

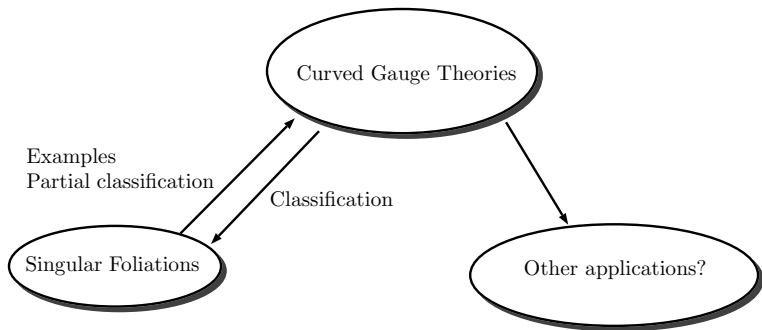
# Curved Yang-Mills gauge theories and their recent applications

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## Curved Yang-Mills gauge theory



# Motivation 1 by Thomas Strobl and Alexei Kotov

Classical formalism	CYMH GT
Lie algebra $\mathfrak{g}$ as $L \times \mathfrak{g}$	Lie algebroid $E \rightarrow N$
$\mathfrak{g}$ -action $\gamma$	Anchor $\rho$ of $E$ & $E$ -connections
Canonical flat connection $\nabla^0$ on $L \times \mathfrak{g}$	General connection $\nabla$ on $E$

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## Remarks (Why a "curved theory"?)

Usually, the field strength  $F$  is given by (abelian, for simplicity)

$$F := dA = d^{\nabla^0} A.$$

$\rightsquigarrow$  We will use a general connection  $\nabla$  instead of  $\nabla^0$ , and  $\nabla$  may not be flat.

# Motivation 2 by S.-R. F.

Consider a semisimple Lie group  $G$  and a principal  $G$ -bundle  $P \rightarrow L$ :

$$(P \times \mathfrak{g})/G \hookrightarrow TP/G \xrightarrow{\quad A \quad} TL$$

## Gedankenexperiment

- ① Adjoint connection  $\leftrightarrow$  Ehresmann connection on  $P$ .
- ② Adjoint connection:

$$\nabla_X \nu := [A(X), \nu]_{TP/G}$$

for all  $X \in \mathfrak{X}(L)$  and  $\nu \in \Gamma((P \times \mathfrak{g})/G)$ .

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- ② As parallel transport:

$$PT_{\gamma}^{\text{Ad}(P)}([p, v]) = [PT_{\gamma}^P(p), v]$$

for all  $[p, v] \in (P \times \mathfrak{g})/G$ .

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$$\text{PT}_{\gamma}^{\text{Ad}(P)}([p, v]) = \left[ \text{PT}_{\gamma}^P(p) \cdot \kappa_{\gamma}, \kappa_{\gamma}^{-1} \cdot \text{PT}_{\gamma}^0(v) \right],$$

Lie algebra  $\mathfrak{g}$  as trivial bundle w/ canonical flat connection,  
 $\kappa_{\gamma}$  values in  $G$  & "suitable"

## Theorem (Field Redefinitions S.-R. F.)

*This leads to an equivalence relation of gauge theories, preserving dynamics and kinematics.*

*But: In the **curved** sense! Curvature terms appear.*

## Motivation (S.-R. F.)

- 1 How to formulate gauge theory such that it is invariant under field redefinitions?
- 2 Are there curved theories which are **not** equivalent to classical ones?

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We will only focus on Yang-Mills theories:

	Classical	Curved
Infinitesimal	Lie algebra $\mathfrak{g}$	LAB <sup>1</sup> $\mathcal{G}$
Integrated	Lie group $G$	LGB <sup>2</sup> $\mathcal{G}$

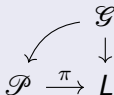
$$\begin{array}{ccc}
 G & \longrightarrow & \mathcal{G} \\
 & & \downarrow \\
 & & M
 \end{array}$$

---

<sup>1</sup>LAB = Lie algebra bundle

<sup>2</sup>LGB = Lie group bundle

## Definition (LGB actions, simplified)



$\mathcal{P} \xrightarrow{\pi} L$  a fibre bundle. A **right-action of  $\mathcal{G}$  on  $\mathcal{P}$**  is a smooth map  $\mathcal{P} * \mathcal{G} := \pi^* \mathcal{G} = \mathcal{P} \times_L \mathcal{G} \rightarrow \mathcal{P}$ ,  $(p, g) \mapsto p \cdot g$ , satisfying the following properties:

$$\pi(p \cdot g) = \pi(p), \quad (1)$$

$$(p \cdot g) \cdot h = p \cdot (gh), \quad (2)$$

$$p \cdot e_{\pi(p)} = p \quad (3)$$

for all  $p \in \mathcal{P}$  and  $g, h \in \mathcal{G}_{\pi(p)}$ , where  $e_{\pi(p)}$  is the neutral element of  $\mathcal{G}_{\pi(p)}$ .

## Definition (Principal bundle)

Still a fibre bundle

$$\begin{array}{ccc} G & \longrightarrow & \mathcal{P} \\ & & \downarrow \pi \\ & & L \end{array}$$

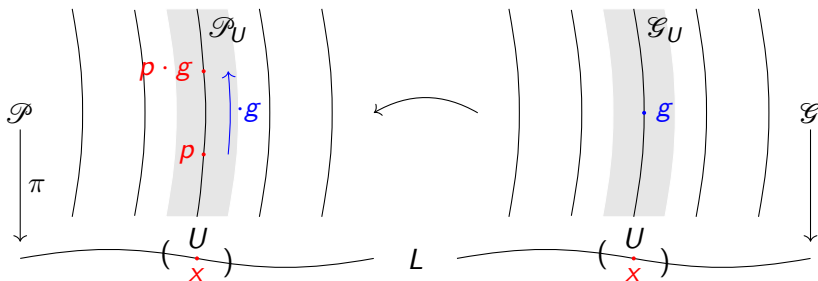
but with  $\mathcal{G}$ -action

$$\begin{array}{ccc} \cancel{\mathcal{P}} \times \cancel{G} & \longrightarrow & \mathcal{P} \\ \mathcal{P} * \mathcal{G} & & \end{array}$$

simply transitive on fibres of  $\mathcal{P}$ , and "suitable" atlas.



# Connection on $\mathcal{P}$ : Idea



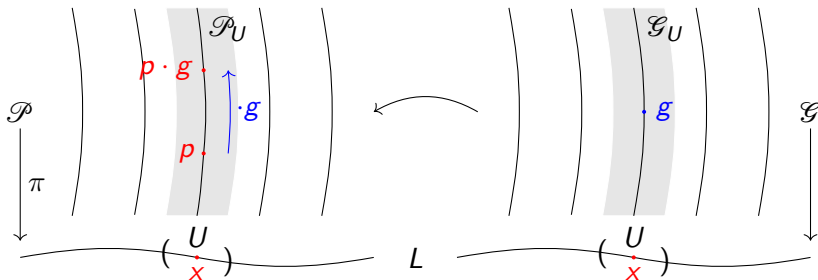
But:

$$r_g : \mathcal{P}_x \rightarrow \mathcal{P}_x$$

$\Rightarrow$

$D_p r_g$  only defined on vertical structure

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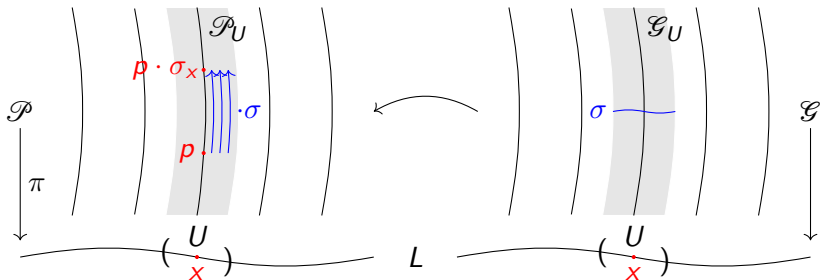
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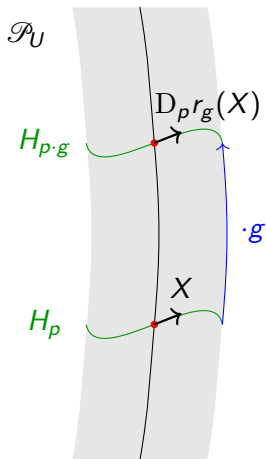
# Connection on $\mathcal{P}$ : Idea



Use  $\sigma \in \Gamma(\mathcal{G}) : r_\sigma(p) := p \cdot \sigma_x$

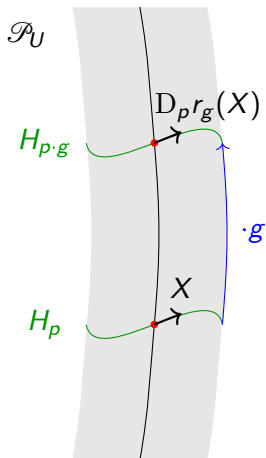
# Connection on $\mathcal{P}$ : Revisiting the classical setup

If  $\mathcal{P}$  a typical principal bundle  
 ( $\mathcal{G}$  trivial,  $\sigma \equiv g$  constant),  
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# Connection on $\mathcal{P}$ : Revisiting the classical setup

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## Remarks (Integrated case)

Parallel transport  $\text{PT}_\gamma^{\mathcal{P}}$  in  $\mathcal{P}$ :

$$\text{PT}_\gamma^{\mathcal{P}}(p \cdot g) = \text{PT}_\gamma^{\mathcal{P}}(p) \cdot g$$

where  $\gamma : I \rightarrow L$  is a base path

# Connection on $\mathcal{P}$ : General case

## Remarks (Integrated case)

Ansatz: Introduce connection on  $\mathcal{G}$ ,

$$\mathrm{PT}_{\gamma}^{\mathcal{P}}(p \cdot g) = \mathrm{PT}_{\gamma}^{\mathcal{P}}(p) \cdot \mathrm{PT}_{\gamma}^{\mathcal{G}}(g).$$

## Back to the roots

- ①  $\mathcal{G} \cong L \times G$
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## Definition (Ehresmann/Yang-Mills connection, [C. L.-G., S.-R. F.]

A surjective submersion  $\pi_{\mathcal{T}}: \mathcal{T} \rightarrow L$  so that one has a commuting diagram

$$\begin{array}{ccc}
 & \mathcal{G} & \\
 \swarrow & \downarrow \pi_{\mathcal{G}} & \\
 \mathcal{T} & \xrightarrow{\pi_{\mathcal{T}}} & L
 \end{array}$$

### 1 Ehresmann connection:

$$\text{PT}_{\gamma}^{\mathcal{T}}(t \cdot g) = \text{PT}_{\gamma}^{\mathcal{T}}(t) \cdot \text{PT}_{\gamma}^{\mathcal{G}}(g)$$

### 2 Yang-Mills connection: Additionally

$$\text{PT}_{\gamma_0}^{\mathcal{T}}(t) = t \cdot g_{\gamma_0}$$

for some  $g_{\gamma_0} \in \mathcal{G}_{\pi_{\mathcal{T}}(t)}$ , where  $\gamma_0$  is a contractible loop.



### Definition (Multiplicative YM connection, [S.-R. F.]

On  $\mathcal{G}$  there is also the notion of **multiplicative Yang-Mills connections**, that is,

$$\begin{aligned} \mathrm{PT}_{\gamma}^{\mathcal{G}}(q \cdot g) &= \mathrm{PT}_{\gamma}^{\mathcal{G}}(q) \cdot \mathrm{PT}_{\gamma}^{\mathcal{G}}(g), \\ \mathrm{PT}_{\gamma_0}^{\mathcal{G}}(q) &= g_{\gamma_0} \cdot q \cdot g_{\gamma_0}^{-1} \end{aligned}$$

## Remarks

There is a simplicial differential  $\delta$  on  $\mathcal{G} \xrightarrow{\pi_{\mathcal{G}}} L$  with Lie algebra bundle  $\mathcal{G}$

$$\delta : \Omega^{\bullet}(\underbrace{\mathcal{G} * \dots * \mathcal{G}}_{k \text{ times}}; \pi_{\mathcal{G}}^* \mathcal{G}) \rightarrow \Omega^{\bullet}(\underbrace{\mathcal{G} * \dots * \mathcal{G}}_{k+1 \text{ times}}; \pi_{\mathcal{G}}^* \mathcal{G})$$

such that the definition of the multiplicative Yang-Mills connection is equivalent to the **compatibility conditions**

- Connection closed
- Curvature exact ([S.-R. F.] )

## Remarks

On the Lie algebra bundle  $\mathcal{g}$  we have a connection  $\nabla$  with

$$\begin{aligned}\nabla([\mu, \nu]_{\mathcal{g}}) &= [\nabla\mu, \nu]_{\mathcal{g}} + [\mu, \nabla\nu]_{\mathcal{g}}, \\ R_{\nabla} &= \text{ad} \circ \zeta.\end{aligned}$$

## Example

Given a short exact sequence of algebroids

$$\mathcal{g} \hookrightarrow E \twoheadrightarrow TL$$

with splitting  $\chi: TL \rightarrow E$ , then

$$\begin{aligned}\nabla_X \nu &= [\chi(X), \nu]_E, \\ \zeta(X, X') &= [\chi(X), \chi(X')]_E - \chi([X, X']).\end{aligned}$$

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# Integrating Alexei Kotov's and Thomas Strobl's idea

Definition (Principal bundle connection, [S.-R. F.])

- On  $\mathcal{G}$ : Multiplicative Yang-Mills connection
- On  $\mathcal{P}$ : Ehresmann connection

Definition (Generalized curvature/field strength  $F$  of  $A$ , [S.-R. F.])

We define

$$F := d^{\pi^*\nabla} A + \frac{1}{2}[A \wedge A]_{\pi^*\mathcal{G}} + \pi^!\zeta.$$

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## Theorem (Lagrangian, [S.-R. F.])

- $\kappa$  be an  $\text{Ad}$ -invariant fibre metric on  $\mathfrak{g}$ ,
- $L$  a spacetime, and  $*$  its Hodge star operator,
- $(U_i)_i$  open covering of  $L$  with subordinate gauges  $s_i \in \Gamma(\mathcal{P}|_{U_i})$ .

Then the Lagrangian  $\mathfrak{L}_{\text{CYM}}[A]$ , defined locally by

$$(\mathfrak{L}_{\text{CYM}}[A])|_{U_i} := -\frac{1}{2}\kappa(F_{s_i} \wedge *F_{s_i}),$$

is well-defined, and

$$\mathfrak{L}_{\text{CYM}}[K^!A] = \mathfrak{L}_{\text{CYM}}[A]$$

for all principal bundle automorphisms  $K$ .

## Back to the roots

- 1  $\mathcal{G} \cong L \times G$
- 2 Equip  $\mathcal{G}$  with canonical flat connection
- 3  $\zeta \equiv 0$



## Example (Hopf fibration $S^7 \rightarrow S^4$ , [S.-R. F.] )

Let  $P$  be the Hopf bundle

$$\begin{array}{ccc} \mathrm{SU}(2) \cong S^3 & \longrightarrow & S^7 \\ & & \downarrow \\ & & S^4 \end{array}$$

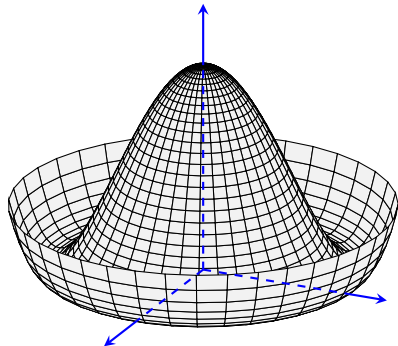
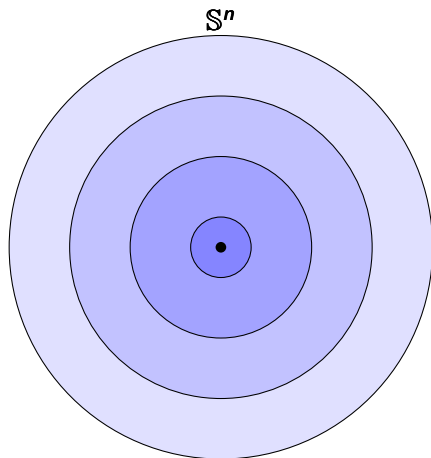
Define  $\mathcal{P} := \mathcal{G}$  as the inner group bundle of  $P$ ,

$$\mathcal{G} := c_{\mathrm{SU}(2)}(P) := (P \times \mathrm{SU}(2)) / \mathrm{SU}(2).$$

This principal  $c_{\mathrm{SU}(2)}(P)$ -bundle admits the structure as curved Yang-Mills gauge theory; there is no description as classical gauge theory.

Applications: Classifying singular foliations  
(joint work w/ Camille Laurent-Gengoux)

## Why foliations?



## Singular Foliations:

- Gauge Theory  
(Ex.: Singular foliation  $\leftrightarrow$  Symmetry breaking  $\rightarrow$  Higgs mechanism)
- Poisson Geometry  
(Singular foliation of symplectic leaves)
- Lie groupoids and algebroids
- Dirac structures
- Generalised complex manifolds
- Non-commutative geometry
- ...

### Definition (Smooth singular foliation)

A **smooth singular foliation**  $\mathcal{F}$  on a smooth manifold is a subspace of  $\mathfrak{X}_c(M)$  so that

- it is **involutive**,
- it is **stable under**  $C^\infty(M)$ -**multiplication**,
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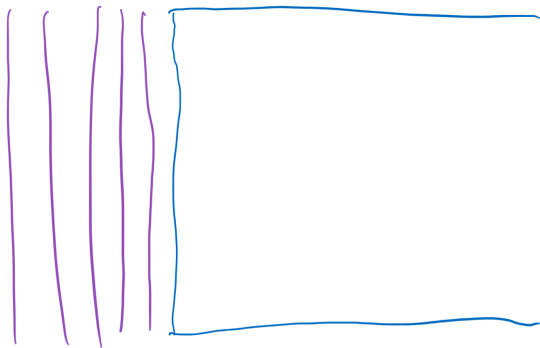
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- it is **stable under  $C^\infty(M)$ -multiplication**, i.e.  $fX \in \mathcal{F}$  for all  $f \in C^\infty(M)$  and  $X \in \mathcal{F}$ ,
- it is **locally finitely generated**, i.e. around each  $p \in M$  there is an open neighbourhood  $U$  and a finite family  $(X^i)_i^r$  ( $X^i \in \mathcal{F}$ ) such that for all  $X \in \mathcal{F}$  there are  $f_i \in C^\infty(M)$  satisfying on  $U$ .

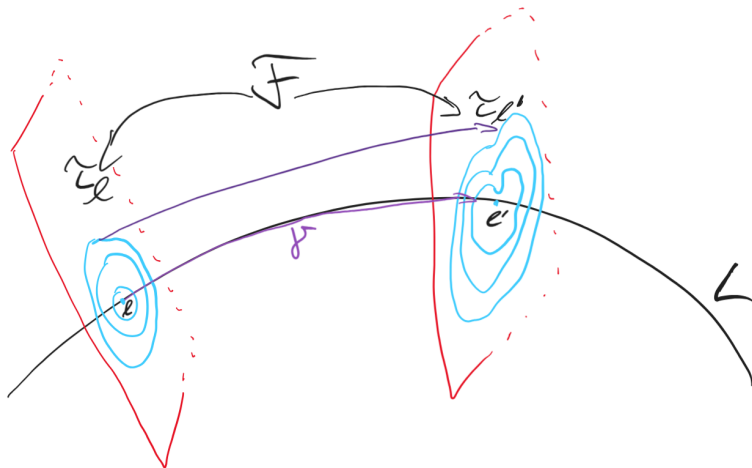
$$X = \sum_i f_i X^i.$$

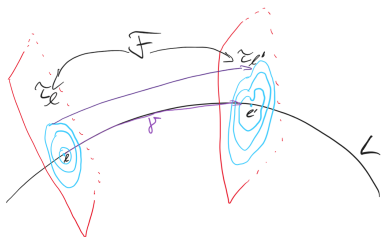


## Remarks (Leaves)

Following the flows in  $\mathcal{F}$ , this gives rise to a partition of connected immersed submanifolds in  $M$ .







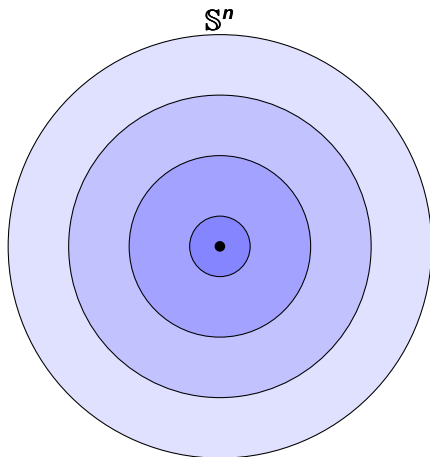
### Theorem ( $\mathcal{F}$ -connections)

There is a connection on the normal bundle of a leaf  $L$ :

- Horizontal vector fields are in  $\mathcal{F}$ .
- Parallel transport  $PT_\gamma$  has values in  $\text{Sym}(\tau_l, \tau_{l'})$ .
- For a contractible loop  $\gamma_0$  at  $l$ :  $PT_{\gamma_0}$  values in  $\text{Inner}(\tau_l)$ .

Idea: Relation to gauge theory

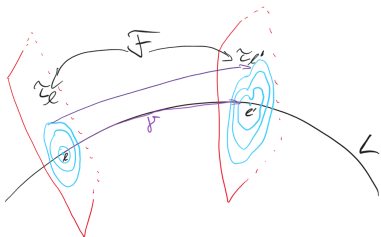
# Example of a transverse foliation $\tau$ :



## Remarks

- $\text{Inner}(\tau_I)$  maps each circle to itself
- $\text{Sym}(\tau_I)$  allows to exchange circles
- Both preserve  $\tau_I$  and fix the origin

# Idea

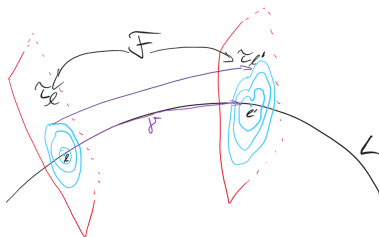


## Idea

Generators of  $\mathcal{F}$  given by  $\mathcal{F}_{\text{projectable}}$ :

$$\mathbb{H}(X) + \bar{\nu},$$

where  $X \in \mathfrak{X}(L)$ ,  $\mathbb{H}(X)$  its projectable horizontal lift,  
 $\nu \in \Gamma(\text{inner}(\tau))$  and  $\bar{\nu}$  its fundamental vector field.



## Idea

Fix  $I$  and given  $\tau_I$ : Reconstruct  $\mathcal{F}$ .

$$\begin{aligned}
 [\mathbb{H}(X) + \bar{\nu}, \mathbb{H}(X') + \bar{\mu}] &= \mathbb{H}([X, X']) + \overline{\dots} \\
 &= \underbrace{[\mathbb{H}(X), \mathbb{H}(X')]}_{\rightsquigarrow \text{curvature}} \\
 &\quad + \underbrace{[\mathbb{H}(X), \bar{\mu}] - [\mathbb{H}(X'), \bar{\nu}]}_{\rightsquigarrow \text{connection}} + \overline{[\nu, \mu]}
 \end{aligned}$$

## Theorem ([C. L.-G., S.-R. F.])

*Given a multiplicative Yang-Mills connection on  $\mathcal{G}$  and a Yang-Mills connection  $\mathbb{H}$  on  $\mathcal{T}$ , then there is a natural foliation on  $\mathcal{T}$  generated by*

$$\mathbb{H}(X) + \bar{\nu},$$

*where  $X \in \mathfrak{X}(L)$  and  $\nu \in \Gamma(\mathfrak{g})$ .*

## Proof.

We have

$$\begin{aligned} [\mathbb{H}(X), \bar{\nu}] &= \overline{\nabla_X \nu}, \\ [\mathbb{H}(X), \mathbb{H}(X')] &= \mathbb{H}([X, X']) + \overline{\zeta(X, X')}, \end{aligned}$$

where  $\zeta \in \Omega^2(L; \mathfrak{g})$ .

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### Idea (Leaf $L$ simply connected)

Fix a point  $l \in L$  with transverse model  $(\mathbb{R}^d, \tau_l)$ :

- 1  $G = \text{Inn}(\tau_l)$
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- $\mathcal{G}$  acts on  $\mathcal{T}$  (canonically from the left).

### Proposition ([C. L.-G., S.-R. F.])

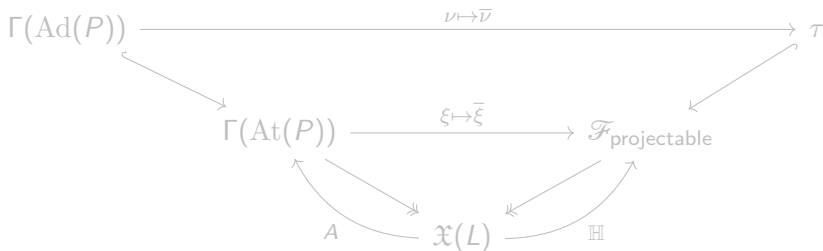
*The associated connection on  $\mathcal{G}$  is a multiplicative Yang-Mills connection and the one on  $\mathcal{T}$  is a corresponding Yang-Mills connection.*

### Remarks

Thus, we have a singular foliation on  $\mathcal{T}$ , which, by construction, admits  $L$  as a leaf and  $\tau_l$  as transverse data.

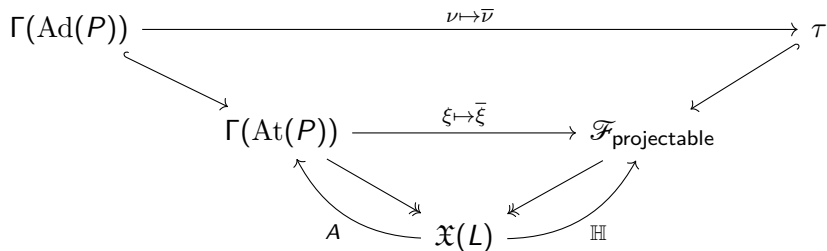
Proposition ([C. L.-G., S.-R. F.])

*The reconstructed foliation is independent of the choice of connection on  $P$ .*



# Proposition ([C. L.-G., S.-R. F.])

*The reconstructed foliation is independent of the choice of connection on  $P$ .*





# Summary

## Remarks ([C. L.-G., S.-R. F.])

In the simply connected case, the following are equivalent:

- Singular foliations with leaf  $L$  and transverse model  $(\mathbb{R}^d, \tau_l)$
- Principal  $\text{Inner}(\tau_l)$ -bundles over  $L$

## Remarks (Classification of curved Yang-Mills gauge theories)

If  $\mathcal{G}$  acts faithfully on  $\mathcal{T}$ , preserving  $L$ , then a curved Yang-Mills gauge theory can be flattened if and only if  $P$  is flat.

Curved YM Gauge Theory

Singular Foliations  $\mathcal{F}$

Multiplicative Yang-Mills connection

$\mathcal{F}$ -connection

Flat gauge theory

Flat singular foliation

Field redefinition of connection on  $\mathcal{G}$

Different choice of  $\mathcal{F}$ -connection

**Thank you!**