Machine Learning II
Homework #3
Due: *Sep-28-2020*

Homework 3:

- **Show ALL Work, Neatly and in Order.**
- **No** credit for Answers Without Work.
- Submit a single pdf file includes all of your solutions.
- **DO NOT** submit individual files or images.
- For coding questions, submit **ONE** .py file and include your comments.

E.1:

Using the following basis vectors, find an orthogonal set using Gram-Schmidt orthogonalization.

$$y_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}, y_2 = \begin{bmatrix}1\\1\\0\end{bmatrix}, y_3 = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

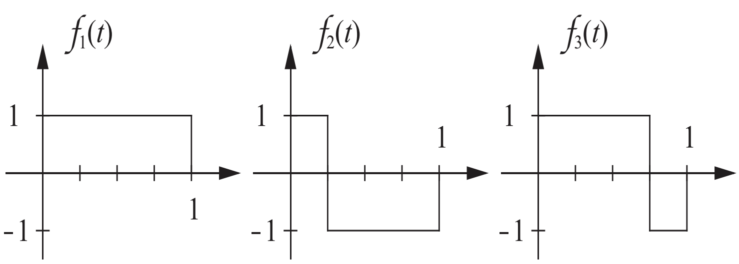
E.2:

Expand $x = \begin{bmatrix}1 & 2 & 2\end{bmatrix}^T$ in terms of the following basis set. (Verify your answer using Python.)

$$v_1 = \begin{bmatrix}-1\\1\\0\end{bmatrix}, v_2 = \begin{bmatrix}1\\1\\-2\end{bmatrix}, v_3 = \begin{bmatrix}1\\1\\0\end{bmatrix}$$

E.3:

Consider the vector space of all piecewise continuous functions on the interval $[0,1]$. The set f_1, f_2, f_3 which is defined in below , contains three vectors from this vector space.



i. Show that this set is linearly independent.

ii. Generate an orthogonal set using the Gram-Schmidt procedure. The inner product is defined to be

$$(f \cdot g) = \int_0^1 f(t)g(t)dt$$

E.4:

Find the value of a that makes $||x - ay||$ a minimum. (Use $||x|| = (x,x)^{\frac{1}{2}}$. Show that for this value of a the vector $z = x - ay$ is orthogonal to y and that

$$||x - ay||^2 + ||ay||^2 = ||x||^2$$

E.5:

Consider the space of complex numbers. Let this be the vector space , and let the basis for be X be $\{1 + j, 1 - j\}$. Let $A : X \to Y$ be the operation of multiplication by $(1 + j)$ (i.e.,$A(X) = (1 + j)X$
i.Find the matrix of the transformation A relative to the basis set given above.

ii. Find the eigenvalues and eigenvectors of the transformation.

iii. Find the matrix representation for A relative to the eigenvectors as the basis vectors.

iv. Check your answers to parts (ii) and (iii) using Python.

E.6:

Consider the space of functions of the form $\alpha sin(t + \phi)$. One basis set for this space is $V = \{sin(t), cos(t)\}$. Consider the differentiation transformation D.
i.Find the matrix of the transformation D relative to the basis set V.

ii. Find the eigenvalues and eigenvectors of the transformation. Show the eigenvectors as columns of numbers and as functions of $\{sin(t), cos(t)\}$.

iii. Find the matrix of the transformation relative to the eigenvectors as basis vectors.

E.7:

We know that a certain linear transformation $A : R^2 \to R^2$ has eigenvalues and eigenvectors given by

$$\lambda_1 = 1, z_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \lambda_2 = 2, z_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(The eigenvectors are represented relative to the standard basis set.)
i.Find the matrix of the transformation A relative to the standard basis set.

ii. Find the matrix representation relative to the new basis

$$V = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$$

E.8:

Consider the following basis set for R^2

$$V = v_1, v_2, \qquad v1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

(The eigenvectors are represented relative to the standard basis set.)
i.Find the reciprocal basis vectors for this basis set.

ii. Consider a transformation $A : R^2 \to R^2$. The matrix representation for A relative to the standard basis in R^2 is

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Find the expansion of Av_1 in terms of the basis set V (Use the reciprocal basis vectors.)

iii. Find the expansion of Av_2 in terms of the basis set V (

iv. Find the matrix representation for A relative to the basis V. (This step should require no further computation.)