctural information. These are the variables used by Stock and Watson (2005a) for forecasting based on principal components, but contrary to the factor literature we model variables in levels to retain information in the trends. We also compare the results of Bayesian VARs with those from the factor augmented VAR (FAVAR) of Bernanke *et al.* (2005).

We find that the largest specification outperforms the small models in forecast accuracy and produces credible impulse responses, but that this performance is already obtained with the medium-size system containing the 20 key macroeconomic indicators. This suggests that for the purpose of forecasting and structural analysis it is not necessary to go beyond the model containing only the aggregated variables. On the other hand, this also shows that the Bayesian VAR is an appropriate tool for forecasting and structural analysis when it is desirable to condition on a large information set.

Given the progress in computing power (see Hamilton, 2006, for a discussion), estimation does not present any numerical problems. More subtly, shrinkage acts as a regularization solution of the problem of inverting an otherwise unstable large covariance matrix (approximately 2000×2000 for the largest model of our empirical application).

The paper is organized as follows. In Section 2 we describe the priors for the baseline Bayesian VAR model and the data. In Section 3 we perform the forecast evaluation for all the specifications and in Section 4 the structural analysis on the effect of the monetary policy shocks. Section 5 concludes and the Appendix provides some more details on the dataset and the specifications. Finally, the Annex available as online supporting information or in the working paper version contains results for a number of alternative specifications to verify the robustness of our findings.

2. SETTING THE PRIORS FOR THE VAR

Let $Y_t = (y_{1,t} \ y_{2,t} \ \dots \ y_{n,t})'$ be a potentially large vector of random variables. We consider the following VAR(p) model:

$$Y_t = c + A_1 Y_{t-1} + \dots + A_p Y_{t-p} + u_t \quad (1)$$

where u_t is an n -dimensional Gaussian white noise with covariance matrix $\mathbb{E}u_t u_t' = \Psi$, $c = (c_1, \dots, c_n)'$ is an n -dimensional vector of constants and A_1, \dots, A_p are $n \times n$ autoregressive matrices.

We estimate the model using the Bayesian VAR (BVAR) approach which helps to overcome the curse of dimensionality via the imposition of prior beliefs on the parameters. In setting the prior distributions, we follow standard practice and use the procedure developed in Litterman (1996a) with modifications proposed by Kadiyala and Karlsson (1997) and Sims and Zha (1998).

Litterman (1996a) suggests using a prior often referred to as the Minnesota prior. The basic principle behind it is that all the equations are 'centered' around the random walk with drift; i.e., the prior mean can be associated with the following representation for Y_t :

$$Y_t = c + Y_{t-1} + u_t$$

This amounts to shrinking the diagonal elements of A_1 toward one and the remaining coefficients in A_1, \dots, A_p toward zero. In addition, the prior specification incorporates the belief that

the more recent lags should provide more reliable information than the more distant ones and that own lags should explain more of the variation of a given variable than the lags of other variables in the equation.

These prior beliefs are imposed by setting the following moments for the prior distribution of the coefficients:

$$\mathbb{E}[(A_k)_{ij}] = \begin{cases} \delta_i, & j = i, k = 1 \\ 0, & \text{otherwise} \end{cases}, \quad \mathbb{V}[(A_k)_{ij}] = \begin{cases} \frac{\lambda^2}{k^2}, & j = i \\ \vartheta \frac{\lambda^2}{k^2} \frac{\sigma_i^2}{\sigma_j^2}, & \text{otherwise} \end{cases} \quad (2)$$

The coefficients A_1, \dots, A_p are assumed to be a priori independent and normally distributed. As for the covariance matrix of the residuals, it is assumed to be diagonal, fixed and known: $\Psi = \Sigma$, where $\Sigma = \text{diag}(\sigma_1^2, \dots, \sigma_n^2)$. Finally, the prior on the intercept is diffuse.

Originally, Litterman sets $\delta_i = 1$ for all i , reflecting the belief that all the variables are characterized by high persistence. However, this prior is not appropriate for variables believed to be characterized by substantial mean reversion. For those we impose the prior belief of white noise by setting $\delta_i = 0$.

The hyperparameter λ controls the overall tightness of the prior distribution around the random walk or white noise and governs the relative importance of the prior beliefs with respect to the information contained in the data. For $\lambda = 0$ the posterior equals the prior and the data do not influence the estimates. If $\lambda = \infty$, on the other hand, posterior expectations coincide with the ordinary least squares (OLS) estimates. We argue that the overall tightness governed by λ should be chosen in relation to the size of the system. As the number of variables increases, the parameters should be shrunk more in order to avoid over-fitting. This point has been shown formally by De Mol *et al.* (2008).

The factor $1/k^2$ is the rate at which prior variance decreases with increasing lag length and σ_i^2/σ_j^2 accounts for the different scale and variability of the data. The coefficient $\vartheta \in (0, 1)$ governs the extent to which the lags of other variables are ‘less important’ than the own lags.

In the context of the structural analysis we need to take into account possible correlation among the residual of different variables. Consequently, Litterman’s assumption of fixed and diagonal covariance matrix is somewhat problematic. To overcome this problem we follow Kadiyala and Karlsson (1997) and Robertson and Tallman (1999) and impose a normal inverted Wishart prior which retains the principles of the Minnesota prior. This is possible under the condition that $\vartheta = 1$, which will be assumed in what follows. Let us write the VAR in (1) as a system of multivariate regressions (see, for example, Kadiyala and Karlsson, 1997):

$$Y_{T \times n} = X_{T \times k} B_{k \times n} + U_{T \times n} \quad (3)$$

where $Y = (Y_1, \dots, Y_T)'$, $X = (X_1, \dots, X_T)'$ with $X_t = (Y'_{t-1}, \dots, Y'_{t-p}, 1)'$, $U = (u_1, \dots, u_T)'$, and $B = (A_1, \dots, A_p, c)'$ is the $k \times n$ matrix containing all coefficients and $k = np + 1$. The normal inverted Wishart prior has the form

$$\text{vec}(B)|\Psi \sim N(\text{vec}(B_0), \Psi \otimes \Omega_0) \quad \text{and} \quad \Psi \sim iW(S_0, \alpha_0) \quad (4)$$

where the prior parameters B_0 , Ω_0 , S_0 and α_0 are chosen so that prior expectations and variances of B coincide with those implied by equation (2) and the expectation of Ψ is equal to the fixed residual covariance matrix Σ of the Minnesota prior; for details see Kadiyala and Karlsson (1997).

We implement the prior (4) by adding dummy observations. It can be shown that adding T_d dummy observations Y_d and X_d to the system (3) is equivalent to imposing the normal inverted Wishart prior with $B_0 = (X_d'X_d)^{-1}X_d'Y_d$, $\Omega_0 = (X_d'X_d)^{-1}$, $S_0 = (Y_d - X_dB_0)'(Y_d - X_dB_0)$ and $\alpha_0 = T_d - k$. In order to match the Minnesota moments, we add the following dummy observations:

$$Y_d = \begin{pmatrix} \text{diag}(\delta_1\sigma_1, \dots, \delta_n\sigma_n)/\lambda \\ 0_{n(p-1) \times n} \\ \dots\dots\dots \\ \text{diag}(\sigma_1, \dots, \sigma_n) \\ \dots\dots\dots \\ 0_{1 \times n} \end{pmatrix} \quad X_d = \begin{pmatrix} J_p \otimes \text{diag}(\sigma_1, \dots, \sigma_n)/\lambda & 0_{np \times 1} \\ \dots\dots\dots & \dots\dots\dots \\ 0_{n \times np} & 0_{n \times 1} \\ \dots\dots\dots & \dots\dots\dots \\ 0_{1 \times np} & \varepsilon \end{pmatrix} \quad (5)$$

where $J_p = \text{diag}(1, 2, \dots, p)$. Roughly speaking, the first block of dummies imposes prior beliefs on the autoregressive coefficients, the second block implements the prior for the covariance matrix and the third block reflects the uninformative prior for the intercept (ε is a very small number). Although the parameters should in principle be set using only prior knowledge we follow common practice (see, for example, Litterman, 1986a; Sims and Zha, 1998) and set the scale parameters σ_i^2 equal to the variance of a residual from a univariate autoregressive model of order p for the variables y_{it} .

Consider now the regression model (3) augmented with the dummies in (5):

$$Y_{*} = X_{*} B + U_{*} \quad (6)$$

$T_* \times n \quad T_* \times k \quad k \times n \quad T_* \times n$

where $T_* = T + T_d$, $Y_* = (Y', Y_d)'$, $X_* = (X', X_d)'$ and $U_* = (U', U_d)'$. To ensure the existence of the prior expectation of Ψ it is necessary to add an improper prior $\Psi \sim |\Psi|^{-(n+3)/2}$. In that case the posterior has the form

$$\text{vec}(B)|\Psi, Y \sim N(\text{vec}(\tilde{B}), \Psi \otimes (X_*'X_*)^{-1}) \quad \text{and} \quad \Psi|Y \sim iW(\tilde{\Sigma}, T_d + 2 + T - k) \quad (7)$$

with $\tilde{B} = (X_*'X_*)^{-1}X_*'Y_*$ and $\tilde{\Sigma} = (Y_* - X_*\tilde{B})'(Y_* - X_*\tilde{B})$. Note that the posterior expectation of the coefficients coincides with the OLS estimates of the regression of Y_* on X_* . It can be easily checked that it also coincides with the posterior mean for the Minnesota setup in (2). From the computational point of view, estimation is feasible since it only requires the inversion of a square matrix of dimension $k = np + 1$. For the large dataset of 130 variables and 13 lags k is smaller than 2000. Adding dummy observations works as a regularization solution to the matrix inversion problem.

The dummy observation implementation will prove useful for imposing additional beliefs. We will exploit this feature in Section 3.3.

2.1. Data

We use the dataset of Stock and Watson (2005a). This dataset contains 131 monthly macro indicators covering a broad range of categories including, among others, income, industrial production,

capacity, employment and unemployment, consumer prices, producer prices, wages, housing starts, inventories and orders, stock prices, interest rates for different maturities, exchange rates and money aggregates. The time span is from January 1959 to December 2003. We apply logarithms to most of the series, with the exception of those already expressed in rates. For non-stationary variables, considered in first differences by Stock and Watson (2005a), we use the random walk prior; i.e., we set $\delta_i = 1$. For stationary variables, we use the white noise prior, i.e., $\delta_i = 0$. A description of the dataset, including the information on the transformations and the specification of δ_i for each series, is provided in the Appendix.

We analyse VARs of different sizes. We first look at the forecast performance. Then we identify the monetary policy shock and study impulse response functions as well as variance decompositions. The variables of special interest include a measure of real economic activity, a measure of prices and a monetary policy instrument. As in Christiano *et al.* (1999), we use employment as an indicator of real economic activity measured by the number of employees on non-farm payrolls (EMPL). The level of prices is measured by the consumer price index (CPI) and the monetary policy instrument is the Federal Funds Rate (FFR).

We consider the following VAR specifications:

- **SMALL.** This is a small monetary VAR including the three key variables.
- **CEE.** This is the monetary model of Christiano *et al.* (1999). In addition to the key variables in **SMALL**, this model includes the index of sensitive material prices (COMM PR) and monetary aggregates: non-borrowed reserves (NBORR RES), total reserves (TOT RES) and M2 money stock (M2).
- **MEDIUM.** This VAR extends the CEE model by the following variables: Personal Income (INCOME), Real Consumption (CONSUM), Industrial Production (IP), Capacity Utilization (CAP UTIL), Unemployment Rate (UNEMPL), Housing Starts (HOUS START), Producer Price Index (PPI), Personal Consumption Expenditures Price Deflator (PCE DEFL), Average Hourly Earnings (HOUR EARN), M1 Monetary Stock (M1), Standard and Poor's Stock Price Index (S&P); Yields on 10 year US Treasury Bond (TB YIELD) and effective exchange rate (EXR). The system contains, in total, 20 variables.
- **LARGE.** This specification includes all the 131 macroeconomic indicators of Stock and Watson's dataset.

It is important to stress that since we compare models of different size we need to have a strategy for how to choose the shrinkage hyperparameter as models become larger. As the dimension increases, we want to shrink more, as suggested by the analysis in De Mol *et al.* (2008) in order to control for over-fitting. A simple solution is to set the tightness of the prior so that all models have the same in-sample fit as the smallest VAR estimated by OLS. By ensuring that the in-sample fit is constant, i.e., independent of the model size, we can meaningfully compare results across models.

3. FORECAST EVALUATION

In this section we compare empirically forecasts resulting from different VAR specifications.

We compute point forecasts using the posterior mean of the parameters. We write $\hat{A}_j^{(\lambda, m)}$, $j = 1, \dots, p$ and $\hat{c}^{(\lambda, m)}$ for the posterior mean of the autoregressive coefficients and the constant

term of a given model (m) obtained by setting the overall tightness equal to λ . The point estimates of the h -step-ahead forecasts are denoted by $Y_{t+h|t}^{(\lambda,m)} = (y_{1,t+h|t}^{(\lambda,m)}, \dots, y_{n,t+h|t}^{(\lambda,m)})'$, where n is the number of variables included in model m . The point estimate of the one-step-ahead forecast is computed as $\hat{Y}_{t+1|t}^{(\lambda,m)} = \hat{c}^{(\lambda,m)} + \hat{A}_1^{(\lambda,m)} Y_t + \dots + \hat{A}_p^{(\lambda,m)} Y_{t-p+1}$. Forecasts h steps ahead are computed recursively.

In the case of the benchmark model the prior restriction is imposed exactly, i.e., $\lambda = 0$. Corresponding forecasts are denoted by $Y_{t+h|t}^{(0)}$ and are the same for all the specifications. Hence we drop the superscript m .

To simulate real-time forecasting we conduct an out-of-sample experiment. Let us denote by H the longest forecast horizon to be evaluated, and by T_0 and T_1 the beginning and the end of the evaluation sample, respectively. For a given forecast horizon h , in each period $T = T_0 + H - h, \dots, T_1 - h$, we compute h -step-ahead forecasts, $Y_{T+h|T}^{(\lambda,m)}$, using only the information up to time T .

Out-of-sample forecast accuracy is measured in terms of mean squared forecast error (MSFE):

$$\text{MSFE}_{i,h}^{(\lambda,m)} = \frac{1}{T_1 - T_0 - H + 1} \sum_{T=T_0+H-h}^{T_1-h} (y_{i,T+h|T}^{(\lambda,m)} - y_{i,T+h})^2$$

We report results for MSFE relative to the benchmark, i.e.,

$$\text{RMSFE}_{i,h}^{(\lambda,m)} = \frac{\text{MSFE}_{i,h}^{(\lambda,m)}}{\text{MSFE}_{i,h}^{(0)}}$$

Note that a number smaller than one implies that the VAR model with overall tightness λ performs better than the naive prior model.

We evaluate the forecast performance of the VARs for the three key series included in all VAR specifications (Employment, CPI and the Federal Funds Rate) over the period going from $T_0 = \text{January } 70$ until $T_1 = \text{December } 03$ and for forecast horizons up to one year ($H = 12$). The order of the VAR is set to $p = 13$ and parameters are estimated using for each T the observations from the most recent 10 years (rolling scheme).¹

The overall tightness is set to yield a desired average fit for the three variables of interest in the pre-evaluation period going from January 1960 ($t = 1$) until December 1969 ($t = T_0 - 1$) and then kept fixed for the entire evaluation period. In other words, for a desired Fit, λ is chosen as

$$\lambda_m(\text{Fit}) = \arg \min_{\lambda} \left| \text{Fit} - \frac{1}{3} \sum_{i \in \mathcal{I}} \frac{\text{msfe}_i^{(\lambda,m)}}{\text{msfe}_i^{(0)}} \right|$$

¹ Using all the available observations up to time T (recursive scheme) does not change the qualitative results. Qualitative results remain the same also if we set $p = 6$.

where $\mathcal{I} = \{\text{EMPL}, \text{CPI}, \text{FFR}\}$ and $\text{msfe}_i^{(\lambda, m)}$ is an in-sample one-step-ahead mean squared forecast error evaluated using the training sample $t = 1, \dots, T_0 - 1$.² More precisely:

$$\text{msfe}_i^{(\lambda, m)} = \frac{1}{T_0 - p - 1} \sum_{t=p}^{T_0-2} (y_{i,t+1|t}^{(\lambda, m)} - y_{i,t+1})^2$$

where the parameters are computed using the same sample $t = 1, \dots, T_0 - 1$.

In the main text we report the results where the desired fit coincides with the one obtained by OLS estimation on the small model with $p = 13$, i.e., for

$$\text{Fit} = \frac{1}{3} \sum_{i \in \mathcal{I}} \frac{\text{msfe}_i^{(\lambda, m)}}{\text{msfe}_i^{(0)}} \Bigg|_{\lambda=\infty, m=\text{SMALL}}$$

In the online Annex we present the results for a range of in-sample fits and show that they are qualitatively the same provided that the fit is not below 50%.

Table I presents the relative MSFE for forecast horizons $h = 1, 3, 6$ and 12 . The specifications are listed in order of increasing size and the last row indicates the value of the shrinkage hyperparameter λ . This has been set so as to maintain the in-sample fit fixed, which requires the degree of shrinkage, $1/\lambda$, to be larger the larger is the size of the model.

Three main results emerge from the table. First, adding information helps to improve the forecast for all variables included in the table and across all horizons. However, and this is a second important result, good performance is already obtained with the medium-size model containing 20 variables. This suggests that for macroeconomic forecasting there is no need to use much sectoral

Table I. BVAR, Relative MSFE, 1971–2003

		<i>SMALL</i>	<i>CEE</i>	<i>MEDIUM</i>	<i>LARGE</i>
$h = 1$	EMPL	1.14	0.67	0.54	0.46
	CPI	0.89	0.52	0.50	0.50
	FFR	1.86	0.89	0.78	0.75
$h = 3$	EMPL	0.95	0.65	0.51	0.38
	CPI	0.66	0.41	0.41	0.40
	FFR	1.77	1.07	0.95	0.94
$h = 6$	EMPL	1.11	0.78	0.66	0.50
	CPI	0.64	0.41	0.40	0.40
	FFR	2.08	1.30	1.30	1.29
$h = 12$	EMPL	1.02	1.21	0.86	0.78
	CPI	0.83	0.57	0.47	0.44
	FFR	2.59	1.71	1.48	1.93
λ		∞	0.262	0.108	0.035

Notes: The table reports MSFE relative to that from the benchmark model (random walk with drift) for employment (EMPL), CPI and federal funds rate (FFR) for different forecast horizons h and different models. *SMALL*, *CEE*, *MEDIUM* and *LARGE* refer to VARs with 3, 7, 20 and 131 variables, respectively. λ is the shrinkage hyperparameter.

² To obtain the desired magnitude of fit the search is performed over a grid for λ . Division by $\text{msfe}_i^{(0)}$ accounts for differences in scale between the series.

and conjunctural information beyond the 20 important macroeconomic variables since results do not improve significantly, although they do not get worse.³ Third, the forecast of the federal funds rate does not improve over the simple random walk model beyond the first quarter. We will see later that by adding additional priors on the sum of the coefficients these results, and in particular those for the federal funds rate, can be substantially improved.

3.1. Parsimony by Lags Selection

In VAR analysis there are alternative procedures to obtain parsimony. One alternative method to the BVAR approach is to implement information criteria for lag selection and then estimate the model by OLS. In what follows we will compare results obtained using these criteria with those obtained from the BVARs.

Table II presents the results for *SMALL* and *CEE*. We report results for $p = 13$ lags and for the number of lags p selected by the BIC criterion. For comparison, we also recall from Table I the results for the Bayesian estimation of the model of the same size. We do not report estimates for $p = 13$ and BIC selection for the large model since for that size the estimation by OLS and $p = 13$ is unfeasible. However, we recall in the last column the results for the large model estimated by the Bayesian approach.

These results show that for the model *SMALL* BIC selection results in the best forecast accuracy. For the larger *CEE* model, the classical VAR with lags selected by BIC and the BVAR perform similarly. Both specifications are, however, outperformed by the large Bayesian VAR.

Table II. OLS and BVAR, relative MSFE, 1971–2003

		<i>SMALL</i>			<i>CEE</i>			<i>LARGE</i>
		$p = 13$	$p = \text{BIC}$	BVAR	$p = 13$	$p = \text{BIC}$	BVAR	BVAR
$h = 1$	EMPL	1.14	0.73	1.14	7.56	0.76	0.67	0.46
	CPI	0.89	0.55	0.89	5.61	0.55	0.52	0.50
	FFR	1.86	0.99	1.86	6.39	1.21	0.89	0.75
$h = 3$	EMPL	0.95	0.76	0.95	5.11	0.75	0.65	0.38
	CPI	0.66	0.49	0.66	4.52	0.45	0.41	0.40
	FFR	1.77	1.29	1.77	6.92	1.27	1.07	0.94
$h = 6$	EMPL	1.11	0.90	1.11	7.79	0.78	0.78	0.50
	CPI	0.64	0.51	0.64	4.80	0.44	0.41	0.40
	FFR	2.08	1.51	2.08	15.9	1.48	1.30	1.29
$h = 12$	EMPL	1.02	1.15	1.02	22.3	0.82	1.21	0.78
	CPI	0.83	0.56	0.83	21.0	0.53	0.57	0.44
	FFR	2.59	1.59	2.59	47.1	1.62	1.71	1.93

Notes: The table reports MSFE relative to that from the benchmark model (random walk with drift) for employment (EMPL), CPI and federal funds rate (FFR) for different forecast horizons h and different models. *SMALL*, *CEE* refer to the VARs with 3 and 7 variables, respectively. Those systems are estimated by OLS with number of lags fixed to 13 or chosen by the BIC. For comparison, the results of Bayesian estimation of the two models and of the large model are also provided.

³ However, due to their timeliness, conjunctural information may be important for improving early estimates of variables in the current quarter, as argued by Giannone *et al.* (2008). This is an issue which we do not explore here.

3.2. The Bayesian VAR and the Factor Augmented VAR (FAVAR)

Factor models have been shown to be successful at forecasting macroeconomic variables with a large number of predictors. It is therefore natural to compare forecasting results based on the Bayesian VAR with those produced by factor models where factors are estimated by principal components.

A comparison of forecasts based, alternatively, on Bayesian regression and principal components regression has recently been performed by De Mol *et al.* (2008) and Giacomini and White (2006). In those exercises, variables are transformed to stationarity, as is standard practice in the principal components literature. Moreover, the Bayesian regression is estimated as a single equation.

Here we want to perform an exercise in which factor models are compared with the standard VAR specification in the macroeconomic literature, where variables are treated in levels and the model is estimated as a system rather than as a set of single equations. Therefore, for comparison with the VAR, rather than considering principal components regression, we will use a small VAR (with variables in levels) augmented by principal components extracted from the panel (in differences). This is the FAVAR method advocated by Bernanke *et al.* (2005) and discussed by Stock and Watson (2005b).

More precisely, principal components are extracted from the large panel of 131 variables. Variables are first made stationary by taking first differences wherever we have imposed a random walk prior $\delta_i = 1$. Then, as principal components are not scale invariant, variables are standardized and the factors are computed on standardized variables, recursively at each point T in the evaluation sample.

We consider specifications with one and three factors and look at different lag specification for the VAR. We set $p = 13$, as in Bernanke *et al.* (2005) and we also consider the p selected by the BIC criterion. Moreover, we consider Bayesian estimation of the FAVAR (BFAVAR), taking $p = 13$ and choosing the shrinkage hyperparameter λ that results in the same in-sample fit as in the exercise summarized in Table I.

Results are reported in Table III (the last column recalls results from the large Bayesian VAR for comparison).

The table shows that the FAVAR is in general outperformed by the BVAR of large size and that therefore Bayesian VAR is a valid alternative to factor-based forecasts, at least to those based on the FAVAR method.⁴ We should also note that BIC lag selection generates the best results for the FAVAR, while the original specification of Bernanke *et al.* (2005) with $p = 13$ performs very poorly due to its lack of parsimony.

3.3. Prior on the Sum of Coefficients

The literature has suggested that improvement in forecasting performance can be obtained by imposing additional priors that constrain the sum of coefficients (see, for example, Sims, 1992; Sims and Zha, 1998; Robertson and Tallman, 1999). This is the same as imposing ‘inexact differencing’ and it is a simple modification of the Minnesota prior involving linear combinations of the VAR coefficients (cf. Doan *et al.*, 1984).

⁴ De Mol *et al.* (2008) show that for regressions based on stationary variables principal components and Bayesian approach lead to comparable results in terms of forecast accuracy.

Table III. FAVAR, relative MSFE, 1971–2003

		FAVAR 1 factor			FAVAR 3 factors			<i>LARGE</i>
		$p = 13$	$p = \text{BIC}$	BVAR	$p = 13$	$p = \text{BIC}$	BVAR	BVAR
$h = 1$	EMPL	1.36	0.54	0.70	3.02	0.52	0.65	0.46
	CPI	1.10	0.57	0.65	2.39	0.52	0.58	0.50
	FFR	1.86	0.98	0.89	2.40	0.97	0.85	0.75
$h = 3$	EMPL	1.13	0.55	0.68	2.11	0.50	0.61	0.38
	CPI	0.80	0.49	0.55	1.44	0.44	0.49	0.40
	FFR	1.62	1.12	1.03	3.08	1.16	0.99	0.94
$h = 6$	EMPL	1.33	0.73	0.87	2.52	0.63	0.77	0.50
	CPI	0.74	0.52	0.55	1.18	0.46	0.50	0.40
	FFR	2.07	1.31	1.40	3.28	1.45	1.27	1.29
$h = 12$	EMPL	1.15	0.98	0.92	3.16	0.84	0.83	0.78
	CPI	0.95	0.58	0.70	1.98	0.54	0.64	0.44
	FFR	2.69	1.43	1.93	7.09	1.46	1.69	1.93

Notes: The table reports MSFE for the FAVAR model relative to that from the benchmark model (random walk with drift) for employment (EMPL), CPI and federal funds rate (FFR) for different forecast horizons h . FAVAR includes 1 or 3 factors and the three variables of interest. The system is estimated by OLS with number of lags fixed to 13 or chosen by the BIC and by applying Bayesian shrinkage. For comparison the results from large Bayesian VAR are also provided.

Let us rewrite the VAR of equation (1) in its error correction form:

$$\Delta Y_t = c - (I_n - A_1 - \dots - A_p)Y_{t-1} + B_1 \Delta Y_{t-1} + \dots + B_{p-1} \Delta Y_{t-p+1} + u_t \quad (8)$$

A VAR in first differences implies the restriction $(I_n - A_1 - \dots - A_p) = 0$. We follow Doan *et al.* (1984) and set a prior that shrinks $\Pi = (I_n - A_1 - \dots - A_p)$ to zero. This can be understood as ‘inexact differencing’ and in the literature it is usually implemented by adding the following dummy observations (cf. Section 2):

$$Y_d = \text{diag}(\delta_1 \mu_1, \dots, \delta_n \mu_n) / \tau \quad X_d = ((1_{1 \times p}) \otimes \text{diag}(\delta_1 \mu_1, \dots, \delta_n \mu_n) / \tau \quad 0_{n \times 1}) \quad (9)$$

The hyperparameter τ controls for the degree of shrinkage: as τ goes to zero we approach the case of exact differences and, as τ goes to ∞ , we approach the case of no shrinkage. The parameter μ_i aims at capturing the average level of variable y_{it} . Although the parameters should in principle be set using only prior knowledge, we follow common practice⁵ and set the parameter equal to the sample average of y_{it} . Our approach is to set a loose prior with $\tau = 10\lambda$. The overall shrinkage λ is again selected so as to match the fit of the small specification estimated by OLS.

Table IV reports results from the forecast evaluation of the specification with the sum of coefficient prior. They show that, qualitatively, results do not change for the smaller models, but improve significantly for the *MEDIUM* and *LARGE* specifications. In particular, the poor results for the federal funds rate discussed in Table I are now improved. Both the *MEDIUM* and *LARGE* models outperform the random walk forecasts at all the horizons considered. Overall, the sum of coefficient prior improves forecast accuracy, confirming the findings of Robertson and Tallman (1999).

⁵ See, for example, Sims and Zha (1998).

Table IV. BVAR, relative MSFE, 1971–2003 (with the prior on the sum of coefficients)

		<i>SMALL</i>	<i>CEE</i>	<i>MEDIUM</i>	<i>LARGE</i>
$h = 1$	EMPL	1.14	0.68	0.53	0.44
	CPI	0.89	0.57	0.49	0.49
	FFR	1.86	0.97	0.75	0.74
$h = 3$	EMPL	0.95	0.60	0.49	0.36
	CPI	0.66	0.44	0.39	0.37
	FFR	1.77	1.28	0.85	0.82
$h = 6$	EMPL	1.11	0.65	0.58	0.44
	CPI	0.64	0.45	0.37	0.36
	FFR	2.08	1.40	0.96	0.92
$h = 12$	EMPL	1.02	0.65	0.60	0.50
	CPI	0.83	0.55	0.43	0.40
	FFR	2.59	1.61	0.93	0.92

Notes to Table I apply. The difference is that the prior on the sum of coefficients has been added. The tightness of this prior is controlled by the hyperparameter $\tau = 10\lambda$, where λ controls the overall tightness.

4. STRUCTURAL ANALYSIS: IMPULSE RESPONSE FUNCTIONS AND VARIANCE DECOMPOSITION

We now turn to the structural analysis and estimate, on the basis of BVARs of different size, the impulse responses of different variables to a monetary policy shock.

To this purpose, we identify the monetary policy shock by using a recursive identification scheme adapted to a large number of variables. We follow Bernanke *et al.* (2005), Christiano *et al.* (1999) and Stock and Watson (2005b) and divide the variables in the panel into two categories: slow- and fast-moving. Roughly speaking, the former group contains real variables and prices, while the latter consists of financial variables (the precise classification is given in the Appendix). The identifying assumption is that slow-moving variables do not respond contemporaneously to a monetary policy shock and that the information set of the monetary authority contains only past values of the fast-moving variables.

The monetary policy shock is identified as follows. We order the variables as $Y_t = (X_t, r_t, Z_t)'$, where X_t contains the n_1 slowly moving variables, r_t is the monetary policy instrument and Z_t contains the n_2 fast-moving variables, and we assume that the monetary policy shock is orthogonal to all other shocks driving the economy. Let $B = CD^{1/2}$ be the $n \times n$ lower diagonal Cholesky matrix of the covariance of the residuals of the reduced form VAR, that is, $CDC' = \mathbb{E}[u_t u_t'] = \Psi$ and $D = \text{diag}(\Psi)$.

Let e_t be the following linear transformation of the VAR residuals: $e_t = (e_{1t}, \dots, e_{nt})' = C^{-1}u_t$. The monetary policy shock is the row of e_t corresponding to the position of r_t , that is, $e_{n_1+1,t}$.

The structural VAR can hence be written as

$$\mathcal{A}_0 Y_t = v + \mathcal{A}_1 Y_{t-1} + \dots + \mathcal{A}_p Y_{t-p} + e_t, \quad e_t \sim WN(0, D)$$

where $v = C^{-1}c$, $\mathcal{A}_0 = C^{-1}$ and $\mathcal{A}_j = C^{-1}A_j$, $j = 1, \dots, p$.

Our experiment consists in increasing contemporaneously the federal funds rate by 100 basis points.

Since we have just identification, the impulse response functions are easily computed following Canova (1991) and Gordon and Leeper (1994) by generating draws from the posterior of

(A_1, \dots, A_p, Ψ) . For each draw Ψ we compute B and C and we can then calculate \mathcal{A}_j , $j = 0, \dots, p$.

We report the results for the same overall shrinkage as given in Table IV. Estimation is based on the sample 1961–2002. The number of lags remains 13. Results are reported for the specification including sum of coefficients priors since it is the one providing the best forecast accuracy and also because, for the *LARGE* model, without sum of coefficients prior, the posterior coverage intervals of the impulse response functions become very wide for horizons beyond two years, eventually becoming explosive (cf. the online Annex). For the other specifications, the additional prior does not change the results.

Figure 1 displays the impulse response functions for the four models under consideration and for the three key variables. The shaded regions indicate the posterior coverage intervals corresponding to 90% and 68% confidence levels. Table V reports the percentage share of the monetary policy shock in the forecast error variance for chosen forecast horizons.

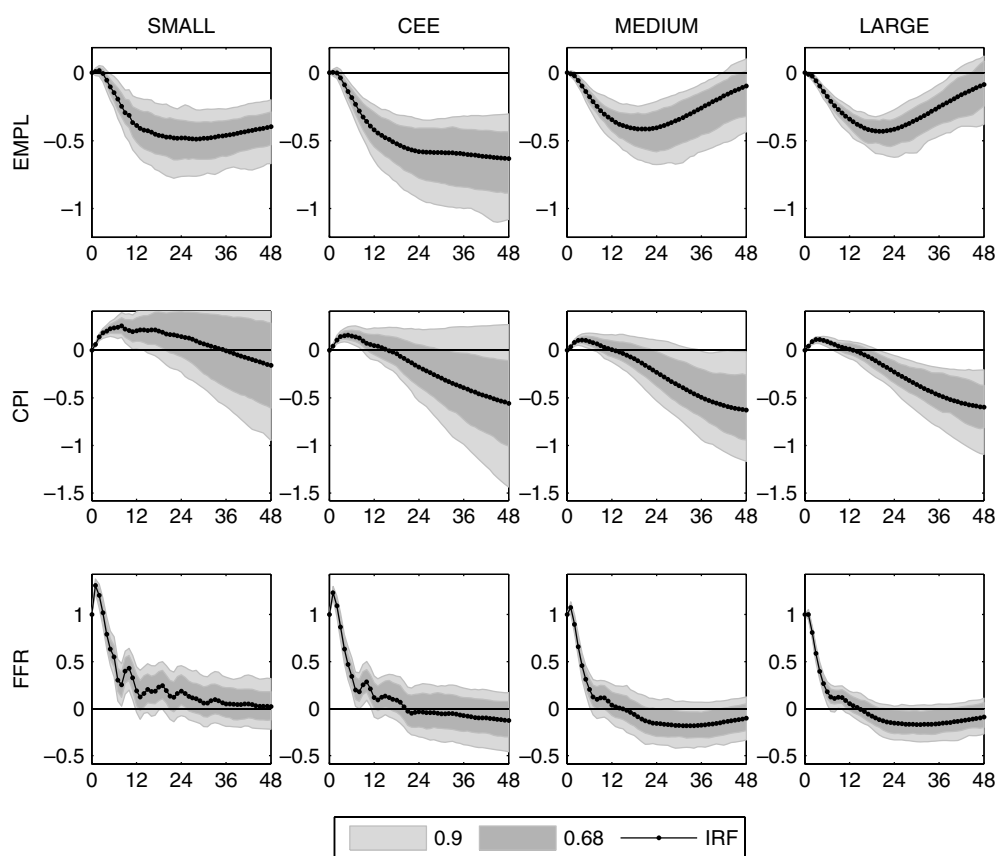


Figure 1. BVAR, impulse response functions. The figure presents the impulse response functions to a monetary policy shock and the corresponding posterior coverage intervals at 0.68 and 0.9 level for employment (EMPL), CPI and federal funds rate (FFR). *SMALL*, *CEE*, *MEDIUM* and *LARGE* refer to VARs with 3, 7, 20 and 131 variables, respectively. The prior on the sum of coefficients has been added with the hyperparameter $\tau = 10\lambda$.

Table V. BVAR, variance decomposition, 1961–2002

	Hor	<i>SMALL</i>	<i>CEE</i>	<i>MEDIUM</i>	<i>LARGE</i>
EMPL	1	0	0	0	0
	3	0	0	0	0
	6	1	1	2	2
	12	5	7	7	5
	24	12	14	13	8
	36	18	19	14	7
	48	23	23	12	6
CPI	1	0	0	0	0
	3	3	2	1	2
	6	7	5	3	3
	12	6	3	1	1
	24	2	1	1	1
	36	1	2	3	2
	48	1	3	5	3
FFR	1	99	97	93	51
	3	90	84	71	33
	6	74	66	49	21
	12	46	39	30	14
	24	26	21	18	9
	36	21	17	16	7
	48	18	15	16	7

Notes: The table reports the percentage share of the monetary policy shock in the forecast error variance for chosen forecast horizons for employment (EMPL), CPI and federal funds rate (FFR). *SMALL*, *CEE*, *MEDIUM* and *LARGE* refer to VARs with 3, 7, 20 and 131 variables, respectively. The prior on the sum of coefficients has been added with the hyperparameter $\tau = 10\lambda$.

Results show that, as we add information, impulse response functions slightly change in shape which suggests that conditioning on realistic informational assumptions is important for structural analysis as well as for forecasting. In particular, it is confirmed that adding variables helps in resolving the price puzzle (on this point see also Bernanke and Boivin, 2003; Christiano *et al.*, 1999). Moreover, for larger models the effect of monetary policy on employment becomes less persistent, reaching a peak at about one year horizon. For the large model, the non-systematic component of monetary policy becomes very small, confirming results in Giannone *et al.* (2004) obtained on the basis of a factor model. It is also important to stress that impulse responses maintain the expected sign for all specifications.

The same features can be seen from the variance decomposition, reported in Table V. As the size of the model increases, the size of the monetary policy shock decreases. This is not surprising, given the fact that the forecast accuracy improves with size, but it highlights an important point. If realistic informational assumptions are not taken into consideration, we may mix structural shocks with misspecification errors. Clearly, the assessment of the importance of the systematic component of monetary policy depends on the conditioning information set used by the econometrician and this may differ from that which is relevant for policy decisions. Once the realistic feature of large information is taken into account by the econometrician, the estimate of the size of the non-systematic component decreases.

Let us now comment on the impulse response functions of the monetary policy shock on all the 20 variables considered in the *MEDIUM* model. Impulse responses and variance decomposition for all the variables and models are reported in the online Annex.

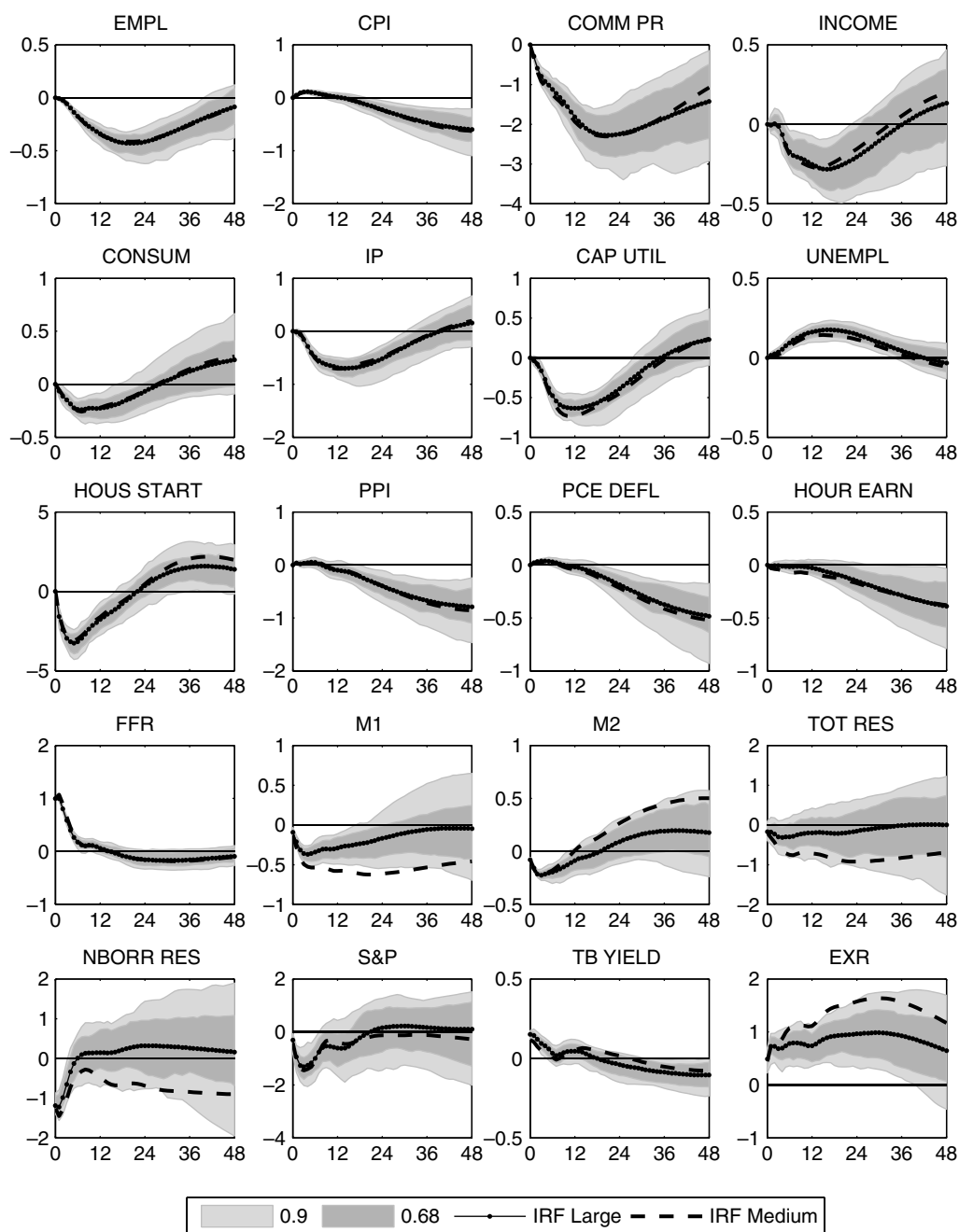


Figure 2. BVAR, impulse response functions for model *MEDIUM* and *LARGE*. The figure presents the impulse response functions to a monetary policy shock and the corresponding posterior coverage intervals at 0.68 and 0.9 level from *MEDIUM* and *LARGE* specifications for all the variables included in *MEDIUM*. The coverage intervals correspond to the *LARGE* specification. The prior on the sum of coefficients has been added with the hyperparameter $\tau = 10\lambda$.

Figure 2 reports the impulses for both the *MEDIUM* and *LARGE* model as well as the posterior coverage intervals produced by the *LARGE* model.

Let us first remark that the impulse responses are very similar for the two specifications and in most cases those produced by the *MEDIUM* model are within the coverage intervals of the *LARGE* model. This reinforces our conjecture that a VAR with 20 variables is sufficient to capture the relevant shocks and the extra information is redundant.

Responses have the expected sign. First of all, a monetary contraction has a negative effect on real economic activity. Besides employment, consumption, industrial production and capacity utilization respond negatively for two years and beyond. By contrast, the effect on all nominal variables is negative. Since the model contains more than the standard nominal and real variables, we can also study the effect of monetary shocks on housing starts, stock prices and exchange rate. The impact on housing starts is very large and negative and it lasts about one year. The effect on stock prices is significantly negative for about one year. Lastly, the exchange rate appreciation is persistent in both nominal and real terms as found in Eichenbaum and Evans (1995).

5. CONCLUSION

This paper assesses the performance of Bayesian VAR for monetary models of different size. We consider standard specifications in the literature with three and seven macroeconomic variables and also study VARs with 20 and 130 variables. The latter considers sectoral and conjunctural information in addition to macroeconomic information. We examine both forecasting accuracy and structural analysis of the effect of a monetary policy shock.

The setting of the prior follows standard recommendations in the Bayesian literature, except for the fact that the overall tightness hyperparameter is set in relation to the model size. As the model becomes larger, we increase the overall shrinkage so as to maintain the same in-sample fit across models and guarantee a meaningful model comparison.

Overall, results show that a standard Bayesian VAR model is an appropriate tool for large panels of data. Not only a Bayesian VAR estimated over 100 variables is feasible, but it produces better forecasting results than the typical seven variables VAR considered in the literature. The structural analysis on the effect of the monetary shock shows that a VAR based on 20 variables produces results that remain robust when the model is enlarged further.

ACKNOWLEDGEMENTS

We would like to thank Simon Price, Barbara Rossi, Bob Rasche and Shaun Vahey and seminar participants at the New York Fed, the Cleveland Fed, the Chicago Fed, the ECB, the Bank of England, Queen Mary University, University of Rome 'Tor Vergata', Universitat Autònoma de Barcelona, University of Warwick, the second forecasting conference at Duke University, the 38th Konstanz seminar in monetary theory and policy, CEF 2007, ISF 2007, EEA 2008, SED 2008 and Forecasting in Rio 2008. Support by the grant 'Action de Recherche Concertée' (no. 02/07-281) is gratefully acknowledged. The opinions expressed in this paper are those of the authors and do not necessarily reflect the views of the European Central Bank.

REFERENCES

- Bernanke BS, Boivin J. 2003. Monetary policy in a data-rich environment. *Journal of Monetary Economics* **50**(3): 525–546.
- Bernanke B, Boivin J, Elias P. 2005. Measuring monetary policy: a factor augmented autoregressive (FAVAR) approach. *Quarterly Journal of Economics* **120**: 387–422.
- Boivin J, Ng S. 2005. Understanding and comparing factor-based forecasts. *International Journal of Central Banking* **3**: 117–151.
- Canova F. 1991. The sources of financial crisis: pre- and post-Fed evidence. *International Economic Review* **32**(3): 689–713.
- Canova F, Ciccarelli M. 2004. Forecasting and turning point predictions in a Bayesian panel VAR model. *Journal of Econometrics*, **120**(2): 327–359.
- Christiano LJ, Eichenbaum M, Evans C. 1996. The effects of monetary policy shocks: evidence from the flow of funds. *Review of Economics and Statistics* **78**(1): 16–34.
- Christiano LJ, Eichenbaum M, Evans CL. 1999. Monetary policy shocks: what have we learned and to what end? In *Handbook of Macroeconomics*, Vol. 1, ch. 2, Taylor JB, Woodford M (eds). Elsevier: Amsterdam; 65–148.
- D'Agostino A, Giannone D. 2006. *Comparing alternative predictors based on large-panel factor models*. Working paper series 680, European Central Bank.
- De Mol C, Giannone D, Reichlin L. 2008. Forecasting using a large number of predictors: is Bayesian regression a valid alternative to principal components? *Journal of Econometrics* **146**: 318–328.
- Dees S, di Mauro F, Pesaran MH, Smith LV. 2007. Exploring the international linkages of the euro area: a global VAR analysis. *Journal of Applied Econometrics* **22**(1): 1–38.
- Doan T, Litterman R, Sims CA. 1984. Forecasting and conditional projection using realistic prior distributions. *Econometric Reviews* **3**: 1–100.
- Eichenbaum M, Evans CL. 1995. Some empirical evidence on the effects of shocks to monetary policy on exchange rates. *Quarterly Journal of Economics* **110**(4): 975–1009.
- Forni M, Hallin M, Lippi M, Reichlin L. 2000. The generalized dynamic factor model: identification and estimation. *Review of Economics and Statistics* **82**: 540–554.
- Forni M, Hallin M, Lippi M, Reichlin L. 2003. Do financial variables help forecasting inflation and real activity in the Euro Area? *Journal of Monetary Economics* **50**: 1243–1255.
- Forni M, Hallin M, Lippi M, Reichlin L. 2005. The generalized dynamic factor model: one-sided estimation and forecasting. *Journal of the American Statistical Association* **100**: 830–840.
- Forni M, Giannone D, Lippi M, Reichlin L. 2009. Opening the black box: structural factor models with large cross sections. *Econometric Theory* **25**(5): 1319–1347.
- Giacomini R, White H. 2006. Tests of conditional predictive ability. *Econometrica* **74**(6): 1545–1578.
- Giannone D, Reichlin L. 2006. Does information help recovering structural shocks from past observations? *Journal of the European Economic Association* **4**(2–3): 455–465.
- Giannone D, Reichlin L, Sala L. 2004. Monetary policy in real time. In *NBER Macroeconomics Annual*, Gertler M, Rogoff K (eds). MIT Press: Cambridge, MA; 161–200.
- Giannone D, Reichlin L, Small D. 2008. Nowcasting: the real-time informational content of macroeconomic data. *Journal of Monetary Economics* **55**(4): 665–676.
- Gordon DB, Leeper EM. 1994. The dynamic impacts of monetary policy: an exercise in tentative identification. *Journal of Political Economy* **102**(6): 1228–1247.
- Hamilton JD. 2006. *Computing power and the power of econometrics*. Manuscript, University of California, San Diego.
- Kadiyala KR, Karlsson S. 1997. Numerical methods for estimation and inference in Bayesian VAR-models. *Journal of Applied Econometrics* **12**(2): 99–132.
- Kim S. 2001. International transmission of U.S. monetary policy shocks: evidence from VAR's. *Journal of Monetary Economics* **48**(2): 339–372.
- Leeper EM, Sims C, Zha T. 1996. What does monetary policy do? *Brookings Papers on Economic Activity* **1996**(2): 1–78.
- Litterman R. 1986a. Forecasting With Bayesian vector autoregressions: five years of experience. *Journal of Business and Economic Statistics* **4**: 25–38.
- Litterman R. 1986b. A statistical approach to economic forecasting. *Journal of Business and Economic Statistics* **4**(1): 1–4.

- Mackowiak B. 2006. What does the Bank of Japan do to East Asia?. *Journal of International Economics* **70**(1): 253–270.
- Maćkowiak B. 2007. External shocks, U.S. monetary policy and macroeconomic fluctuations in emerging markets. *Journal of Monetary Economics* **54**(8): 2512–2520.
- Marcellino M, Stock JH, Watson MW. 2003. Macroeconomic forecasting in the euro area: country specific versus area-wide information. *European Economic Review* **47**(1): 1–18.
- Robertson JC, Tallman EW. 1999. Vector autoregressions: forecasting and reality. *Economic Review* **Q1**: 4–18.
- Sims CA. 1992. *Bayesian inference for multivariate time series with trend*. Mimeo Yale University.
- Sims CA, Zha T. 1998. Bayesian methods for dynamic multivariate models. *International Economic Review* **39**(4): 949–968.
- Stock JH, Watson MW. 2002a. Forecasting using principal components from a large number of predictors. *Journal of the American Statistical Association* **97**: 1167–1179.
- Stock JH, Watson MW. 2002b. Macroeconomic forecasting using diffusion indexes. *Journal of Business and Economics Statistics* **20**: 147–162.
- Stock JH, Watson MW. 2005a. *An empirical comparison of methods for forecasting using many predictors*. Manuscript, Princeton University.
- Stock JH, Watson MW. 2005b. *Implications of dynamic factor models for VAR analysis*. Manuscript, Princeton University.
- Uhlig H. 2004. What moves GNP? In *Econometric Society 2004 North American Winter Meetings 636*, Econometric Society.

Appendix: Description of the Dataset

Mnemonic	Series	Slow/Fast	SMALL	CEE	MEDIUM	Log	RW	prior
CES002	EMPLOYEES ON NONFARM PAYROLLS - TOTAL PRIVATE	S	X	X	X	X		X
PUNEW	CPI-U; ALL ITEMS (82-84=100,SA)	S	X	X	X	X		X
PSM99Q	INDEX OF SENSITIVE MATERIALS PRICES (1990=100)(BCI-99A)	S		X	X	X		X
A0M051	PERSONAL INCOME LESS TRANSFER PAYMENTS (AR, BIL. CHAIN 2000 \$)	S			X	X		X
A0M224_R	REAL CONSUMPTION (AC) A0M224/GMDC	S			X	X		X
IPS10	INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX	S			X	X		X
A0M082	CAPACITY UTILIZATION (MFG)	S			X			X
LHUR	UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%SA)	S			X			X
HSFR	HOUSING STARTS: NONFARM(1947-58); TOTAL FARM&NONFARM(1959-)(THOUS.,SA)	S			X			X
PWFSA	PRODUCER PRICE INDEX: FINISHED GOODS (1982=100,SA)	S			X	X		X
GMDC	PCE,IMPL PR DEFL: PCE (1987=100)	S			X	X		X
CES275	AVG HRLY EARNINGS OF PROD OR NONSUP WORKERS ON PRIV NONFARM PAYROLLS - GOODS PRODUCING	S			X	X		X
A0M052	PERSONAL INCOME (AR, BIL. CHAIN 2000 \$)	S				X		X
A0M057	MANUFACTURING AND TRADE SALES (MIL. CHAIN 1996 \$)	S				X		X
A0M059	SALES OF RETAIL STORES (MIL. CHAIN 2000 \$)	S				X		X
IPS11	INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL	S				X		X
IPS299	INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS	S				X		X
IPS12	INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS	S				X		X
IPS13	INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS	S				X		X
IPS18	INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS	S				X		X
IPS25	INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT	S				X		X
IPS32	INDUSTRIAL PRODUCTION INDEX - MATERIALS	S				X		X
IPS34	INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS	S				X		X
IPS38	INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS	S				X		X
IPS43	INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC)	S				X		X
IPS307	INDUSTRIAL PRODUCTION INDEX - RESIDENTIAL UTILITIES	S				X		X
IPS306	INDUSTRIAL PRODUCTION INDEX - FUELS	S				X		X
PMP	NAPM PRODUCTION INDEX (PERCENT)	S						X
LHEL	INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)	S						X
LHEHX	EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF	S						X
LHEM	CIVILIAN LABOR FORCE; EMPLOYED, TOTAL (THOUS,SA)	S				X		X
LHNAG	CIVILIAN LABOR FORCE; EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA)	S				X		X
LHU680	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)	S						X
LHU5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)	S				X		X

Appendix: (Continued)

Mnemonic	Series	Slow/Fast	SMALL	CEE	MEDIUM	Log	RW	prior
LHU14	UNEMPLOY BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)	S				X		X
LHU15	UNEMPLOY BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)	S				X		X
LHU26	UNEMPLOY BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)	S				X		X
LHU27	UNEMPLOY BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS.,SA)	S				X		X
A0M005	AVERAGE WEEKLY INITIAL CLAIMS, UNEMPLOY. INSURANCE (THOUS.)	S				X		X
CES003	EMPLOYEES ON NONFARM PAYROLLS - GOODS-PRODUCING	S				X		X
CES006	EMPLOYEES ON NONFARM PAYROLLS - MINING	S				X		X
CES011	EMPLOYEES ON NONFARM PAYROLLS - CONSTRUCTION	S				X		X
CES015	EMPLOYEES ON NONFARM PAYROLLS - MANUFACTURING	S				X		X
CES017	EMPLOYEES ON NONFARM PAYROLLS - DURABLE GOODS	S				X		X
CES033	EMPLOYEES ON NONFARM PAYROLLS - NONDURABLE GOODS	S				X		X
CES046	EMPLOYEES ON NONFARM PAYROLLS - SERVICE-PROVIDING	S				X		X
CES048	EMPLOYEES ON NONFARM PAYROLLS - TRADE, TRANSPORTATION, AND UTILITIES	S				X		X
CES049	EMPLOYEES ON NONFARM PAYROLLS - WHOLESALE TRADE	S				X		X
CES053	EMPLOYEES ON NONFARM PAYROLLS - RETAIL TRADE	S				X		X
CES088	EMPLOYEES ON NONFARM PAYROLLS - FINANCIAL ACTIVITIES	S				X		X
CES140	EMPLOYEES ON NONFARM PAYROLLS - GOVERNMENT	S				X		X
A0M048	EMPLOYEE HOURS IN NONAG. ESTABLISHMENTS (AR, BIL. HOURS)	S				X		X
CES151	AVG WEEKLY HRS OF PROD OR NONSUP WORKERS ON PRIV NONFAR PAYROLLS - GOODS PRODUCING	S						
CES155	AVG WEEKLY HRS OF PROD OR NONSUP WORKERS ON PRIV NONFAR PAYROLLS - MFG OVERTIME HRS	S						X
A0M001	AVERAGE WEEKLY HOURS, MFG. (HOURS)	S						
PMEMP	NAPM EMPLOYMENT INDEX (PERCENT)	S						
HSNE	HOUSING STARTS : NORTHEAST (THOUS.U.,S.A.)	S				X		
HSMW	HOUSING STARTS : MIDWEST(THOUS.U.,S.A.)	S				X		
HSSOU	HOUSING STARTS : SOUTH (THOUS.U.,S.A.)	S				X		
HSWST	HOUSING STARTS : WEST (THOUS.U.,S.A.)	S				X		
HSBR	HOUSING AUTHORIZED: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)	S				X		
HSBNE	HOUSES AUTHORIZED BY BUILD. PERMITS : NORTHEAST(THOUS.U.,S.A.)	S				X		
HSBMW	HOUSES AUTHORIZED BY BUILD. PERMITS : MIDWEST(THOUS.U.,S.A.)	S				X		
HSBSOU	HOUSES AUTHORIZED BY BUILD. PERMITS : SOUTH(THOUS.U.,S.A.)	S				X		
HSBWST	HOUSES AUTHORIZED BY BUILD. PERMITS : WEST(THOUS.U.,S.A.)	S				X		
PMI	PURCHASING MANAGERS' INDEX (SA)	S						
PMNO	NAPM NEW ORDERS INDEX (PERCENT)	S						
PMDEL	NAPM VENDOR DELIVERIES INDEX (PERCENT)	S						

Appendix: (Continued)

Mnemonic	Series	Slow/Fast	SMALL	CEE	MEDIUM	Log	RW	prior
PMNV	NAPM INVENTORIES INDEX (PERCENT)	S						
A0M008	MFRS' NEW ORDERS, CONSUMER GOODS AND MATERIALS (BIL. CHAIN 1982 \$)	S				X		X
A0M007	MFRS' NEW ORDERS, DURABLE GOODS INDUSTRIES (BIL. CHAIN 2000 \$)	S						X
A0M027	MFRS' NEW ORDERS, NONDEFENCE CAPITAL GOODS (MIL. CHAIN 1982 \$)	S				X		X
A1M092	MFRS' UNFILLED ORDERS, DURABLE GOODS INDUS. (BIL. CHAIN 2000 \$)	S				X		X
A0M070	MANUFACTURING AND TRADE INVENTORIES (BIL. CHAIN 2000 \$)	S				X		X
A0M077	RATIO, MFG. AND TRADE INVENTORIES TO SALES BASED ON CHAIN 2000 \$)	S						X
PWFCSA	PRODUCER PRICE INDEX: FINISHED CONSUMER GOODS (1982=100,SA)	S				X		X
PWIMSA	PRODUCER PRICE INDEX: INTERMED MAT.SUPPLIES &COMPONENTS(1982=100,SA)	S				X		X
PWCM5A	PRODUCER PRICE INDEX: CRUDE MATERIALS (1982=100,SA)	S				X		X
PMCP	NAPM COMMODITY PRICES INDEX (PERCENT)	S						
PUS3	CPI-U: APPAREL &UPKEEP (1982-84=100,SA)	S				X		X
PUS4	CPI-U: TRANSPORTATION (1982-84=100,SA)	S				X		X
PUS5	CPI-U: MEDICAL CARE (1982-84=100,SA)	S				X		X
PUC	CPI-U: COMMODITIES (1982-84=100,SA)	S				X		X
PUCD	CPI-U: DURABLES (1982-84=100,SA)	S				X		X
PUS	CPI-U: SERVICES (1982-84=100,SA)	S				X		X
PUXF	CPI-U: ALL ITEMS LESS FOOD (1982-84=100,SA)	S				X		X
PUXHS	CPI-U: ALL ITEMS LESS SHELTER (1982-84=100,SA)	S				X		X
PUXM	CPI-U: ALL ITEMS LESS MIDICAL CARE (1982-84=100,SA)	S				X		X
GMDCD	PCE:IMPL PR DEFL: PCE; DURABLES (1987=100)	S				X		X
GMDCN	PCE:IMPL PR DEFL: PCE; NONDURABLES (1996=100)	S				X		X
GMDCS	PCE:IMPL PR DEFL: PCE; SERVICES (1987=100)	S				X		X
CES277	AVG HRLY EARNINGS OF PROD OR NONSUP WORKERS ON PRIV NONFARM PAYROLLS - CONSTRUCTION	S				X		X
CES278	AVG HRLY EARNINGS OF PROD OR NONSUP WORKERS ON PRIV NONFARM PAYROLLS - MANUFACTURING	S				X		X
HHSNTN	U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)	S						
FYFF	INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA)	R	X	X	X			
FM2	MONEY STOCK: M2(MI+O+NITE RPS,EUROS,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL.\$,SA)	F		X	X			
FMRA	DEPOSITORY INST RESERVES: TOTAL,ADJ FOR RESERVE REQ	F		X	X			
FMNBA	CHGS(MIL.\$,SA) DEPOSITORY INST RESERVES: NONBORROWED,ADJ RES REQ CHGS(MIL.\$,SA)	F		X	X			

Appendix: (Continued)

Mnemonic	Series	Slow/Fast	SMALL	CEE	MEDIUM	Log	RW	prior
FM1	MONEY STOCK: M1(CURR,TRAV,CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL,\$.SA)	F			X	X		X
FSPCOM	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)	F			X	X		X
FYGT10	INTEREST RATE: U.S.TREASURY CONST MATURITIES, 10-YR.(% PER ANN,NSA)	F			X			X
EXRUS	UNITED STATES:EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)	F			X	X		X
FM3	MONEY STOCK: M3(M2+LG TIME DEP,TERM RP'S&INST ONLY MMMFS)(BIL,\$.SA)	F			X	X		X
FM2DQ	MONEY SUPPLY - M2 IN 1996 DOLLARS (BCI)	F				X		X
FMFBA	MONEY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL,\$.SA)	F				X		X
FCLNQ	COMMERCIAL & INDUSTRIAL LOANS OUTSTANDING IN 1996 DOLLARS (BCI)	F				X		X
FCLBMC	WKLY RP LG COM'L BANKS: NET CHANGE COM'L & INDUS LOANS(BIL,\$.SAAR)	F						
CCINRV	CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)	F				X		X
A0M095	RATIO, CONSUMER INSTALLMENT CREDIT TO PERSONAL INCOME (PERCENT)	F						X
FSPIN	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)	F				X		X
FSDXP	S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)	F						X
FSPXE	S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (% NSA)	F				X		X
CP90	COMMERCIAL PAPER RATE (AC)	F						X
FYGM6	INTEREST RATE: U.S.TREASURY BILLS, SEC MKT, 6-MO. (% PER ANN,NSA)	F						X
FYGT1	INTEREST RATE: U.S.TREASURY CONST MATURITIES, 1-YR. (% PER ANN,NSA)	F						X
FYGT5	INTEREST RATE: U.S.TREASURY CONST MATURITIES, 5-YR. (% PER ANN,NSA)	F						X
FYGM3	INTEREST RATE: U.S.TREASURY BILLS, SEC MKT, 3-MO. (% PER ANN,NSA)	F						X
FYAAAC	BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)	F						X
FYBAAC	BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)	F						X
SCP90	CP90-FYFF	F						X
SFYGM3	FYGM3-FYFF	F						X
SFYGM6	FYGM6-FYFF	F						X
sFYGT1	FYGT1-FYFF	F						X
sFYGT5	FYGT5-FYFF	F						X
sFYGT10	FYGT10-FYFF	F						X
sFYAAAC	FYAAAC-FYFF	F						X
sFYBAAC	FYBAAC-FYFF	F						X
EXRSW	FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.\$)	F				X		X
EXRIAN	FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.\$)	F				X		X
EXRUK	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)	F				X		X
EXRCAN	FOREIGN EXCHANGE RATE: CANADA (CANADIAN \$ PER U.S.\$)	F				X		X