Curved Yang-Mills-Higgs theories

Simon-Raphael Fischer, *based on joint works with Camille Laurent-Gengoux, and with Mehran Jalali Farahani, Hyungrok Kim (金炯錄), Christian Sämann*

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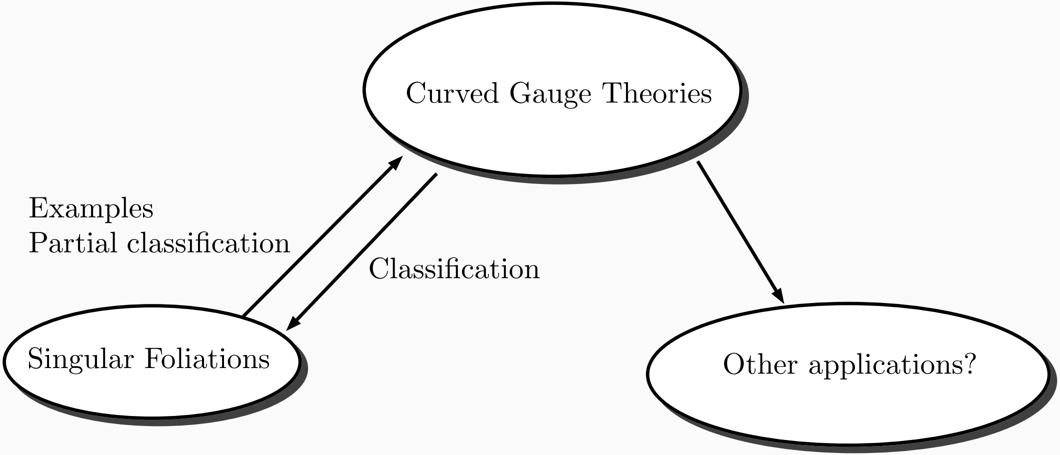
Curved Gauge Theories
Examples
Partial classification
Classification
Singular Foliations
Other applications?

Curved Yang-Mills-Higgs theory

Motivation (sort of historical)

We will mainly focus on Yang-Mills theories:

	Classical	Curved
Infinitesimal	Lie algebra $\mathfrak{g}$	LAB[1] $\mathcal{g}$
Integrated	Lie group G	LGB[2] $\mathscr{G}$

$$\begin{tikzcd} G \arrow[r] & \mathscr{G} \arrow[d] \\ & L \end{tikzcd}$$

Remarks (Why curved?)

For gauge invariance and closure of gauge transformations:

- Generalize Maurer-Cartan form $\to$ **Multiplicative Yang-Mills connection**.
- Generalize Field Strength.

[1] LAB = Lie algebra bundle
[2] LGB = Lie group bundle

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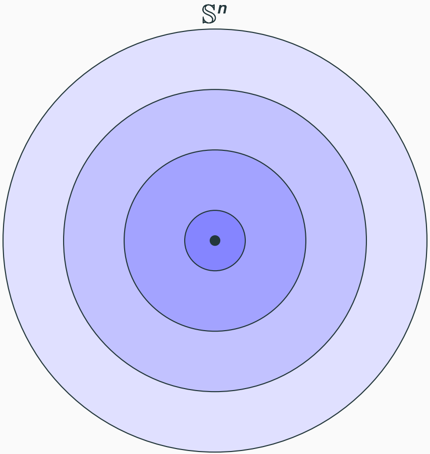
Singular Foliations

Example of a singular foliation

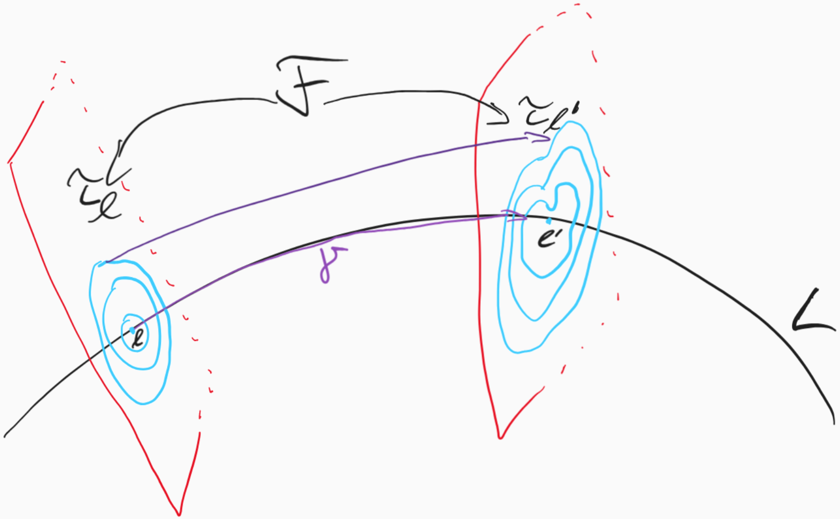


Foliations are widespread

Singular Foliations:

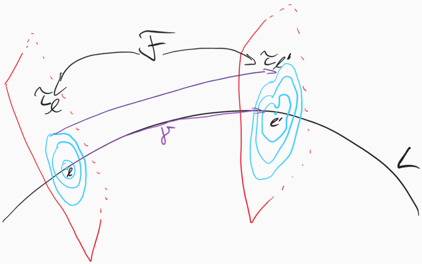
- Gauge Theory
- Poisson Geometry
  (Singular foliation of symplectic leaves)
- Lie groupoids and algebroids
- Dirac structures
- Generalised complex manifolds
- Non-commutative geometry
- . . .

Unique transverse structure



Camille Laurent-Gengoux and Leonid Ryvkin, The holonomy of a singular leaf, *Selecta Mathematica 28*, no. 2, 45, 2022.

How to classify singular foliations?



Remarks ([C. L.-G., S.-R. F.])

There is a connection on the normal bundle of a leaf L preserving the foliation!

- Transverse structure is unique: Classify singular foliation $\mathscr{F}$ like a bundle!
- Connection is a multiplicative Yang-Mills connection: Use curved gauge theory!

Source of the existence of connection on normal bundle: Camille Laurent-Gengoux and Leonid Ryvkin, The holonomy of a singular leaf, *Selecta Mathematica 28*, no. 2, 45, 2022.

Thank you!

Classification of singular foliations

Theorem ([C. L.-G., S.-R. F.])

Formal singular foliations with leaf L and transverse model $(\mathbb{R}^d, \tau_I)$ are equivalent to:

- A Galois cover L' over L with structural group K
- A short exact sequence of groups

$$\mathrm{Inner}(\tau_I) \hookrightarrow H \twoheadrightarrow K$$

- A principal H-bundle P over L