

Optimizing ROC Curves with a Sort-Based Surrogate Loss for Binary Classification and Changepoint Detection, arXiv:2107.01285

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Problem Setting 1: ROC curves for evaluating supervised binary classification algorithms

Problem setting 2: ROC curves for evaluating supervised changepoint algorithms

Proposed surrogate loss for ROC curve optimization: Area Under Min{FP,FN} (AUM)

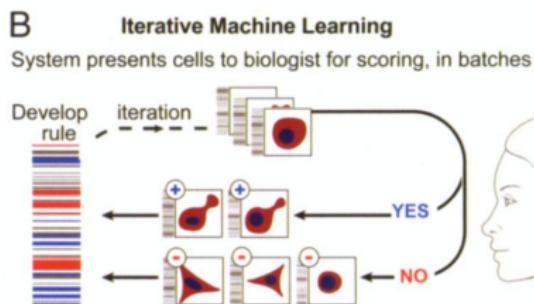
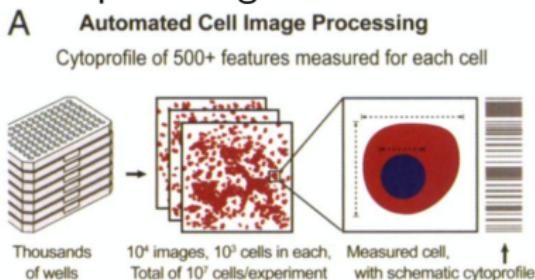
Empirical results: minimizing AUM results in optimized ROC curves

Discussion and Conclusions

Appendix: Non-monotonic ROC curves in changepoint detection

Problem: supervised binary classification

- Given pairs of inputs $\mathbf{x} \in \mathbb{R}^p$ and outputs $y \in \{0, 1\}$ can we learn $f(\mathbf{x}) = y$?
- Example: email, \mathbf{x} = bag of words, y = spam or not.
- Example: images. Jones *et al.* PNAS 2009.



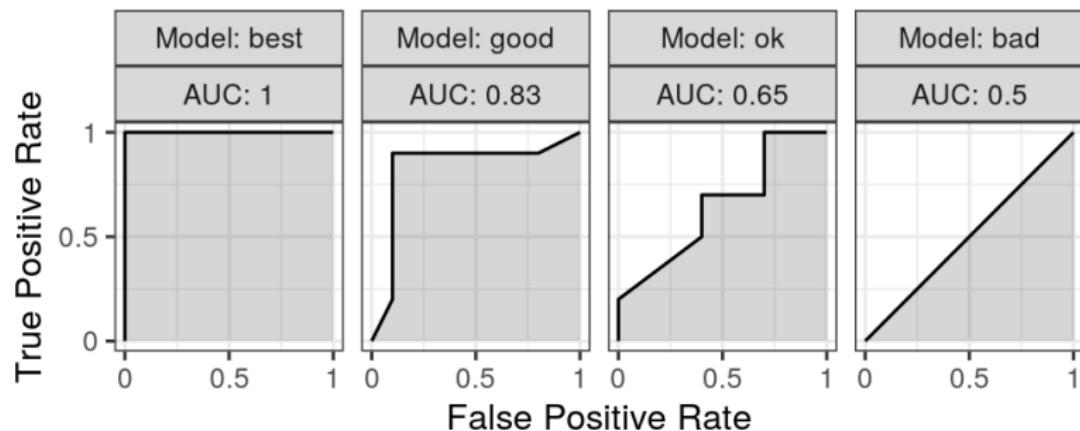
Most algorithms (SVM, Logistic regression, etc) minimize a differentiable surrogate of zero-one loss = sum of:

False positives: $f(\mathbf{x}) = 1$ but $y = 0$ (predict budding, but cell is not).

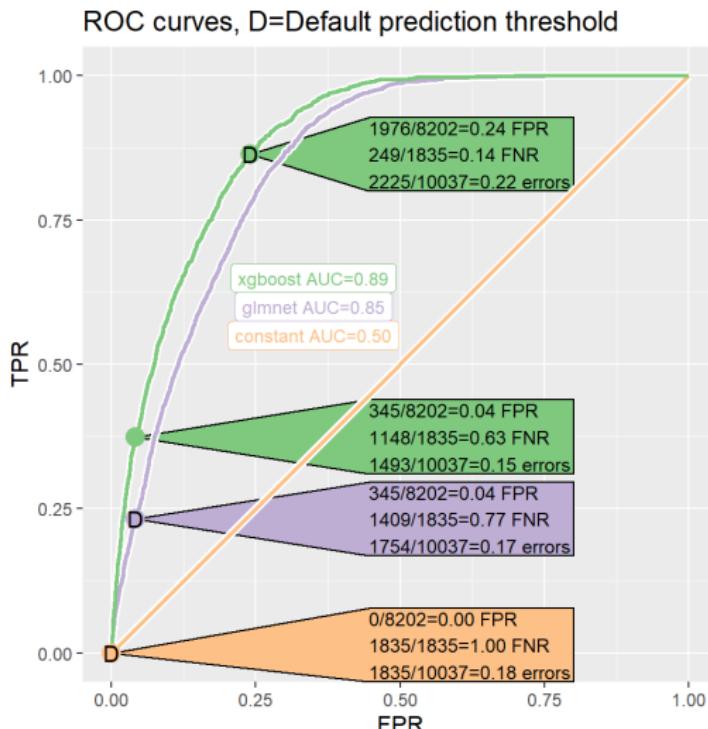
False negatives: $f(\mathbf{x}) = 0$ but $y = 1$ (predict not budding but cell is).

Receiver Operating Characteristic (ROC) Curves

- ▶ Classic evaluation method from the signal processing literature (Egan and Egan, 1975).
- ▶ For a given set of predicted scores, plot True Positive Rate vs False Positive Rate, each point on the ROC curve is a different threshold of the predicted scores.
- ▶ Best classifier has a point near upper left ($\text{TPR}=1$, $\text{FPR}=0$), with large Area Under the Curve (AUC).



ROC curves useful for imbalanced problems

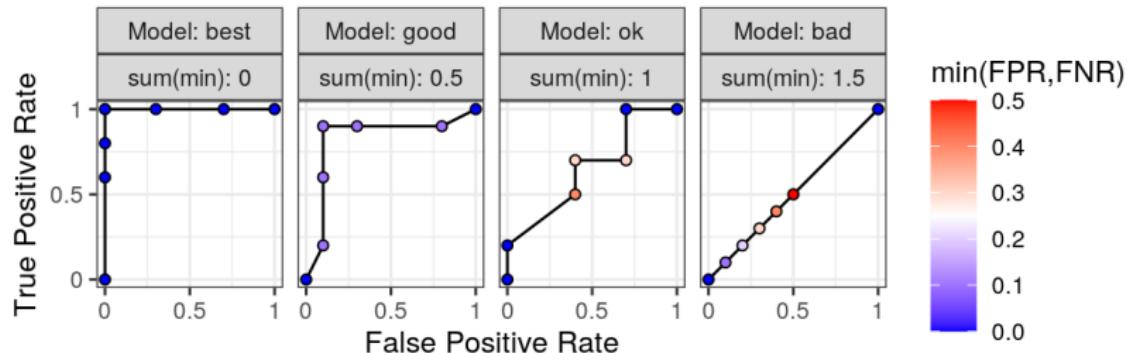


- ▶ At default prediction threshold (D), glmnet has fewer errors.
- ▶ At FPR=4%, xgboost has fewer errors.

Research question and new idea

Can we learn a binary classification function f which directly optimizes the ROC curve?

- ▶ Most algorithms involve minimizing a differentiable surrogate of the zero-one loss, which is not the same.
- ▶ The Area Under the ROC Curve (AUC) is piecewise constant (gradient zero almost everywhere), so can not be used with gradient descent algorithms.
- ▶ We propose to encourage points to be in the upper left of ROC space, using a loss function which is a differentiable surrogate of the sum of $\min(\text{FP}, \text{FN})$.



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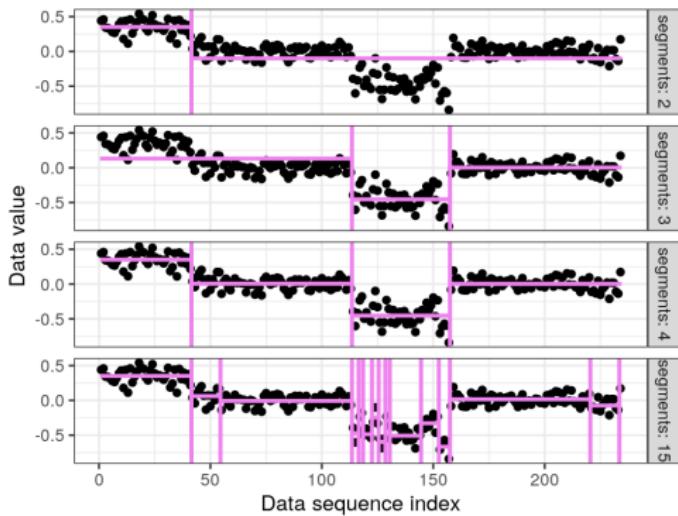
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Appendix: Non-monotonic ROC curves in changepoint detection

Problem: unsupervised changepoint detection

- ▶ Data sequence z_1, \dots, z_T at T points over time/space.
- ▶ Ex: DNA copy number data for cancer diagnosis, $z_t \in \mathbb{R}$.
- ▶ The penalized changepoint problem (Maidstone *et al.* 2017)

$$\arg \min_{u_1, \dots, u_T \in \mathbb{R}} \sum_{t=1}^T (u_t - z_t)^2 + \lambda \sum_{t=2}^T I[u_{t-1} \neq u_t].$$

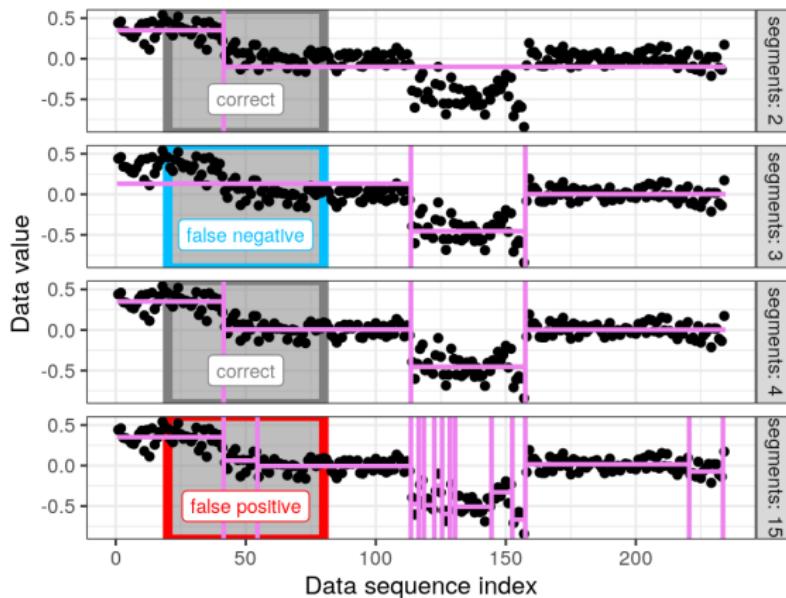


Larger penalty λ results in fewer changes/segments.

Smaller penalty λ results in more changes/segments.

Problem: weakly supervised changepoint detection

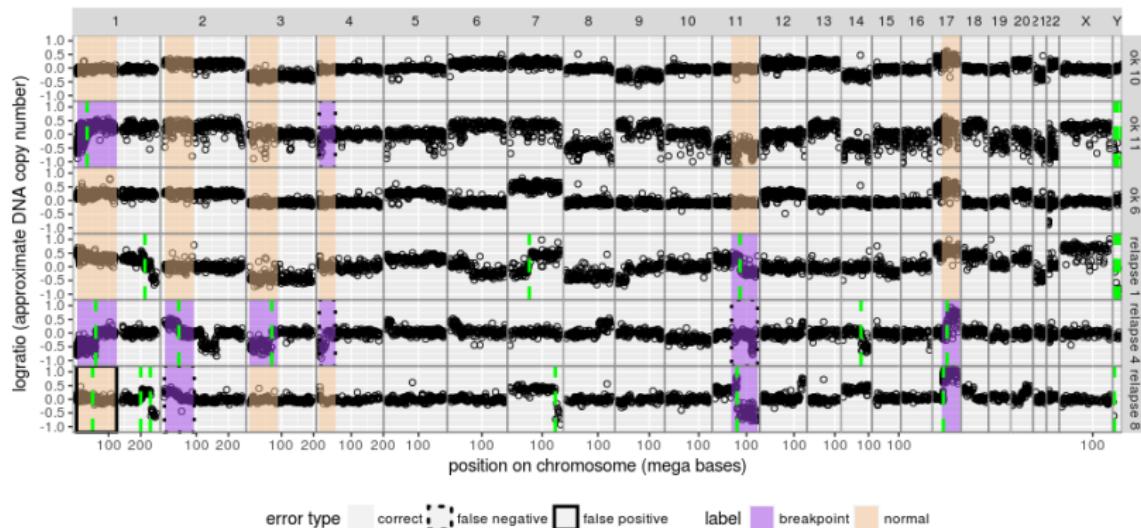
- ▶ First described by Hocking *et al.* ICML 2013.
- ▶ We are given a data sequence \mathbf{z} with labeled regions L .
- ▶ We compute features $\mathbf{x} = \phi(\mathbf{z}) \in \mathbb{R}^P$ and want to learn a function $f(\mathbf{x}) = -\log \lambda \in \mathbb{R}$ that minimizes label error (sum of false positives and false negatives), or maximizes AUC.



Weakly supervised changepoint detection problem setting

Hocking TD. Introduction to supervised changepoint detection. International useR2017 conference tutorial.

Number of incorrect labels: 1 FP + 4 FN



- ▶ Black dots are data sequences in which we want to find changepoints (each panel is a separate sequence).
- ▶ Colored rectangles are weak/partial labels from an expert.
- ▶ Want accurate predictions on new/unlabeled regions.

Empirical test error rates in 10-fold cross-validation

Hocking TD, Rigaill G, Bach F, Vert J-P. Learning Sparse Penalties for Change-point Detection using Max Margin Interval Regression. ICML'13.

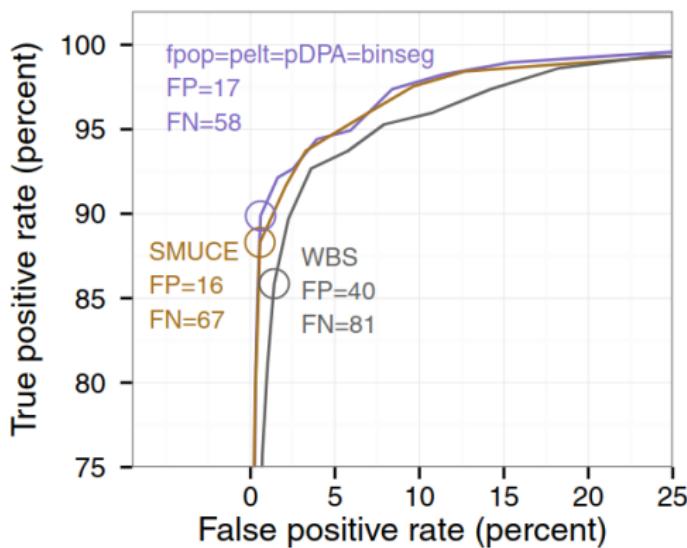
model	features m	original	high.density	low.density	simulation
BIC	0	7.99 ± 0.00	19.52 ± 0.00	13.64 ± 0.00	11.97 ± 0.00
mBIC	0	40.99 ± 0.00	70.00 ± 0.00	36.88 ± 0.00	2.25 ± 0.00
cghseg.k	0	2.19 ± 0.82	6.64 ± 3.99	6.49 ± 1.16	11.85 ± 3.52
log.d	1	2.40 ± 1.00	7.59 ± 6.43	6.21 ± 1.01	13.13 ± 4.14
log.s.log.d	2	1.90 ± 0.77	8.12 ± 5.62	4.72 ± 0.54	1.50 ± 1.63
L1-reg	117	1.81 ± 0.58	7.66 ± 5.72	4.70 ± 0.88	1.28 ± 1.47

- ▶ Proposed penalty learning methods ($m \geq 1$ features with linear weights to learn, R package `penaltyLearning`) have much smaller error rates than previous unsupervised models (BIC, mBIC) and constant method (cghseg.k).
- ▶ In changepoint detection, evaluation using predicted error rates can be misleading/unfair for the same reasons as in binary classification.

Empirical evaluation using AUC

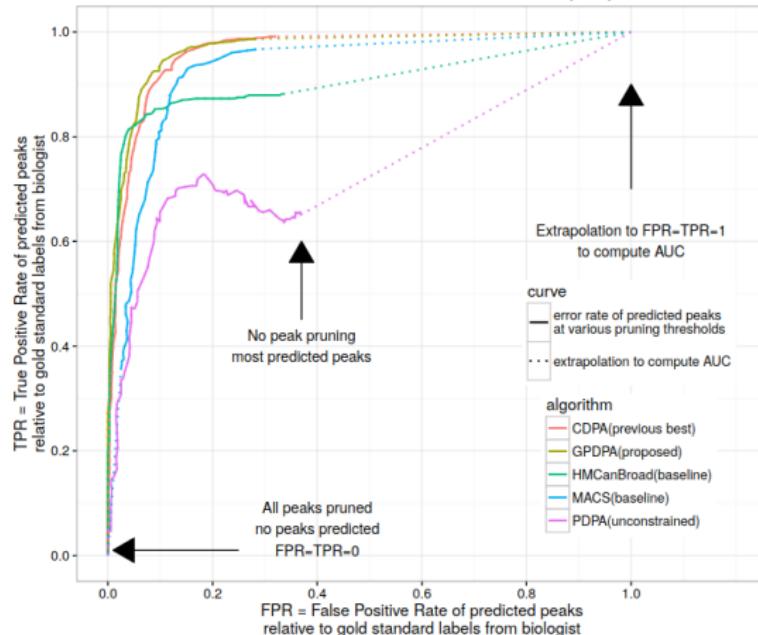
Maidstone R, Hocking TD, Rigaill G, Fearnhead P. On optimal multiple changepoint algorithms for large data. *Statistics and Computing* (2016).

- ▶ Proposed FPOP (R package fpop), computes optimal solution to penalized changepoint problem.
- ▶ ROC curve computed by adding constants to penalty λ (penaltyLearning::ROChange in R).



Evaluating peak detection algorithms using AUC

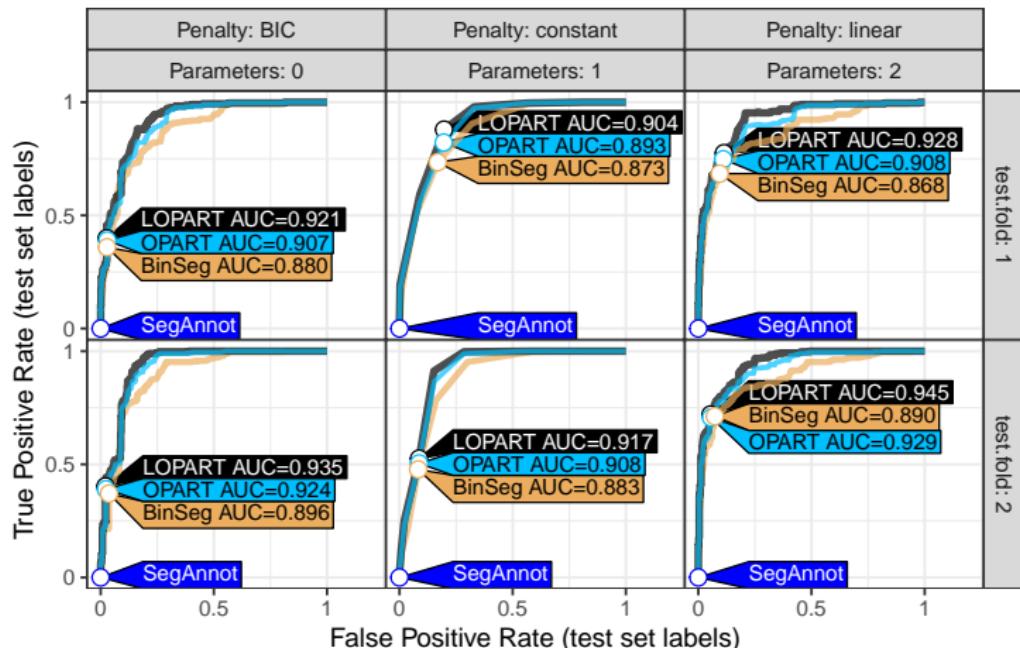
Hocking TD, Rigaill G, Fearnhead P, Bourque G. Constrained Dynamic Programming and Supervised Penalty Learning Algorithms for Peak Detection in Genomic Data. Journal of Machine Learning Research 21(87):1-40, 2020.



Proposed GPDPA (R package PeakSegOptimal) has larger AUC than previous algorithms.

Evaluating a new algorithm with label constraints

Hocking TD, Srivastava A. Labeled Optimal Partitioning. Accepted in Computational Statistics, arXiv:2006.13967.



Proposed LOPART algorithm (R package LOPART) has consistently larger test AUC than previous algorithms.

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Proposed surrogate loss for ROC curve optimization: Area Under Min{FP,FN} (AUM)

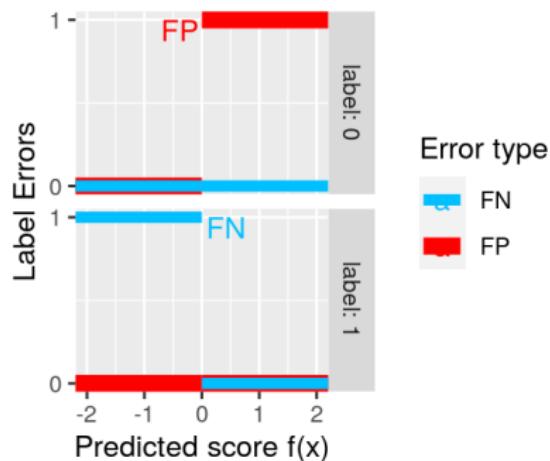
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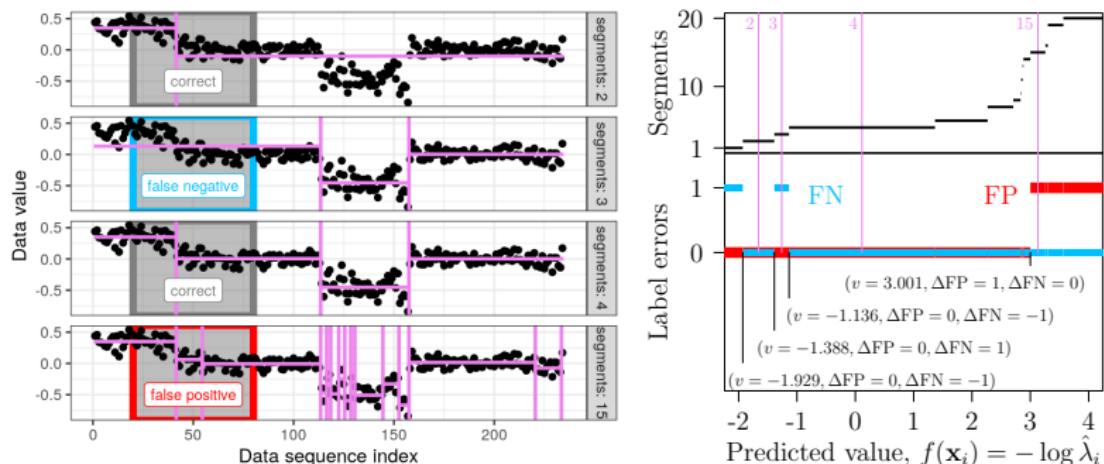
Appendix: Non-monotonic ROC curves in changepoint detection

Binary classification FP/FN functions

- ▶ We assume there are n observations and each observation $i \in \{1, \dots, n\}$ has a predicted score $\hat{y}_i \in \mathbb{R}$ and a corresponding error function.
- ▶ In binary classification each observation i with a negative label has an error function which results in a false positive if $\hat{y}_i > 0$.
- ▶ And each observation with a positive label has an error function which results in a false negative if $\hat{y}_i < 0$.



Changepoint FP/FN functions may be non-monotonic



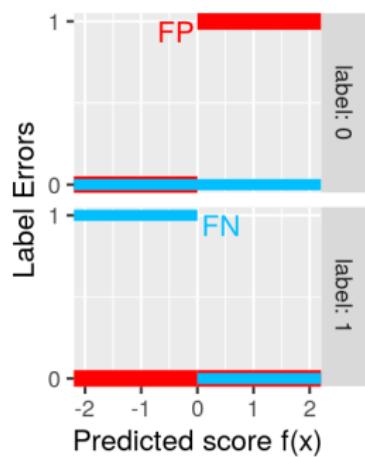
- ▶ Optimal changepoint model may have non-monotonic error (for example FN above), because changepoints at model size s may not be present in model $s + 1$.
- ▶ Penalty values where the FP/FN changes can be efficiently computed, `penaltyLearning::modelSelection` in R.

Hocking TD, Vargovich J. Linear Time Dynamic Programming for Computing Breakpoints in the Regularization Path of Models Selected From a Finite Set. Journal of Computational and Graphical Statistics (2021).

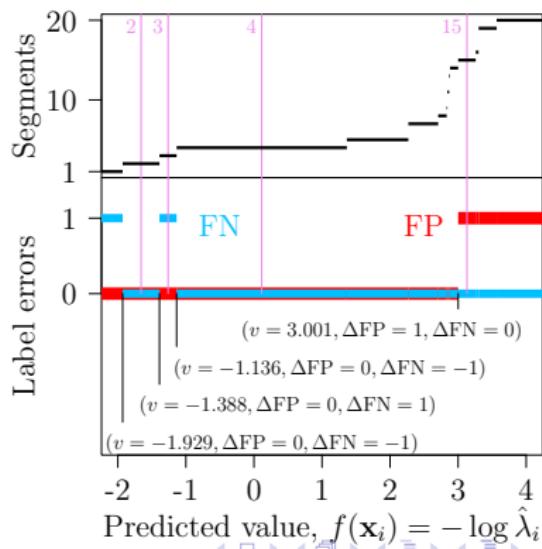
Algorithm inputs: predictions and label error functions

- ▶ Each observation $i \in \{1, \dots, n\}$ has a predicted value $\hat{y}_i \in \mathbb{R}$.
- ▶ Breakpoints $b \in \{1, \dots, B\}$ used to represent label error via tuple $(v_b, \Delta FP_b, \Delta FN_b, \mathcal{I}_b)$.
- ▶ There are changes $\Delta FP_b, \Delta FN_b$ at predicted value $v_b \in \mathbb{R}$ in error function $\mathcal{I}_b \in \{1, \dots, n\}$.

Binary classification



Changepoint detection

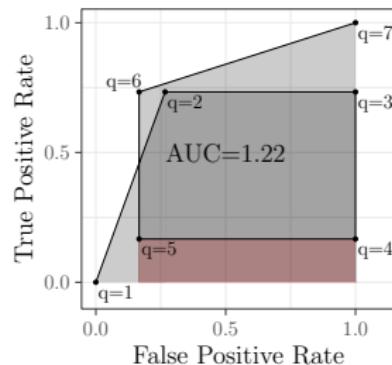
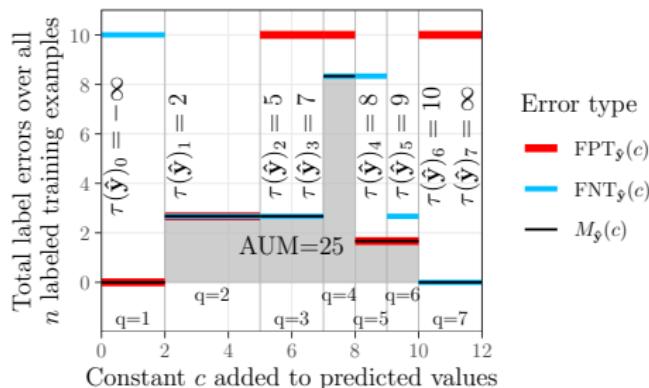


Proposed surrogate loss, Area Under Min (AUM)

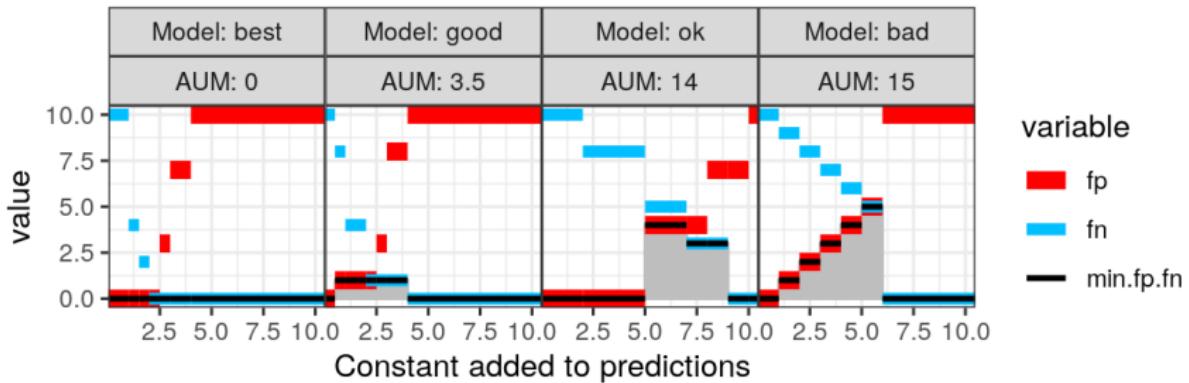
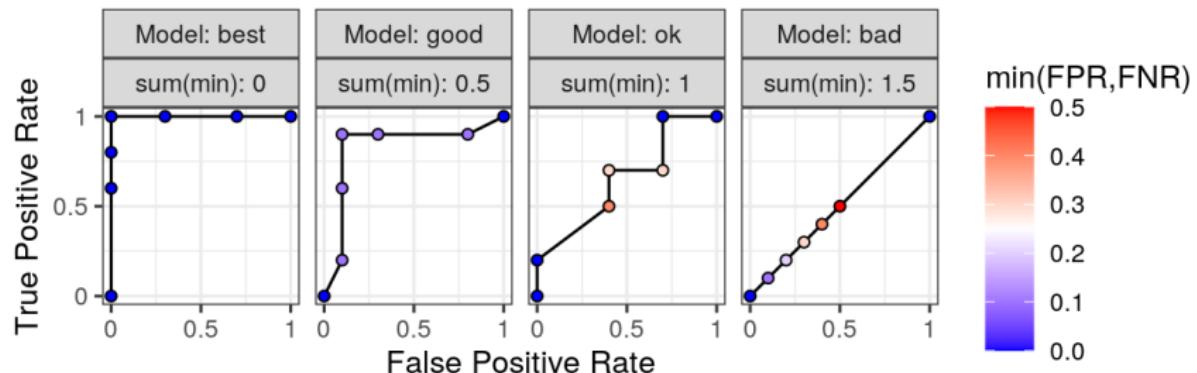
- ▶ Threshold $t_b = v_b - \hat{y}_{\mathcal{I}_b} = \tau(\hat{y})_q$ is largest constant you can add to predictions and still be on ROC point q .
- ▶ Proposed surrogate loss, Area Under Min (AUM) of total FP/FN, computed via sort and modified cumsum:

$$\underline{\text{FP}}_b = \sum_{j:t_j < t_b} \Delta \text{FP}_j, \quad \overline{\text{FP}}_b = \sum_{j:t_j \leq t_b} \Delta \text{FP}_j,$$

$$\underline{\text{FN}}_b = \sum_{j:t_j \geq t_b} -\Delta \text{FN}_j, \quad \overline{\text{FN}}_b = \sum_{j:t_j > t_b} -\Delta \text{FN}_j.$$



Small AUM is correlated with large AUC



Proposed algorithm computes two directional derivatives

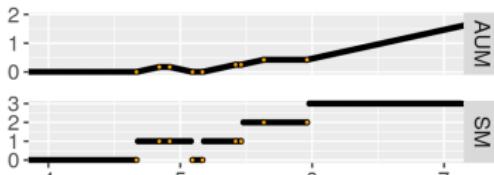
- ▶ Gradient only defined when function is differentiable, but AUM is not differentiable everywhere (see below).
- ▶ Directional derivatives always computable (R package aum),

$$\nabla_{\mathbf{v}(-1,i)} \text{AUM}(\hat{\mathbf{y}}) =$$

$$\sum_{b: \mathcal{I}_b=i} \min\{\overline{\text{FP}}_b, \overline{\text{FN}}_b\} - \min\{\overline{\text{FP}}_b - \Delta \text{FP}_b, \overline{\text{FN}}_b - \Delta \text{FN}_b\},$$

$$\nabla_{\mathbf{v}(1,i)} \text{AUM}(\hat{\mathbf{y}}) =$$

$$\sum_{b: \mathcal{I}_b=i} \min\{\underline{\text{FP}}_b + \Delta \text{FP}_b, \underline{\text{FN}}_b + \Delta \text{FN}_b\} - \min\{\underline{\text{FP}}_b, \underline{\text{FN}}_b\}.$$



Prediction difference, $f(\text{negative}) - f(\text{positive})$

Proposed learning algo uses mean of these two directional derivatives as “gradient.”

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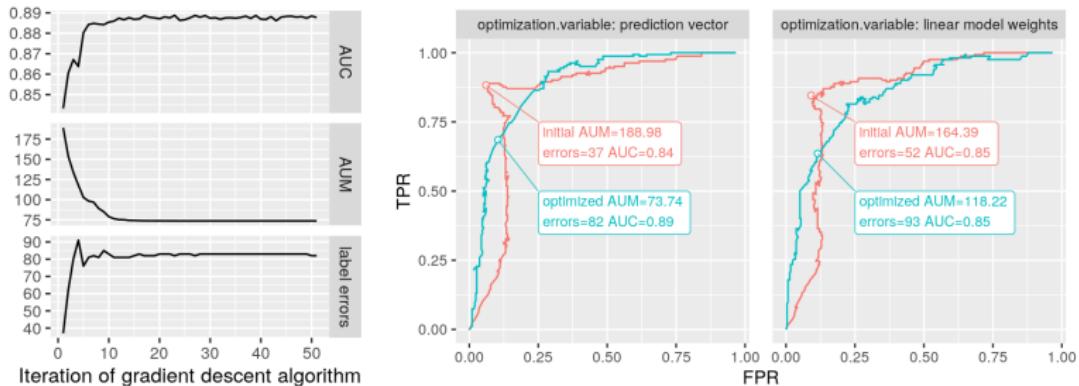
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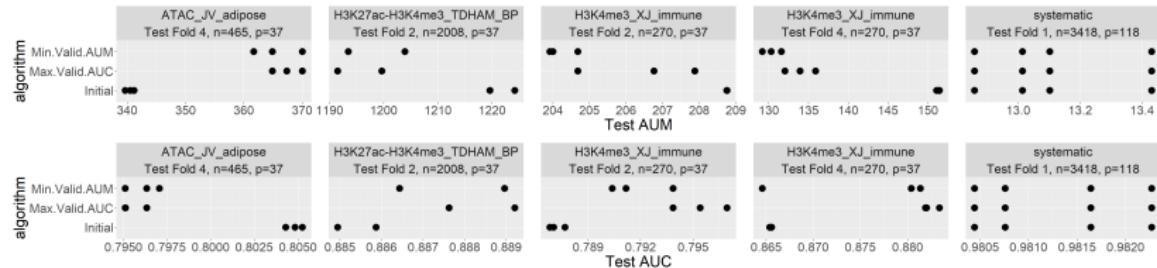
Appendix: Non-monotonic ROC curves in changepoint detection

AUM gradient descent results in increased train AUC for a real changepoint problem



- ▶ Left/middle: changepoint problem initialized to prediction vector with min label errors, gradient descent on prediction vector.
- ▶ Right: linear model initialized by minimizing regularized convex loss (surrogate for label error, Hocking *et al.* ICML 2013), gradient descent on weight vector.

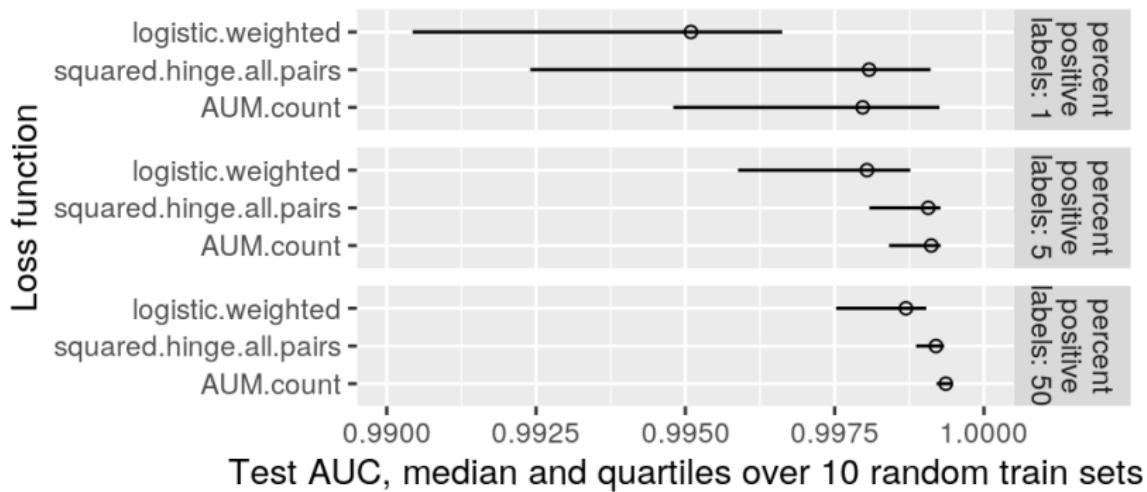
Learning algorithm results in better test AUC/AUM for changepoint problems



- ▶ Five changepoint problems (panels from left to right).
- ▶ Two evaluation metrics (AUM=top, AUC=bottom).
- ▶ Three algorithms (Y axis), Initial=Min regularized convex loss (surrogate for label error, Hocking *et al.* ICML 2013), Min.Valid.AUM/Max.Valid.AUC=AUM gradient descent with early stopping regularization.
- ▶ Four points = Four random initializations.

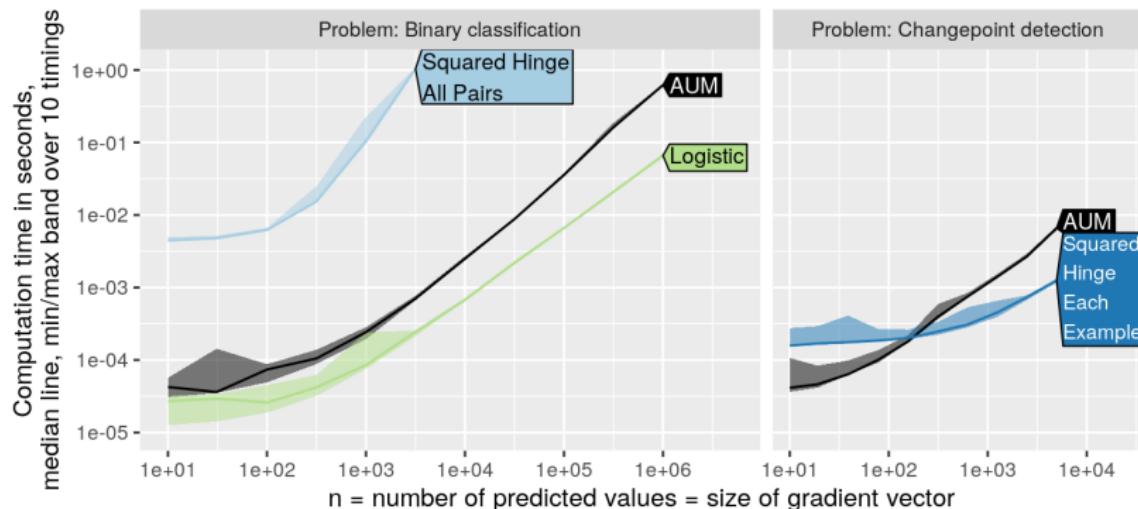
Learning algorithm competitive for unbalanced binary classification

(b) AUM compared to baselines



- ▶ Squared hinge all pairs is a classic/popular surrogate loss function for AUC optimization. (Yan *et al.* ICML 2003)
- ▶ All linear models with early stopping regularization.

Comparable computation time to other loss functions



- ▶ Logistic $O(n)$.
- ▶ AUM $O(n \log n)$. (proposed)
- ▶ Squared Hinge All Pairs $O(n^2)$. (Yan *et al.* ICML 2003)
- ▶ Squared Hinge Each Example $O(n)$. (Hocking *et al.* ICML 2013)

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- ▶ ROC curves are used to evaluate binary classification and changepoint detection algorithms.
- ▶ We propose a new loss function, AUM=Area Under Min(FP,FN), which is a differentiable surrogate of the sum of Min(FP,FN) over all points on the ROC curve.
- ▶ We propose new algorithm for efficient AUM and directional derivative computation.
- ▶ Implementations available in R and python/torch:
<https://cloud.r-project.org/web/packages/aum/>
<https://tdhock.github.io/blog/2022/aum-learning/>
- ▶ Empirical results provide evidence that learning using AUM minimization results in ROC curve optimization (encourages monotonic/regular curves with large AUC).
- ▶ Future work: other model classes, sort-based surrogates for other problems/objectives such as information retrieval.

Thanks to co-author Jonathan Hillman! (second from left)



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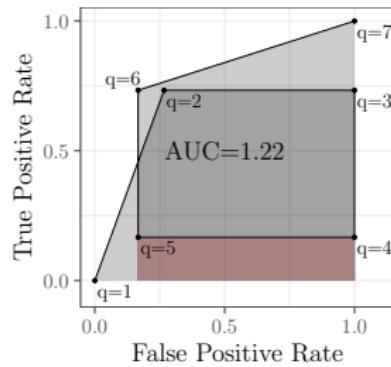
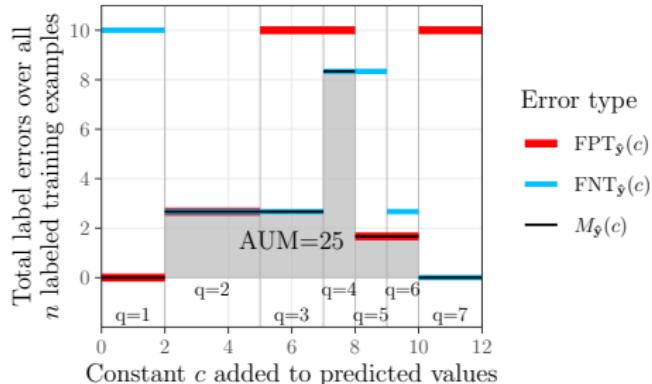
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Looping ROC curve, simple synthetic example

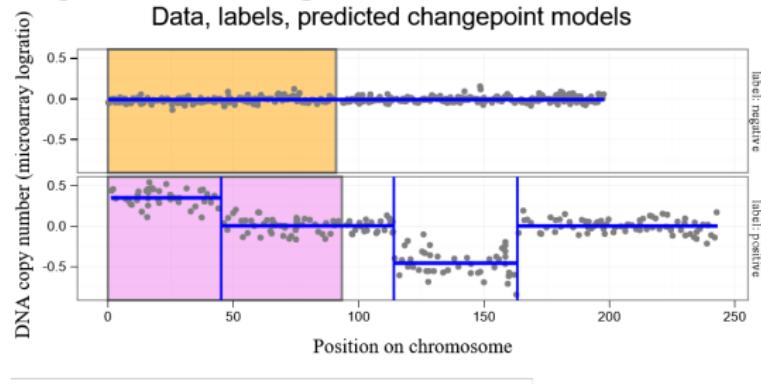
- ▶ Non-monotonic FP/FN can result in looping ROC curve.
- ▶ AUC can be greater than one (dark grey area double counted, red area negative counted).
- ▶ Loops have very sub-optimal points (large min error, for example $q=4$), so do we want to maximize AUC?
- ▶ Minimize Area Under Min (AUM) instead, which encourages monotonic ROC curve with points in upper left (small min error, for example $q=1,6,7$).



Two real changepoint data sets

<https://bl.ocks.org/tdhock/raw/545d76ea8c0678785896e7dbe5ff5510/>

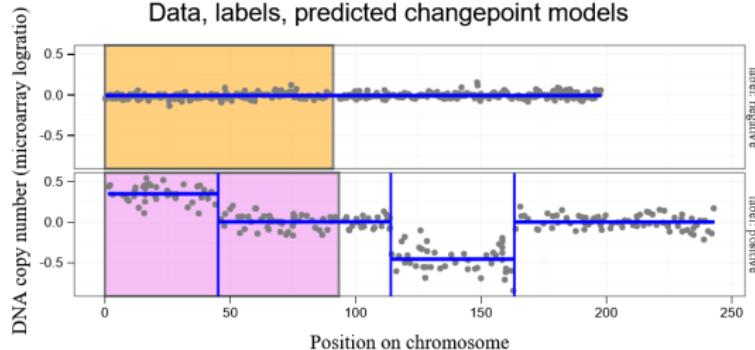
Data, labels, predicted changepoint models



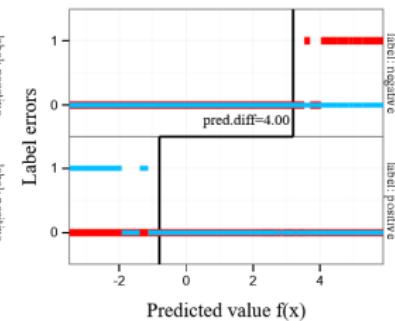
Two real changepoint error functions

<https://bl.ocks.org/tdhock/raw/545d76ea8c0678785896e7dbe5ff5510/>

Data, labels, predicted changepoint models



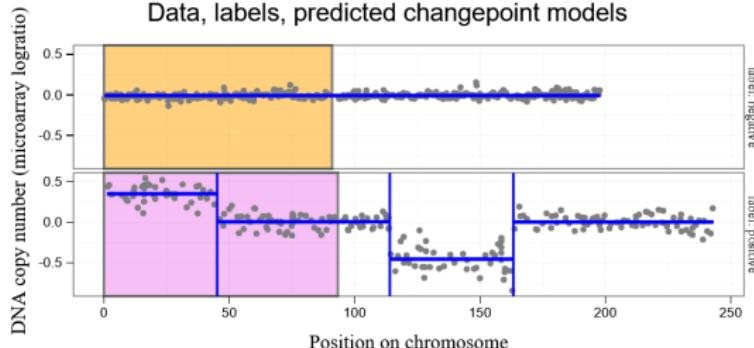
Example error functions



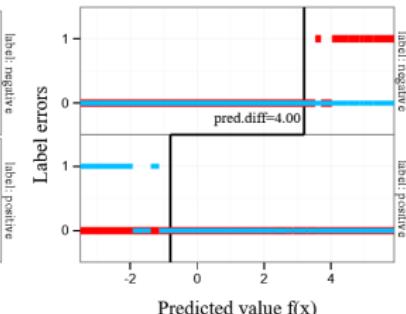
Total error as a function of constant added to predictions

<https://bl.ocks.org/tdhock/raw/545d76ea8c0678785896e7dbe5ff5510/>

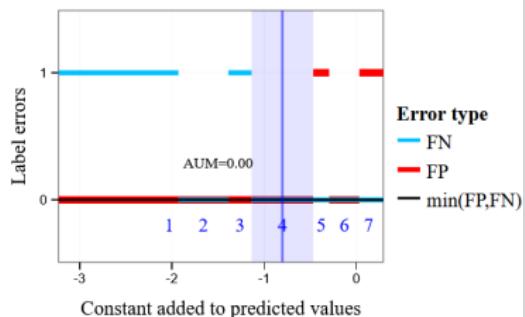
Data, labels, predicted changepoint models



Example error functions



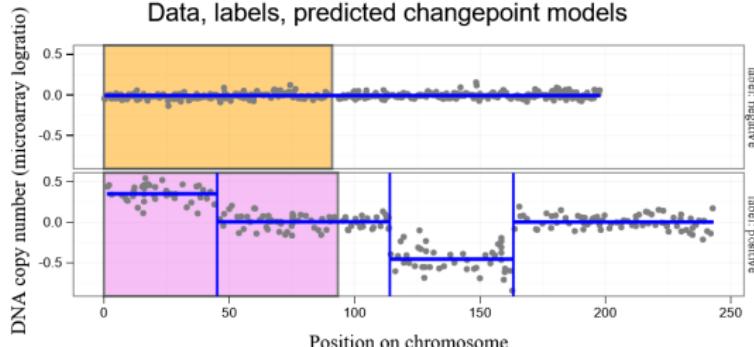
Total error, select interval



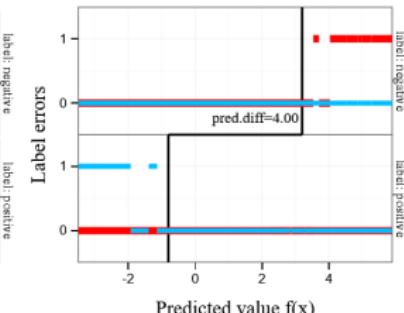
Corresponding ROC curves

<https://bl.ocks.org/tdhock/raw/545d76ea8c0678785896e7dbe5ff5510/>

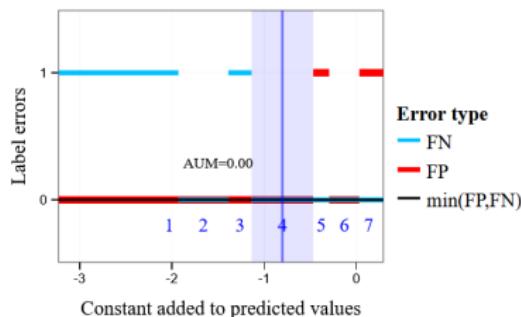
Data, labels, predicted changepoint models



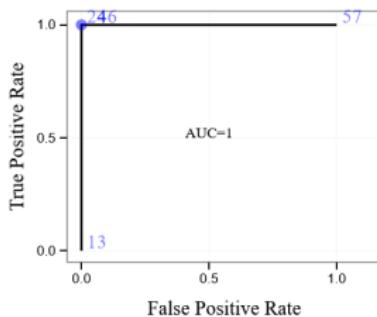
Example error functions



Total error, select interval



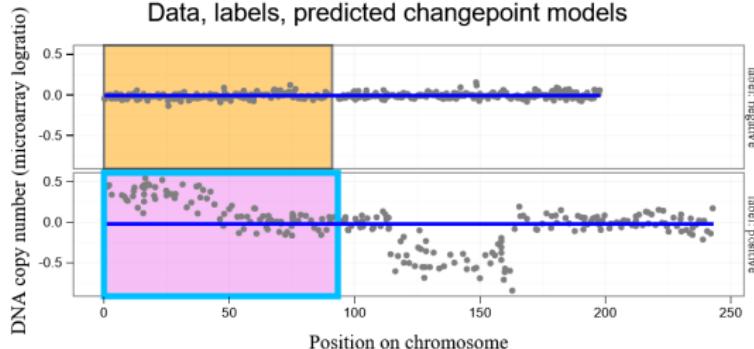
ROC curve, select point



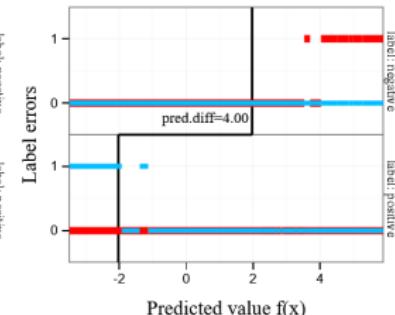
Demonstration of AUC/AUM computation

<https://bl.ocks.org/tdhock/raw/545d76ea8c0678785896e7dbe5ff5510/>

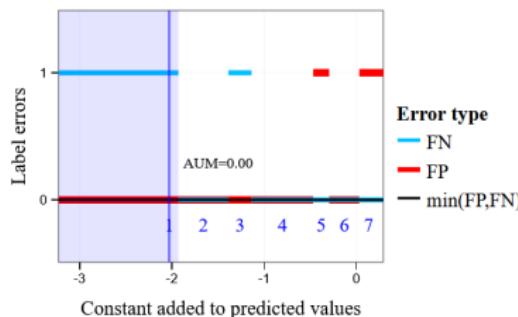
Data, labels, predicted changepoint models



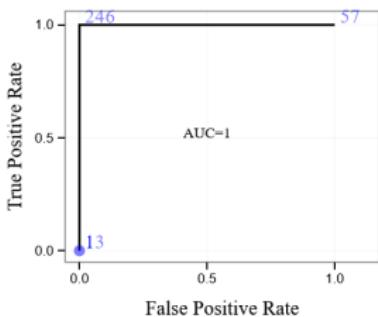
Example error functions



Total error, select interval



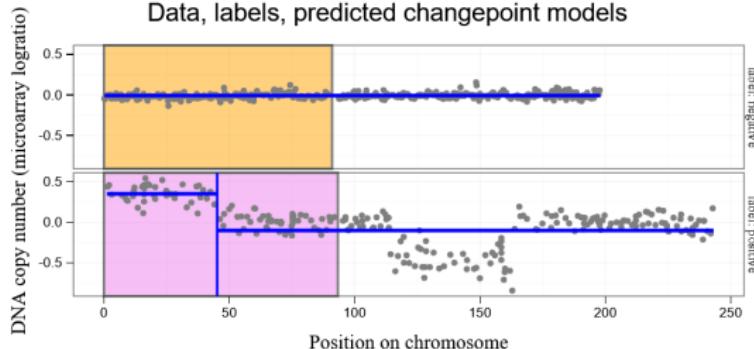
ROC curve, select point



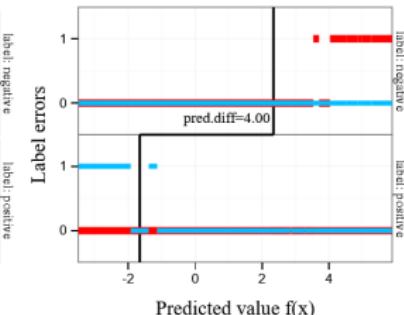
Demonstration of AUC/AUM computation

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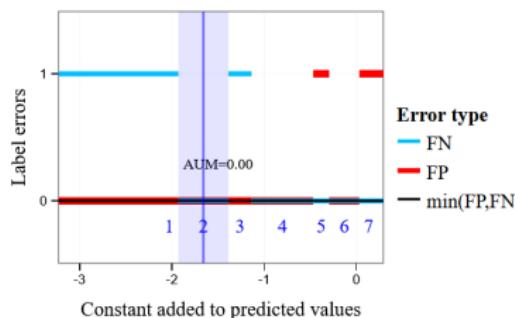
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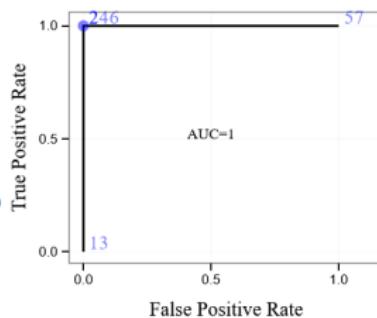
Example error functions



Total error, select interval



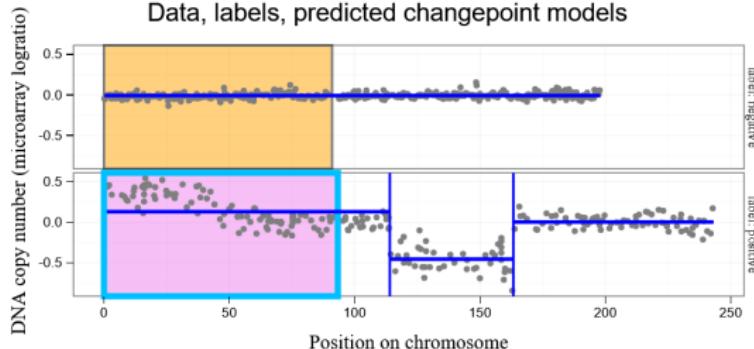
ROC curve, select point



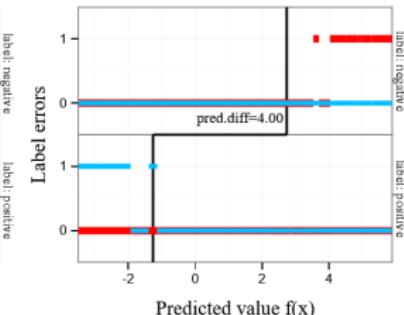
Demonstration of AUC/AUM computation

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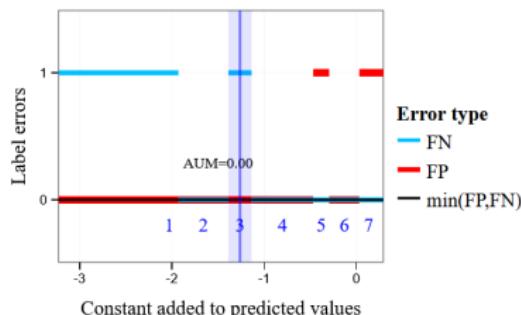
Data, labels, predicted changepoint models



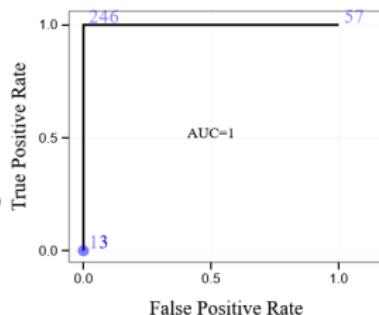
Example error functions



Total error, select interval



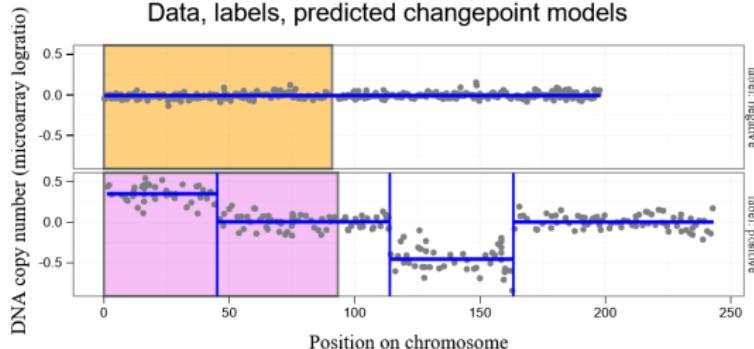
ROC curve, select point



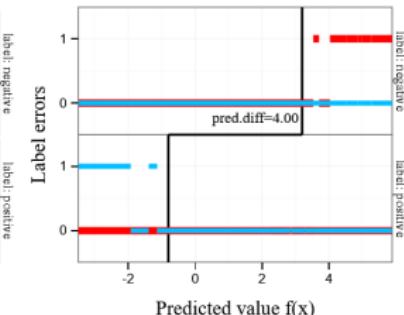
Demonstration of AUC/AUM computation

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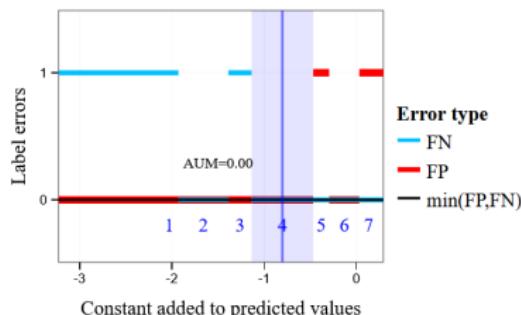
Data, labels, predicted changepoint models



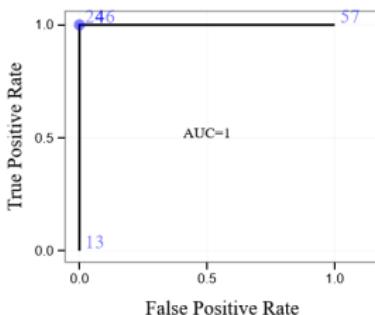
Example error functions



Total error, select interval



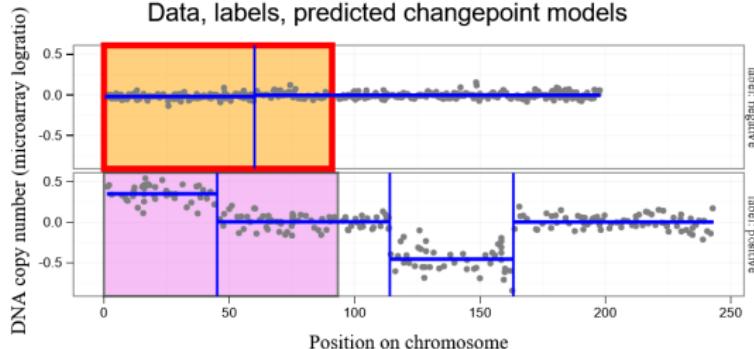
ROC curve, select point



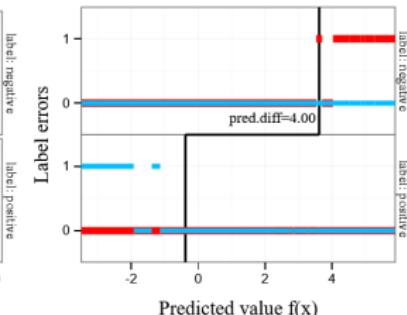
Demonstration of AUC/AUM computation

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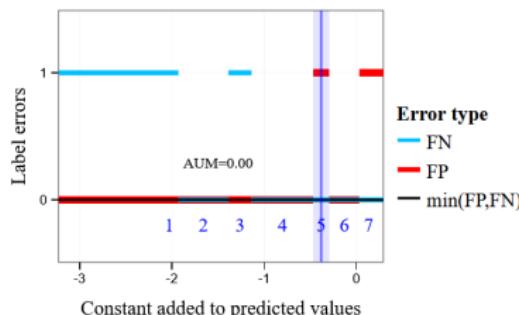
Data, labels, predicted changepoint models



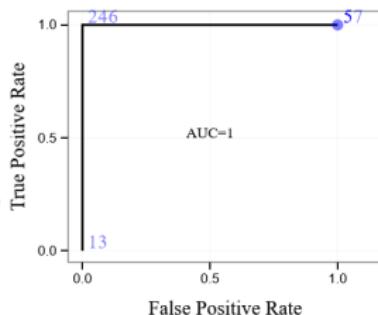
Example error functions



Total error, select interval



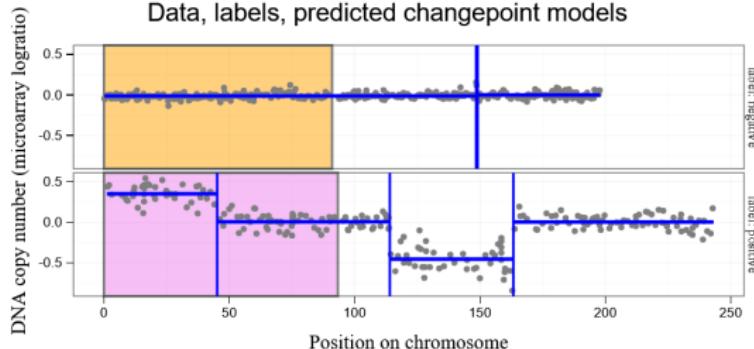
ROC curve, select point



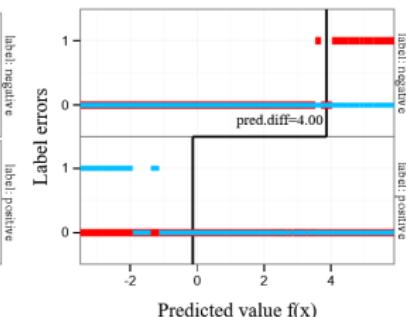
Demonstration of AUC/AUM computation

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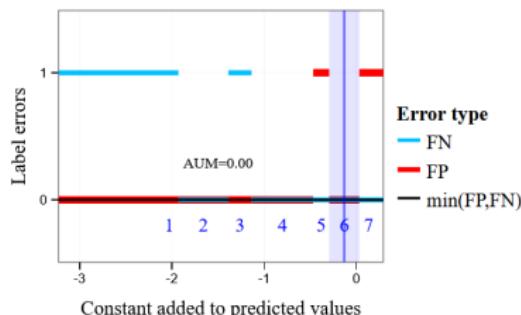
Data, labels, predicted changepoint models



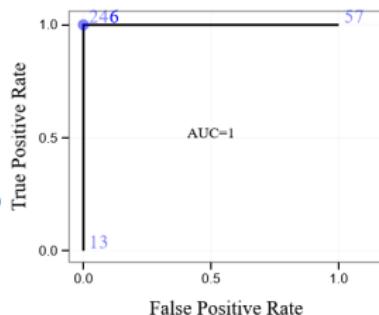
Example error functions



Total error, select interval



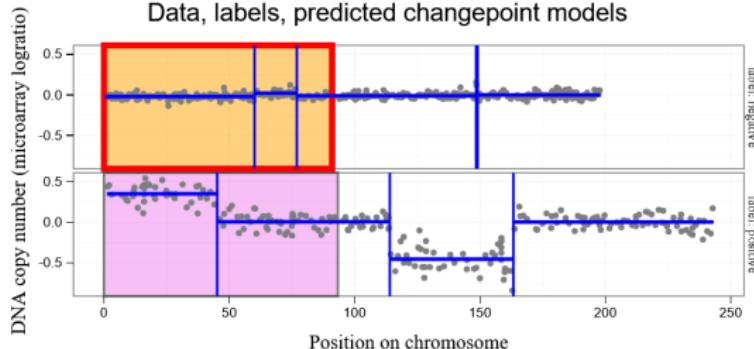
ROC curve, select point



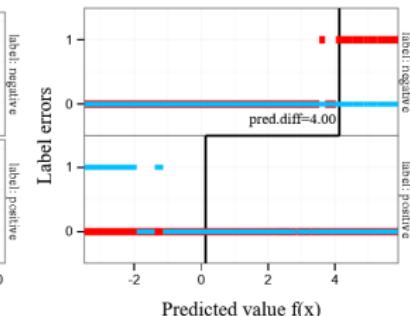
Demonstration of AUC/AUM computation

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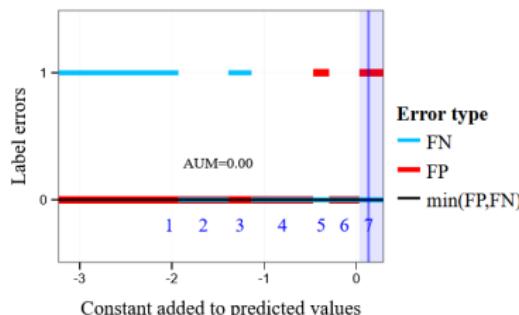
Data, labels, predicted changepoint models



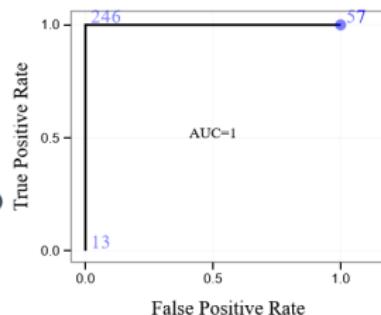
Example error functions



Total error, select interval



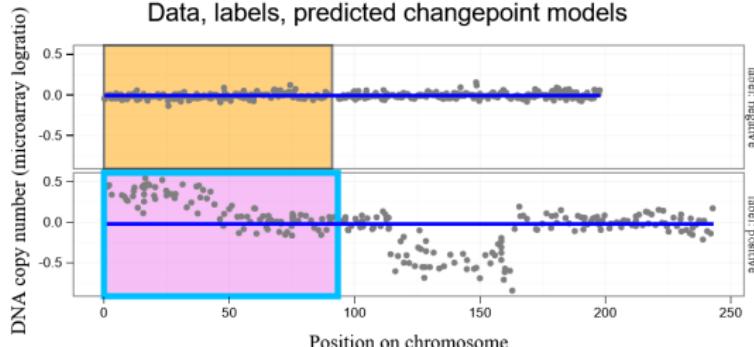
ROC curve, select point



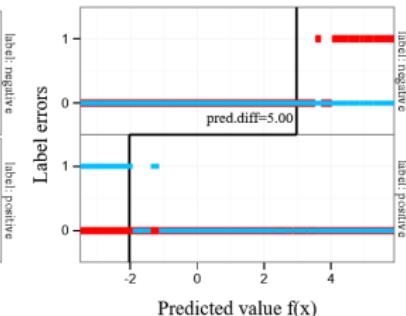
Demonstration of AUC/AUM computation

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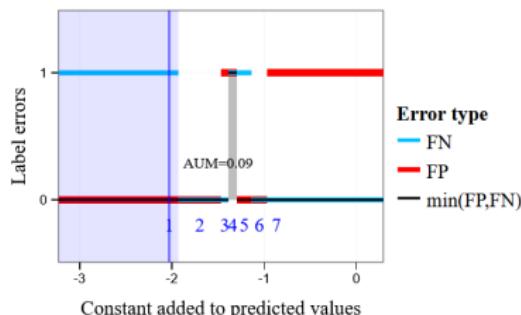
Data, labels, predicted changepoint models



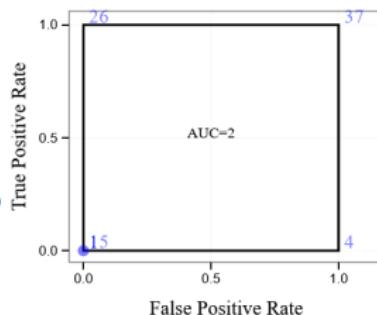
Example error functions



Total error, select interval



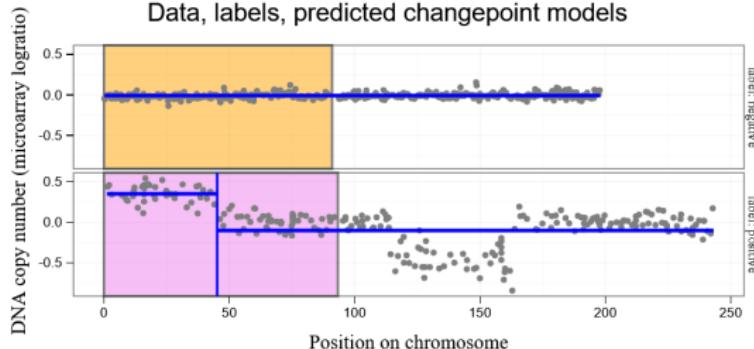
ROC curve, select point



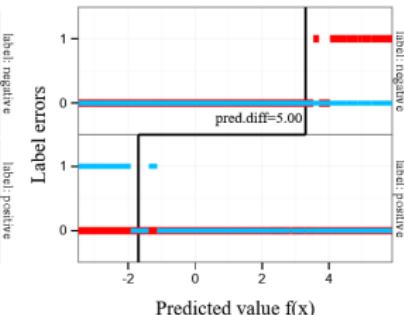
Demonstration of AUC/AUM computation

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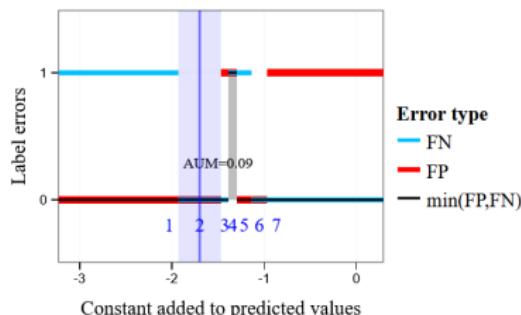
Data, labels, predicted changepoint models



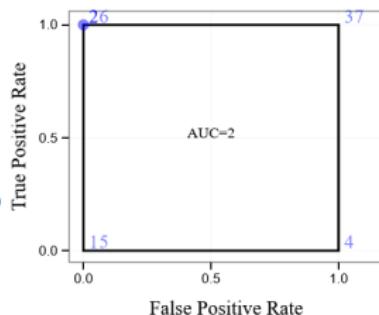
Example error functions



Total error, select interval



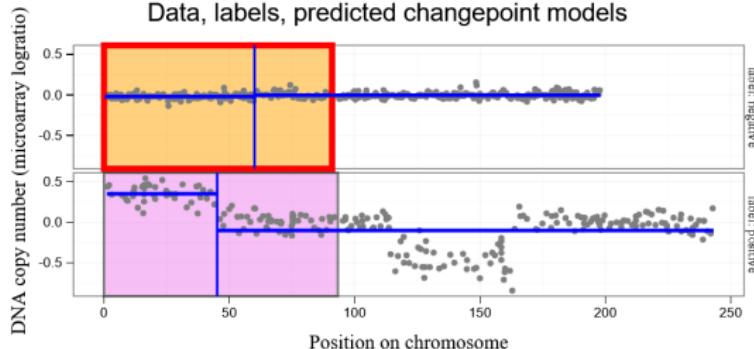
ROC curve, select point



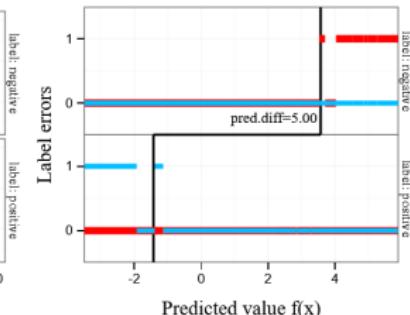
Demonstration of AUC/AUM computation

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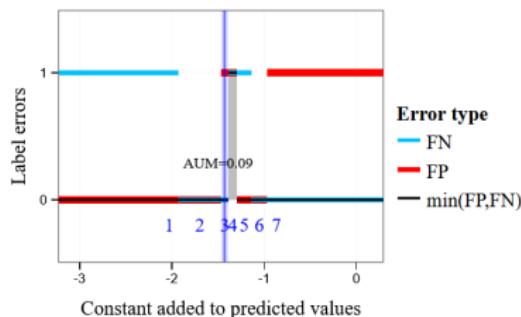
Data, labels, predicted changepoint models



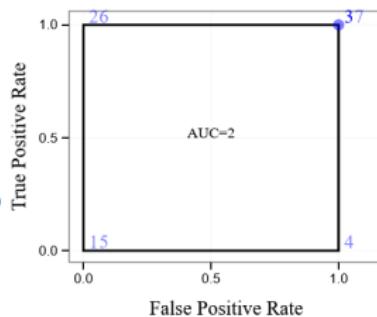
Example error functions



Total error, select interval



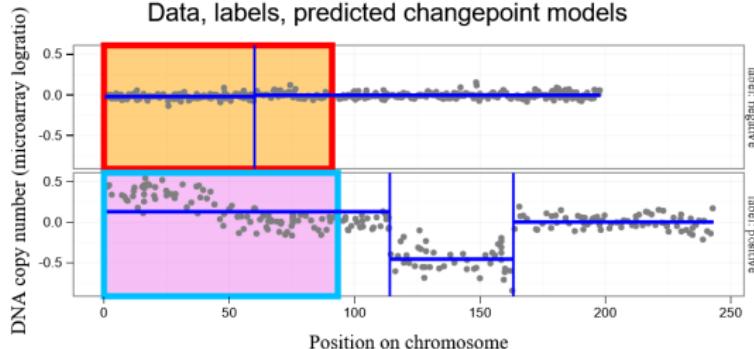
ROC curve, select point



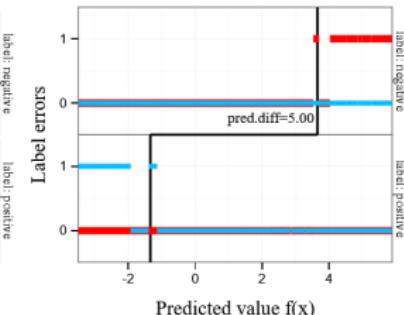
Demonstration of AUC/AUM computation

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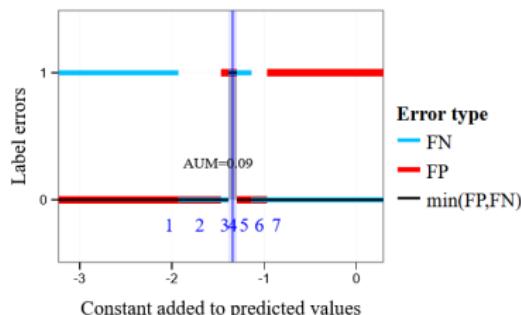
Data, labels, predicted changepoint models



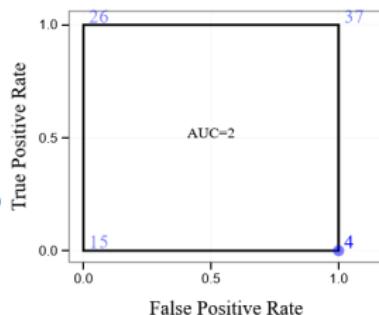
Example error functions



Total error, select interval



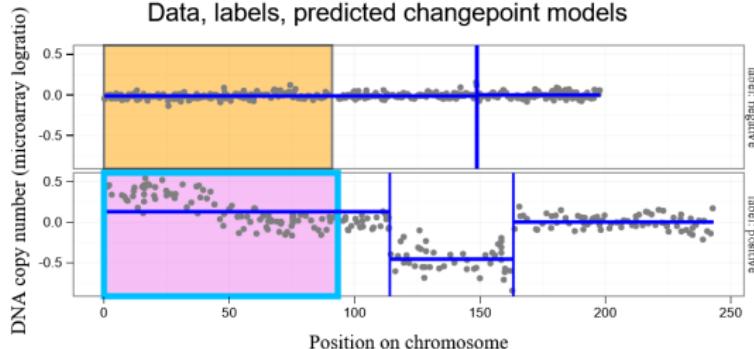
ROC curve, select point



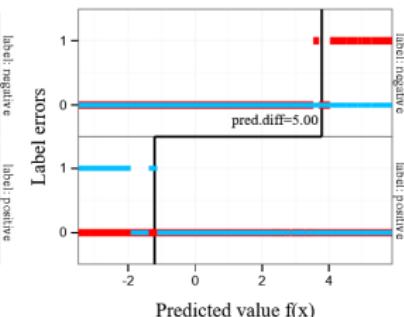
Demonstration of AUC/AUM computation

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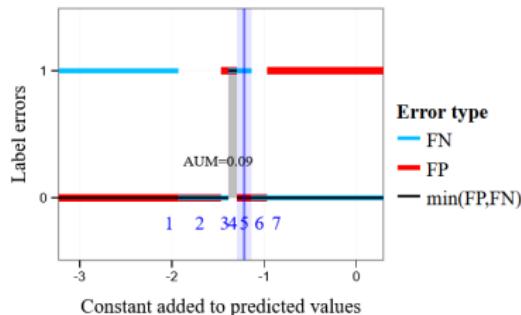
Data, labels, predicted changepoint models



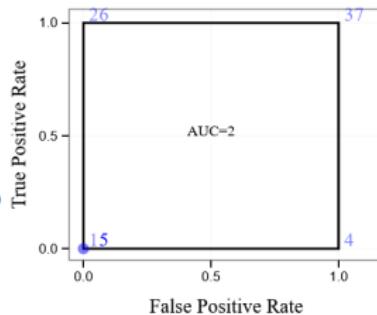
Example error functions



Total error, select interval



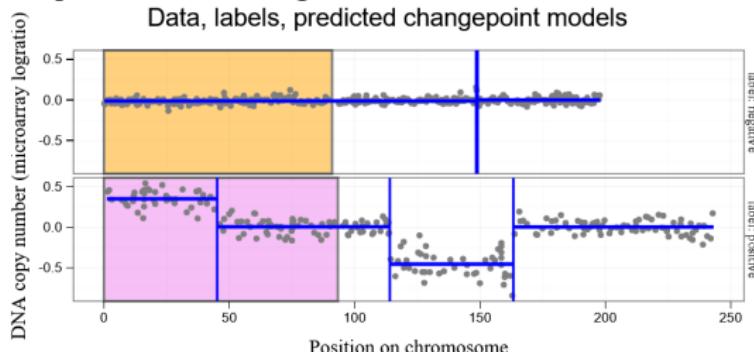
ROC curve, select point



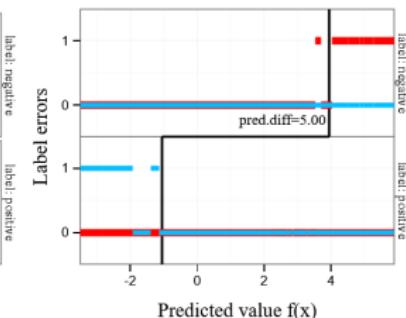
Demonstration of AUC/AUM computation

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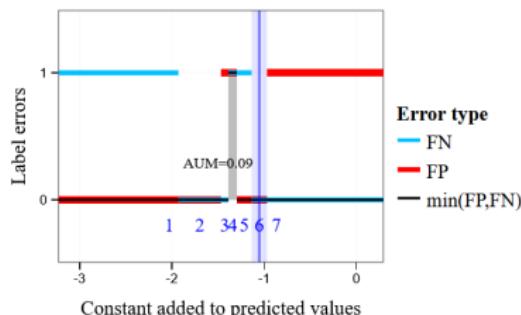
Data, labels, predicted changepoint models



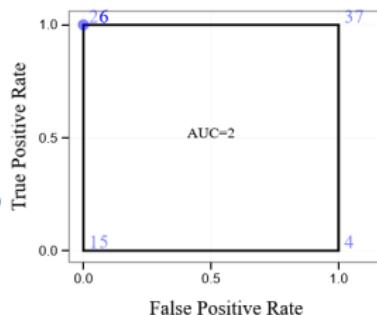
Example error functions



Total error, select interval



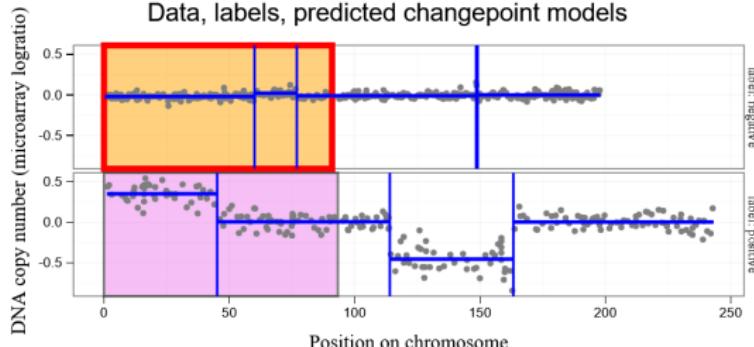
ROC curve, select point



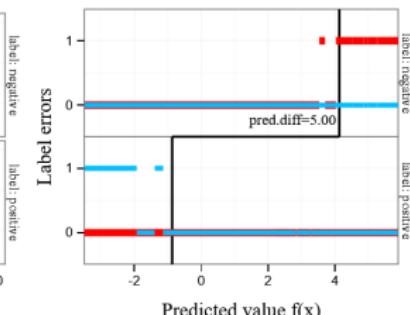
Demonstration of AUC/AUM computation

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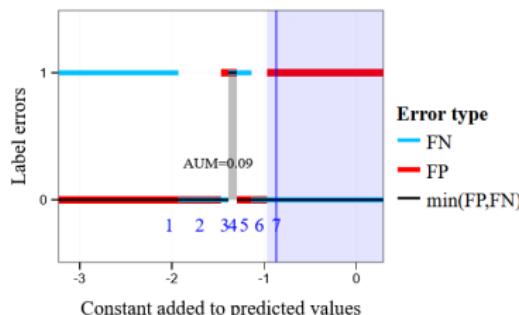
Data, labels, predicted changepoint models



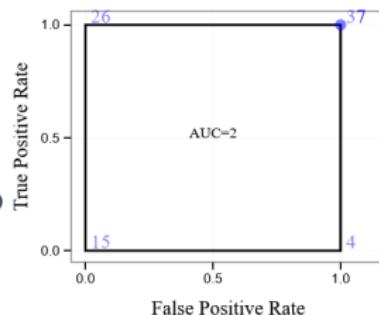
Example error functions



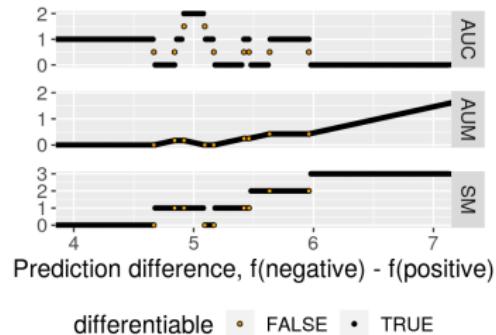
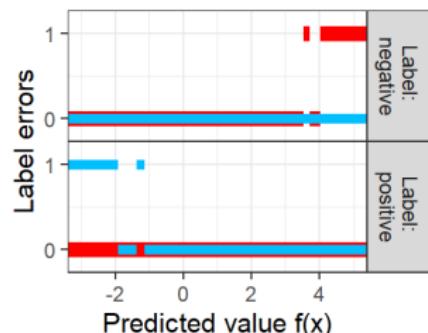
Total error, select interval



ROC curve, select point



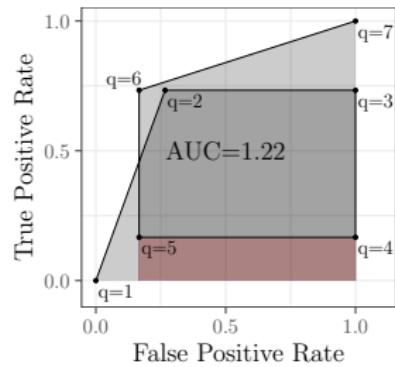
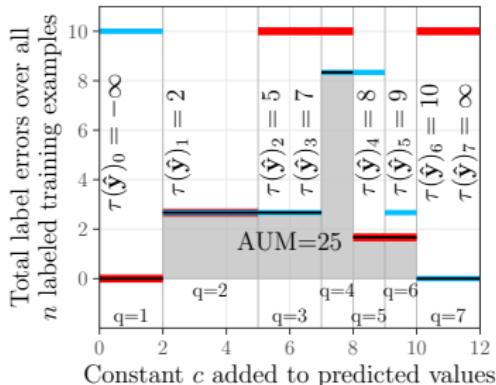
Real data example with AUC greater than one



- ▶ $n = 2$ labeled changepoint problems.
- ▶ Prediction difference=4 \Rightarrow AUC=1 and AUM=0.
- ▶ Prediction difference=5 \Rightarrow AUC=2 and AUM> 0.
- ▶ AUM is continuous L1 relaxation of piecewise constant Sum of Min (SM).
- ▶ AUM is differentiable almost everywhere.
- ▶ Main new idea: compute the gradient of this function and use it for learning.

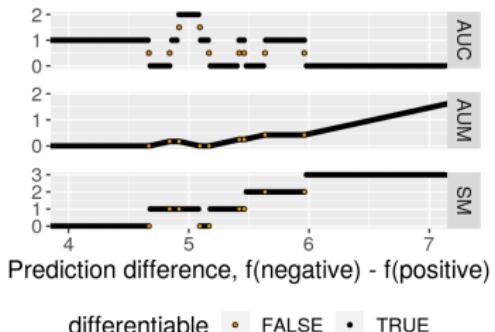
More notation

- ▶ First let $\{(\text{fpt}(\hat{\mathbf{y}})_q, \text{fnt}(\hat{\mathbf{y}})_q, \tau(\hat{\mathbf{y}})_q)\}_{q=1}^Q$ be a sequence of Q tuples, each of which corresponds to a point on the ROC curve (and an interval on the fn/fp error plot).
- ▶ For each q the $\text{fpt}(\hat{\mathbf{y}})_q, \text{fnt}(\hat{\mathbf{y}})_q$ are false positive/negative totals at that point (in that interval) whereas $\tau(\hat{\mathbf{y}})_q$ is the upper limit of the interval.
- ▶ The limits are increasing, $-\infty = \tau(\hat{\mathbf{y}})_0 < \dots < \tau(\hat{\mathbf{y}})_Q = \infty$.
- ▶ Then we define $m(\hat{\mathbf{y}})_q = \min\{\text{fpt}(\hat{\mathbf{y}})_q, \text{fnt}(\hat{\mathbf{y}})_q\}$ as the min of fp and fn totals in that interval.



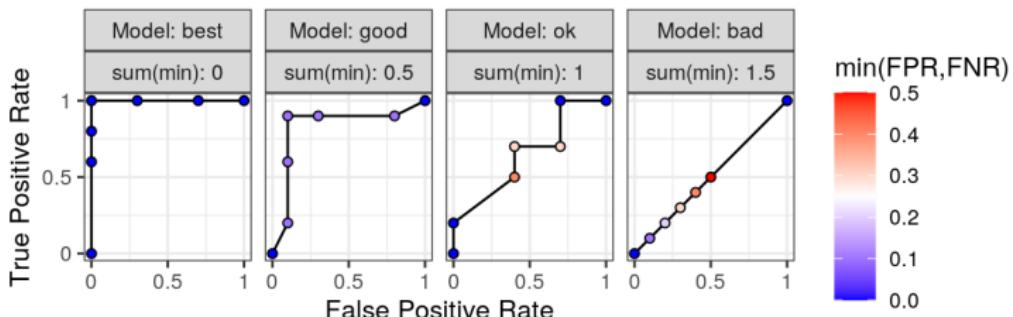
Our proposed loss function is

$$\text{AUM}(\hat{\mathbf{y}}) = \sum_{q=2}^{Q-1} [\tau(\hat{\mathbf{y}})_q - \tau(\hat{\mathbf{y}})_{q-1}] m(\hat{\mathbf{y}})_q.$$



It is a continuous L1 relaxation of the following non-convex **Sum of Min(FP,FN)** function,

$$\text{SM}(\hat{\mathbf{y}}) = \sum_{q=2}^{Q-1} I[\tau(\hat{\mathbf{y}})_q \neq \tau(\hat{\mathbf{y}})_{q-1}] m(\hat{\mathbf{y}})_q = \sum_{q: \tau(\hat{\mathbf{y}})_q \neq \tau(\hat{\mathbf{y}})_{q-1}} m(\hat{\mathbf{y}})_q.$$



Definition of data set, notations

- ▶ Let there be a total of B breakpoints in the error functions over all n labeled training examples.
- ▶ Each breakpoint $b \in \{1, \dots, B\}$ is represented by the tuple $(v_b, \Delta FP_b, \Delta FN_b, \mathcal{I}_b)$, where the $\mathcal{I}_b \in \{1, \dots, n\}$ is an example index, and there are changes $\Delta FP_b, \Delta FN_b$ at predicted value $v_b \in \mathbb{R}$ in the error functions.
- ▶ For example in binary classification, there are $B = n$ breakpoints (same as the number of labeled training examples); for each breakpoint $b \in \{1, \dots, B\}$ we have $v_b = 0$ and $\mathcal{I}_b = b$. For breakpoints b with positive labels $y_b = 1$ we have $\Delta FP = 0, \Delta FN = -1$, and for negative labels $y_b = -1$ we have $\Delta FP = 1, \Delta FN = 0$.
- ▶ In changepoint detection we have more general error functions, which may have more than one breakpoint per example.

Proposed algorithm uses sort to compute AUM and directional derivatives

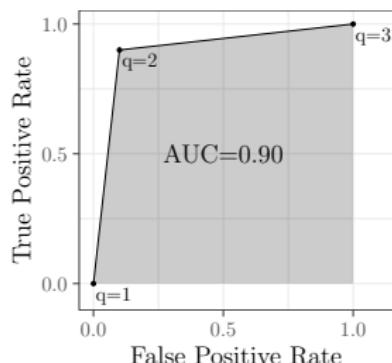
- 1: **Input:** Predictions $\hat{\mathbf{y}} \in \mathbb{R}^n$, breakpoints in error functions $v_b, \Delta\text{FP}_b, \Delta\text{FN}_b, \mathcal{I}_b$ for all $b \in \{1, \dots, B\}$.
- 2: Zero the AUM $\in \mathbb{R}$ and directional derivatives $\mathbf{D} \in \mathbb{R}^{n \times 2}$.
- 3: $t_b \leftarrow v_b - \hat{y}_{\mathcal{I}_b}$ for all b .
- 4: $s_1, \dots, s_B \leftarrow \text{SORTEDINDICES}(t_1, \dots, t_B)$.
- 5: Compute $\underline{\text{FP}}_b, \overline{\text{FP}}_b, \underline{\text{FN}}_b, \overline{\text{FN}}_b$ for all b using s_1, \dots, s_B .
- 6: **for** $b \in \{2, \dots, B\}$ **do**
- 7: AUM $+= (t_{s_b} - t_{s_{b-1}}) \min\{\underline{\text{FP}}_b, \overline{\text{FN}}_b\}$.
- 8: **for** $b \in \{1, \dots, B\}$ **do**
- 9: $\mathbf{D}_{\mathcal{I}_b, 1} += \min\{\overline{\text{FP}}_b, \overline{\text{FN}}_b\} - \min\{\overline{\text{FP}}_b - \Delta\text{FP}_b, \overline{\text{FN}}_b - \Delta\text{FN}_b\}$
- 10: $\mathbf{D}_{\mathcal{I}_b, 2} += \min\{\underline{\text{FP}}_b + \Delta\text{FP}_b, \underline{\text{FN}}_b + \Delta\text{FN}_b\} - \min\{\underline{\text{FP}}_b, \underline{\text{FN}}_b\}$
- 11: **Output:** AUM and matrix \mathbf{D} of directional derivatives.

- ▶ Overall $O(B \log B)$ time due to sort.

Receiver Operating Characteristic (ROC) curve

Classic evaluation method from the signal processing literature
(Egan and Egan, 1975).

- ▶ Binary classification algo gives predictions $[\hat{y}_1, \hat{y}_2, \hat{y}_3, \hat{y}_4]$.
- ▶ Each point on the ROC curve is the FPR/TPR if you add c to the predictions, $[\hat{y}_1 + c, \hat{y}_2 + c, \hat{y}_3 + c, \hat{y}_4 + c]$.
- ▶ Best point in ROC space is upper left (0% FPR, 100% TPR).
- ▶ Maximizing Area Under the ROC curve (AUC) is a common objective for binary classification, especially for imbalanced data (example: 99% positive, 1% negative labels).

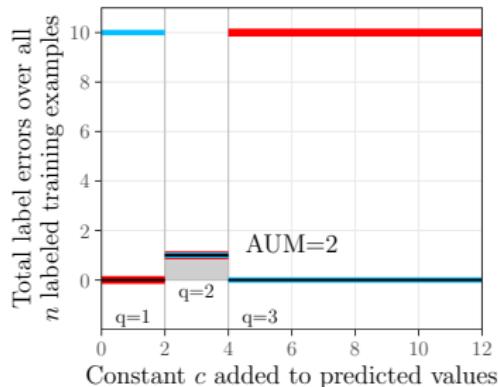


In binary classification, ROC curve is monotonic increasing.

- ▶ AUC=1 best.
- ▶ AUC=0.5 for constant prediction (usually worst).

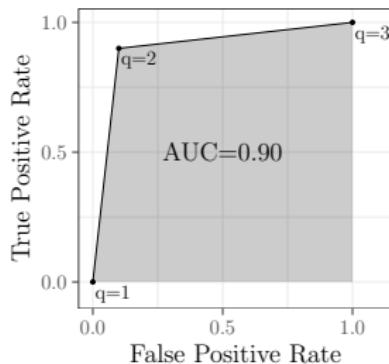
Area Under ROC curve, synthetic example

- ▶ Labels = [1,0,0,...,1,1,0] (20 labels, 10 positive, 10 negative).
- ▶ Predictions = [-4, -4, -4, ..., -2, -2, -2].
- ▶ No constant added $c = 0$, $q = 1$, everything predicted negative, so no false positives, but no true positives.
- ▶ Add $c = 3 \Rightarrow [-1, -1, -1, \dots, 1, 1, 1]$, 1 FP and 9 TP, $q = 2$.
- ▶ Add $c = 5 \Rightarrow [1, 1, 1, \dots, 3, 3, 3]$, all FP and TP, $q = 3$.



Error type

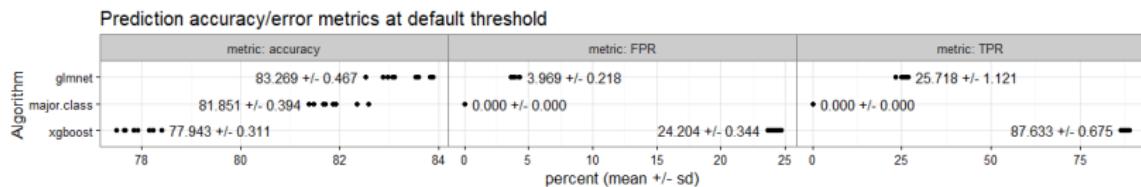
- FPT $_{\hat{y}}(c)$
- FNT $_{\hat{y}}(c)$
- $M_{\hat{y}}(c)$



Real data example when ROC curves are useful

Data from collaboration with SICCS professor Patrick Jantz, about predicting presence/absence of trees in different locations.

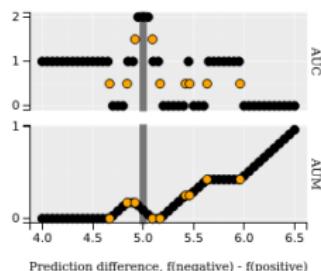
- ▶ glmnet: L1-regularized linear model.
- ▶ major.class: featureless baseline (ignores inputs, always predicts most frequent class label in train set)
- ▶ xgboost: gradient boosted decision trees.
- ▶ Which algorithm is the most accurate?



<https://bl.ocks.org/tdhock/raw/172d0f68a51a8de5d6f1bed7f23f5f82/>

Real data example, interactive AUC/AUM demo

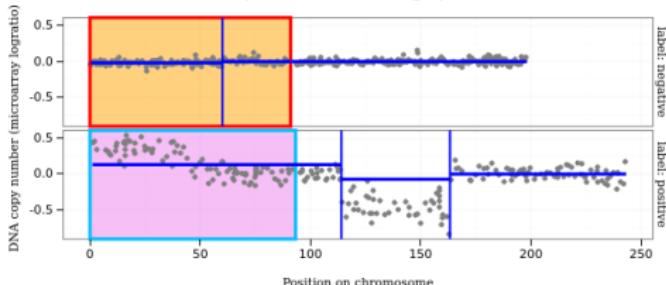
Overview, select difference



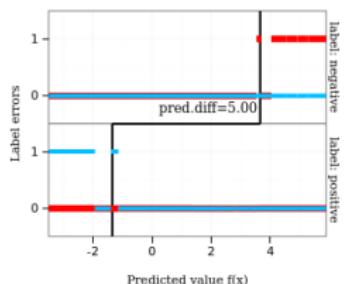
differentiable

- FALSE
- TRUE

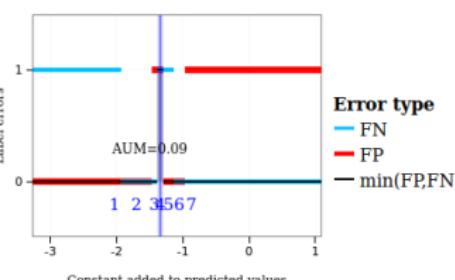
Data, labels, predicted changepoint models



Example error functions



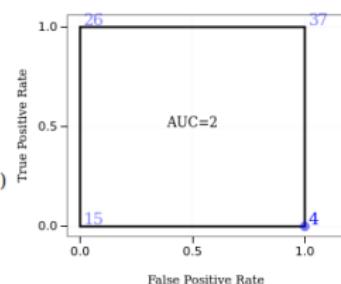
Total error, select interval



Error type

- FN
- FP
- min(FP,FN)

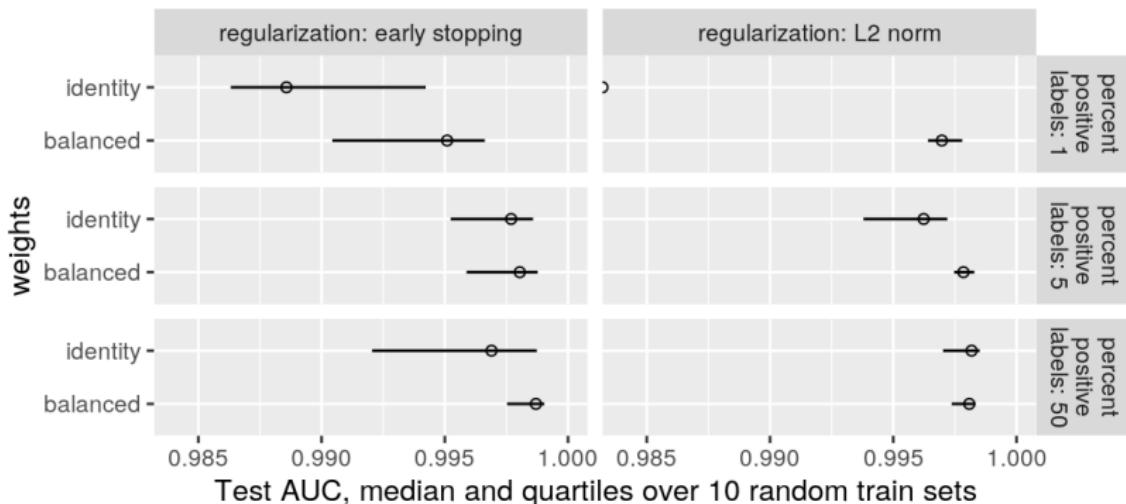
ROC curve, select point



[http://bl.ocks.org/tdhock/raw/
e3f56fa419a6638f943884a3abe1dc0b/](http://bl.ocks.org/tdhock/raw/e3f56fa419a6638f943884a3abe1dc0b/)

Standard logistic loss fails for highly imbalanced labels

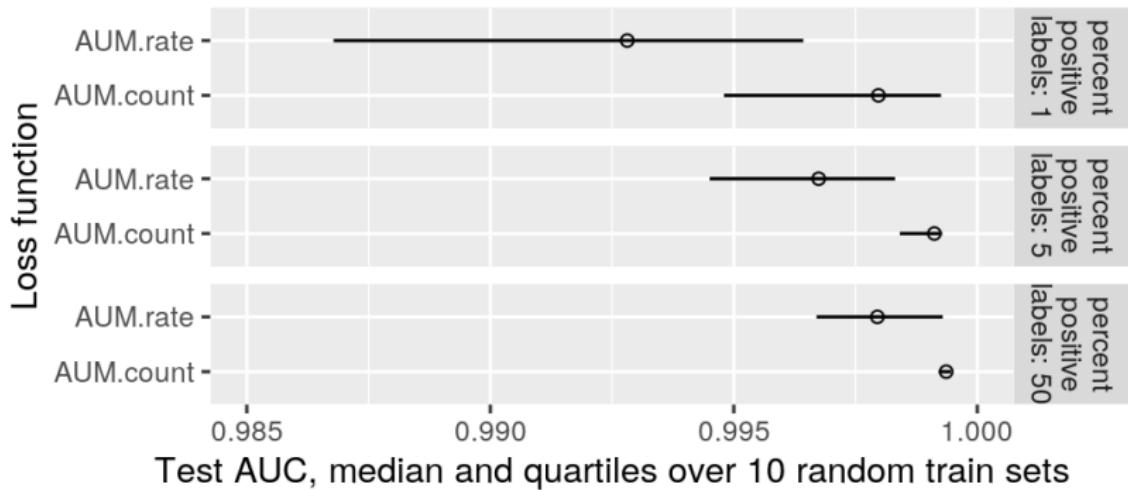
Comparing logistic regression models (control experiment)



- ▶ Subset of zip.train/zip.test data (only 0/1 labels).
- ▶ Test set size 528 with balanced labels (50%/50%).
- ▶ Train set size 1000 with variable class imbalance.
- ▶ Loss is $\ell[f(x_i), y_i]w_i$ with $w_i = 1$ for identity weights, $w_i = 1/N_{y_i}$ for balanced, ex: 1% positive means $w_i \in \{1/10, 1/990\}$.

Error rate loss is not as useful as error count loss

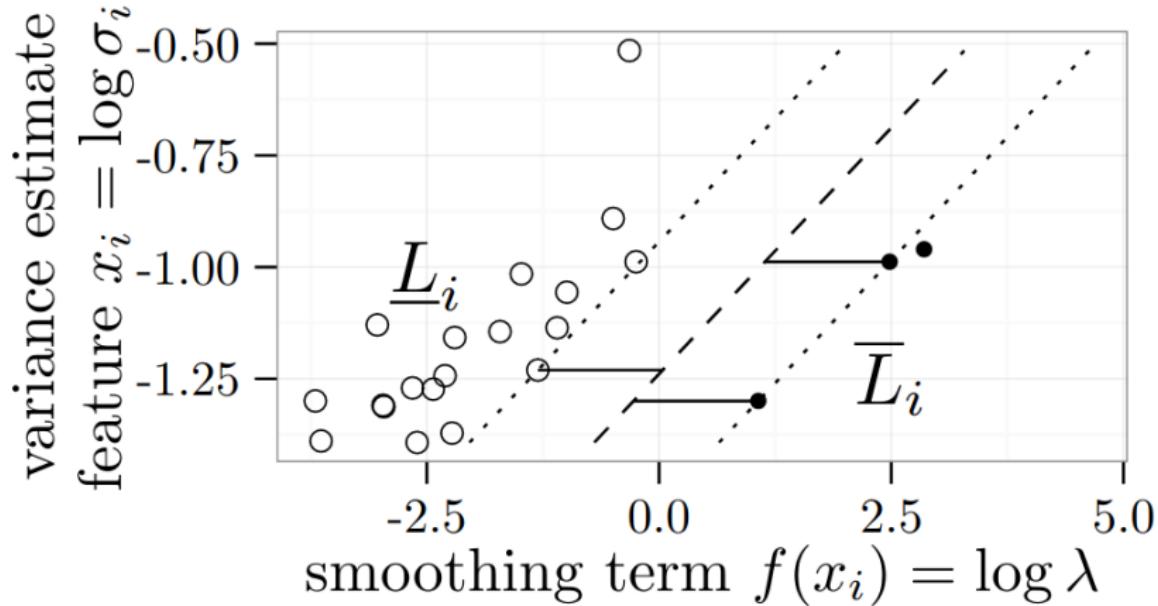
(a) Comparing AUM variants



- ▶ AUM.count is as described previously: error functions used to compute $\text{Min}(\text{FP}, \text{FN})$ are absolute label counts.
- ▶ AUM.rate is a variant which uses normalized error functions, $\text{Min}(\text{FPR}, \text{FNR})$.
- ▶ Both linear models with early stopping regularization.

New max-margin loss function for penalty learning

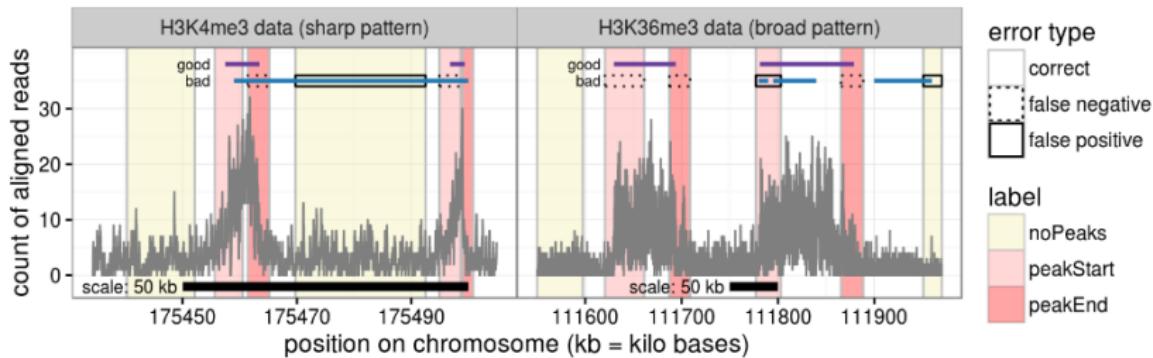
Hocking TD, Rigaill G, Bach F, Vert J-P. Learning Sparse Penalties for Change-point Detection using Max Margin Interval Regression. ICML'13.



Main new idea: learning a penalty/smoothing by minimizing a margin-based differentiable loss function (surrogate for label error), similar to Support Vector Machine and censored regression.

Weakly supervised peak detection in genomic data

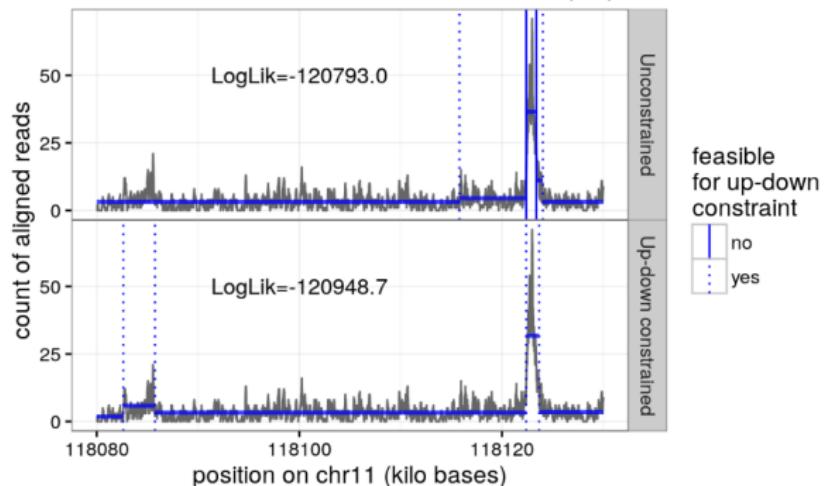
Hocking TD, Rigaill G, Fearnhead P, Bourque G. Constrained Dynamic Programming and Supervised Penalty Learning Algorithms for Peak Detection in Genomic Data. Journal of Machine Learning Research 21(87):1-40, 2020.



Problem setting: weakly supervised peak detection in genomic data
(want to learn peak pattern from partial labels, and predict consistently/accurately in unlabeled regions).

New up-down constraints on adjacent segment means

Hocking TD, Rigaill G, Fearnhead P, Bourque G. Constrained Dynamic Programming and Supervised Penalty Learning Algorithms for Peak Detection in Genomic Data. Journal of Machine Learning Research 21(87):1-40, 2020.



Proposed fast dynamic programming algorithm for computing optimal changepoints subject to up-down constraints on adjacent segment means.