Receding Horizon Approach to Informative Seafloor Exploration using Linearised Entropy of Gaussian Process Classifiers

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Abstract

While seafloor bathymetry have been mapped extensively over the last century, geological and ecological observations of seafloor benthic zones only began recently. Unlike bathymetric mapping, data collection of benthic imagery requires *in situ* exploration - a significantly slower and costly endeavour. An efficient exploration policy would therefore require solving the informative path planning problem. This paper investigates a receding horizon approach to the informative path planning problem using linearised entropy as the proposed acquisition function. We model the benthic environment upon five bathymetric features through Gaussian process classifiers, whose linearised entropy would be defined and derived. We compare our method to a monte carlo approach for estimating joint entropy under a prediction accuracy criterion, demonstrating advantages of the linearised entropy approach. Under the same acquisition criterion, we also show the benefits of a receding horizon approach over simpler approaches such as greedy and open loop methods. Finally, we test our method on collected benthic datasets from past AUV missions to Scott Reef, Western Australia.

1 Introduction

2 Background

3 Mapping Benthic Habitats with Gaussian Process Classifiers

4 Linearised Entropy of Gaussian Process Classifiers

4.1 Binary Classification

For binary classification, linearisation is performed on the sigmoid, or response, function.

Suppose we have trained our Gaussian process classifier using Laplace approximation with respect to a training set $\mathcal{D} = \{X, \boldsymbol{y}\} = \{[\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n]^T, [y_1, y_2, \ldots, y_n]\}$ with n training points. We know that the latent function $f(\boldsymbol{x})$ is distributed as a GP with a particular predictive mean $m(\boldsymbol{x})$ and covariance $k(\boldsymbol{x}, \boldsymbol{x}')$ once conditioned on the training data (1).

$$f(\boldsymbol{x}) \sim \mathcal{GP}(m(\boldsymbol{x}), k(\boldsymbol{x}, \boldsymbol{x}')) \tag{1}$$

Let $X^\star = [\boldsymbol{x}_1^\star, \boldsymbol{x}_2^\star, \ldots, \boldsymbol{x}_{n^\star}^\star]^T$ denote the collection of $n^\star$ query points for which inference is to be performed. Denote $\boldsymbol{f}^\star$ the vector of latent function values $f_i^\star = f(\boldsymbol{x}_i^\star)$ at each query point. We have by definition of a GP that $\boldsymbol{f}^\star$ is multivariate Gaussian distributed with a corresponding means $\mu_i^\star = m(\boldsymbol{x}_i^\star)$ and covariances $\Sigma_{ij}^\star = k(\boldsymbol{x}_i^\star, \boldsymbol{x}_j^\star)$ (2).

$$\boldsymbol{f}^\star = [f_1^\star, f_2^\star, \ldots, f_{n^\star}^\star]^T \sim \mathcal{N}(\boldsymbol{\mu}^\star, \Sigma^\star) \tag{2}$$

The binary prediction probability $\boldsymbol{\pi}^\star$ at the query points is obtained through passing the queried latent function random vector $\boldsymbol{f}^\star$ through a sigmoid in a component wise fashion (3).

$$\pi_i^\star = \sigma(f_i^\star) \qquad \forall i \in \{1, 2, \ldots, n^\star\} \tag{3}$$

As a straightforward transformation of the latent vector, the predictive probability vector $\boldsymbol{\pi}^\star$ is thus a random vector itself. The usual procedure is then to treat the expected predition probabilities $\mathbb{E}(\boldsymbol{\pi}^\star)$ as the posterior class probabilities for further inference. However, this discards any information regarding the joint behaviour at the query points. As a result, a measure of mutual information shared amongst the query points cannot be obtained.

One straightforward approach to address this problem is to perform Monte Carlo estimation of the posterior joint distribution for class predictions via jointly sampling latent vectors from the GP and assigning class label 1 for positive latent values and -1 otherwise. Aside from the relatively long computational time required for

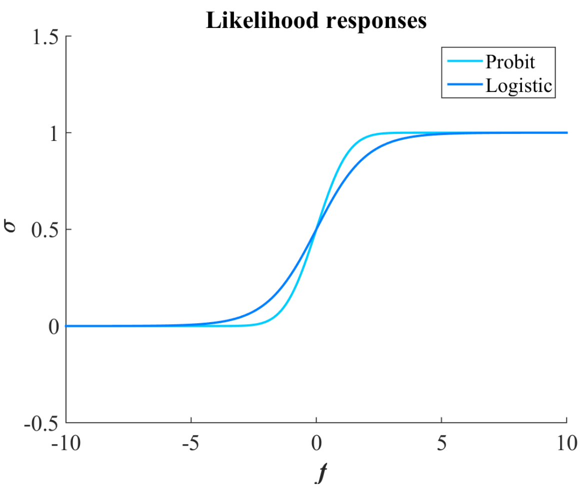

Figure 1: Likelihood Responses

sampling enough draws for accurate joint distribution estimation, the Monte Carlo approach also has the tendency to overestimate variances at locations of low densities of training observations.

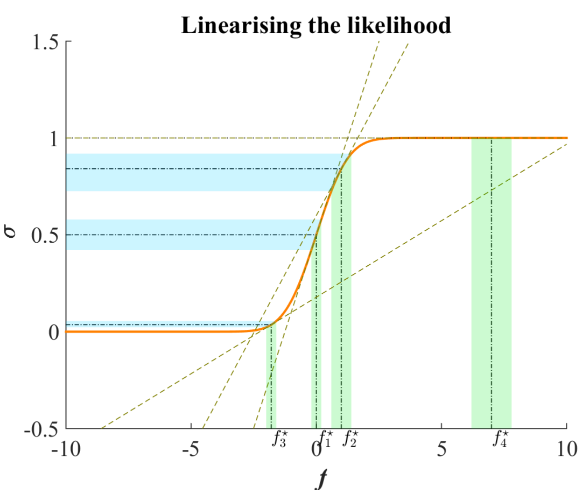

Figure 2: Linearisation Accuracy

Instead, we propose using the joint distribution of the predictive probabilities $\boldsymbol{\pi}^\star$ itself as a basis of constructing a measure of mutual information. As the predictive probabilities are nonlinear transformations (Figure 1) of the Gaussian distributed latent vector, they are no longer Gaussian distributed. Nevertheless, there are two situations for which Gaussian approximation through linearised transformations about the mean predictive probabilities would provide accurate estimates:

1. $\mathbb{E}(f_i^\star)$ is sufficiently far away from zero
2. $\mathbb{V}(f_i^\star)$ is sufficiently small

In fact, when the above two conditions do not hold such that $\mathbb{E}(f_i^\star)$ is close to zero and $\mathbb{V}(f_i^\star)$ is relatively large, we can show that the variances and hence uncertainty estimated from the linearised transformation almost always overestimates the actual variances.

Derivation

We proceed to derive the linearisation which also serves to construct the definition of linearised entropy. Using first order Taylor's expansion, we linearise the sigmoid function about a linearisation point $\bar{f}_i^\star$ for each query location $i \in \{1, 2, \dots, n^\star\}$ (4). We choose the linearisation point to be the expected latent value at each query point $\bar{f}_i^\star = \mathbb{E}(f_i^\star)$.

$$\sigma(f_i^\star) \approx \sigma_L(f_i^\star) := \sigma(\bar{f}_i^\star) + \sigma'(\bar{f}_i^\star)(f_i^\star - \bar{f}_i^\star) \tag{4}$$

The prediction probabilities are now approximated as a linear transformation $\sigma_L(f)$ of the latent vector, so that it is also multivariate Gaussian distributed with expectance and covariance available in analytical form (5).

$$\begin{aligned} \sigma_L(\boldsymbol{f}^\star) &\sim \mathcal{N}(\boldsymbol{\mu}_L^\star, \Sigma_L^\star) \\ \mathbb{E}(\sigma_L(f_i^\star)) &= \mathbb{E}(\sigma(f_i^\star)) = (\mu_L^\star)_i \\ \mathbb{C}\mathrm{ov}(\sigma_L(f_i^\star), \sigma_L(f_j^\star)) &= \mathbb{C}\mathrm{ov}(\sigma(\bar{f}_i^\star) + \sigma'(\bar{f}_i^\star)(f_i^\star - \bar{f}_i^\star), \\ &\qquad \sigma(\bar{f}_j^\star) + \sigma'(\bar{f}_j^\star)(f_j^\star - \bar{f}_j^\star)) \\ &= \sigma'(\bar{f}_i^\star)\sigma'(\bar{f}_j^\star)\mathbb{C}\mathrm{ov}(f_i^\star, f_j^\star) = (\Sigma_L^\star)_{ij} \end{aligned} \tag{5}$$

We then define the linearised entropy $H_L^\star$ at the query points $X^\star$ to be the differential entropy for which the random vector $\sigma_L(\boldsymbol{f}^\star)$ holds. Since $\sigma_L(\boldsymbol{f}^\star)$ is multivariate Gaussian distributed, H_L exhibits a closed form (6).

$$H_L^\star := \frac{1}{2}\log\left((2\pi e)^{n^\star}|\Sigma_L|\right) \tag{6}$$

Properties

4.2 Multiclass Classification

5 Receding Horizon Approach to Informative Path Planning

6 Conclusions and Future Work

Acknowledgments

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