3.1. $f(x) = [1+(x_1+x_2-5)^2][1+(3x_1-2x_2)^2]$

(i) $\Delta x_{k+1} = x_k - A_k^{-1} \cdot g_k$ $\quad x_0 = \begin{bmatrix}10\\10\end{bmatrix}$ $\quad \nabla f(x) = \begin{bmatrix}\frac{\partial}{\partial x_1} f(x) \\ \frac{\partial}{\partial x_2} f(x)\end{bmatrix}$

$$\frac{\partial}{\partial x_1} f(x) = \frac{\partial}{\partial x_1}(1+(x_1+x_2-5)^2) \times (1+(3x_1-2x_2)^2) + \frac{\partial}{\partial x_1}(1+(3x_1-2x_2)^2) \times (1+(x_1+x_2-5)^2)$$

$$= \left[\frac{\partial}{\partial x_1}(1) + \frac{\partial}{\partial x_1}(x_1+x_2-5)^2\right] \times (1+(3x_1-2x_2)^2) + \left[\frac{\partial}{\partial x_1}(1) + \frac{\partial}{\partial x_1}(3x_1-2x_2)^2\right] \times (1+(x_1+x_2-5)^2)$$

$$= \left[0 + 2\cdot(x_1+x_2-5)\cdot\frac{\partial}{\partial x_1}(x_1+x_2-5)\right] \times (1+(3x_1-2x_2)^2) + \left[0 + 2\cdot(3x_1-2x_2)\cdot\frac{\partial}{\partial x_1}(3x_1-2x_2)\right] \times (1+(x_1+x_2-5)^2)$$

$$= 2(x_1+x_2-5) \times 1 \times (1+(3x_1-2x_2)^2) + 2(3x_1-2x_2) \times 3 \times (1+(x_1+x_2-5)^2)$$

$$= 2(x_1+x_2-5) \times (1+(3x_1-2x_2)^2) + 6(3x_1-2x_2) \times (1+(x_1+x_2-5)^2)$$

$$\frac{\partial}{\partial x_2} f(x) = \frac{\partial}{\partial x_2}(1+(x_1+x_2-5)^2) \times (1+(3x_1-2x_2)^2) + \frac{\partial}{\partial x_2}(1+(3x_1-2x_2)^2) \times (1+(x_1+x_2-5)^2)$$

$$= \left[\frac{\partial}{\partial x_2}(1) + \frac{\partial}{\partial x_2}(x_1+x_2-5)^2\right] \times (1+(3x_1-2x_2)^2) + \left[\frac{\partial}{\partial x_2}(1) + \frac{\partial}{\partial x_2}(3x_1-2x_2)^2\right] \times (1+(x_1+x_2-5)^2)$$

$$= \left[0 + 2\cdot(x_1+x_2-5)\cdot\frac{\partial}{\partial x_2}(x_1+x_2-5)\right] \times (1+(3x_1-2x_2)^2) + \left[0 + 2(3x_1-2x_2)\cdot\frac{\partial}{\partial x_2}(3x_1-2x_2)\right] \times (1+(x_1+x_2-5)^2)$$

$$= 2 \times (x_1+x_2-5) \times 1 \times (1+(3x_1-2x_2)^2) + 2 \times (3x_1-2x_2) \times (-2) \times (1+(x_1+x_2-5)^2)$$

$$= 2(x_1+x_2-5) \times (1+(3x_1-2x_2)^2) + (-4)(3x_1-2x_2) \times (1+(x_1+x_2-5)^2)$$

$$g_0 = \nabla f(x)\Big|_{x=x_0} = \begin{bmatrix} 2 \times (10+10-5) \times (1+(3\times 10-2\times 10)^2) + 6 \times (3\times 10-2\times 10) \times (1+(10+10-5)^2) \\ 2 \times (10+10-5) \times (1+(3\times 10-2\times 10)^2) - 4 \times (3\times 10-2\times 10) \times (1+(10+10-5)^2) \end{bmatrix} = \begin{bmatrix}16590 \\ -6010\end{bmatrix}$$

$$A_0 = \nabla^2 f(x)\Big|_{x=x_0} = \begin{bmatrix} \frac{\partial\left(\frac{\partial f(x)}{\partial x_1}\right)}{\partial x_1} & \frac{\partial\left(\frac{\partial f(x)}{\partial x_1}\right)}{\partial x_2} \\ \frac{\partial\left(\frac{\partial f(x)}{\partial x_2}\right)}{\partial x_1} & \frac{\partial\left(\frac{\partial f(x)}{\partial x_2}\right)}{\partial x_2} \end{bmatrix}\Bigg|_{x=x_0}$$

$$\frac{\partial^2}{\partial x_1^2} f(x) = \frac{\partial}{\partial x_1}(2(x_1+x_2-5)) \times (1+(3x_1-2x_2)^2) + \frac{\partial}{\partial x_1}(1+(3x_1-2x_2)^2) \times (2(x_1+x_2-5)) +$$
$$\frac{\partial}{\partial x_1}(6(3x_1-2x_2)) \times (1+(x_1+x_2-5)^2) + \frac{\partial}{\partial x_1}(1+(x_1+x_2-5)^2) \times (6(3x_1-2x_2))$$

$$= 2(1+(3x_1-2x_2)^2) + 6(3x_1-2x_2) \times (2(x_1+x_2-5)) + 18(1+(x_1+x_2-5)^2) + 2(x_1+x_2-5) \times (6(3x_1-2x_2))$$

$$\frac{\partial^2}{\partial x_1^2} f(x)\Big|_{x=x_0} = 2(1+(3\times 10-2\times 10)^2) + 6(3\times 10-2\times 10) \times (2(10+10-5)) + 18(1+(10+10-5)^2) + 2(10+10-5) \times (6(3\times 10-2\times 10))$$
$$= 7870$$

$$\frac{\partial^2 f(x)}{\partial x_2 \partial x_1} = \frac{\partial}{\partial x_2} 2(x_1+x_2-5)\times(1+(3x_1-2x_2)^2) + \frac{\partial}{\partial x_2}(1+(3x_1-2x_2)^2)\times 2(x_1+x_2-5) + \frac{\partial}{\partial x_2} 6(3x_1-2x_2)\times(1+(x_1+x_2-5)^2)$$
$$+ \frac{\partial}{\partial x_2}(1+(x_1+x_2-5)^2)\times 6(3x_1-2x_2)$$
$$= 2(1+(3x_1-2x_2)^2) - 4(3x_1-2x_2)\times 2(x_1+x_2-5) - 12(1+(x_1+x_2-5)^2) + 2(x_1+x_2-5)\times 6(3x_1-2x_2)$$

$$\frac{\partial f(x)}{\partial x_2 \partial x_1}\Big|_{x=x_0} = 2(1+100) - 4\times 10\times 30 - 12\times 226 + 2\times 15\times 6\times 10 = -1910$$

$$\frac{\partial^2 f(x)}{\partial x_1 \partial x_2}\Big|_{x=x_0} = \frac{\partial}{\partial x_1} 2(x_1+x_2-5)\times(1+(3x_1-2x_2)^2) + \frac{\partial}{\partial x_1}(1+(3x_1-2x_2)^2)\times 2(x_1+x_2-5) + \frac{\partial}{\partial x_1}(-4)(3x_1-2x_2)\times(1+(x_1+x_2-5)^2)$$
$$+ \frac{\partial}{\partial x_1}(1+(x_1+x_2-5)^2)\times(-4)(3x_1-2x_2)$$
$$= 2(1+(3x_1-2x_2)^2) + 6(3x_1-2x_2)\cdot 2(x_1+x_2-5) - 12(1+(x_1+x_2-5)^2) + 2(x_1+x_2-5)\cdot(-4)(3x_1-2x_2)$$
$$= 2\times 101 + 6\times 10\times 30 - 12\times 226 - 30\times 4\times 10 = -1910$$

$$\frac{\partial^2 f(x)}{\partial x_2^2}\Big|_{x=x_0} = \frac{\partial}{\partial x_2} 2(x_1+x_2-5)\times(1+(3x_1-2x_2)^2) + \frac{\partial}{\partial x_2}(1+(3x_1-2x_2)^2)\times 2(x_1+x_2-5) + \frac{\partial}{\partial x_2}(-4)(3x_1-2x_2)\times(1+(x_1+x_2-5)^2)$$
$$+ \frac{\partial}{\partial x_2}(1+(x_1+x_2-5)^2)\times(-4)(3x_1-2x_2)$$
$$= 2(1+(3x_1-2x_2)^2) - 4(3x_1-2x_2)\times 2(x_1+x_2-5) + 8(1+(x_1+x_2-5)^2) + 2(x_1+x_2-5)\cdot(-4)\cdot(3x_1-2x_2)$$
$$= 2\times 101 - 8\times 10\times 15 + 8\times 226 - 8\times 15\times 10 = -390$$

$$\therefore A_0 = \begin{bmatrix} 7870 & -1910 \\ -1910 & -390 \end{bmatrix} \qquad \therefore A_0^{-1} = \frac{1}{-6717400}\begin{bmatrix} -390 & 1910 \\ 1910 & 7870 \end{bmatrix} = \frac{1}{-671740}\begin{bmatrix} -39 & 191 \\ 191 & 787 \end{bmatrix}$$

$$\therefore x_1 = x_0 - A_0^{-1} g_0 = \begin{bmatrix} 10 \\ 10 \end{bmatrix} - \left(-\frac{1}{671740}\begin{bmatrix} -39 & 191 \\ 191 & 787 \end{bmatrix}\right)\cdot\begin{bmatrix} 16590 \\ -6010 \end{bmatrix} \approx \begin{bmatrix} 7.33 \\ 7.68 \end{bmatrix}$$

(ii) $x_0 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ Known from (i), $\frac{\partial}{\partial x_1} f(x)\big|_{x=x_0} = 14$, $\frac{\partial}{\partial x_2} f(x)\big|_{x=x_0} = -26$

$\therefore g_0 = \begin{bmatrix} 14 \\ -26 \end{bmatrix}$ $\quad \frac{\partial^2}{\partial x_1^2} f(x)\big|_{x=x_0} = -2$, $\frac{\partial^2}{\partial x_2 \partial x_1} f(x)\big|_{x=x_0} = -22$

$\frac{\partial^2}{\partial x_1 \partial x_2} f(x)\big|_{x=x_0} = -22$, $\frac{\partial^2}{\partial x_2^2} f(x)\big|_{x=x_0} = 58$

$$A_0 = \begin{bmatrix} -2 & -22 \\ -22 & 58 \end{bmatrix} \Rightarrow A_0^{-1} = \frac{1}{-600}\begin{bmatrix} 58 & 22 \\ 22 & -2 \end{bmatrix}$$

$$\therefore x_1 = x_0 - A_0^{-1} g_0 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \left(-\frac{1}{600}\begin{bmatrix} 58 & 22 \\ 22 & -2 \end{bmatrix}\right)\cdot\begin{bmatrix} 14 \\ -26 \end{bmatrix} = \begin{bmatrix} 2.4 \\ 2.6 \end{bmatrix}$$

(iii) $\nabla F(x) = \begin{bmatrix} \frac{\partial}{\partial x_1} F(x) \\ \frac{\partial}{\partial x_2} F(x) \end{bmatrix} = 0 \Rightarrow \begin{cases} \frac{\partial}{\partial x_1} F(x) = 0 \\ \frac{\partial}{\partial x_2} F(x) = 0 \end{cases} \Rightarrow \begin{cases} 3x_1 - 2x_2 = 0 \\ x_1 + x_2 - 5 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 2 \\ x_2 = 3. \end{cases} \quad X_0 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$A|_{X=X_0} = \begin{bmatrix} 20 & -10 \\ -10 & 10 \end{bmatrix} \quad \begin{vmatrix} 20-\lambda & -10 \\ -10 & 10-\lambda \end{vmatrix} = 0 \quad \begin{matrix} \lambda_1 = -5\sqrt{5} + 15 \\ \lambda_2 = 5\sqrt{5} + 15. \end{matrix}$

$\because \lambda_1 > 0,\ \lambda_2 > 0 \quad \therefore$ point $(2,3)$ is the strong minimum point.

$F(2,3) = 1.$

Comparing with part (i) and part (ii), the value $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is closer to $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ than $\begin{bmatrix} 10 \\ 10 \end{bmatrix}$. So, when the initial point is $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$, we can use Newton's method to perform two iterations to easily reach the strong minimum point. However, when the initial point is $\begin{bmatrix} 10 \\ 10 \end{bmatrix}$, it needs more iterations. Because the Newton's method is sensitive to the initial point.

2.2.

(i) $f(x) = \frac{7}{2}x_1^2 - 6x_1x_2 - x_2^2$ $\qquad p = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad X_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\nabla f(x) = \begin{bmatrix} \frac{\partial}{\partial x_1} f(x) \\ \frac{\partial}{\partial x_2} f(x) \end{bmatrix} = \begin{bmatrix} 7x_1 - 6x_2 \\ -6x_1 - 2x_2 \end{bmatrix} \qquad \nabla f(x)\Big|_{x=x_0} = \begin{bmatrix} 1 \\ -8 \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2}{\partial x_1^2} f(x) & \frac{\partial^2}{\partial x_2 \partial x_1} f(x) \\ \frac{\partial^2}{\partial x_1 \partial x_2} f(x) & \frac{\partial^2}{\partial x_2^2} f(x) \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ -6 & -2 \end{bmatrix}$$

$$\frac{p^T \nabla f(x)}{\|p\|} = \frac{\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -8 \end{bmatrix}}{\sqrt{(-1)^2 + (1)^2}} = \frac{-9}{\sqrt{2}}$$

$$\frac{p^T \nabla^2 f(x)\, p}{\|p\|^2} = \frac{\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 7 & -6 \\ -6 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}}{2} = \frac{17}{2}$$

(ii) $f(x) = 5x_1^2 - 6x_1x_2 + 5x_2^2 + 4x_1 + 4x_2$. $\qquad p = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad X_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\nabla f(x) = \begin{bmatrix} \frac{\partial}{\partial x_1} f(x) \\ \frac{\partial}{\partial x_2} f(x) \end{bmatrix} = \begin{bmatrix} 10x_1 - 6x_2 + 4 \\ -6x_1 + 10x_2 + 4 \end{bmatrix} \qquad \nabla f(x)\Big|_{x=x_0} = \begin{bmatrix} 8 \\ 8 \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2}{\partial x_1^2} f(x) & \frac{\partial^2}{\partial x_2 \partial x_1} f(x) \\ \frac{\partial^2}{\partial x_1 \partial x_2} f(x) & \frac{\partial^2}{\partial x_2^2} f(x) \end{bmatrix} = \begin{bmatrix} 10 & -6 \\ -6 & 10 \end{bmatrix}$$

$$\frac{p^T \nabla f(x)}{\|p\|} = \frac{\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 8 \end{bmatrix}}{\sqrt{2}} = 0$$

$$\frac{p^T \nabla^2 f(x)\, p}{\|p\|^2} = \frac{\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 10 & -6 \\ -6 & 10 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}}{2} = 16$$

(iii) $f(x) = \frac{9}{2}x_1^2 - 2x_1x_2 + 3x_2^2 + 2x_1 - x_2$ $\qquad p = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad X_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\nabla f(x) = \begin{bmatrix} \frac{\partial}{\partial x_1} f(x) \\ \frac{\partial}{\partial x_2} f(x) \end{bmatrix} = \begin{bmatrix} 9x_1 - 2x_2 + 2 \\ -2x_1 + 6x_2 - 1 \end{bmatrix} \qquad \nabla f(x)\Big|_{x=x_0} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2}{\partial x_1^2} f(x) & \frac{\partial^2}{\partial x_2 \partial x_1} f(x) \\ \frac{\partial^2}{\partial x_1 \partial x_2} f(x) & \frac{\partial^2}{\partial x_2^2} f(x) \end{bmatrix} = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$$

$$\frac{p^T \nabla f(x)}{\|p\|} = \frac{\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 3 \end{bmatrix}}{\sqrt{2}} = \frac{-6}{\sqrt{2}}$$

$$\frac{p^T \nabla^2 f(x)\, p}{\|p\|^2} = \frac{\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}}{2} = \frac{19}{2}$$

(iv)

$$f(x) = -\frac{1}{2}(7x_1^2 + 12x_1x_2 - 2x_2^2)$$
$$= -\frac{7}{2}x_1^2 - 6x_1x_2 + x_2^2$$

$$\nabla f(x) = \begin{bmatrix} \frac{\partial}{\partial x_1} f(x) \\ \frac{\partial}{\partial x_2} f(x) \end{bmatrix} = \begin{bmatrix} 7x_1 - 6x_2 \\ -6x_1 + 2x_2 \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2}{\partial x_1^2} f(x) & \frac{\partial^2}{\partial x_2 \partial x_1} f(x) \\ \frac{\partial^2}{\partial x_1 \partial x_2} f(x) & \frac{\partial^2}{x_2^2} f(x) \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ -6 & 2 \end{bmatrix}$$

$$\frac{p^T \nabla^2 f(x) p}{\|p\|^2} = \frac{[-1 \ 1]\begin{bmatrix} 7 & -6 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix}}{2} = \frac{21}{2}$$

$$p = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\nabla f(x)\Big|_{x=x_0} = \begin{bmatrix} -13 \\ -4 \end{bmatrix}$$

$$\frac{p^T \nabla f(x)}{\|p\|} = \frac{[-1 \ 1]\begin{bmatrix} -13 \\ -4 \end{bmatrix}}{\sqrt{2}} = \frac{9}{\sqrt{2}}$$

2.3.

① i) $\nabla F(x) = Ax + d = \begin{bmatrix} 7 & -6 \\ -6 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + d = \begin{bmatrix} 7x_1 - 6x_2 \\ -6x_1 - 2x_2 \end{bmatrix}$ $\quad d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

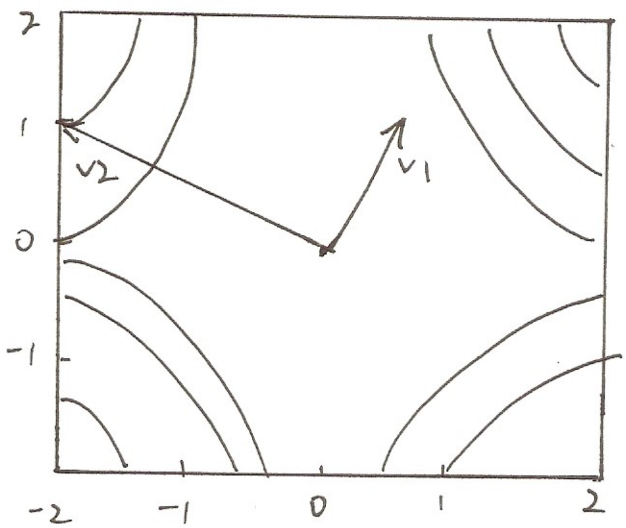
$x^* = A^{-1}d = \begin{bmatrix} 7 & -6 \\ -6 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

ii) $\nabla^2 F(x) = \begin{bmatrix} 7 & -6 \\ -6 & -2 \end{bmatrix}$ $\quad \begin{vmatrix} 7-\lambda & -6 \\ -6 & -2-\lambda \end{vmatrix} = (\lambda+5)(\lambda-10)$ $\quad \therefore \lambda_1 = -5$, $\lambda_2 = 10$

when $\lambda_1 = -5$, $v_1 = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$. $\lambda_2 = 10$, $v_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$\because \lambda_1 < 0$, $\lambda_2 > 0$ $\quad \therefore$ point $(0,0)$ is a saddle point. $F(0,0) = 0$

(iii)



② i) $\nabla F(x) = Ax + d = \begin{bmatrix} 10 & -6 \\ -6 & 10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + d = \begin{bmatrix} 10x_1 - 6x_2 + 4 \\ -6x_1 + 10x_2 + 4 \end{bmatrix}$ $\quad d = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$x^* = A^{-1}d = \begin{bmatrix} 10 & -6 \\ -6 & 10 \end{bmatrix}\begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

ii) $\nabla^2 F(x) = \begin{bmatrix} 10 & -6 \\ -6 & 10 \end{bmatrix}$ $\quad \begin{vmatrix} 10-\lambda & -6 \\ -6 & 10-\lambda \end{vmatrix} = (\lambda-16)(\lambda-4) = 0$ $\quad \lambda_1 = 4$, $\lambda_2 = 16$

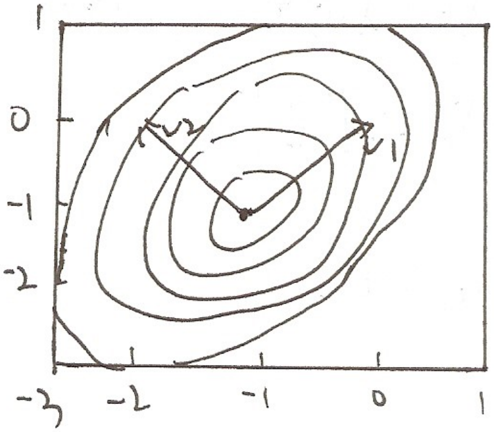
when $\lambda_1 = 4$, $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\quad \lambda_2 = 16$, $v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\because \lambda_1 > 0$, $\lambda_2 > 0$

$\therefore$ point $(-1,-1)$ is the strong minimum point

$F(-1,-1) = -4$

(iii)



③ i) $\nabla f(x) = Ax + d = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + d = \begin{bmatrix} 9x_1 - 2x_2 + 2 \\ -2x_1 + 6x_2 - 1 \end{bmatrix}$ $\quad d = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$x^* = A^{-1}d = \begin{bmatrix} -\frac{1}{5} \\ \frac{1}{10} \end{bmatrix}$

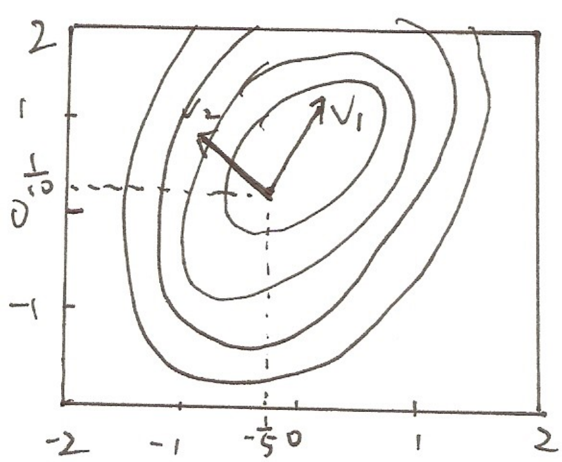
ii) $\nabla^2 f(x) = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$ $\quad \begin{vmatrix} 9-\lambda & -2 \\ -2 & 6-\lambda \end{vmatrix} = (\lambda - 5)(\lambda - 10) = 0$ $\quad \lambda_1 = 5,\ \lambda_2 = 10$

when $\lambda_1 = 5$, $v_1 = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$, $\lambda_2 = 10$, $v_2 = \begin{bmatrix} -1 \\ \frac{1}{2} \end{bmatrix}$

$\because \lambda_1 > 0, \lambda_2 > 0$

$\therefore$ point $(-\frac{1}{5}, \frac{1}{10})$ is a strong minimum point, $f(x) = -0.25$

(iii)



④ i) $\nabla f(x) = Ax + d = \begin{bmatrix} -7 & -6 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + d = \begin{bmatrix} -7x_1 - 6x_2 \\ -6x_1 + 2x_2 \end{bmatrix}$ $\quad d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x^* = A^{-1}d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(ii) $\nabla^2 f(x) = \begin{bmatrix} -7 & -6 \\ -6 & 2 \end{bmatrix}$ $\begin{vmatrix} -7-\lambda & -6 \\ -6 & 2-\lambda \end{vmatrix} = (\lambda+10)(\lambda-5) = 0$ $\lambda_1 = -10$, $\lambda_2 = 5$

when $\lambda_1 = -10$, $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\lambda_2 = 5$, $v_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$\because \lambda_1 < 0$, $\lambda_2 > 0$

$\therefore$ point $(0,0)$ is a saddle point. $f(0,0) = 0$.

(iii)