2.1. $W^1=\begin{bmatrix}-1\\1\end{bmatrix}$ $b^1=\begin{bmatrix}0.5\\1\end{bmatrix}$ $W^2=\begin{bmatrix}1\\1\end{bmatrix}$ $b^2=[-1]$ $p\in(-2,2)$

$$n_1^1=w_{1,1}^1 p+b_1^1=-p+0.5 \qquad a_1^1=\text{poslin}(n_1^1)=\begin{cases}n_1^1 & n_1^1\geq 0\\ 0 & \text{else}\end{cases} \Rightarrow \begin{cases}-p+0.5, & p\in(-2,0.5]\\ 0, & p\in(0.5,2)\end{cases}$$

$$n_2^1=w_{2,1}^1 p+b_2^1=p+1 \qquad a_2^1=\text{poslin}(n_2^1)=\begin{cases}n_2^1 & n_2^1\geq 0\\ 0 & \text{else}\end{cases} \Rightarrow \begin{cases}p+1, & p\in[-1,2)\\ 0, & p\in(-2,-1)\end{cases}$$

$$n_1^2=a_1^1\cdot w_{1,1}^2+a_2^1\cdot w_{2,1}^2+b^2=a_1^1+a_2^1-1$$

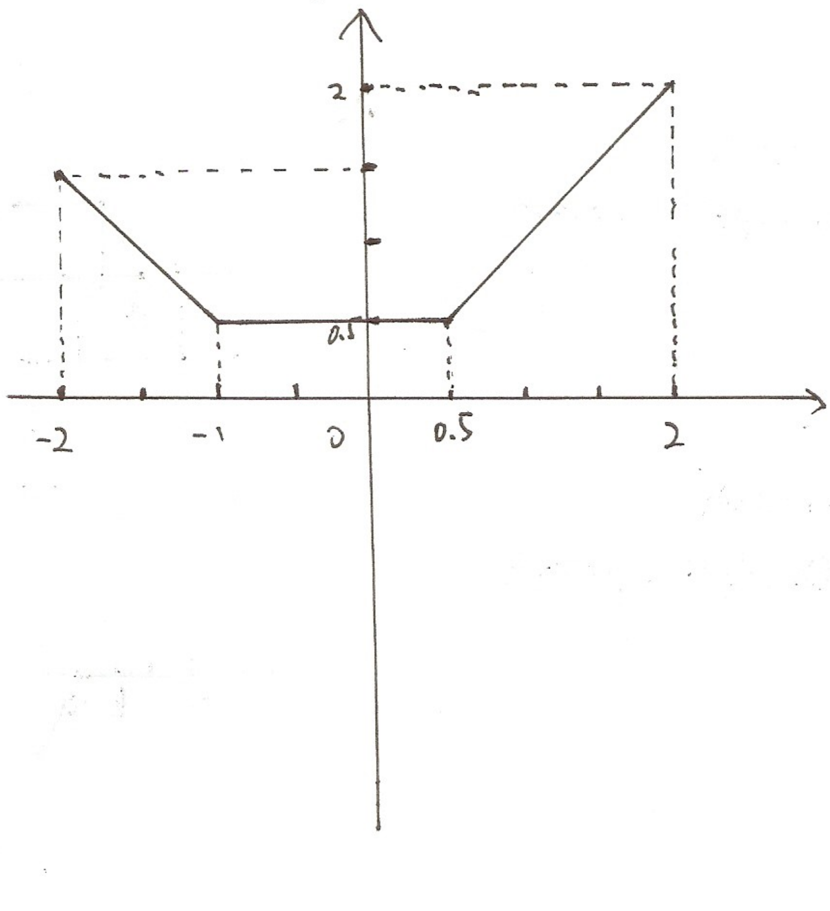
$$a_1^2=\text{purelin}(n_1^2)=a_1^1+a_2^1-1.$$

when: $p\in(-2,-1)$ $\begin{cases}a_1^1=-p+0.5\\ a_2^1=0\end{cases}$ $\Rightarrow a_1^2=-p-0.5$

$p\in[-1,0.5]$ $\begin{cases}a_1^1=-p+0.5\\ a_2^1=p+1\end{cases}$ $\Rightarrow a_1^2=0.5$

$p\in(0.5,2)$ $\begin{cases}a_1^1=0\\ a_2^1=p+1\end{cases}$ $\Rightarrow a_1^2=p$

$$\Rightarrow \begin{cases}-p-0.5 & p\in(-2,-1)\\ 0.5 & p\in[-1,0.5]\\ p & p\in(0.5,2)\end{cases}$$



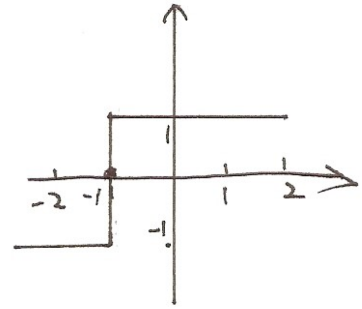
2.2.

(i) $w=1,\ b=1$. $f=\text{hardlims}$

$a=f(wp+b)=\text{hardlims}(p+1)$. $p\in(-2,2)$

$p+1>0 \qquad p+1<0 \qquad p+1=0$

$p>-1 \qquad p<-1 \qquad p=-1$

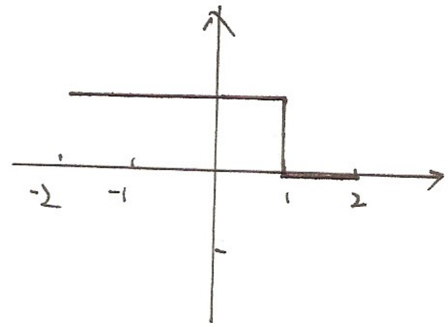


(ii) $w=-1,\ b=1,\ f=\text{hardlim}$

$a=f(wp+b)=\text{hardlim}(-p+1)$, $p\in(-2,2)$

$-p+1<0 \qquad -p+1=0$

$p>1 \qquad p=1.$

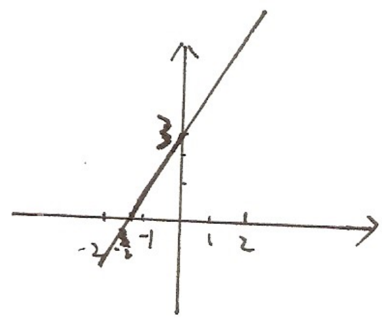


(iii) $w=2,\ b=3$, $f=\text{purelin}$

$a=f(wp+b)=\text{purelin}(2p+3)$, $p\in(-2,2)$

$2p+3=0$

$p=-\frac{3}{2}$

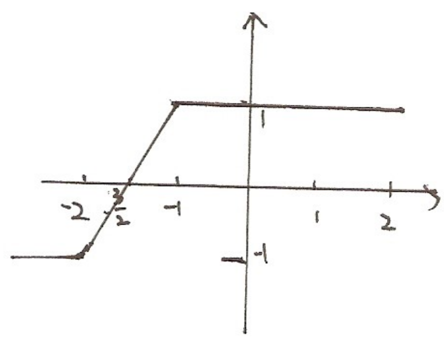


(iv) $w=2,\ b=3.\ f=\text{satlins}$

$a=f(wp+b)=\text{satlins}(2p+3)$, $p\in(-2,2)$

$2p+3=0 \qquad 2p+3<-1 \qquad 2p+3>1$

$p=-\frac{3}{2} \qquad \cancel{p>2}\ p<-2 \qquad p>-1$

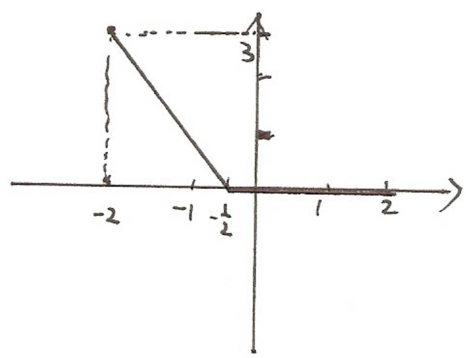


(v) $w=-2,\ b=-1\ f=\text{poslin}$

$a=f(wp+b)=\text{poslin}(-2p-1)$, $p\in(-2,2)$

$-2p-1<0 \qquad -2p-1=0$

$p>-\frac{1}{2} \qquad p=-\frac{1}{2}$



2.3. $w^1_{1,1}=2$, $w^1_{2,1}=1$, $b^1_1=2$, $b^1_2=-1$, $w^2_{1,1}=1$, $w^2_{1,2}=-1$, $b^2_1=0$

i) $n^1_1 = w_{1,1} p + b^1_1 = 2p+2$

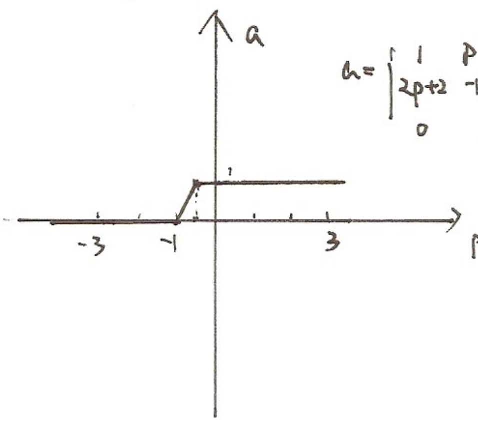
ii) $a^1_1 = \text{satlin}(n^1_1) = \text{satlin}(2p+2)$, $-3<p<3$

(iii) $n^1_2 = w^1_{2,1} p + b^1_2 = p-1$

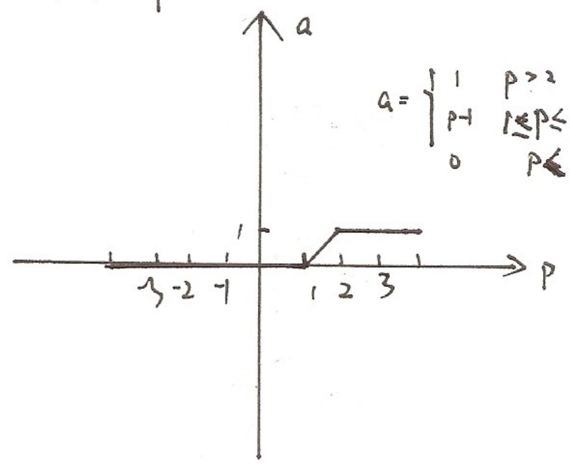
(iv) $a^1_2 = \text{satlin}(n^1_2) = \text{satlin}(p-1)$, $-3<p<3$

(v) $n^2_1 = a^1_1 \cdot w^2_{1,1} + a^1_2 \cdot w^2_{1,2} + b^2_1 = a^1_1 + (-1)a^1_2 + 0 = a^1_1 - a^1_2$

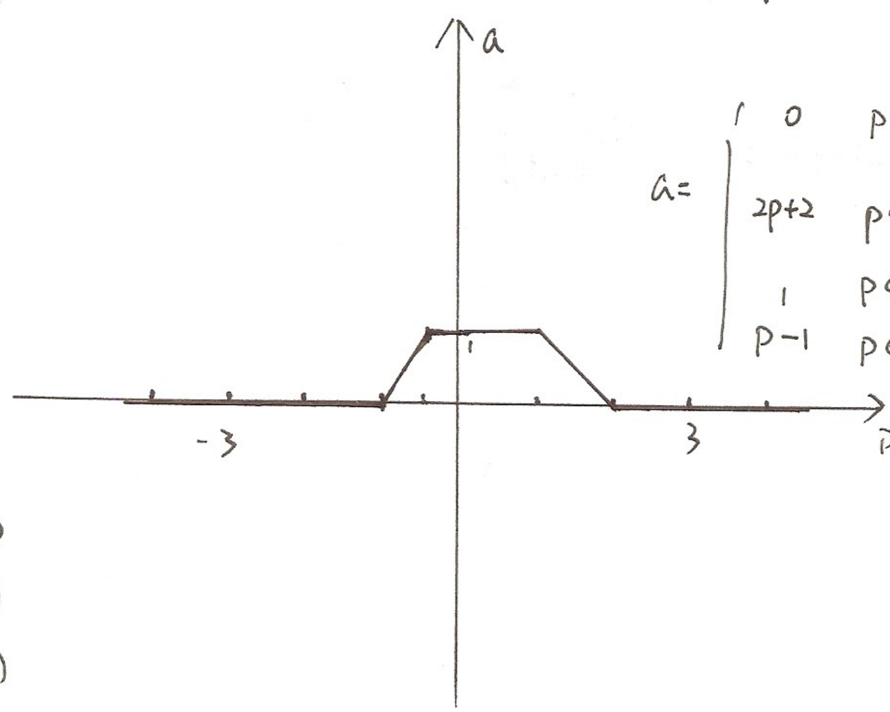
(vi) $a^2_1 = \text{purelin}(n^2_1) = \text{purelin}(\text{satlin}(2p+2) - \text{satlin}(p-1))$



$$a = \begin{cases} 1 & p > -\frac{1}{2} \\ 2p+2 & -1 \le p \le -\frac{1}{2} \\ 0 & p < -1 \end{cases}$$



$$a = \begin{cases} 1 & p > 2 \\ p-1 & 1 \le p \le 2 \\ 0 & p < 1 \end{cases}$$



$$a = \begin{cases} 0 & p \in [2,3) \cup (-3,-1) \\ 2p+2 & p \in [-1, -\frac{1}{2}] \\ 1 & p \in (-\frac{1}{2}, 1) \\ p-1 & p \in [1, 2] \end{cases}$$

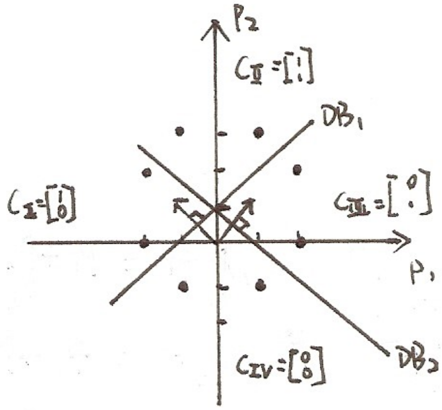
$p=1 \Rightarrow a = (1-0)$

$p=2 \Rightarrow a = (1-1)$

$p=-1 \Rightarrow a = (0-0)$

2.4.

(i)



$[-1 \;\; 1]\begin{bmatrix}0\\1\end{bmatrix} + b = 0 \qquad b = -1$

$[1 \;\; 1]\begin{bmatrix}0\\1\end{bmatrix} + b = 0 \qquad b = -1$

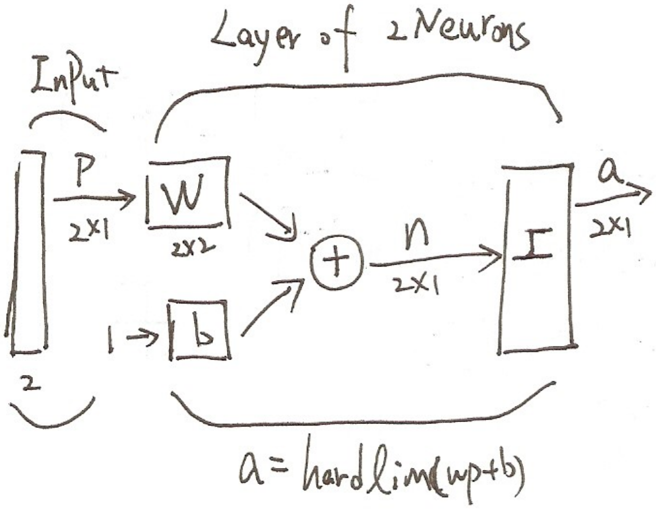
As shown in the figure, the distances from every point to the corresponding boundary lines in figure are all equal. Therefore, those two boundary lines are the BEST, they equally divide the four classesor regions.

$DB_1 : P_2 = P_1 + 1 \qquad DB_2 : P_2 = -P_1 + 1$

$\therefore \; W = \begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix} \qquad b = \begin{bmatrix}-1\\-1\end{bmatrix}$

$\Rightarrow wp + b = 0$

(ii)



(iii) $P = \begin{bmatrix}-3\\1\end{bmatrix} \qquad C_{IV} = t = \begin{bmatrix}0\\0\end{bmatrix} \qquad a = \text{hardlim}(wp+b)$

Initial weight $W = W^{(1)}$

$b = \begin{bmatrix}-1\\-1\end{bmatrix}$

~~$W^{new} = W^{old} p + b$~~

$\Rightarrow \; = \text{hardlim}\left(\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix} \cdot \begin{bmatrix}-3\\1\end{bmatrix} + \begin{bmatrix}-1\\-1\end{bmatrix}\right)$

$= \text{hardlim}\left(\begin{bmatrix}3\\-3\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$

$$e = t - a = \begin{bmatrix}0\\0\end{bmatrix} - \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-1\\0\end{bmatrix}$$

$$W^{new} = W^{old} + e \cdot p^T = \begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix} + \begin{bmatrix}-1\\0\end{bmatrix}[-3, 1] = \begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix} + \begin{bmatrix}3 & -1\\ 0 & 0\end{bmatrix} = \begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

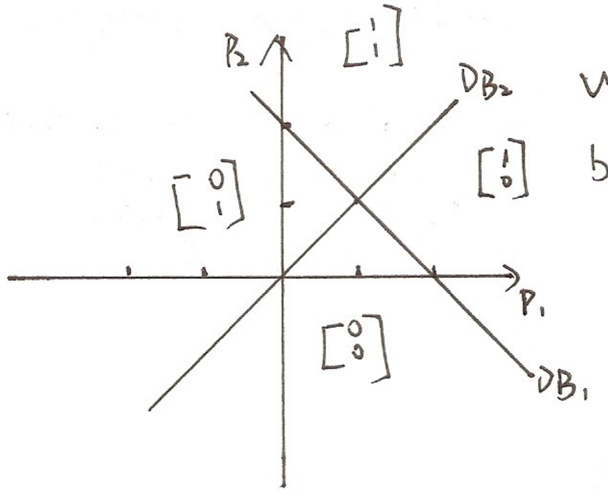
$$b^{new} = b^{old} + e = \begin{bmatrix}-1\\-1\end{bmatrix} + \begin{bmatrix}-1\\0\end{bmatrix} = \begin{bmatrix}-2\\-1\end{bmatrix}$$

$$\therefore \; W^{new} = \begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix} \qquad b^{new} = \begin{bmatrix}-2\\-1\end{bmatrix}$$

3.5.

(i) Four classes

(ii)



$W = \begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$ $\quad b = \begin{bmatrix}-2\\0\end{bmatrix}$

$w p_1 + b = p_1 + p_2 - 2 = 0$

$p_2 = 2 - p_1$

$\Rightarrow w p_2 + b = -p_1 + p_2 + 0 = 0$

$p_2 = p_1$

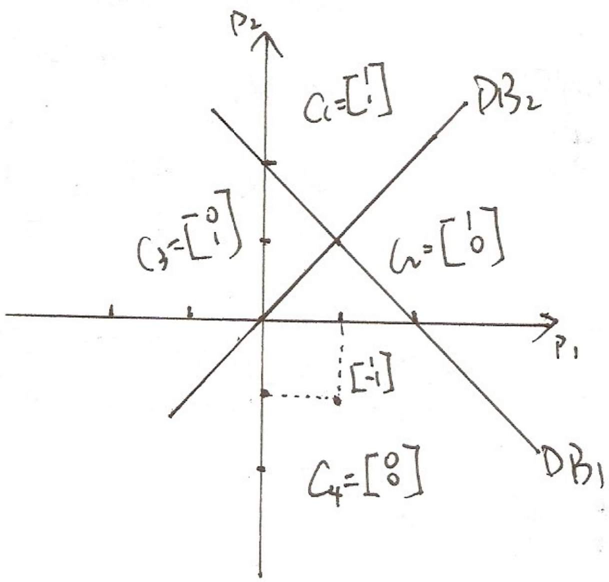
(iii) $a = \text{hardlims}(wp+b) = \text{hardlims}\left(\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix} \cdot \begin{bmatrix}1\\-1\end{bmatrix} + \begin{bmatrix}-2\\0\end{bmatrix}\right)$

$= \text{hardlims}\left(\begin{bmatrix}0\\-2\end{bmatrix} + \begin{bmatrix}-2\\0\end{bmatrix}\right)$

$= \text{hardlims}\left(\begin{bmatrix}-2\\-2\end{bmatrix}\right)$

$= \begin{bmatrix}0\\0\end{bmatrix}$

(iv).



$\therefore$ Input $\begin{bmatrix}1\\-1\end{bmatrix}$ has target $\begin{bmatrix}0\\0\end{bmatrix}$.

It is in region IV.