4.

$$f(x) = \frac{1}{2} x^T \begin{bmatrix} 1 & -3 \\ -3 & 1 \end{bmatrix} x + \begin{bmatrix} -4 & 4 \end{bmatrix} x$$

$$x = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore f(x,y) = \frac{1}{2} \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} x - 3y \\ -3x + y \end{bmatrix} + \begin{bmatrix} -4x + 4y \end{bmatrix}$$

$$= \frac{1}{2}(x^2 - 6xy + y^2) - 4x + 4y$$

$$\nabla f(x,y) = \begin{bmatrix} \frac{\partial}{\partial x} f(x,y) \\ \frac{\partial}{\partial y} f(x,y) \end{bmatrix} = \begin{bmatrix} x - 3y - 4 \\ -3x + y + 4 \end{bmatrix} = Ax + d$$

$$A = \begin{bmatrix} 1 & -3 \\ -3 & 1 \end{bmatrix} \quad d = \begin{bmatrix} -4 \\ 4 \end{bmatrix}$$

$$x^* = A^{-1} d = \begin{bmatrix} 1 & -3 \\ -3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -4 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\nabla^2 f(x,y) = \begin{bmatrix} \frac{\partial^2}{\partial x^2} f(x,y) & \frac{\partial^2}{\partial y \partial x} f(x,y) \\ \frac{\partial^2}{\partial x \partial y} f(x,y) & \frac{\partial^2}{\partial y^2} f(x,y) \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ -3 & 1 \end{bmatrix}$$

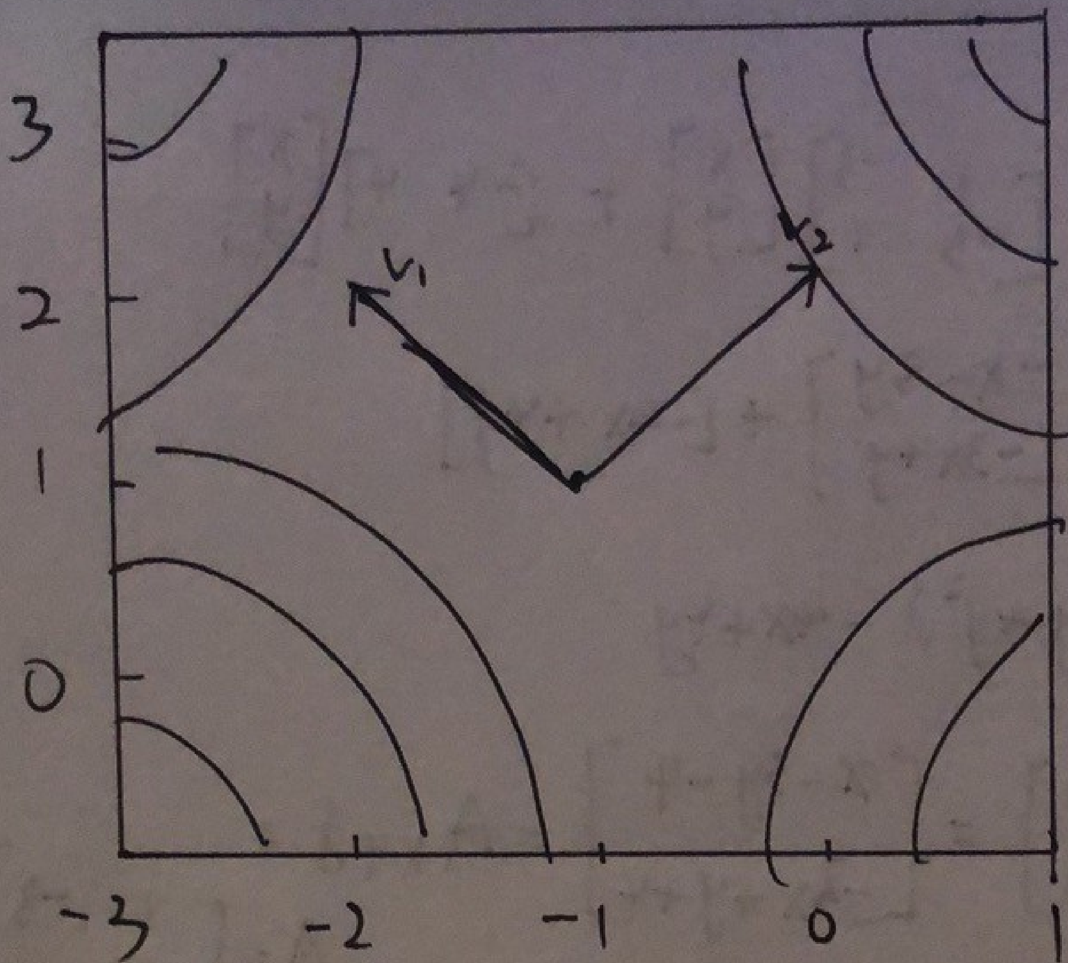
$$\begin{vmatrix} 1-\lambda & -3 \\ -3 & 1-\lambda \end{vmatrix} = 1 - 2\lambda + \lambda^2 - 9 = \lambda^2 - 2\lambda - 8 = (\lambda - 4)(\lambda + 2)$$

$\lambda_1 = 4$, $v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\lambda_2 = -2$, $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\because \lambda_1 > 0 \quad \lambda_2 < 0 \quad \therefore (-1, 1)$ is a saddle point

$f(-1, 1) = 12$



3
2
1
0
-3
-2
-1
0
1
v1
v2