

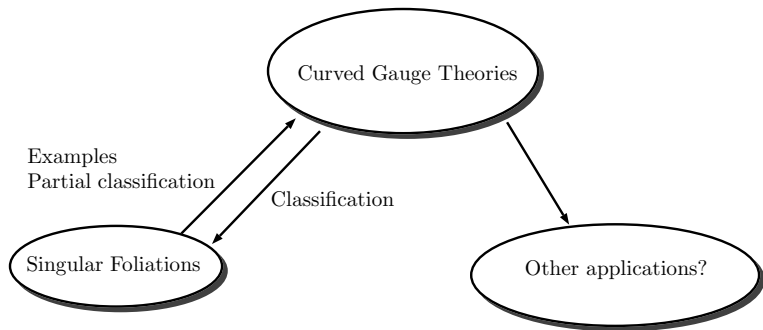
Curved gauge theories and their applications

Simon-Raphael Fischer

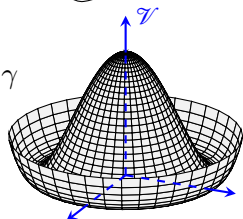


國家理論科學研究中心

National Center for Theoretical Sciences (National Taiwan University)



Curved gauge theory



Motivation 1 by Thomas Strobl and Alexei Kotov

Classical formalism	CYMH GT
Lie algebra \mathfrak{g} as $L \times \mathfrak{g}$	Lie algebroid $E \rightarrow N$
\mathfrak{g} -action γ	Anchor ρ of E & E -connections
Canonical flat connection ∇^0 on $L \times \mathfrak{g}$	General connection ∇ on E

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Remarks (Why a "curved theory"?)

Usually, the field strength F is given by (abelian, for simplicity)

$$F := dA = d^{\nabla^0} A.$$

\leadsto We will use a general connection ∇ instead of ∇^0 , and ∇ may not be flat.

Motivation 2 by S.-R. F.

Consider a semisimple Lie group G and a principal G -bundle $P \rightarrow L$:

$$(P \times \mathfrak{g})/G \hookrightarrow TP/G \xrightarrow{\quad A \quad} TL$$

Gedankenexperiment

- ① Adjoint connection \leftrightarrow Ehresmann connection on P .
- ② Adjoint connection:

$$\nabla_X \nu := [A(X), \nu]_{TP/G}$$

for all $X \in \mathfrak{X}(L)$ and $\nu \in \Gamma((P \times \mathfrak{g})/G)$.

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- ② As parallel transport:

$$PT_{\gamma}^{\text{Ad}(P)}([p, v]) = [PT_{\gamma}^P(p), v]$$

for all $[p, v] \in (P \times \mathfrak{g})/G$.

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$$PT_{\gamma}^{\text{Ad}(P)}([p, v]) = [PT_{\gamma}^P(p) \cdot \kappa_{\gamma}, \kappa_{\gamma}^{-1} \cdot PT_{\gamma}^0(v)],$$

Lie algebra \mathfrak{g} as trivial bundle w/ canonical flat connection,
 κ_{γ} values in G & "suitable"

Theorem (Field Redefinitions S.-R. F.)

This leads to an equivalence relation of gauge theories, preserving dynamics and kinematics.

*But: In the **curved** sense! Curvature terms appear.*

Motivation (S.-R. F.)

- ① How to formulate gauge theory such that it is invariant under field redefinitions?
- ② Are there curved theories which are **not** equivalent to classical ones?

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- ① How to formulate gauge theory such that it is invariant under field redefinitions?
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We will only focus on Yang-Mills theories:

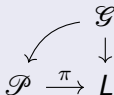
	Classical	Curved
Infinitesimal	Lie algebra \mathfrak{g}	LAB ¹ \mathcal{G}
Integrated	Lie group G	LGB ² \mathcal{G}

$$\begin{array}{ccc}
 G & \longrightarrow & \mathcal{G} \\
 & & \downarrow \\
 & & M
 \end{array}$$

¹LAB = Lie algebra bundle

²LGB = Lie group bundle

Definition (LGB actions, simplified)



$\mathcal{P} \xrightarrow{\pi} L$ a fibre bundle. A **right-action of \mathcal{G} on \mathcal{P}** is a smooth map $\mathcal{P} * \mathcal{G} := \pi^* \mathcal{G} = \mathcal{P} \times_L \mathcal{G} \rightarrow \mathcal{P}$, $(p, g) \mapsto p \cdot g$, satisfying the following properties:

$$\pi(p \cdot g) = \pi(p), \quad (1)$$

$$(p \cdot g) \cdot h = p \cdot (gh), \quad (2)$$

$$p \cdot e_{\pi(p)} = p \quad (3)$$

for all $p \in \mathcal{P}$ and $g, h \in \mathcal{G}_{\pi(p)}$, where $e_{\pi(p)}$ is the neutral element of $\mathcal{G}_{\pi(p)}$.

Definition (Principal bundle)

Still a fibre bundle

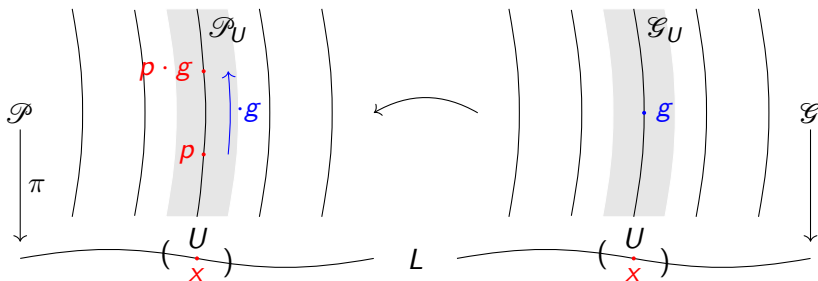
$$\begin{array}{ccc} G & \longrightarrow & \mathcal{P} \\ & & \downarrow \pi \\ & & L \end{array}$$

but with \mathcal{G} -action

$$\begin{array}{ccc} \cancel{\mathcal{P}} \times \cancel{G} & \longrightarrow & \mathcal{P} \\ \mathcal{P} * \mathcal{G} & & \end{array}$$

simply transitive on fibres of \mathcal{P} , and "suitable" atlas.

Connection on \mathcal{P} : Idea



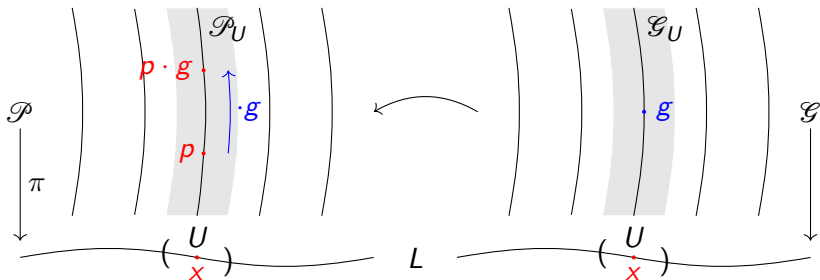
But:

$$r_g : \mathcal{P}_x \rightarrow \mathcal{P}_x$$

\Rightarrow

$D_p r_g$ only defined on vertical structure

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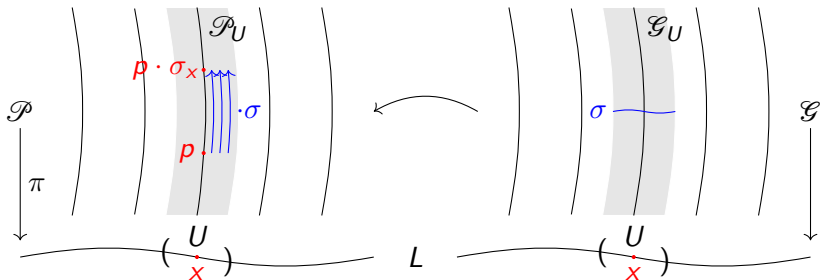
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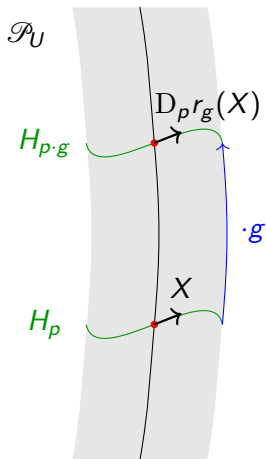
Connection on \mathcal{P} : Idea



Use $\sigma \in \Gamma(\mathcal{G}) : r_\sigma(p) := p \cdot \sigma_x$

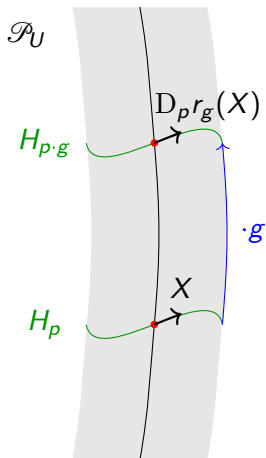
Connection on \mathcal{P} : Revisiting the classical setup

If \mathcal{P} a typical principal bundle
 (\mathcal{G} trivial, $\sigma \equiv g$ constant),
 and H a connection:



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Remarks (Integrated case)

Parallel transport $\text{PT}_\gamma^{\mathcal{P}}$ in \mathcal{P} :

$$\text{PT}_\gamma^{\mathcal{P}}(p \cdot g) = \text{PT}_\gamma^{\mathcal{P}}(p) \cdot g$$

where $\gamma : I \rightarrow L$ is a base path

Connection on \mathcal{P} : General case

Remarks (Integrated case)

Ansatz: Introduce connection on \mathcal{G} ,

$$\mathrm{PT}_{\gamma}^{\mathcal{P}}(p \cdot g) = \mathrm{PT}_{\gamma}^{\mathcal{P}}(p) \cdot \mathrm{PT}_{\gamma}^{\mathcal{G}}(g).$$

Back to the roots

- ① $\mathcal{G} \cong L \times G$
- ② Equip \mathcal{G} with canonical flat connection

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- 2 Equip \mathcal{G} with canonical flat connection

Definition (Ehresmann/Yang-Mills connection, [C. L.-G., S.-R. F.]

A surjective submersion $\pi_{\mathcal{T}}: \mathcal{T} \rightarrow L$ so that one has a commuting diagram

$$\begin{array}{ccc} & \mathcal{G} & \\ & \downarrow \pi_{\mathcal{G}} & \\ \mathcal{T} & \xrightarrow{\pi_{\mathcal{T}}} & L \end{array}$$

(Note: A curved arrow also points from \mathcal{G} to \mathcal{T} in the original diagram.)

1 Ehresmann connection:

$$\text{PT}_{\gamma}^{\mathcal{T}}(t \cdot g) = \text{PT}_{\gamma}^{\mathcal{T}}(t) \cdot \text{PT}_{\gamma}^{\mathcal{G}}(g)$$

2 Yang-Mills connection: Additionally

$$\text{PT}_{\gamma_0}^{\mathcal{T}}(t) = t \cdot g_{\gamma_0}$$

for some $g_{\gamma_0} \in \mathcal{G}_{\pi_{\mathcal{T}}(t)}$, where γ_0 is a contractible loop.

Definition (Multiplicative YM connection, [S.-R. F.]

On \mathcal{G} there is also the notion of **multiplicative Yang-Mills connections**, that is,

$$\begin{aligned} \mathrm{PT}_{\gamma}^{\mathcal{G}}(q \cdot g) &= \mathrm{PT}_{\gamma}^{\mathcal{G}}(q) \cdot \mathrm{PT}_{\gamma}^{\mathcal{G}}(g), \\ \mathrm{PT}_{\gamma_0}^{\mathcal{G}}(q) &= g_{\gamma_0} \cdot q \cdot g_{\gamma_0}^{-1} \end{aligned}$$

Remarks

There is a simplicial differential δ on $\mathcal{G} \xrightarrow{\pi_{\mathcal{G}}} L$ with Lie algebra bundle \mathcal{G}

$$\delta : \Omega^{\bullet}(\underbrace{\mathcal{G} * \dots * \mathcal{G}}_{k \text{ times}}; \pi_{\mathcal{G}}^* \mathcal{G}) \rightarrow \Omega^{\bullet}(\underbrace{\mathcal{G} * \dots * \mathcal{G}}_{k+1 \text{ times}}; \pi_{\mathcal{G}}^* \mathcal{G})$$

such that the definition of the multiplicative Yang-Mills connection is equivalent to the **compatibility conditions**

- Connection closed
- Curvature exact ([S.-R. F.])

Remarks

On the Lie algebra bundle \mathcal{g} we have a connection ∇ with

$$\begin{aligned}\nabla([\mu, \nu]_{\mathcal{g}}) &= [\nabla\mu, \nu]_{\mathcal{g}} + [\mu, \nabla\nu]_{\mathcal{g}}, \\ R_{\nabla} &= \text{ad} \circ \zeta.\end{aligned}$$

Example

Given a short exact sequence of algebroids

$$\mathcal{g} \hookrightarrow E \begin{array}{c} \xleftarrow{A} \\ \longrightarrow \twoheadrightarrow \end{array} TL$$

with splitting $A: TL \rightarrow E$, then

$$\begin{aligned}\nabla_X \nu &= [A(X), \nu]_E, \\ \zeta(X, X') &= [A(X), A(X')]_E - A([X, X']).\end{aligned}$$

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Integrating Alexei Kotov's and Thomas Strobl's idea

Definition (Principal bundle connection, [S.-R. F.])

- On \mathcal{G} : Multiplicative Yang-Mills connection
- On \mathcal{P} : Ehresmann connection

Definition (Generalized curvature/field strength F of A , [S.-R. F.])

We define

$$F := d^{\pi^*\nabla} A + \frac{1}{2}[A \wedge A]_{\pi^*\mathcal{G}} + \pi^!\zeta.$$

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Theorem (Lagrangian, [S.-R. F.])

- κ be an Ad -invariant fibre metric on \mathfrak{g} ,
- L a spacetime, and $*$ its Hodge star operator,
- $(U_i)_i$ open covering of L with subordinate gauges $s_i \in \Gamma(\mathcal{P}|_{U_i})$.

Then the Lagrangian $\mathfrak{L}_{\text{CYM}}[A]$, defined locally by

$$(\mathfrak{L}_{\text{CYM}}[A])|_{U_i} := -\frac{1}{2}\kappa(F_{s_i} \wedge *F_{s_i}),$$

is well-defined, and

$$\mathfrak{L}_{\text{CYM}}[K^!A] = \mathfrak{L}_{\text{CYM}}[A]$$

for all principal bundle automorphisms K .

Back to the roots

- 1 $\mathcal{G} \cong L \times G$
- 2 Equip \mathcal{G} with canonical flat connection
- 3 $\zeta \equiv 0$

Example (Hopf fibration $S^7 \rightarrow S^4$, [S.-R. F.])

Let P be the Hopf bundle

$$\begin{array}{ccc} \mathrm{SU}(2) \cong S^3 & \longrightarrow & S^7 \\ & & \downarrow \\ & & S^4 \end{array}$$

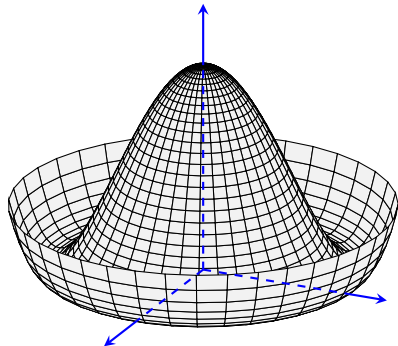
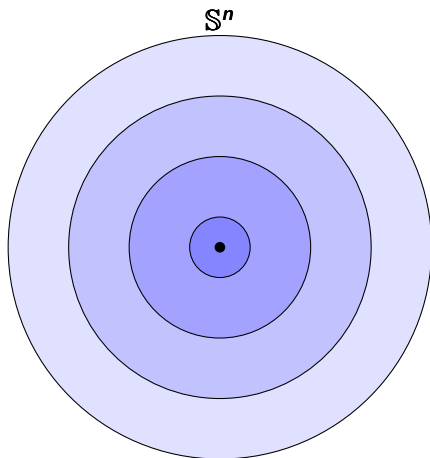
Define $\mathcal{P} := \mathcal{G}$ as the inner group bundle of P ,

$$\mathcal{G} := c_{\mathrm{SU}(2)}(P) := (P \times \mathrm{SU}(2)) / \mathrm{SU}(2).$$

This principal $c_{\mathrm{SU}(2)}(P)$ -bundle admits the structure as curved Yang-Mills gauge theory; there is no description as classical gauge theory.

Applications: Classifying singular foliations
(joint work w/ Camille Laurent-Gengoux)

Why foliations?



Singular Foliations:

- Gauge Theory
- Poisson Geometry
(Singular foliation of symplectic leaves)
- Lie groupoids and algebroids
- Dirac structures
- Generalised complex manifolds
- Non-commutative geometry
- ...

Definition (Smooth singular foliation)

A **smooth singular foliation** \mathcal{F} on a smooth manifold is a subspace of $\mathfrak{X}_c(M)$ so that

- it is **involutive**,
- it is **stable under** $C^\infty(M)$ -**multiplication**,
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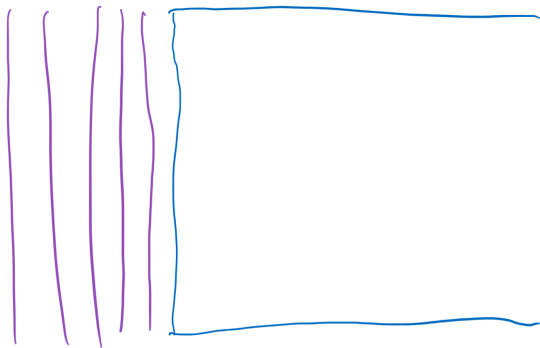
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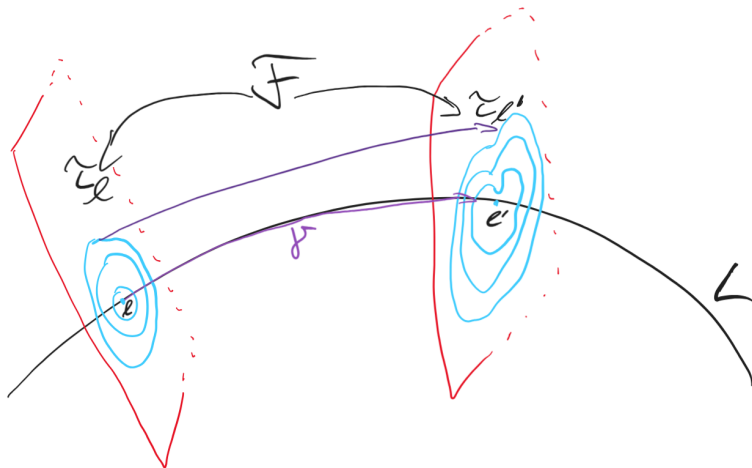
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- it is **stable under $C^\infty(M)$ -multiplication**, i.e. $fX \in \mathcal{F}$ for all $f \in C^\infty(M)$ and $X \in \mathcal{F}$,
- it is **locally finitely generated**, i.e. around each $p \in M$ there is an open neighbourhood U and a finite family $(X^i)_i^r$ ($X^i \in \mathcal{F}$) such that for all $X \in \mathcal{F}$ there are $f_i \in C^\infty(M)$ satisfying on U .

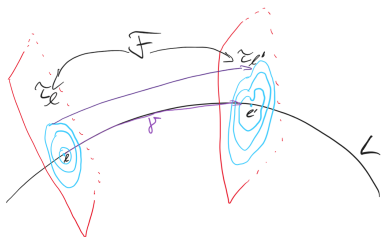
$$X = \sum_i f_i X^i.$$

Remarks (Leaves)

Following the flows in \mathcal{F} , this gives rise to a partition of connected immersed submanifolds in M .







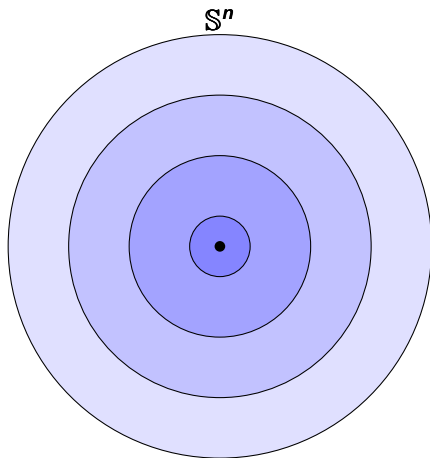
Theorem (\mathcal{F} -connections)

There is a connection on the normal bundle of a leaf L :

- *Horizontal vector fields are in \mathcal{F} .*
- *Parallel transport PT_γ has values in $\text{Sym}(\tau_l, \tau_{l'})$.*
- *For a contractible loop γ_0 at l : PT_{γ_0} values in $\text{Inner}(\tau_l)$.*

Idea: Relation to gauge theory

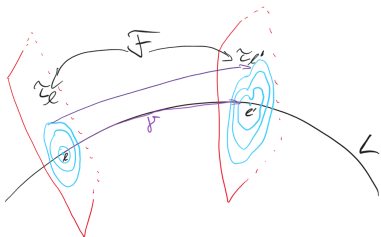
Example of a transverse foliation τ :



Remarks

- $\text{Inner}(\tau_I)$ maps each circle to itself
- $\text{Sym}(\tau_I)$ allows to exchange circles
- Both preserve τ_I and fix the origin

Idea

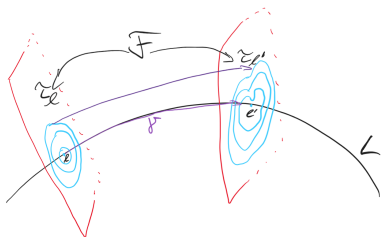


Idea

Generators of \mathcal{F} given by $\mathcal{F}_{\text{projectable}}$:

$$\mathbb{H}(X) + \bar{\nu},$$

where $X \in \mathfrak{X}(L)$, $\mathbb{H}(X)$ its projectable horizontal lift,
 $\nu \in \Gamma(\text{inner}(\tau))$ and $\bar{\nu}$ its fundamental vector field.



Idea

Fix I and given τ_I : Reconstruct \mathcal{F} .

$$\begin{aligned}
 [\mathbb{H}(X) + \bar{\nu}, \mathbb{H}(X') + \bar{\mu}] &= \mathbb{H}([X, X']) + \overline{\dots} \\
 &= \underbrace{[\mathbb{H}(X), \mathbb{H}(X')]}_{\rightsquigarrow \text{curvature}} \\
 &\quad + \underbrace{[\mathbb{H}(X), \bar{\mu}] - [\mathbb{H}(X'), \bar{\nu}]}_{\rightsquigarrow \text{connection}} + \overline{[\nu, \mu]}
 \end{aligned}$$

Theorem ([C. L.-G., S.-R. F.])

Given a multiplicative Yang-Mills connection on \mathcal{G} and a Yang-Mills connection \mathbb{H} on \mathcal{T} , then there is a natural foliation on \mathcal{T} generated by

$$\mathbb{H}(X) + \bar{\nu},$$

where $X \in \mathfrak{X}(L)$ and $\nu \in \Gamma(\mathfrak{g})$.

Proof.

We have

$$\begin{aligned} [\mathbb{H}(X), \bar{\nu}] &= \overline{\nabla_X \nu}, \\ [\mathbb{H}(X), \mathbb{H}(X')] &= \mathbb{H}([X, X']) + \overline{\zeta(X, X')}, \end{aligned}$$

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Idea (Leaf L simply connected)

Fix a point $l \in L$ with transverse model (\mathbb{R}^d, τ_l) :

- 1 $G = \text{Inn}(\tau_l)$
- 2 P a principal G -bundle, equipped with an ordinary connection

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Remarks

- Think of the induced connection on \mathcal{T} as the \mathcal{F} -connection.
- \mathcal{G} acts on \mathcal{T} (canonically from the left).

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Proposition ([C. L.-G., S.-R. F.])

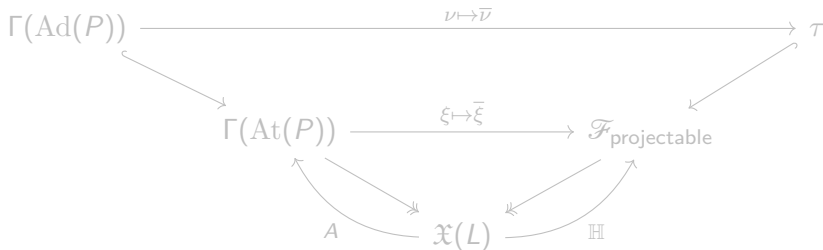
The associated connection on \mathcal{G} is a multiplicative Yang-Mills connection and the one on \mathcal{T} is a corresponding Yang-Mills connection.

Remarks

Thus, we have a singular foliation on \mathcal{T} , which, by construction, admits L as a leaf and τ_l as transverse data.

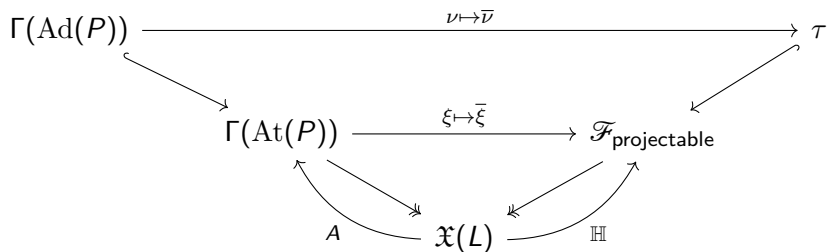
Proposition ([C. L.-G., S.-R. F.])

The reconstructed foliation is independent of the choice of connection on P .



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Summary

Remarks ([C. L.-G., S.-R. F.])

In the simply connected case, the following are equivalent:

- Singular foliations with leaf L and transverse model (\mathbb{R}^d, τ_l)
- Principal $\text{Inner}(\tau_l)$ -bundles over L

Remarks (Classification of curved Yang-Mills gauge theories)

If \mathcal{G} acts faithfully on \mathcal{T} , preserving L , then a curved Yang-Mills gauge theory can be flattened if and only if P is flat.

Curved YM Gauge Theory

Singular Foliations \mathcal{F}

Multiplicative Yang-Mills connection

\mathcal{F} -connection

Flat gauge theory

Flat singular foliation

Field redefinition of connection on \mathcal{G}

Different choice of \mathcal{F} -connection

Thank you!