Quiz.3.

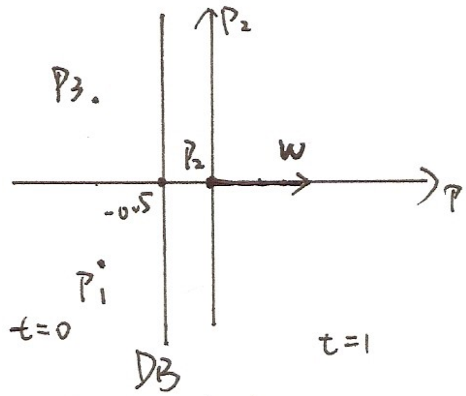
Given $\left\{P_1=\begin{bmatrix}-1\\-1\end{bmatrix}, t_1=0\right\}$, $\left\{P_2=\begin{bmatrix}0\\0\end{bmatrix}, t_2=0\right\}$, $\left\{P_3=\begin{bmatrix}-1\\1\end{bmatrix}, t_3=1\right\}$.

Initial weight matrix and bias are $w_0=[1\ 0]$, $b_0=0.5$

(i) $w_1 P_1 + w_2 P_2 + b = 0$

$P_1 + 0.5 = 0$

$P_1 = -0.5$



As shown in figure, only P_1 is correctly classified.

(ii) ① $a_1 = \text{hardlim}(wp+b) = \text{hardlim}\left([1\ 0]\begin{bmatrix}-1\\-1\end{bmatrix}+[0.5]\right) = \text{hardlim}(-0.5) = 0$

$e_1 = t_1 - a_1 = 0 \quad \therefore w^{new} = w^{old}, \ b^{new} = b^{old}$

② $a_2 = \text{hardlim}(wp+b) = \text{hardlim}\left([1\ 0]\begin{bmatrix}0\\0\end{bmatrix}+[0.5]\right) = \text{hardlim}(0.5) = 1$

$e_2 = t_2 - a_2 = -1 \quad \therefore w^{new} = w^{old} + e_2 \cdot P_2^T = [1\ 0] - [0\ 0] = [1\ 0]$

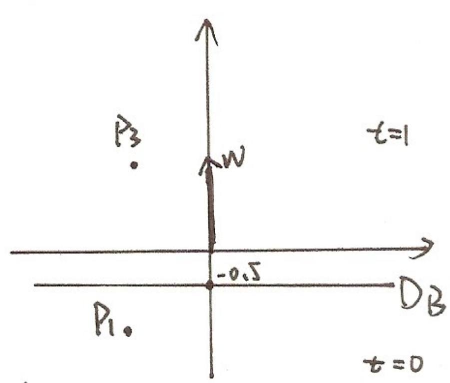
$b^{new} = b^{old} + e_2 = 0.5 - 1 = -0.5$

③ $a_3 = \text{hardlim}(wp+b) = \text{hardlim}\left([1\ 0]\begin{bmatrix}-1\\1\end{bmatrix}-[0.5]\right) = \text{hardlim}(-1.5) = 0$

$e_3 = t_3 - a_3 = 1 \quad \therefore w^{new} = w^{old} + e_3 P_3^T = [1\ 0] + [-1\ 1] = [0\ 1]$

$b^{new} = b^{old} + e_3 = -0.5 + 1 = 0.5$

(iii)



$w_1 P_1 + w_2 P_2 + b = 0$

$P_2 + 0.5 = 0$

$P_2 = -0.5$

P_3 and P_1 are correctly classified. P_2 not.

(iv) Yes. Because in this training set, two classes composed of those three points are linearly separable. Therefore, whatever initial weight we give, we can get a final weight which can correctly classify the patterns in this training set through perceptron rule with enough iterations.