SUPPORT VECTOR MULTI-REGRESSION AND EQUIVALENT 2D MODELLING FOR 3D ANTENNA ARRAY SYNTHESIS

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Abstract

A novel 3D antenna array modelling and synthesis technique based on the Support Vector Regression (SVR) framework and equivalent 2D modelling of 3D systems is presented. It allows taking into account all the real properties and behaviour of an antenna array, including coupling effects or non-idealities, and developing accurate models to be included in the synthesis schemes without increase of the complexity. Both 2D and 3D problems can be solved.

1 Introduction

Support Vector Machines (SVM) [1] are powerful state-of-the-art data-learning techniques based on the Structural Risk Minimization (SRM) principle. The traditional Empirical Risk Minimization (ERM) principle aims finding a function that relates the available input and output data, while the SRM intends that such function does not only fit the available data but also any data not taken into account in the regression, what increases its generalization capabilities. Recently, the support vector theory has been extended to the domain of regression problems [2] becoming a powerful base to solve different problems, such as filter design [3], direction of arrival estimation [4] or ideal-element antenna array synthesis [5] with excellent results.

In this paper we propose the use of a multiple Support Vector Regression (SVR), a recently developed multi-regression technique [6] and an equivalent 2D modelling of 3D problems to develop accurate 3D models of a given antenna array from sets of feeding values/radiated field data obtained from measurement or simulation. The use of data-learning techniques leads to real models able to account for the real properties and behaviour of the antenna array, including coupling effects, non-idealities, non-uniformities, etc.

Once such models have been developed, they can be easily included in different synthesis schemes increasing their accuracy and without any additional complexity, from the designer's point of view, or computational cost.

2 Support vector characterization

Let $\mathbf{v}$ be a column vector containing the M feeding values applied to an antenna array. The value of the radiation pattern in the direction given by the θ, φ-coordinates can be expressed as:

$$E(\theta,\varphi) = \mathbf{v}^T \mathbf{g}(\theta,\varphi) \tag{1}$$

where $\mathbf{g}(\theta,\varphi)=(g_0(\theta,\varphi), g_1(\theta,\varphi),\ldots, g_{M\text{-}1}(\theta,\varphi))^T$; $(.)^T$ denotes the transpose, and $g_i(\theta,\varphi)$ is a term that indicates the influence of the i-th radiating element in the direction $\{\theta,\varphi\}$. For each φ-plane, a matrix $\mathbf{G}(\varphi)=(\mathbf{g}(\theta_1,\varphi), \mathbf{g}(\theta_2,\varphi),\ldots\ \mathbf{g}(\theta_N,\varphi))$ can be defined, allowing to calculate the field distribution radiated by the structure in the directions θ_i, $i=1\ldots N$, being N the number of θ-planes, when the voltage set is applied, as

$$\mathbf{e}^T(\varphi) = \mathbf{v}^T \mathbf{G}(\varphi) \tag{2}$$

where $\mathbf{e}(\varphi)=(E(\theta_1,\varphi), E(\theta_2,\varphi),\ldots, E(\theta_N,\varphi))^T$ is a vector containing samples of the radiated field distribution at all the directions of interest. Other φ-planes can be considered defining a set of matrices $\mathbf{G}(\varphi_i)$, $i=1,\ldots,L$, being L the number of considered φ-planes. Then, the matrix model can now be expressed as a 3D matrix $\underline{\mathbf{G}}$ with dimensions $M\times N\times L$ defined as the combination of the L matrices with dimensions $M\times N$ corresponding to each φ-plane.

Let us consider that P voltage sets and their corresponding field distributions $\{\mathbf{v}_n, E_n(\theta,\varphi)\}$, $n=1,\ldots,P$, are available for training purposes. For a given direction of the space $\{\theta_i,\varphi_j\}$, the SVR theory establishes that $\mathbf{g}(\theta_i,\varphi_j)$ can be obtained through the minimization of the following cost function:

$$J(\mathbf{g}(\theta_i,\varphi_j)) = \frac{1}{2}\left\|\mathbf{g}(\theta_i,\varphi_j)\right\|^2 + C\sum_{n=1}^{P}\left|E_n(\theta_i,\varphi_j) - \mathbf{g}(\theta_i,\varphi_j)^T\mathbf{v}_n\right|_{\varepsilon} \tag{3}$$

where

$$\left|E_n(\theta,\varphi)-\mathbf{g}(\theta,\varphi)^T\mathbf{v}_n\right|_\varepsilon=\max\left(0,\left|E_n(\theta,\varphi)-\mathbf{g}(\theta,\varphi)^T\mathbf{v}_n\right|-\varepsilon\right) \tag{4}$$

is the so-called Vapnik's ε-insensitive loss function, and $C>0$ is a penalty value which establishes a trade-off between the model complexity and the cost of deviations larger than ε. Each vector $\mathbf{v}_n$ is considered as the input to the regressor, while $E_n(\theta_i)$ is the desired output. The P training patterns are used for the regression introducing a set of positive slack variables ξ_n and $\tilde{\xi}_n$ in order to deal with the fact that the data used for training could be noisy and the linear function considered for the regression could not be feasible. Then, the minimization of (3) can be rewritten as the constrained optimization problem of minimizing

$$L(\mathbf{g}(\theta_i,\varphi),\xi,\tilde{\xi})=\frac{1}{2}\left\|\mathbf{g}(\theta_i,\varphi)\right\|^2+C\sum_{n=1}^{P}(\xi_n+\tilde{\xi}_n) \tag{5}$$

subject to

$$\mathbf{g}(\theta_i,\varphi)^T\mathbf{v}_n-E_n(\theta_i,\varphi)\le\varepsilon+\xi_n \tag{6}$$

$$E_n(\theta_i,\varphi)-\mathbf{g}(\theta_i,\varphi)^T\mathbf{v}_n\le\varepsilon+\tilde{\xi}_n \tag{7}$$

$$\xi_n,\tilde{\xi}_n\ge 0 \tag{8}$$

for all $n=1\ldots P$.

This problem can be solved through the definition of a Lagrangian function [1]. The Lagrange technique establishes that the first derivative of this Lagrangian function with respect to all the variables of interest mush vanish, what leads to an equivalent dual problem given by the maximization of

$$\begin{aligned}W(\alpha,\tilde{\alpha})=&-\sum_{n=1}^{P}\varepsilon(\tilde{\alpha}_n+\alpha_n)+\sum_{n=1}^{P}E_n(\theta_i,\varphi)(\tilde{\alpha}_n+\alpha_n)\\&-\frac{1}{2}\sum_{n,m=1}^{P}(\tilde{\alpha}_n+\alpha_n)(\tilde{\alpha}_m+\alpha_m)\langle\mathbf{v}_n,\mathbf{v}_m\rangle\end{aligned} \tag{9}$$

subject to $0\le\tilde{\alpha}_n,\alpha_n\le C$, and where $\langle\mathbf{v}_n,\mathbf{v}_m\rangle$ denotes the inner product between the input voltage sets. This is a convex quadratic programming (QP) problem and, therefore, has a globally optimal solution that can be efficiently found using [7]. Then, the optimal regressor can be obtained as

$$\mathbf{g}_{QP}(\theta_i,\varphi)=\sum_{n=1}^{P}(\tilde{\alpha}_n-\alpha_n)\mathbf{v}_n \tag{10}$$

According to support vector theory, the input patterns (voltage sets) that appear in the expansion (10) are points where exactly one of the Lagrange multipliers is greater than zero. These input patterns (typically less than P) are called *support vectors*.

Repeating this regression for each of the considered N directions of the space, an estimation of the 3D matrix model $\underline{\mathbf{G}}$ is obtained. If the training patterns have been obtained including the real antenna characteristics (mutual coupling, presence of parasitic elements, etc.) the obtained model will also include them.

This approach to SVR-based modelling of the array must be performed through as many optimization problems as directions of the space are considered, as SVR is limited to vector calculation. Recently Support Vector Multi-Regression (SVMR) has been proposed [6] for calculation of a matrix relating input and output. SVMR can be used to calculate the matrix $\mathbf{G}(\varphi_i)$ instead of $\mathbf{g}(\theta_i,\varphi_j)$, reducing the number of optimization problems from $N\cdot L$ to L. Furthermore, a proper formulation of the problem using an equivalent 2D problem for 3D problems can require only a single optimization. This equivalent 2D formulation can be stated as follows.

Let us define a matrix $\tilde{\mathbf{G}}=(\mathbf{G}(\varphi_1),\ \mathbf{G}(\varphi_2),\ldots,\ \mathbf{G}(\varphi_L))$, with dimensions $M\times NL$. An extended field vector can also be defined as $\tilde{\mathbf{e}}=(\mathbf{e}(\varphi_1),\ \mathbf{e}(\varphi_2),\ldots,\ \mathbf{e}(\varphi_L))^T$ and it can be calculated as

$$\tilde{\mathbf{e}}^T=\mathbf{v}^T\tilde{\mathbf{G}} \tag{11}$$

Equations (2) and (11) represent a 2D problem, but (11) includes information about all the considered directions of the space, so it can be considered an equivalent 2D formulation of a 3D problem. Once this 2D equivalent formulation has been defined, SVMR can be used to calculate $\tilde{\mathbf{G}}$ and then the 3D matrix $\underline{\mathbf{G}}$ in a single optimization problem.

In SVMR, equation (5) becomes

$$L(\tilde{\mathbf{g}},\xi,\tilde{\xi})=\frac{1}{2}\sum_{j=1}^{NL}\left\|\tilde{\mathbf{g}}_j\right\|^2+C\sum_{n=1}^{P}L(\xi_n+\tilde{\xi}_n) \tag{12}$$

where $\tilde{\mathbf{g}}_i$ represents the i-th column of the matrix $\tilde{\mathbf{G}}$, and $L(u)$ is a function defined as:

$$L(u)=\begin{cases}0, & u<\varepsilon\\ u^2-2u\varepsilon+\varepsilon^2 & u\ge\varepsilon\end{cases} \tag{13}$$

The use of this function instead of Vapnik's ε-insensitive loss function reduces the accuracy of this regression, but allows considering only one optimization because the use of the $l2$ norm couples all the regressions.

Once more, if the training data have been obtained taking into account coupling effects in the antenna array, the matrix model will also take them into account and can be included in different synthesis schemes providing accurate solutions without any increase of the complexity.

3 Support vector synthesis

After the matrix $\underline{\mathbf{G}}$ has been calculated, the field radiated by the modeled antenna, when a specific voltage set is applied to it, can be calculated using the expression (11). If a radiation pattern is specified in terms of its amplitude and phase, it is possible to directly calculate the corresponding voltage set using the expression

$$\mathbf{v}^T = \mathbf{e}^T \underline{\mathbf{G}}^{-1} \tag{14}$$

where $\underline{\mathbf{G}}^{-1}$ is named the pseudoinverse matrix of $\underline{\mathbf{G}}$ and is defined as 3D-matrix combination of the pseudoinverse matrices $\mathbf{G}^{-1}(\varphi_i)$.

Typically, information about the phase distribution of the radiation pattern is not available or is irrelevant for synthesis and the expression (14) cannot be considered valid to obtain the voltages corresponding to a specified radiation pattern. Additional constrains or information must be included in the synthesis process to complete the information. An iterative phase retrieval scheme can be used to reconstruct a coherent phase distribution according to the properties of the specified radiating structure.

The process consists on the use of the model of the radiating structure previously calculated, involving the matrix $\underline{\mathbf{G}}$. Let $|\mathbf{e}|=(|\mathbf{e}(\varphi_1)|, |\mathbf{e}(\varphi_2)|,\ldots, |\mathbf{e}(\varphi_N)|)^T$ be a vector containing the desired radiation pattern specified in amplitude along $N\times L$ directions of the space θ_i, i=1,…,N and φ_i, i=1,…,L. A null initial phase distribution is considered together with the specified radiation pattern amplitude, generating an *extended* radiation pattern. Through (14) an approximation to the necessary voltages is obtained. This approximation is used in (11) to obtain the corresponding radiation pattern, whose amplitude is compared with the specified one. If the error does not comply with a predefined stop criterion, the phase of the reconstructed radiation pattern is used together with the specified amplitude, generating another extended radiation pattern used in a new iteration. This process is graphically represented in figure 1.

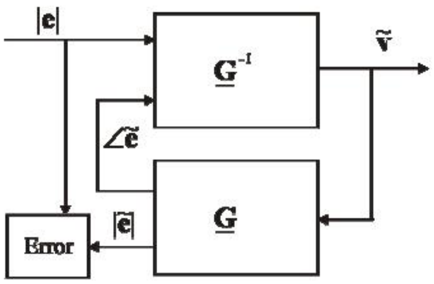

Figure 1: Iterative scheme for synthesis from field amplitude.

In most practical applications, specifications for synthesis problems are given as a set of parameters that can be translated into a template defining the maximum and minimum value of the radiation pattern amplitude at each aspect angle, $Bmax(\theta,\varphi)$ and $Bmin(\theta,\varphi)$ respectively. Vapnik's *ε-insensitive* loss function (4) adopts a form that allows adapting the SVR problem to the resolution of this kind of problem [5] once the 3D matrix $\underline{\mathbf{G}}$ has been approximated. Making use again of the SRM principle, the new cost function to be minimized is

$$J(\mathbf{v}) = \frac{1}{2}\|\mathbf{v}\|^2 + C\sum_{i=1}^{NL}\left|M(\theta_i,\varphi_j) - \mathbf{v}^T\mathbf{g}(\theta_i,\varphi_j)\right|_{\varepsilon(\theta_i,\varphi_j)} \tag{15}$$

where $M(\theta,\varphi)=[Bmax(\theta,\varphi)+Bmin(\theta,\varphi)]/2$, is the medium value between the maximum and minimum allowed values, and $\varepsilon(\theta,\varphi)$ is the difference between $M(\theta,\varphi)$ and $Bmax(\theta,\varphi)$ (or $Bmin(\theta,\varphi)$). The regression will try to provide a radiation pattern equal to $M(\theta,\varphi)$ with a maximum error $\varepsilon(\theta,\varphi)$ guaranteeing that the obtained field will comply with the specified 3D template. It is also important to notice that the "fictitious field" $M(\theta,\varphi)$ is only specified in terms of amplitude. As developed in [-], field amplitude-only data can be expressed through the $l2$-norm as

$$\|M(\theta_i)\|^2 = \|\mathbf{v}^T\mathbf{g}(\theta_i)\|^2 \tag{16}$$

Taking into account that $\mathbf{v}$, $\mathbf{g}(\theta,\varphi)$ and, $M(\theta,\varphi)$ are, in general, complex values, (14) can be written as

$$\|\mathbf{v}^T\mathbf{g}(\theta_i,\varphi_j)\|^2 = \mathrm{Re}\{\mathbf{v}^T\mathbf{g}(\theta_i,\varphi_j)\}^2 + \mathrm{Im}\{\mathbf{v}^T\mathbf{g}(\theta_i,\varphi_j)\}^2 \tag{17}$$

This expression can also be rewritten as

$$\begin{aligned}\|\mathbf{v}^T\mathbf{g}(\theta_i)\|^2 &= \mathrm{Re}\{M(\theta_i)\}\begin{bmatrix}\mathrm{Re}\{\mathbf{v}^T\} & \mathrm{Im}\{\mathbf{v}^T\}\end{bmatrix}\begin{bmatrix}\mathrm{Re}\{\mathbf{g}(\theta_i)\}\\ -\mathrm{Im}\{\mathbf{g}(\theta_i)\}\end{bmatrix}\\ &+\mathrm{Im}\{M(\theta_i)\}\begin{bmatrix}\mathrm{Re}\{\mathbf{v}^T\} & \mathrm{Im}\{\mathbf{v}^T\}\end{bmatrix}\begin{bmatrix}\mathrm{Im}\{\mathbf{g}(\theta_i)\}\\ \mathrm{Re}\{\mathbf{g}(\theta_i)\}\end{bmatrix}\end{aligned} \tag{18}$$

or, in a more compact formulation, as

$$\|\mathbf{v}^T\mathbf{g}(\theta_i,\varphi_j)\|^2 = \hat{\mathbf{v}}^T\hat{\mathbf{g}}(\theta_i,\varphi_j) \tag{19}$$

where the following vectors have been defined:

$$\hat{\mathbf{v}} = \begin{bmatrix}\mathrm{Re}\{\mathbf{v}\}\\ \mathrm{Im}\{\mathbf{v}\}\end{bmatrix} \tag{20}$$

and

$$\begin{aligned}\hat{\mathbf{g}}(\theta_i,\varphi_j) &= \mathrm{Re}\{M(\theta_i,\varphi_j)\}\begin{bmatrix}\mathrm{Re}\{\mathbf{g}(\theta_i,\varphi_j)\}\\ -\mathrm{Im}\{\mathbf{g}(\theta_i,\varphi_j)\}\end{bmatrix}\\ &+\mathrm{Im}\{M(\theta_i,\varphi_j)\}\begin{bmatrix}\mathrm{Im}\{\mathbf{g}(\theta_i,\varphi_j)\}\\ \mathrm{Re}\{\mathbf{g}(\theta_i,\varphi_j)\}\end{bmatrix} = \begin{bmatrix}\mathrm{Re}\{M(\theta_i,\varphi_j)\mathbf{g}^*(\theta_i,\varphi_j)\}\\ \mathrm{Im}\{M(\theta_i,\varphi_j)\mathbf{g}^*(\theta_i,\varphi_j)\}\end{bmatrix}\end{aligned} \tag{21}$$

where the superscript (.)* denotes complex conjugate. Since $M(\theta_i,\varphi_j)$ is a function of template values, for this particular problem, $\mathrm{Im}\{M(\theta_i,\varphi_j)\}=0$. Then, (1´5) can be rewritten for this problem as

$$L(\hat{\mathbf{v}},\xi,\tilde{\xi}) = \frac{1}{2}\left\|\hat{\mathbf{v}}\right\|^2 + C\sum_{i=1}^{N}(\xi_i + \tilde{\xi}_i) \tag{22}$$

subject to

$$\hat{\mathbf{v}}^T\hat{\mathbf{g}}(\theta_i,\varphi_j) - \hat{M}(\theta_i,\varphi_j) \le \hat{\varepsilon}(\theta_i,\varphi_j) + \xi_i \tag{23}$$

$$\hat{M}(\theta_i,\varphi_j) - \hat{\mathbf{v}}^T\hat{\mathbf{g}}(\theta_i,\varphi_j) \le \hat{\varepsilon}(\theta_i,\varphi_j) + \tilde{\xi}_i \tag{24}$$

$$\xi_i, \tilde{\xi}_i \ge 0 \tag{25}$$

where $\hat{M}(\theta_i,\varphi_j) = \left\|M(\theta_i,\varphi_j)\right\|^2$ and $\hat{\varepsilon}(\theta_i,\varphi_j) = \varepsilon^2(\theta_i,\varphi_j)$.

The optimal set of feeding values can then be derived using (22) from

$$\hat{\mathbf{v}} = \sum_{i=1}^{N}(\tilde{\alpha}_i - \alpha_i)M(\theta_i,\varphi_j)\mathbf{g}^*(\theta_i,\varphi_j) \tag{26}$$

where $\tilde{\alpha}_i$ and α_i are the positive Lagrange multipliers obtained in this case by solving the dual optimization problem of maximizing the expression

$$\begin{aligned} W(\alpha,\tilde{\alpha}) = &-\sum_{i=1}^{N}\varepsilon(\tilde{\alpha}_i + \alpha_i) + \sum_{i=1}^{N}M(\theta_i,\varphi_j)(\tilde{\alpha}_i + \alpha_i) \\ &-\frac{1}{2}\sum_{i,j=1}^{N}(\tilde{\alpha}_i + \alpha_i)(\tilde{\alpha}_j + \alpha_j)\left\langle \hat{\mathbf{g}}(\theta_i,\varphi_j), \hat{\mathbf{g}}(\theta_j,\varphi_j)\right\rangle \end{aligned} \tag{27}$$

subject to $0 \le \tilde{\alpha}_i, \alpha_i \le C$.

4 Results

To evaluate the performance of the proposed methods two examples are presented. In the first example, a 16 half-wavelength parallel dipole array, with elements placed along the x-axis and y-axis oriented and with centers uniformly separated 0.7λ, (being λ the wavelength), is considered. In order to obtain the 3D model of the antenna, 50 sets of random voltages have been generated and applied to the array, obtaining the co-polar component of the radiation pattern at 91 directions of the space angularly equispaced between θ=-90° and θ=90° and at 91 directions also angularly equispaced between φ=0° and φ=180°, using a full-wave analysis tool based on the Method of Moments [8] and wire modeling [9]. The parameters for the SVMR have been chosen C=1 and ε=0 obtaining the matrix $\underline{\mathbf{G}}$. A nominal voltage distribution with hamming window-type amplitude and linear phase is applied to the antenna, obtaining the radiation pattern plotted in figure 2. The amplitude of this target radiation pattern is proposed as input to the proposed amplitude-only synthesis iterative method. The reconstructed voltages are obtained after 25 iterations corresponding to the synthesized radiation pattern also plotted in figure 2.

In the second example, a 16 half-wavelength collinear dipole array, with elements placed along the x-axis and with centers uniformly separated 0.7λ is considered. The same 91 directions of the space angularly equispaced between θ=-90° and θ=90° have been considered, but for the sake of simplicity, only one φ-plane (φ=0°) is observed. In order to model the antenna, 50 set of random voltages are generated and analyzed. The regression parameters are chosen C=1 and ε=0, obtaining the antenna matrix model. The target radiation pattern is specified by a set of templates determining the required maximum and minimum radiation pattern values at each aspect angle, as represented in figure 3. Using the proposed SVR-based method with C=1, a set of voltages is obtained through (24). These voltages generate a radiation pattern, also plotted in figure 3, which perfectly complies with the specifications.

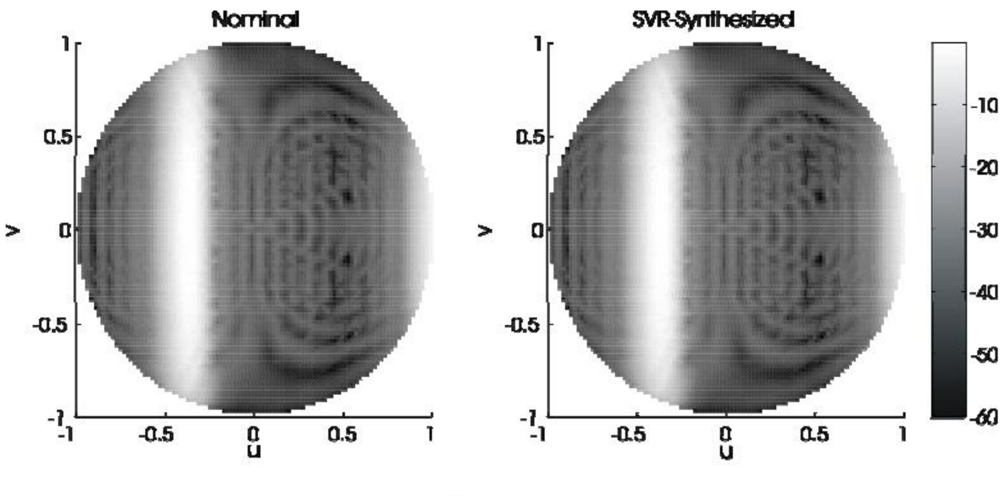

Figure 2: Nominal and synthesized radiation patterns.

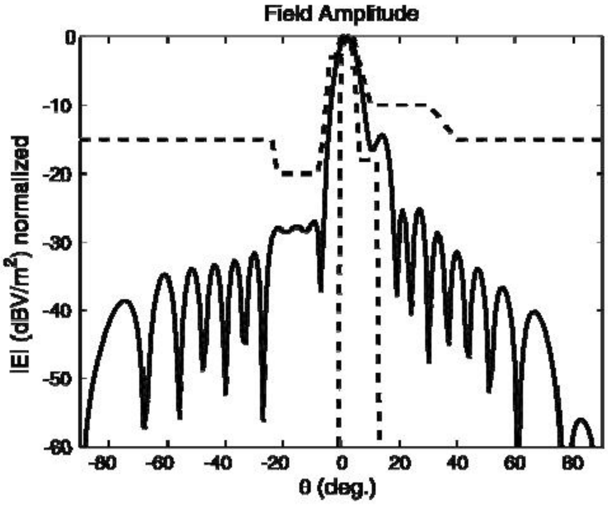

Figure 3: Template and synthesized radiation pattern.

5 Conclusions

A new technique for accurate antenna array modelling based on multiple support vector regression (SVMR) has been presented. This method is able to extract the 3D model of the array from voltage/radiation pattern pairs that can be obtained through accurate numerical simulation or experimentally, and used to perform regression leading to a matrix that models the real behaviour of the array. This model accounts for all the real properties of the array, including element coupling effects, non-idealities, etc. This fact represents an important advantage over traditional models used in synthesis methods, usually unable to account for coupling effects or passive elements in a near environment.

The use of SVMR allows considering only one optimization problems through the use of an equivalent 2D model, while traditional SVR requires an optimization problem for each considered direction of the space. The size of the training set necessary to perform a proper and accurate regression is small enough to allow the use of antenna measurement facilities to obtain the training pairs.

Once the array has been properly modeled, the obtained model can be used to solve efficiently synthesis problems specified in different ways and considering all the real properties of the array. When the specifications are given in terms of field amplitude-only data, an iterative phase-retrieval method has been proposed. When specifications are given as a template, a solution also based on support vector regression has been proposed. This SVR method is especially suitable for this kind of template specification, providing accurate solutions.

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