

Ex. 1. Given $w^1(0) = 1$, $b^1(0) = -2$, $w^2(0) = 1$, $b^2(0) = 1$
 $f^1(n) = n^2$, $f^2(n) = \frac{1}{n}$, $p = 1$, $t = 1$, $\alpha = 1$.

① Forward Propagation

$$a^1 = f^1(n^1) = (w^1 p + b^1)^2 = (1 \times 1 - 2)^2 = 1$$

$$a^2 = f^2(n^2) = \frac{1}{(w^2 a^1 + b^2)} = \frac{1}{(1 \times 1 + 1)} = \frac{1}{2}.$$

$$e = t - a = 1 - \frac{1}{2} = \frac{1}{2}$$

② Transfer Function Derivatives

$$f'^1(n) = \frac{d}{dn}(n^2) = 2n. \quad f'^2(n) = \frac{d}{dn}\left(\frac{1}{n}\right) = -\frac{1}{n^2}$$

③ Backpropagation

$$s^2 = -2 f'^2(n^2) \cdot (t - a) = -2 \times \left(-\frac{1}{4}\right) \times \frac{1}{2} = \frac{1}{4}$$

$$s^1 = f'^1(n^1) \cdot (w^2)^T s^2 = 2 \times (-1) \times 1 \times \frac{1}{4} = -\frac{1}{2}$$

④ Weight Update

$$w^2(1) = w^2(0) - \alpha s^2 (a^1)^T = 1 - 1 \times \frac{1}{4} \times 1 = \frac{3}{4}$$

$$b^2(1) = b^2(0) - \alpha s^2 = 1 - 1 \times \frac{1}{4} = \frac{3}{4}$$

$$w^1(1) = w^1(0) - \alpha s^1 (a^0)^T = 1 - 1 \times \left(-\frac{1}{2}\right) \times 1 = \frac{3}{2}$$

$$b^1(1) = b^1(0) - \alpha s^1 = -2 - 1 \times \left(-\frac{1}{2}\right) = -\frac{3}{2}$$