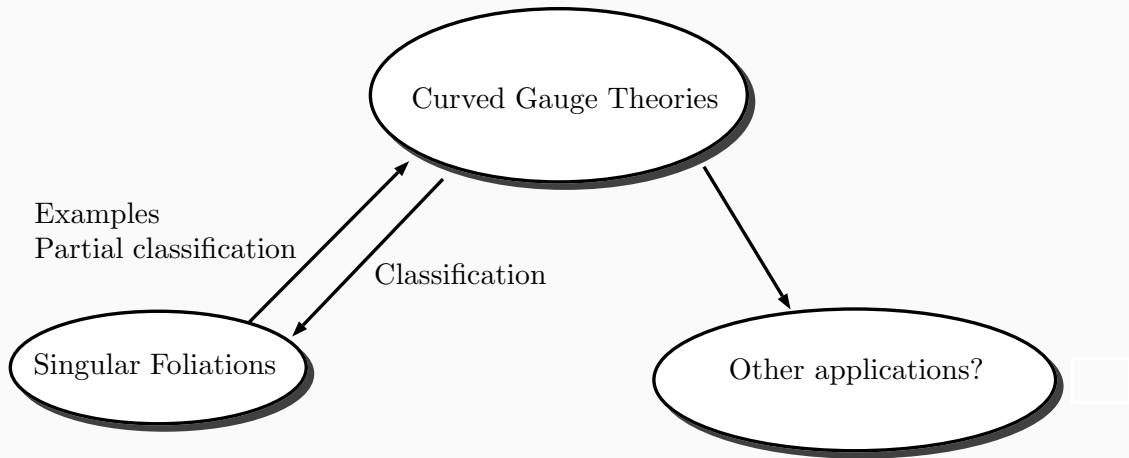


Curved Yang-Mills-Higgs theories

Simon-Raphael Fischer, *based on joint works with Camille Laurent-Gengoux, and with Mehran Jalali Farahani, Hyungrok Kim (金炯錄), Christian Sämann*





Curved Yang-Mills-Higgs theory

Motivation: Covariantisation of Yang-Mills(-Higgs) theory

Covariantization



Classical theory

Covariantised flat theory

Curved Theory

Vector space V

Trivial vector bundle $M \times V$

Vector bundle $V \rightarrow M$

$$\frac{\partial}{\partial x^i}$$

Canonical flat connection ∇^0

Vector bundle connection ∇

Coordinate changes may lead to extra terms

Coordinate expressions form-invariant under coordinate changes

Curved Yang-Mills gauge theory (curved YM theory)

Covariantization



YM theory

Covariantised YM theory

Curved YM theory

Lie group G

Trivial Lie group bundle $M \times G$

Lie group bundle $G \rightarrow M$

Maurer Cartan form

Fibre-wise Maurer-Cartan

Multiplicative YM connection

Field redefinitions lead to extra terms in gauge transformations and field strength

Coordinate expressions form-invariant under coordinate changes

S.-R. Fischer. *Integrating curved Yang–Mills gauge theories*, arXiv: 2210.02924, 2022.

S.-R. Fischer. *Geometry of curved Yang–Mills–Higgs gauge theories*, Ph.D. thesis, Institut Camille Jordan [Villeurbanne], France, U. Geneva, Switzerland, 2021; doi: 10.13097/archive-ouverte/unige:152555

Curved Yang-Mills-Higgs theory (curved YMH theory)

Covariantization



YMH theory	Covariantised YMH theory	Curved YMH theory
Lie group G with right-action on N	Action groupoid $N \times G$	Lie groupoid $G \rightrightarrows N$
Maurer Cartan form	Fibre-wise Maurer-Cartan	Covariant adjustments
Field redefinitions lead to extra terms in gauge transformations and field strength	Coordinate expressions form-invariant under coordinate changes	

S.-R. Fischer, M. Jalali Farahani, H. Kim, and C. Saemann. *Adjusted connections I: Differential cocycles for principal groupoid bundles with connection*, arXiv: 2406.16755, 202.

S.-R. Fischer. *Geometry of curved Yang–Mills–Higgs gauge theories*, Ph.D. thesis, Institut Camille Jordan [Villeurbanne], France, U. Geneva, Switzerland, 2021; doi: 10.13097/archive-ouverte/unige:152555

Classical theory	Covariantised flat theory
Lie algebra \mathfrak{g} as $L \times \mathfrak{g}$	Lie algebroid $E \rightarrow N$
\mathfrak{g} -action γ	Anchor ρ of E & E -connections
Canonical flat connection ∇^0 on $L \times \mathfrak{g}$	General connection ∇ on E

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$$\begin{array}{c} \mathcal{G} \\ \downarrow t \quad \downarrow s \\ M \end{array}$$

Definition

\mathcal{G} a **Lie groupoid** if there are surjective submersions $s, t: \mathcal{G} \rightarrow M$, *source* and *target*, respectively, and a smooth *multiplication map* $\mathcal{G}_{s \times_t \mathcal{G}} \rightarrow \mathcal{G}$ such that

$$s(g'g) = s(g), \quad t(g'g) = t(g')$$

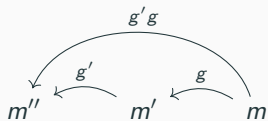
for all $(g', g) \in \mathcal{G}_{s \times_t \mathcal{G}}$ (i.e. $s(g') = t(g)$), satisfying the typical expected properties, that is,

$$\text{Associativity:} \quad (g''g')g = g''(g'g),$$

$$\text{Units:} \quad ge_{s(g)} = g, \quad e_{t(g)}g = g,$$

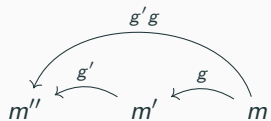
$$\text{Inverse:} \quad g^{-1}g = e_{s(g)}, \quad gg^{-1} = e_{t(g)}$$

for all $(g'', g', g) \in \mathcal{G}_{s \times_t \mathcal{G}} \times_{s \times_t \mathcal{G}} \mathcal{G}$, where one requires the existence of the *unit* e as a global section of both, s and t , and the *inverse* $g^{-1} \in \mathcal{G}$ of g .



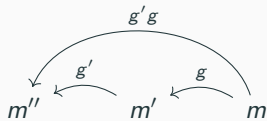
Example
Lie groups G





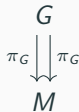
Example
Lie groups G





Example

Lie group bundles (LGBs) $\pi_G: G \rightarrow M$



Motivation (sort of historical)

We will mainly focus on Yang-Mills theories:

	Classical	Curved
Infinitesimal	Lie algebra \mathfrak{g}	LAB ¹ \mathcal{g}
Integrated	Lie group G	LGB ² \mathcal{G}

$$\begin{array}{ccc} G & \longrightarrow & \mathcal{G} \\ & & \downarrow \\ & & L \end{array}$$

Remarks (Why curved?)

For gauge invariance and closure of gauge transformations:

- Generalize Maurer-Cartan form \rightarrow **Multiplicative Yang-Mills connection.**
- Generalize Field Strength.

¹LAB = Lie algebra bundle

²LGB = Lie group bundle

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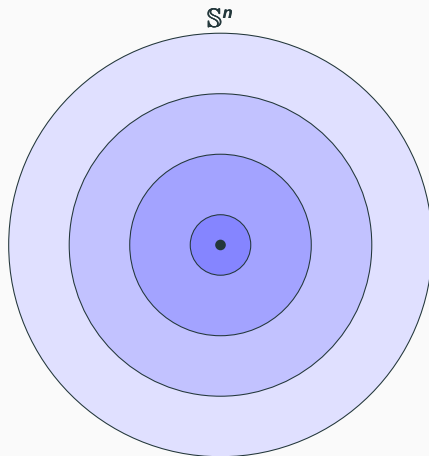
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Singular Foliations

Example of a singular foliation



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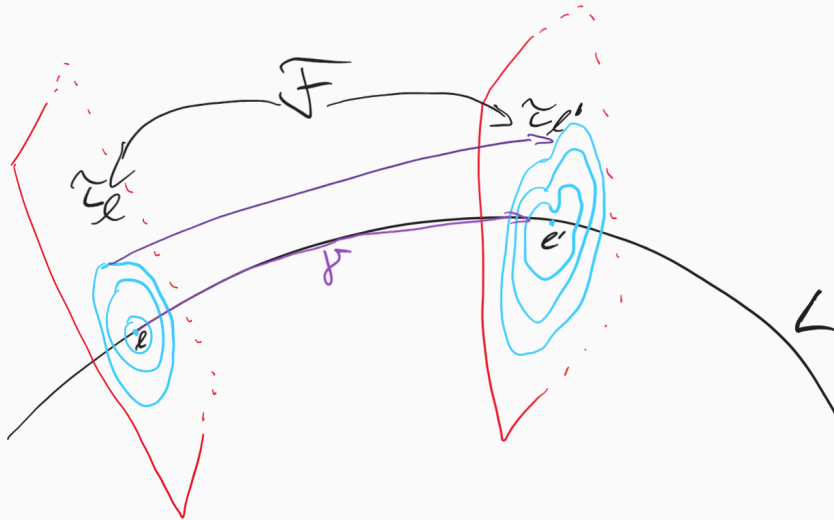
Singular Foliations:

- Gauge Theory
- Poisson Geometry
(Singular foliation of symplectic leaves)
- Lie groupoids and algebroids
- Dirac structures
- Generalised complex manifolds
- Non-commutative geometry
- ...

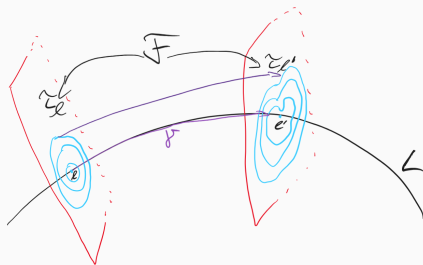
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Unique transverse structure



How to classify singular foliations?



Remarks ([C. L.-G., S.-R. F.])

There is a connection on the normal bundle of a leaf L preserving the foliation!

- Transverse structure is unique: Classify singular foliation \mathcal{F} like a bundle!
- Connection is a multiplicative Yang-Mills connection: Use curved gauge theory!

Thank you!

Classification of singular foliations

Theorem ([C. L.-G., S.-R. F.])

Formal singular foliations with leaf L and transverse model (\mathbb{R}^d, τ_I) are equivalent to:

- A Galois cover L' over L with structural group K
- A short exact sequence of groups

$$\text{Inner}(\tau_I) \hookrightarrow H \twoheadrightarrow K$$

- A principal H -bundle P over L