Z1.

i $f(x) = \sin(6x-1)$

$$\frac{d}{dx}(\sin(6x-1))$$
$$= \cos(6x-1)\frac{d}{dx}(6x-1)$$
$$= \cos(6x-1)\cdot 6$$

ii $f(x) = x^8 + 30 + \frac{1}{x^4}$

$$\frac{d}{dx}(x^8) + \frac{d}{dx}(30) + \frac{d}{dx}\left(\frac{1}{x^4}\right)$$
$$= 8x^7 + 0 + \left(-4\cdot\frac{1}{x^5}\right)$$
$$= 8x^7 - \frac{4}{x^5}$$

iii $f(x) = e^{(\frac{1}{x}) + (\frac{1}{x^2})}$

$$\frac{d}{dx}\left(e^{(\frac{1}{x})+(\frac{1}{x^2})}\right)$$
$$= e^{\frac{1}{x}+\frac{1}{x^2}}\cdot\frac{d}{dx}\left(\frac{1}{x}+\frac{1}{x^2}\right)$$
$$= e^{\frac{1}{x}+\frac{1}{x^2}}\cdot\left(\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{d}{dx}\left(\frac{1}{x^2}\right)\right)$$
$$= e^{\frac{1}{x}+\frac{1}{x^2}}\cdot\left(-\frac{1}{x^2}+(-2)\cdot\frac{1}{x^3}\right)$$
$$= e^{\frac{1}{x}+\frac{1}{x^2}}\cdot\left(-\frac{1}{x^2}-\frac{2}{x^3}\right)$$

iv $f(x) = \sin^2(6x-1)$

$$\frac{d}{dx}(\sin^2(6x-1))$$
$$= 2\sin(6x-1)\cdot\cancel{dx}\,\frac{d}{dx}(\sin(6x-1))$$
$$= 2\sin(6x-1)\cdot\cos(6x-1)\cdot\frac{d}{dx}(6x-1)$$
$$= 2\sin(6x-1)\cdot\cos(6x-1)\cdot 6$$
$$= 6\cdot\sin(2\cdot(6x-1))$$

t_2

$f(x) = 2x^3 + 24x^2 - 54x$

$f'(x) = 6x^2 + 48x - 54$

$\Delta = b^2 - 4ac = 2304 - 1296 > 0$

when $f'(x) = 0$ $\quad x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

$x_1 = -9 \quad x_2 = 1$

$f(-9) = 2 \cdot (-9)^3 + 24 \cdot (-9)^2 - 54 \cdot (-9) = -1458 + 1944 - (-486) = 972$

$f(1) = 2 \cdot (1)^3 + 24 \cdot (1)^2 - 54 \cdot (1) = -28$

when $x < -9$, $f(x)$ is increasing and concave down.

when $-9 < x < 1$, $f(x)$ is decreasing and concave up.

when $x > 1$, $f(x)$ is increasing and concave up.

when $f(x) = 0$. $\quad x \cdot (2x^2 + 24x - 54) = 0$

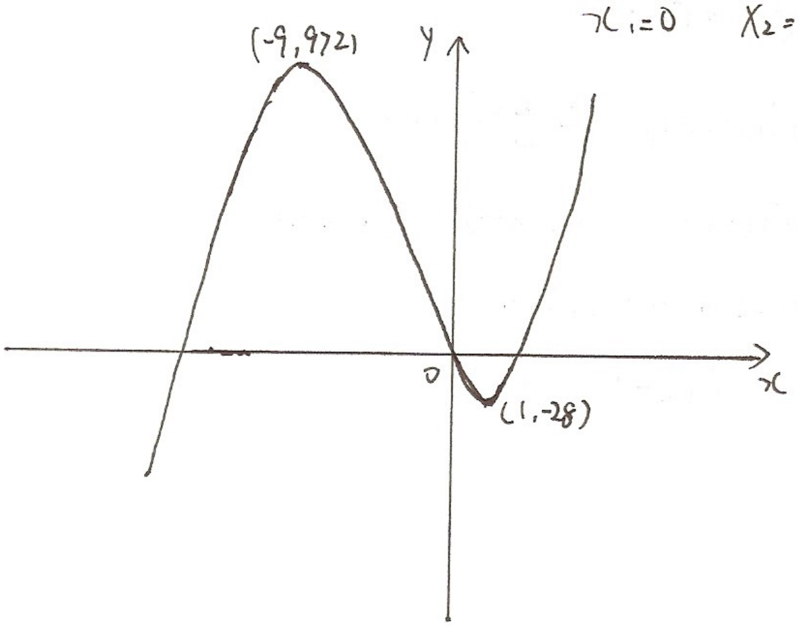
$x_1 = 0 \quad X_2 = -6 - 3\sqrt{7} \quad X_3 = -6 + 3\sqrt{7}$

$\Delta = b^2 - 4ac$
$= 26^2 - 4 \cdot 2(-54)$
$= 576 + 432$
$= \cancel{1444}\ 1008 > 0$

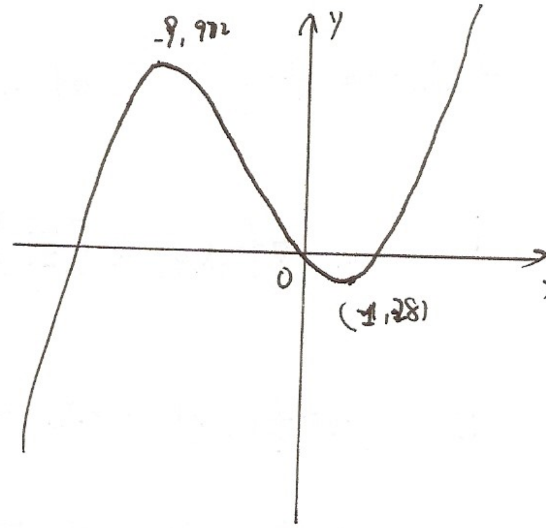
$x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-24 \pm \sqrt{1008}}{4}$

$= -6 \pm 3\sqrt{7}$



E3.



$f(x) = 2x^3 + 24x^2 - 54x$

$f'(x) = 6x^2 + 48x - 54$

when $f'(x) = 0$ $\quad 6x^2 + 48x - 54 = 0$

$x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

$x_1 = -9 \quad x_2 = 1.$

$f(-9) = 2\cdot(-9)^3 + 24\cdot(-9)^2 - 54\cdot(-9) = -1458 + 1944 - (-486) = 972$

$f(1) = 2\cdot(1)^3 + 24\cdot(1)^2 - 54\cdot(1) = -28$

$\therefore$ critical points are $(-9, 972)$ and $(1, -28)$

$(-9, 972)$ is local max point and $(1, -28)$ is local min point;
972 is local max value and -28 is local min value.

when $x \in [-3, 3]$ $\quad f(-3) = 2\cdot(-3)^3 + 24\cdot(-3)^2 - 54(-3) = -54 + 216 - (-162) = 324$

$f(3) = 2\cdot(3)^3 + 24\cdot(3)^2 - 54(3) = 54 + 216 - 162 = 108$

$\because f(-3) > f(3) > f(1)$

$\therefore$ global max point is $(-3, 324)$, value is 324.
global min point is $(1, -28)$, value is -28.

when $x \in (-\infty, 0)$ $\quad \because x \to -\infty \quad f(x) \to -\infty$

$\therefore$ global max point is $(-9, 972)$, value is 972

$f(x)$ doesn't have global min.

24. $f(x,y) = x^2+y^2$ $\nabla f(x,y) = \begin{bmatrix} \frac{d}{dx}(x^2+y^2) \\ \frac{d}{dy}(x^2+y^2) \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

i $\therefore \nabla f(x,y) = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

ii $\because \nabla f(x,y) = \langle 2x, 2y \rangle$

$\therefore \nabla f(1,2) = \langle 2, 4 \rangle = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\nabla f(2,1) = \langle 4, 2 \rangle = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

$\nabla f(0,0) = \langle 0, 0 \rangle = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

5.

$$f(x,y) = 2xy + x^2 + y$$

i $$\nabla f(x,y) = \begin{bmatrix} \frac{d}{dx}(2xy + x^2 + y) \\ \frac{d}{dy}(2xy + x^2 + y) \end{bmatrix} = \begin{bmatrix} 2y + 2x \\ 2x + 1 \end{bmatrix}$$

ii $$\because \nabla f(x,y) = \langle 2y+2x, 2x+1 \rangle$$

$$\therefore \nabla f(1,1) = \langle 4, 3 \rangle = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\nabla f(0,-1) = \langle -2, 1 \rangle = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\nabla f(0,0) = \langle 0, 1 \rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

iii $$f(x_1, x_2) = x_1^2 + 2x_1x_2 + x_2^2 + x_1x_2^2$$

$$\nabla f(x_1, x_2) = \begin{bmatrix} \frac{d}{dx_1}(x_1^2 + 2x_1x_2 + x_2^2 + x_1x_2^2) \\ \frac{d}{dx_2}(x_1^2 + 2x_1x_2 + x_2^2 + x_1x_2^2) \end{bmatrix}$$

$\frac{d}{dx_1} x_1^2 = 2x_1$ $\quad$ $\frac{d}{dx_2} 2x_1x_2 = 2x_1$

$\frac{d}{dx_1} 2x_1x_2 = 2x_2$ $\quad$ $\frac{d}{dx_1} x_2^2 = 0$

$\frac{d}{dx_2} x_2^2 = 2x_2$ $\quad$ $\frac{d}{dx_1} x_1x_2^2 = x_2^2$

$\frac{d}{dx_2} x_1x_2^2 = x_1 2x_2$ $\quad$ $\frac{d}{dx_2} x_1^2 = 0$

$$= \begin{bmatrix} 2x_1 + 2x_2 + x_2^2 \\ 2x_1 + 2x_2 + 2x_1x_2 \end{bmatrix}$$

Z6.

i $y = kx + b$

$\because$ y-intercept is $(0, -0.5)$

$\therefore b = -0.5$

$\because$ slope $= 3$

$\therefore k = 3$

$\cancel{\therefore y = 3x - 0.5}$ $\boxed{\therefore y = 3x - 0.5}$

ii $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

$\because$ line passes through $(4, 8)$ and $(6, 14)$

$\therefore \dfrac{y-8}{14-8} = \dfrac{x-4}{6-4}$

$\dfrac{y-8}{6} = \dfrac{x-4}{2}$

$y - 8 = 3x - 12$

$\boxed{y = 3x - 4}$

iii $y = kx + b$

$\because l_1 \perp l_2$

$\therefore k_1 \cdot k_2 = -1$

$\therefore k \cdot 5 = -1$

$k = -\frac{1}{5}$

$\because$ line passes through $(3, 2)$

$\therefore 2 = -\frac{3}{5} + b$

$b = \frac{13}{5}$

$\boxed{\therefore y = -\frac{1}{5}x + \frac{13}{5}}$

iv $y = kx + b$

$\because b = 3$, line passes through $(2, 1)$

$\therefore 1 = 2k + 3$

$k = -1$

$\boxed{\therefore y = -x + 3}$

v. $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

$\because$ line through $(6, 4)$ and $(1, -1)$

$\therefore \dfrac{y-4}{-1-4} = \dfrac{x-6}{1-6}$

$\dfrac{y-4}{-5} = \dfrac{x-6}{-5}$

$y - 4 = x - 6$

$\boxed{\therefore y = x - 2}$

27.

i. $A = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$ $\quad |\lambda E - A| = \begin{vmatrix} \lambda-2 & 0 \\ 0 & \lambda-5 \end{vmatrix} = (\lambda-2)(\lambda-5) = 0$ $\quad \boxed{\lambda_1 = 2 \quad \lambda_2 = 5}$

when $\lambda = 2$.

$(2E - A)\vec{v_1} = \begin{bmatrix} 0 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{cases} x_1 = 0 \\ -3x_2 = 0 \end{cases}$

$\boxed{\lambda_1 = 2 \quad \vec{v_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$

when $\lambda = 5$

$(5E - A)\vec{v_2} = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{cases} 3x_1 = 0 \\ x_2 = 0 \end{cases}$

$\boxed{\lambda_2 = 5 \quad \vec{v_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$

ii. $A = \begin{bmatrix} 5 & 1 \\ 4 & 5 \end{bmatrix}$ $\quad |\lambda E - A| = \begin{vmatrix} \lambda-5 & -1 \\ -4 & \lambda-5 \end{vmatrix} = (\lambda-5)^2 - (-1)\times(-4)$ $\quad \boxed{\lambda_1 = 3 \quad \lambda_2 = 7}$

$= \lambda^2 - 10\lambda + 25 \cancel{+4} - 4$

$= \lambda^2 - 10\lambda + 21$

$= (\lambda-3)(\lambda-7)$

when $\lambda = 3$

$(3E - A)\vec{v_1} = \begin{bmatrix} -2 & -1 \\ -4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{cases} -2x_1 - x_2 = 0 \\ -4x_1 - 2x_2 = 0 \end{cases} \Rightarrow \begin{cases} 2x_1 + x_2 = 0 \\ 4x_1 + 2x_2 = 0 \end{cases} \Rightarrow 2x_1 = -x_2$

$\begin{cases} x_1 = 1 \\ x_2 = -2 \end{cases}$

$\boxed{\therefore \lambda_1 = 3, \quad \vec{v_1} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}}$

when $\lambda = 7$

$(7E - A)\vec{v_2} = \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{cases} 2x_1 - x_2 = 0 \\ -4x_1 + x_2 = 0 \end{cases} \Rightarrow 2x_1 - x_2 = 0$

$2x_1 = x_2$

$\begin{cases} x_1 = 1 \\ x_2 = 2 \end{cases}$

$\boxed{\therefore \lambda_2 = 7 \quad \vec{v_2} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$

iii. $A = \begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix}$ $\quad |\lambda E - A| = \begin{vmatrix} \lambda-3 & -5 \\ -3 & \lambda-1 \end{vmatrix} = (\lambda^2 - 4\lambda + 3) - (-5)\times(-3)$ $\quad \boxed{\lambda_1 = 6 \quad \lambda_2 = -2}$

$= \lambda^2 - 4\lambda - 12$

$= (\lambda - 6)(\lambda + 2)$

when $\lambda = 6$

$(6E - A)\vec{v_1} = \begin{bmatrix} 3 & -5 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{cases} 3x_1 - 5x_2 = 0 \\ -3x_1 + 5x_2 = 0 \end{cases} \Rightarrow 3x_1 = 5x_2 \Rightarrow \begin{cases} x_1 = 5 \\ x_2 = 3 \end{cases}$

$\boxed{\therefore \lambda_1 = 6, \quad \vec{v_1} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}}$

when $\lambda = -2$

$(-2E - A)\vec{v} = \begin{bmatrix} -5 & -5 \\ -3 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{cases} -5x_1 - 5x_2 = 0 \\ -3x_1 - 3x_2 = 0 \end{cases} \Rightarrow x_1 = -x_2 \begin{cases} x_1 = 1 \\ x_2 = -1 \end{cases}$

$\therefore \boxed{\lambda_2 = -2, \quad \vec{v_2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$

E8.

1. An operation called vector addition is defined such that if $\vec{x} \in X$ and $\vec{y} \in X$, then $\vec{x}+\vec{y} \in X$.

2. $\vec{x}+\vec{y} = \vec{y}+\vec{x}$

3. $(\vec{x}+\vec{y})+\vec{z} = \vec{x}+(\vec{y}+\vec{z})$

4. There is a unique vector $\vec{0} \in X$, called zero vector, such that $\vec{x}+\vec{0}=\vec{x}$ for all $\vec{x} \in X$.

5. For each vector there is a unique vector in X, to be called $(-\vec{x})$, such that $\vec{x}+ -\vec{x} = \vec{0}$.

6. An operation, called multiplication, is defined such that for all scalars $a \in F$, and all vectors $\vec{x} \in X$, $a\vec{x} \in X$.

7. For any $\vec{x} \in X$, $1\vec{x} = \vec{x}$ (for scalar 1)

8. For any two scalars $a \in F$ and $b \in F$, and any $\vec{x} \in X$, $a(b\vec{x}) = (ab)\vec{x}$.

9. $(a+b)\vec{x} = a\vec{x} + b\vec{x}$

10. $a(\vec{x}+\vec{y}) = a\vec{x} + a\vec{y}$.

i. if all continuous functions that satisfy the condition $f(0)=0$ is a vectorspace. Then each function should be a vector.

①②③: The sum of a finite number of continuous functions is a continuous function. Thus, that sum is in vector space, too. All of those function satisfy $f(0)=0$.

④.⑤: Any function $f(x)$ add $f(0)=0$ / $\vec{0}$ is itself. Any function $f(x)$ has a function $-f(x)$ allows $f(x)+(-f(x))=\vec{0}$.

⑥~⑩: if $F=R$, then any a or b times $f(x)$ that satisfy $f(0)=0$ will be a new continuous function satisfy $f(0)=0$.

ii The sum of a finite number of 2x2 matrices is a 2x2 matrix, no matter the order. ①②③

④ Any 2x2 matrix add a zero martix is itself.

⑤ Any 2x2 matrix has a 2x2 matrix, makes the sum be a zero matrix.

⑥~⑩ when $F=R$, any a or b or 1 times a 2x2 matrix is a new 2x2 matrix, which is still in that vector space.

~~Any~~ Any 2x2 matrix multiplied by 1 (scalar) is itself.

Eg.

i. $\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $A = (\vec{v_1}, \vec{v_2}, \vec{v_3}) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix} = 1 \times [(0\times1) - (1\times2)] - 1\times[(2\times1) - (2\times3)] + 1\times[(2\times1) - (3\times0)]$$

$$= 1 \times (-2) - 1\times(-4) + 1 \times 2$$

$$= 4$$

$\because |A| \neq 0 \quad \therefore \text{rank}(A) = n = 3 \quad \therefore \vec{v_1}, \vec{v_2}, \vec{v_3}$ linearly independent.

$\therefore$ Basis of span(S) is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

$\therefore \dim(\text{span}(S)) = 3$.

ii. $S = \{\sin(t), \cos(t), \cos(2t)\}$. $\qquad \therefore \dim(\text{span}(S)) = 3$

$k_1(\sin(t)) + k_2(\cos(t)) + k_3(\cos(2t)) = 0$

$$\begin{cases} \text{when } t = 0, & k_2 + k_3 = 0 \\ \text{when } t = \frac{\pi}{2}, & k_1 - k_3 = 0 \\ \text{when } t = \pi, & -k_2 + k_3 = 0 \end{cases}$$

$\therefore k_1 = k_2 = k_3 = 0$

$\therefore \vec{v_1}, \vec{v_2}, \vec{v_3}$ linearly independent

iii. $S = \{1+t, 1-t\}$ $\qquad \therefore \vec{v_1}, \vec{v_2}$ linearly independent.

$k_1(1+t) + k_2(1-t) = 0$ $\qquad \therefore \dim(\text{span}(S)) = 2$

$(k_1 + k_2) + (k_1 - k_2)t = 0$

$\therefore k_1 + k_2 = 0$

$k_1 - k_2 = 0$

$\therefore k_1 = k_2 = 0$

iv. $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 4 \\ 3 \end{bmatrix} \right\}$. $\quad A = (\vec{v_1}, \vec{v_2}, \vec{v_3}) = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 0 & 4 \\ 2 & 0 & 4 \\ 1 & 1 & 3 \end{bmatrix} \underset{\sim}{\overset{R_1 - R_4 \to R_4}{R_2 - R_3 \to R_3}} \begin{bmatrix} 1 & 1 & 3 \\ 2 & 0 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\overset{R_2 \div 2 \to R_2}{\sim} \begin{bmatrix} 1 & 1 & 3 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \overset{R_1 - R_2 \to R_2}{\sim} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \overset{R_2 - R_1 \to R_1}{\sim} \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \text{rank}(A) = 2 \neq 4$

$\therefore \vec{v_1}, \vec{v_2}, \vec{v_3}$ linearly ~~independent~~ dependent.

$\therefore \dim(\text{span}(S)) = 2$

Z10.

$$\vec{V}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{V}_2 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \quad \vec{V}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & -2 & 0 \end{bmatrix} \qquad [B\ I] = \left[\begin{array}{ccc|ccc} -1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\underset{R_2 \div (-2) \to R_2}{\overset{R_1 \div (-1) \to R_1}{\sim}} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} \\ 1 & 1 & 1 & 0 & 1 & 0 \end{array}\right] \underset{R_3 - R_2 \to R_1}{\overset{R_1 + R_2 \to R_1}{\sim}} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} \\ 1 & 0 & 1 & 0 & 1 & \frac{1}{2} \end{array}\right] \underset{R_3 - R_1 \to R_3}{\overset{R_1 + R_3 \to R_1}{\sim}} \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 2 & 1 & 1 & 1 \end{array}\right]$$

$$\underset{R_3 \div 2 \to R_3}{\overset{R_1 \div 2 \to R_1}{\sim}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}\right] \qquad \therefore B^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\therefore \vec{x}_1^{v_1} = \vec{V}_1^T \cdot \vec{x} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \frac{1}{2}$$

$$\vec{x}_2^{v_2} = \vec{v}_2^T \cdot \vec{x} = \begin{bmatrix} 0 & 0 & -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = -1$$

$$\vec{x}_3^{v_3} = \vec{V}_3^T \cdot \vec{x} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \frac{5}{2}$$

$$\therefore \vec{x} = \frac{1}{2}\vec{V}_1 - \vec{V}_2 + \frac{5}{2}\vec{V}_3$$