NCTS 2023 Postdoc Symposium

Classification of neighbourhoods of leaves of singular foliations

joint work with Camille Laurent-Gengoux

Simon-Raphael Fischer (National Center for Theoretical Sciences)

Abstract:

This talk is about my recent work with Camille Laurent-Gengoux. I will present our results about classifying singular foliations admitting a given leaf L in a manifold M and a given transverse model $(\mathbb{R}^d, \tau)$, where $\mathbb{R}^d$ is the fibre of a normal bundle of L in M, and τ is a singular foliation in $\mathbb{R}^d$ admitting 0 as a leaf. Such a classification is motived by the fact that every foliation $\mathcal{F}$ induces a singular foliation in the fibres of a normal bundle, the *transverse (singular) foliation*, and these transverse foliations at each point in L are canonically isomorphic. These isomorphisms are given by the parallel transport of what one calls $\mathcal{F}$-connections.

The idea of this talk is to recover $\mathcal{F}$ given $(\mathbb{R}^d, \tau)$, and we will see that in a local neighbourhood around L every foliation admitting $(\mathbb{R}^d, \tau)$ as transverse model is given by an associated connection of a curved Yang-Mills gauge theory, a generalised gauge theory I have developed last year. Usually, the horizontal distribution of a flat connection gives rise to a regular foliation, while our condition roughly says that the curvature is related to the field strength of a curved gauge theory. This is a natural enhancement, allowing singular foliations as a consequence, and this construction is naturally invariant of the choice of $\mathcal{F}$-connections.