Ramsey Optimal Monetary Policy

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1 Optimal Monetary Policy

The first question we need ask is what is an optimal monetary policy. The optimality based on some criteria. The most commonly used criteria are the representative agent's or planner's welfare function or welfare loss of the monetary authorities.

Usually we close our model after all the other equilibrium conditions are ready by adding one more equation: the monetary policy rule for nominal rate in the economy. For example, it is usually a Taylor rule. This kind of rules are usually not optimal, and there are often sub-optimal. This is because Taylor rule is a kind of restrictions that limit some endogenous variables (for example, nominal interest rate, inflation and output gap) to some smaller space instead of the whole space. So in this sense, the welfare maximization usually can not be achieved in limited space.

1.1 What is the Ramsey equilibrium?

Ramsey optimal monetary policy is a policy that maximization the policy maker's social welfare function by dropping any given monetary policy in the model. Then collecting all the other equilibrium conditions and setting up Lagrangian problem which is the representation of Ramsey optimal problem. Hence, in Ramsey problem, the equilibrium conditions used is one less than that in usual model.

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Sometimes we call the solution of Ramsey problem to be Ramsey allocation or efficient allocation since it is the first best allocation. Let's look at what is a Ramsey problem. This includes several points in sequence:

1. The equilibrium conditions of a model could be written as

$$E_t \underbrace{f\left(v_{t-1}, v_t, v_{t+1}, u_t, u_{t+1}\right)}_{(N-1)\times N} = 0, \text{for all} \underbrace{v_t}_{N\times 1} (\text{endogenous}), u_t (\text{exogenous})$$
$$u_t = P u_{t-1} + \epsilon_t \tag{1}$$

   Noticing that there are $N-1$ equilibrium conditions for N endogenous variables v_t. The other equilibrium condition the monetary policy rule that we have removed from the equilibrium system.

2. Preferences or Welfare

$$W\left(v_{-1}, v_0, u_0\right) \equiv E_0 \sum_{t=0}^{\infty} \beta^t U\left(v_t, u_t\right)$$

   which could be written as

$$W\left(v_{t-1}, v_t, u_t\right) = U\left(v_t, u_t\right) + \beta E_t W\left(v_t, v_{t+1}, u_{t+1}\right)$$

   Hence it could be included in function f as in (1).

3. Then Ramsey optimal equilibrium is a allocation that maximize the following Lagrangian problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U\left(v_t, u_t\right) + \underbrace{\lambda_t'}_{1\times N-1} E_t \underbrace{f\left(v_{t-1}, v_t, v_{t+1}, u_t, u_{t+1}\right)}_{N-1\times N} \right\}$$

4. The Ramsey optimal problem characterized by $2N-1$ equilibrium equations and $2N-1$ endogenous variables: The N first order conditions w.r.t. N endogenous variables:

$$\begin{aligned} 0 = \frac{\partial L}{\partial v_t} &= \underbrace{U_1\left(v_t, u_t\right)}_{1\times N} + \underbrace{\lambda_t'}_{1\times N-1} E_t \underbrace{f_2\left(v_{t-1}, v_t, v_{t+1}, u_t, u_{t+1}\right)}_{N-1\times N} \\ &+ \beta^{-1} \underbrace{\lambda_{t-1}'}_{1\times N-1} E_t \underbrace{f_3\left(v_{t-1}, v_t, v_{t+1}, u_t, u_{t+1}\right)}_{N-1\times N} \\ &+ \beta \underbrace{\lambda_{t+1}'}_{1\times N-1} E_t \underbrace{f_1\left(v_{t-1}, v_t, v_{t+1}, u_t, u_{t+1}\right)}_{N-1\times N} \end{aligned} \tag{2}$$

and original equilibrium conditions ($\# = N-1$),$E_t f\left(v_{t-1}, v_t, v_{t+1}, u_t, u_{t+1}\right) = 0$. The $2N-1$ endogenous variables are: $v_t(\# = N)$ and multipliers $\lambda_t\ (\# = N-1)$.

1.2 How can we solve it?

We will solve these equilibrium conditions by first or second order perturbation using Dynare.

To apply the perturbation method, we require the non-stochastic steady state values of v. We compute the steady state values in two steps.

1. Steady states of N endogenous variables v_t: Fix one of the endogenous variable, for example inflation rate π or nominal rate of interest (the instrument variable(s)). We solve for the remaining $N-1$ endogenous variables by imposing the $N-1$ equations, (1).

2. Steady states of $N-1$ Lagrangian multipliers: Let's consider the static version or steady state version of (2).

$$0 = U_1 + \lambda'\left(f_2 + \beta^{-1} f_3 + \beta f_1\right)$$

where a function without an explicit argument is understood to mean that it is evaluated in steady state. Write

$$Y = U_1'$$

$$X = \left(f_2 + \beta^{-1} f_3 + \beta f_1\right)'$$

$$\beta_c = \lambda$$

By construction, Y is an $N \times 1$ column vector, X is an $N \times N-1$ matrix and β_c is an $N-1 \times 1$ column vector. As the OLS in the Econometric,

$$\beta_c = \left(X'X\right)^{-1} X'Y$$

$$u = Y - X\beta_c$$

Note that this 'regression' will not in general fit perfectly, because there are $N-1$ explanatory variables and N elements of Y to 'explain'. We vary the value of π until $\max|u| = 0$. This complete the discussion of the calculation of the steady state.

2 Rotemberg Sticky-Price Model

2.1 The Model[1]

Household preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log \left(C_t \right) - \frac{\chi}{2} h_t^2 \right)$$

subject to budget constraint:

$$P_t C_t + B_t = \left(1 + R_{t-1}\right) B_{t-1} + W_t h_t + \Pi_t$$

the first order condition w.r.t consumption, labor and bond yields the intra-temporal and intertemporal FOCS:

$$\chi h_t C_t = \frac{W_t}{P_t}$$

$$\frac{1}{1+R_t} = \beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}}, t = 0, 1, 2, ...$$

Final good firms production technology:

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \epsilon \geq 1$$

where ϵ is the elasticity of substitution between different intermediate inputs j. The demand curve for jth intermediate good is

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\epsilon} Y_t$$

jth intermediate good producer maximize the problem

$$\max_{\left\{P_{jl}\right\}_{l=0}^{\infty}} E_0 \sum_{l=0}^{\infty} \beta^t v_{t+l} \left(\left(1+v\right) P_{jt+l} Y_{jt+l} - \overbrace{s_{t+l} P_{t+l} Y_{jt+l}}^{\text{labor costs of production}} \overbrace{\underbrace{- \frac{\phi}{2} \left(\frac{P_{j,t+l}}{P_{j,t+l-1}} - 1 \right)^2 P_{t+l} C_{t+l}}}^{\text{cost of adjusting prices in final good}} \right)$$

[1] Rotemberg J. J. Monopolistic Price Adjustment and Aggregate Output[J]. Review of Economic Studies, 1982(4):517-531.

The first term in the parenthesis is the firm's revenues including a tax subsidy, v received from government which is financed by a lump-sum tax on the household. The tax subsidy is introduced so that we have a option to offset the distortion effects from monopolistic competition in model setting. That is to say that we could choose the value for the tax subsidy so that we could achieve efficient level where there are full price flexibility.

The term after the first minus sign corresponds to the labor costs incurred in producing $Y_{j,t+l}$. The state-contingent value $v_t = \frac{1}{P_t C_t}$ to the households of profits is taken as exogenous by the firm. s_t is the real marginal cost

$$s_t = \frac{\frac{d(\text{wage cost})}{d(\text{labor})}}{\frac{d(\text{output})}{d(\text{labor})}} / P_t = \frac{W_t}{P_t A_t} \overbrace{=}^{\text{household FOC}} \frac{\chi h_t C_t}{A_t}$$

The maximization problem subject to demand curve

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\epsilon} Y_t.$$

The FOC w.r.t P_{jt}:

$$v_t \left((1+v)(1-\epsilon) + \epsilon s_t \frac{P_t}{P_{jt}} \right) Y_{jt} - \phi \left(\frac{P_{jt}}{P_{jt-1}} - 1 \right) \frac{1}{P_{jt-1}} + \beta E_t \phi \left(\frac{P_{jt+1}}{P_{jt}} - 1 \right) \frac{P_{jt+1}}{P_{jt}^2} = 0$$

which can be further simplified as

$$\overbrace{(1+v)\frac{P_{jt}}{P_t}}^{\text{real price earned}}$$

$$= \overbrace{\frac{\epsilon}{\epsilon - 1}}^{\text{markup}} \times \overbrace{s_t}^{\text{real marginal cost}}$$

$$+ \frac{1}{\epsilon - 1} \phi \left(\frac{P_{jt}}{P_t}\right)^{\epsilon} \frac{C_t}{Y_t} \left[\left(\overbrace{- \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right) \frac{P_{j,t}}{P_{j,t-1}}}^{\text{a rise in today price will rise not that much since this term is negative}} \right) \right.$$

$$\left. + \beta E_t \overbrace{\left(\frac{P_{j,t+1}}{P_{j,t}} - 1 \right) \frac{P_{j,t+1}}{P_{j,t}}}^{\text{a rise in tomorrow price will rise much more sice this term is positive}} \right]$$

When there are no adjustment cost, i.e., $\phi = 0$, we have the classical result: price equals markup over marginal cost. And Let's us impose the restriction that all firm charge the same price which equal to overall price level, i.e. in symmetric equilibrium, $P_{jt} = P_t$ for all j, the price setting equation reads as:i

$$\frac{1}{\phi}\left(\left(1+v\right)\left(\epsilon-1\right)-\epsilon s_t\right)\frac{Y_t}{C_t} = -\left(\pi_t - 1\right)\pi_t + \beta E_t\left(\pi_{t+1}-1\right)\pi_{t+1} \qquad (3)$$

If we log-linearized on the efficient steady states ($\pi_t = 1$,$(1+v) = \frac{\epsilon}{\epsilon-1}$), the above pricing equation can be written as

$$\hat{\pi}_t = \frac{\epsilon}{\phi}s_t + \beta E_t \hat{\pi}_{t+1}$$

Note that this is the Rotemberg version of NKPC with different interpretation of the slope on the real marginal cost.

The production function

$$Y_{jt} = A_t h_{jt}$$

where A_t is a standard technology shock whose log follows a AR(1) process whose persistence is $\rho \in [0,1]$.

The resource constraint

$$C_t\left(1+\frac{\phi}{2}\left(\pi_t - 1\right)^2\right) = A_t h_t = Y_t$$

where $h_t = \int_0^1 h_{jt}dj$, $Y_t = \int_0^1 Y_{jt}dj$[2] while imposing all price are equal to the overall price level.

2.2 The Equilibrium Conditions

Let's summarize the overall equilibrium conditions as follows for 5 endogenous variables:$A_t, C_t, R_t\pi_t, h_t$. We only have 3 private sector equilibrium conditions plus one exogenous shock process. Absent money policy rule, we do not have enough relations to determine all three variables.

1. households Euler equation

$$\frac{1}{1+R_t} = \beta E_t \frac{C_t}{\pi_{t+1}C_{t+1}}, t = 0, 1, 2, ... \qquad (4)$$

[2] This equation will not contradict with the demand curve.

2. firm efficiency condition for price

$$\begin{aligned}\left[(1+v)(1-\epsilon)+\epsilon\left(\frac{\chi h_t C_t}{A_t}\right)\right]\left(1+\frac{\phi}{2}(\pi_t-1)^2\right)& \\ -\phi(\pi_t-1)\pi_t+\phi\beta E_t(\pi_{t+1}-1)\pi_{t+1} &= 0 \qquad (5)\end{aligned}$$

This condition could be written as

$$\begin{aligned}\left[\left((1+v)-\frac{\epsilon}{\epsilon-1}\right)(1-\epsilon)+\epsilon\left(\frac{\chi h_t C_t}{A_t}-1\right)\right]\left(1+\frac{\phi}{2}(\pi_t-1)^2\right)& \\ -\phi(\pi_t-1)\pi_t+\phi\beta E_t(\pi_{t+1}-1)\pi_{t+1} &= 0\end{aligned}$$

The efficient steady state of real marginal cost is unity, i.e. $s_t = 1$, And this means that $(1+v) = \frac{\epsilon}{\epsilon-1}$, i.e., $v = \frac{1}{\epsilon-1}$. The tax subsidy is chosen so that we offset the negative effect from monopolist competition.

3. The resource constraint

$$C_t\left(1+\frac{\phi}{2}(\pi_t-1)^2\right) = A_t h_t \qquad (6)$$

4. Technology shock

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t \qquad (7)$$

2.3 The Ramsey Optimal Problem

Considering the following Lagrangian representation of Ramsey problem with the equilibrium conditions as the constraints:

$$\begin{aligned}&\max_{\{C_t,h_t,\pi_t,R_t\}} E_0\sum_{t=0}^{\infty}\beta_p^t\left\{\left(\log(C_t)-\frac{\chi}{2}h_t^2\right)\right.\\ &+\lambda_{1t}\left(\frac{1}{1+R_t}-\beta E_t\frac{C_t}{\pi_{t+1}C_{t+1}}\right)\\ &+\lambda_{2t}\left\{\left[\left((1+v)-\frac{\epsilon}{\epsilon-1}\right)(1-\epsilon)+\epsilon\left(\frac{\chi h_t C_t}{A_t}-1\right)\right]\left(1+\frac{\phi}{2}(\pi_t-1)^2\right)\right.\\ &\left.-\phi(\pi_t-1)\pi_t+\phi\beta E_t(\pi_{t+1}-1)\pi_{t+1}\right\}\\ &\left.+\lambda_{3t}\left(C_t\left(1+\frac{\phi}{2}(\pi_t-1)^2\right)-A_t h_t\right)\right\}\end{aligned}$$

where β_p is the planner's discount factor which maybe be the same as the household's discount factor β. [3]

Note that if we we set

$$C_t = A_t h_t$$

$$\pi_t = 1$$

$$h_t = \left(\frac{1}{\chi}\right)^{\frac{1}{2}}$$

$$v = \frac{1}{\epsilon - 1}$$

then (5) and (6) are satisfied. If we let the intertemporal Euler equation (4) define the nominal rate, R_t, then (4) is satisfied too. This is the Ramsey equilibrium because this setting of labor h_t and consumption C_t solves the planner's problem given the process for technology in which there are no losses to price adjustment. With these preferences and technology, you cannot do better than to set $\chi h_t^2 = 1$. Note that $v = \frac{1}{\epsilon-1}$ is critical.This is the efficient. Otherwise, if $v \neq \frac{1}{\epsilon-1}$, then (5) indicates that there must be some deviation from $\pi_t = 1, \chi h_t^2 = 1$ given firm profit maximization problem. So let's verify this assumption.

[3]Generally speaking, we should add one more constraint $R_t \geq 0$. But ignoring this condition hoping that it is non-binding. That is to say that the multiplier associated with this extra constraint will be no different than zero all the time.

2.3.1 Case 1: First best, $v = \frac{1}{\epsilon - 1}$

We conjecture that $\lambda_{1t} = \lambda_{2t} = 0$[4] and then verify the conjecture[5]. Under this conjecture, our Ramsey problem boils down to the following problem:

$$\max_{\{C_t, h_t, \pi_t\}} E_0 \sum_{t=0}^{\infty} \beta_p^t \left\{ \left(\log(C_t) - \frac{\chi}{2} h_t^2 \right) + \lambda_{3t} \left(C_t \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right) - A_t h_t \right) \right\}$$

FOC w.r.t inflation π_t is:

$$\lambda_{3t} C_t \phi (\pi_t - 1) = 0$$

hence $\pi_t \equiv 1$ for any t since $\lambda_{3t} = 0$ or $C_t = 0$ for any t is not a solution at all. The resource constraint now is

$$C_t = A_t h_t$$

Hence, under this condition, the Ramsey problem becomes

$$\max_{\{C_t, h_t\}} E_0 \sum_{t=0}^{\infty} \beta_p^t \left\{ \left(\log(C_t) - \frac{\chi}{2} h_t^2 \right) + \lambda_{3t} (C_t - A_t h_t) \right\}$$

FOC w.r.t C_t, h_t are

$$\frac{1}{C_t} = -\lambda_{3t}$$

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[4] The FOC associated with R_t is:

$$\bullet \quad \lambda_{1t} \frac{1}{1 + R_t} = 0, t = 0, 1, 2, \ldots$$

this must be the case that $\lambda_{1t} = 0$ for all t. This means that the Euler condition is kind of non-binding here. Hence, only $\lambda_{2t} = 0$ for all t is a conjecture here. If we consider the parameter v is choice variable here, then v could always be chosen to enforced the price adjustment equation (5). Thus, the price adjustment equation is non-binding here too.

[5] Just think of the R_t, v are a matter of indifference here. Actually there are some intuitions that can help you to understand. First you need understand that what is a non-binding or binding constraint. Constraints whose change in value do not affect the optimal solution are called non-binding. The shadow price,i.e., the multiplier is the amount associated with a unit change of the particular constraint. Nonbinding constraints have a shadow price zero, while binding constraints typically have shadow prices other than zero. Second, households inter-temporal Euler equation is non-binding from the point of view of maximizing utility, because R_t (a variable of no direct interest in utility) can always be chosen to enforce inter-temporal Euler equation). Hence the same is true with tax subsidy v. The treatment of v is crucial here. If we can solve the model without encounter any contradictions, then the conjecture is verified automatically.

$$-\chi h_t - \lambda_{3t} A_t = 0$$

This will produces

$$h_t = \left(\frac{1}{\chi}\right)^{\frac{1}{2}}$$

The price setting equations will now produces

$$1 + v = \frac{\epsilon}{\epsilon - 1}$$

Let's summarize the Ramsey solution which achieves the first best which has no time inconsistency problem[6]:

$$C_t = A_t h_t$$

$$\pi_t = 1$$

$$h_t = \left(\frac{1}{\chi}\right)^{\frac{1}{2}}$$

$$1 + v = \frac{\epsilon}{\epsilon - 1}$$

$$R_t = \frac{1}{\beta E_t \frac{C_t}{\pi_{t+1} C_{t+1}}} - 1$$

It is easy to show that overall social welfare under Ramsey allocation will achieve the maximized welfare

$$\text{Welf} \equiv E_0 \sum_{t=0}^{\infty} \beta_p^t \left\{ \left(\log (C_t) - \frac{\chi}{2} h_t^2 \right) \right\} = \frac{1}{1 - \beta_p} \left(\log A_0 - \frac{1}{2} \log \chi - \frac{1}{2} \right)$$

If $\chi = 1 = A_0$ and $\beta_p = 0.99$, then welfare will achieve the maximum -50. Any allocations differ from this Ramsey allocation will not achieve this utility. That is to say that this is the most efficient allocation of the economy since there is no distortions appear.

[6] The optimal policy is optimal at current time that may not optimal at tomorrow.

2.3.2 Case 2: $v \neq \frac{1}{\epsilon - 1}$

If consider the case where the tax subsidy v does not satisfy $1+v \neq \frac{\epsilon}{\epsilon-1}$ and fix somewhere else. The FOC associated with λ_{1t} is still non-binding, hence $\lambda_{1t} \equiv 0$ for all t. Let's reconsider the Ramsey problem

$$
\begin{aligned}
&\max_{\{C_t, h_t, \pi_t, R_t\}} E_0 \sum_{i=0}^{\infty} \beta_p^t \left\{ \left(\log(C_t) - \frac{\chi}{2} h_t^2 \right) \right. \\
&+\lambda_{1t} \left(\frac{1}{1+R_t} - \beta E_t \frac{C_t}{\pi_{t+1} C_{t+1}} \right) \\
&+\lambda_{2t} \left\{ \left[(1+v)(1-\epsilon) + \epsilon \left(\frac{\chi h_t C_t}{A_t} \right) \right] \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right) \right. \\
&-\phi (\pi_t - 1) \pi_t + \phi \beta E_t (\pi_{t+1} - 1) \pi_{t+1} \} \\
&\left. +\lambda_{3t} \left(C_t \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right) - A_t h_t \right) \right\}
\end{aligned}
$$

The FOCs w.r.t C_t, h_t, π_t are:

$$
\frac{1}{C_t} + \lambda_{2t} \epsilon \frac{\chi h_t}{A_t} \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right) + \lambda_{3_t} \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right) = 0 \tag{8}
$$

$$
-\chi h_t + \lambda_{2t} \epsilon \frac{\chi C_t}{A_t} \left(1 + \frac{\phi}{2} (\pi_t - 1)^2 \right) - \lambda_{3t} A_t = 0 \tag{9}
$$

$$
\lambda_{2t} \left[\left[(1+v)(1-\epsilon) + \epsilon \left(\frac{\chi h_t C_t}{A_t} \right) \right] \phi (\pi_t - 1) - \phi (2\pi_t - 1) \right] \tag{10}
$$

$$
+\lambda_{2t-1} \beta_p^{-1} \beta \phi (2\pi_t - 1) + \lambda_{3t} C_t \phi (\pi_t - 1) = 0 \tag{11}
$$

Now we have three equilibrium conditions from original model and the three FOCs from Ramsey problem, together six equations for four variables C_t, h_t, π_t, R_t plus two Lagrangian multipliers $\lambda_{2t}, \lambda_{3t}$. The system does not has an analytic solution in this case. It has to be solved numerically.

We can solve the system using Dynare now. But before we can proceed, we need find out the steady states of the six variables. With $\beta_p^{-1}\beta = 1$ and steady state technology be unity, for any given target of inflation π^*, it is very easy to show that the steady state value of labor is

$$
h = \left(\frac{1}{\chi \epsilon} \right)^{\frac{1}{2}} \left(\frac{\phi (1-\beta) \pi^* (\pi^* - 1)}{1 + \frac{\phi}{2} (\pi^* - 1)^2} + (1+v)(\epsilon - 1) \right)^{\frac{1}{2}}
$$

consumption is

$$C = \frac{h}{1 + \frac{\phi}{2}(\pi^* - 1)^2}$$

For a simple life here, let assume that $\pi^* = 1$, then we have[7]

$$\pi = 1, C = h = \left(\frac{\gamma}{\chi}\right)^{\frac{1}{2}}, R = \frac{1}{\beta} - 1, \lambda_2 = \frac{\gamma - 1}{2\gamma\epsilon}, \lambda_3 = -\chi h \frac{\gamma + 1}{2\gamma}$$

where

$$\gamma = \frac{1 + v}{\frac{\epsilon}{\epsilon - 1}} = \frac{\text{subsidy}}{\text{steady state of markup absence of subsidy}}$$

. Then if $\gamma = 1$, i.e., $1 + v = \frac{\epsilon}{\epsilon - 1}$, or $v = \frac{1}{\epsilon - 1}$. Then $\lambda_2 = 0$,$\lambda_3 = -\chi^{\frac{1}{2}}$. If $v \neq \frac{1}{\epsilon - 1}$, then $\lambda_2 \neq 0$. There are few points that deserved mentioned here:

1. Steady state hours and consumption is bigger than first best if subsidy is too high, i.e.,$\gamma > 1$ and smaller in case that the subsidy is too low, i.e., $\gamma < 1$.

2. The sign of λ_2 depends on whether the subsidy is too high (then $\lambda_2 > 0$) or too low (then $\lambda_2 < 0$).

3. $\lambda_3 \leq 0$ as expected, always or more accurately $\lambda_3 < 0$. The Kuhn-Tucker Theorem ensures that.

4. We can verify numerically that $\lambda_{2,t-1}$enters policy rules for hours, consumption and nominal rate. In this sense, the Ramsey problem is not time consistent when $\lambda_2 \neq 0$.[8] This could be seen at the DIY approach in section 2.4.2.

5. Although Ramsey has only one degree of freedom (i.e., an excess of only one variable over equilibrium conditions) and two barriers to first best (i.e., monopoly power and inflation), it chooses to neutralize only inflation (i.e., $\pi = 1$), hence there will be no adjustment cost or there will be no loss in welfare to the price adjustment. That is to say that all resources in constraint contribute to the consumption which is essential to welfare improves.

[7]Using (8), (9) to find the steady states of $\lambda_{2t}, \lambda_{3t}$ if $\pi^* = 1$.

[8]As we will see that $\lambda_{2,t}$ will be a constant i.e., on steady state, since we can see that its standard deviation is zero.

2.4 The Code For Rotemberg Model

There are two ways we can find the solution in Dynare. One is easy way by using ramsey_policy command in Dynare; One is harder way, we call it a DIY way. You do not need use ramsey_policy command and what you need to do is to write all equilibrium conditions including the ones in Ramsey optimal problems in model file and use stoch_simul to solve the system as usual. You have to declare your own Lagrangian multipliers in the harder way. But it does give you more advantages than the easy way since you can understand more by coding and know what is going on behind the Dynare command ramsey_policy. We will try both ways to see whether they reach the same results.

The following codes can be easily adapted to both cases above. You are strongly advised to run the codes by yourself and have a close look for the differences between the two cases.

2.4.1 Using ramsey_policy

This way is the easiest way to find Ramsey policy numerically. What you need do is

1. Input model equilibrium conditions, but not include the FOCs of Ramsey problem. Ramsey_policy command will calcualte all the FOCs of Lagrangian problem automatically and use them as equilibrium conditions. This will greatly relief you out of painful jobs from finding the focs and input them in Dynare. At the same time, Dynare will create Lagrangian multipliers automatically;

2. Define the planner's objective by specify the period utility function;

3. Define planner's discount factor β_p, and you may set $\beta_p = \beta$;

4. Use the ramsey_policy command and provide some options for ramsey_policy as in the mod file.

Let's have a look of the Dynare mod file first when $v = \frac{1}{\epsilon-1}$(case 1):

```
//2015-10-21@Beijing
// 6 endogenous variables including period utility
var R //Nominal interest rate
h       //labor
```

```
pie     //gross inflation
C      //consumption
A      //technology shock
Util   //period utility, planner's objective
;

// Innovations
varexo eps_A;

// PARAMETERS
parameters nbeta chi beta epsil phi rho nu pietarget;
parameters as rs pis hs cs Utils;
beta = 0.99;
epsil = 5;
phi = 100;
rho = 0.9;
nbeta = beta;
nu=1/(epsil-1);
//nu=0; %tax subsidy does not satisfiy: nu=1/(epsil-1);
chi=1;
pietarget=1.;

%technology steady state
as=1;

%nominal rate ss from the intertemporal Euler equation
rs=pietarget/beta-1;

%the inflation target if we set it a monetary policy rule
pis=pietarget;

%steady state labor
hs=((1+nu)*(epsil-1)+phi*(pis-1)*pis*(1-beta)/
(1+(phi/2)*(pis-1)^2))/(epsil*chi);
hs=sqrt(hs);

%consumption and utility s.s.
cs=hs/(1+(phi/2)*(pis-1)^2);
```

```
Utils = log(cs) -chi*hs^2/2;

model;
//(1) home Euler equation--intertemporal condition
1/(1+R) = beta*C/(C(+1)*pie(+1));

//(2) new Keynesian Phillips Curve
((1+nu)*(1-epsil)+epsil*(chi*h*C/A))*(1+phi*(pie-1)^2/2)-phi*(pie-1)*pie
   + beta*phi*(pie(+1)-1)*pie(+1);

//(3) resource constraint
C*(1+(phi/2)*(pie-1)^2)-A*h;

//(4) Technology Shocks
log(A) = rho*log(A(-1))+eps_A;

//(5) utlility function
Util=log(C)-chi*h^2/2;

end;

initval;
Util=Utils;
R=rs;
h=hs;
pie=pis;
C=cs;
A=as;
end;

shocks;
var eps_A;
stderr .01;
end;

//steady;

planner_objective Util;
```

```
ramsey_policy(order=1,irf=20, planner_discount=0.99);
```

This is the output when $v = \frac{1}{\epsilon-1}$:

```
Starting Dynare (version 4.4.1).
Starting preprocessing of the model file ...
Ramsey Problem: added 5 Multipliers.
Found 5 equation(s).
Found 11 FOC equation(s) for Ramsey Problem.
Evaluating expressions...done
Computing static model derivatives:
 - order 1
Computing dynamic model derivatives:
 - order 1
 - order 2
Computing static model derivatives:
 - order 1
 - order 2
Processing outputs ...done
Preprocessing completed.
Starting MATLAB/Octave computing.

MODEL SUMMARY

  Number of variables:         11
  Number of stochastic shocks: 1
  Number of state variables:   4
  Number of jumpers:           3
  Number of static variables:  5


MATRIX OF COVARIANCE OF EXOGENOUS SHOCKS

Variables     eps_A
eps_A        0.000100

POLICY AND TRANSITION FUNCTIONS
              R           h           pie         C           A           Util
Constant      0.010101   1.000000    1.000000    1.000000    1.000000    -0.500000
```

A(-1)	-0.090909	0	0	0.900000	0.900000	0.900000
MULT_1(-1)	-0.464647	0.399666	0.030067	0.399666	0	0
MULT_2(-1)	-1.010090	2.508343	0.498331	2.508343	0	0
eps_A	-0.101010	0	0	1.000000	1.000000	1.000000

THEORETICAL MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
R	0.0101	0.0023	0.0000
h	1.0000	0.0000	0.0000
pie	1.0000	0.0000	0.0000
C	1.0000	0.0229	0.0005
A	1.0000	0.0229	0.0005
Util	-0.5000	0.0229	0.0005

MATRIX OF CORRELATIONS

Variables	R	C	A	Util
R	1.0000	-1.0000	-1.0000	-1.0000
C	-1.0000	1.0000	1.0000	1.0000
A	-1.0000	1.0000	1.0000	1.0000
Util	-1.0000	1.0000	1.0000	1.0000

COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
R	0.9000	0.8100	0.7290	0.6561	0.5905
C	0.9000	0.8100	0.7290	0.6561	0.5905
A	0.9000	0.8100	0.7290	0.6561	0.5905
Util	0.9000	0.8100	0.7290	0.6561	0.5905

STEADY-STATE RESULTS:

```
R        0.010101
h        1
pie      1
C        1
A        1
Util     -0.5

Approximated value of planner objective function
    - with initial Lagrange multipliers set to 0: -50
    - with initial Lagrange multipliers set to steady state: -50

Total computing time : 0h00m02s
```

This the part of the output when $v = 0$ (case 2):

```
Configuring Dynare ...
[mex] Generalized QZ.
[mex] Sylvester equation solution.
[mex] Kronecker products.
[mex] Sparse kronecker products.
[mex] Local state space iteration (second order).
[mex] Bytecode evaluation.
[mex] k-order perturbation solver.
[mex] k-order solution simulation.
[mex] Quasi Monte-Carlo sequence (Sobol).
[mex] Markov Switching SBVAR.

Starting Dynare (version 4.4.3).
Starting preprocessing of the model file ...
Ramsey Problem: added 5 Multipliers.
Found 5 equation(s).
Found 11 FOC equation(s) for Ramsey Problem.
Evaluating expressions...done
Computing static model derivatives:
 - order 1
Computing dynamic model derivatives:
 - order 1
 - order 2
Computing static model derivatives:
```

```
 - order 1
 - order 2
Processing outputs ...done
Preprocessing completed.
Starting MATLAB/Octave computing.


MODEL SUMMARY

  Number of variables:         11
  Number of stochastic shocks: 1
  Number of state variables:   4
  Number of jumpers:           3
  Number of static variables:  5


MATRIX OF COVARIANCE OF EXOGENOUS SHOCKS

Variables     eps_A
eps_A       0.000100

POLICY AND TRANSITION FUNCTIONS
                                  R                h                pie
Constant                   0.010101         0.894427         1.000000         0.8944
A(-1)                     -0.090909                0                0         0.8049
MULT_1(-1)                -0.486705         0.392281         0.028171         0.3922
MULT_2(-1)                -0.705597         2.112780         0.454956         2.1127
eps_A                     -0.101010                0                0         0.8944


THEORETICAL MOMENTS

VARIABLE      MEAN    STD. DEV.   VARIANCE
R            0.0101     0.0023     0.0000
h            0.8944     0.0000     0.0000
pie          1.0000     0.0000     0.0000
C            0.8944     0.0205     0.0004
A            1.0000     0.0229     0.0005
```

```
Util          -0.5116     0.0229     0.0005



MATRIX OF CORRELATIONS

Variables       R       C       A     Util
R            1.0000 -1.0000 -1.0000 -1.0000
C           -1.0000  1.0000  1.0000  1.0000
A           -1.0000  1.0000  1.0000  1.0000
Util        -1.0000  1.0000  1.0000  1.0000




COEFFICIENTS OF AUTOCORRELATION

Order      1       2       3       4       5
R        0.9000  0.8100  0.7290  0.6561  0.5905
C        0.9000  0.8100  0.7290  0.6561  0.5905
A        0.9000  0.8100  0.7290  0.6561  0.5905
Util     0.9000  0.8100  0.7290  0.6561  0.5905

STEADY-STATE RESULTS:

R        0.010101
h        0.894427
pie      1
C        0.894427
A        1
Util     -0.511572

Approximated value of planner objective function
    - with initial Lagrange multipliers set to 0: -51.1287
    - with initial Lagrange multipliers set to steady state: -51.1572

Total computing time : 0h00m03s
```

Just focus on the section 'POLICY AND TRANSITION FUNCTIONS'. The first column is the list of the variables appeared in the right-hand side of the policy function in case 1. The 'Constant' stands for the steady state of correspondent variable. 'A(-1)' is the one-period lag of technology shock. 'MULT_1','MULT_2' are two multiplier created by Dynare. Since their one-period lags appear in the FOCs of Ramsey optimal problem, they are believed to be state variables in Dynare and in turn appear in policy function. The last variable is the exogenous technology shock variables.

Since the model is written in level variables, the policy function should be read like this, for example, the nominal rate:

$$R_t = 0.0101 - 0.4867 \times (\lambda_{1t-1} - 0) - 0.7056 \times (\lambda_{2t-1} - 0) + 0.0101\,(A_t - 1)$$

The multipliers are implicitly defined by Dynare and there are not too much information about multipliers displayed in output. Let's us find out all endogenous variables in the model. Command 'M_.endo_names(oo_.dr.order_var,:)' will display all endogenous variables in DR order:

Table 1: All Endogenous Variables in DR order

No.	Name	Steady State
1	R	0.0101
2	h	1
3	Util	-0.5
4	MULT_3	-1
5	MULT_5	-1
6	**A**	**1**
7	**MULT_1**	**0**
8	**MULT_2**	**0**
9	C	1
10	Pie	1
11	MULT_4	-9.1743

There are two more multipliers created by Dynare than we have listed in our note. This is because we have 5 equilibrium conditions (includes law of technology and definition of utility function) in our mod block and Dynare believe them all as the FOCs. And it will create 5 multipliers for each of the equilibrium conditions. The forth and fifth multipliers correspondent to technology shock equation and equation of utility function definition. This two multipliers will not appear in the right-hand side of the policy function along with the multiplier No. 3 which associated with resource constraint.

Since $\lambda_{1t} = \lambda_{2t} = 0$ all the time in the first best, hence nominal rate equation will read like

$$R_t = 0.0101 - 0.0101 \times (A_t - 1)$$

The conjecture will be verified by the following DIY way where the first three multipliers will be displayed in Matlab command window.

2.4.2 DIY

If you want to know more about the background, you can try this way. Now you need to do a little bit more:

1. Find the FOCs of the Ramsey problem by yourself. You can use paper and pencil to find the FOC. The good news is that there are a little bit easy way to go. You can use some specially written codes to help you find out the Ramsey FOCs with Lagrangian multipliers automatically created, such as Andy Levin's[9]. In Appendix, we will show how you can use the Andy Levin's codes to produce the Ramsey FOCs.

2. Define the Lagrangian multipliers and find the steady states of this multiplier;

3. Input focs from both original and Ramsey problem;

4. Use the stoch_simul command as usual;

Here is the Dynare code when $v = \frac{1}{\epsilon - 1}$:

```
//Written By Xiangyang Li 2015-10-21@BJ
// Endogenous variables
var R h pie C A Util Welf lmult1 lmult2 lmult3;
```

[9]Andrew Levin, Lopez-Salido, J.D., 2004. Optimal Monetary Policy with Endogenous Capital Accumulation", manuscript, Federal Reserve Board, and Andrew Levin, Onatski, A., Williams, J., Williams, N., 2005. "Monetary Policy under Uncertainty in Microfounded Macroeconometric Models." In: NBER Macroeconomics Annual 2005,Gertler, M., Rogoff, K., eds. Cambridge, MA: MIT Press

```
// Innovations
varexo eps_A;

// PARAMETERS
parameters nbeta chi beta epsil phi rho alpha nu pietarget;
parameters as rs pis hs cs Utils;
parameters lmult1_SS  lmult2_SS  lmult3_SS;
beta = 0.99;
epsil = 5;
phi = 100;
rho = 0.9;
alpha = 1.5;
nbeta = beta;
//nu=1/(epsil-1); //lmult2 will equal to zero
nu =0;    //lmult2 will not equal to zero
chi=1;
pietarget=1.;

%technology steady state
as=1;
%nominal rate ss from the intertemporal Euler equation
rs=pietarget/beta-1;

%from the monetary policy rule:
pis=pietarget;

hs=(((1+nu)*(epsil-1)+phi*(pietarget-1)*pietarget*(1-beta)/
(1+(phi/2)*(pietarget-1)^2)))/(epsil*chi);

hs=sqrt(hs);
cs=hs/(1+(phi/2)*(pis-1)^2);
Utils = log(cs) -chi*hs^2/2;

% inital values, actually ss under the current parameterization;
lmult1_SS = 0;
lmult2_SS = 0;
lmult3_SS  = -1;
```

```
model;
//(1) period utility
Util=log(C)-chi*h^2/2;

//(2) policymakers' social welfare
Welf=Util + nbeta*Welf(+1);

// Ramsey optimal FOCs
// produced by Andy Levin's codes;
//(3) FOCs w.r.t nomial rate;
-lmult1/(R + 1)^2=0;

//(4) FOCs w.r.t consumption;
lmult3*((phi*(pie - 1)^2)/2 + 1) + 1/C -
(beta*lmult1)/(C(+1)*pie(+1)) + (C(-1)*beta*lmult1(-1))/(C^2*nbeta*pie) +
 (chi*epsil*h*lmult2*((phi*(pie - 1)^2)/2 + 1))/A;

//(5) FOCs w.r.t. labor;
(C*chi*epsil*lmult2*((phi*(pie - 1)^2)/2 + 1))/A - chi*h - A*lmult3;

//(6) FOCs w.r.t. inflation
(lmult2(-1)*(beta*phi*(pie - 1) + beta*phi*pie))/nbeta
- lmult2*(phi*pie + phi*(pie - 1) + (phi*(2*pie - 2)*((epsil - 1)*(nu + 1)
- (C*chi*epsil*h)/A))/2) + (C*lmult3*phi*(2*pie - 2))/2
+ (C(-1)*beta*lmult1(-1))/(C*nbeta*pie^2);

//(7)Original FOCs - Home Euler Equation
1/(1+R) = beta*C/(C(+1)*pie(+1));

//(8)Original FOCs -new Keynesian Phillips Curve
((1+nu)*(1-epsil)+epsil*(chi*h*C/A))*(1+phi*(pie-1)^2/2)-phi*(pie-1)*pie
   + beta*phi*(pie(+1)-1)*pie(+1);

//(9)Original FOCs -resource constraint
C*(1+(phi/2)*(pie-1)^2)-A*h;

//(10)Technology Shocks
log(A) = rho*log(A(-1))+eps_A;
```

```
end;

initval;
R=rs;
h=hs;
pie=pis;
C=cs;
A=1;
lmult1 = lmult1_SS;
lmult2 = lmult2_SS;
lmult3 = lmult3_SS;
Util=log(cs)-chi*hs^2/2;
Welf=log(cs)-chi*hs^2/2/(1-nbeta);
end;

shocks;
var eps_A;
stderr .01;
end;

%steady;

stoch_simul(order=1,irf=20,periods=200) pie h R C A lmult1 lmult2 lmult3 Welf Util
```

Here is part of the output when $v = \frac{1}{\epsilon - 1}$:

POLICY AND TRANSITION FUNCTIONS

	lmult1	lmult2	lmult3	Welf
Constant	0	0	-1.000000	-50.000000
A(-1)	0	0	0.900000	8.256881
lmult1(-1)	0	-0.020067	-0.500000	0
lmult2(-1)	0	0.501669	0	0
eps_A	0	0	1.000000	9.174312

THEORETICAL MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
pie	1.0000	0.0000	0.0000
h	1.0000	0.0000	0.0000
R	0.0101	0.0023	0.0000
C	1.0000	0.0229	0.0005
A	1.0000	0.0229	0.0005
lmult1	0.0000	0.0000	0.0000
lmult2	0.0000	0.0000	0.0000
lmult3	-1.0000	0.0229	0.0005
Welf	-50.0000	0.2105	0.0443

Here is the Dynare output when $v = 0$:

POLICY AND TRANSITION FUNCTIONS

	pie	h	R	C	A
Constant	1.000000	0.894427	0.010101	0.894427	1.0000
A(-1)	0	0	-0.090909	0.804984	0.9000
lmult1(-1)	0.028171	0.392281	-0.486705	0.392281	
lmult2(-1)	0.454956	2.112780	-0.705597	2.112780	
eps_A	0	0	-0.101010	0.894427	1.0000

	lmult1	lmult2	lmult3	Welf	Util
Constant	0	-0.025000	-1.006231	-51.157178	-0.511572
A(-1)	0	0	0.905608	8.256881	0.900000
lmult1(-1)	0	-0.015354	-0.509982	0.070428	0.087717
lmult2(-1)	0	0.590540	0.264098	1.137389	0.472432
eps_A	0	0	1.006231	9.174312	1.000000

THEORETICAL MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
pie	1.0000	0.0000	0.0000

h	0.8944	0.0000	0.0000
R	0.0101	0.0023	0.0000
C	0.8944	0.0205	0.0004
A	1.0000	0.0229	0.0005
lmult1	0.0000	0.0000	0.0000
lmult2	-0.0250	0.0000	0.0000
lmult3	-1.0062	0.0231	0.0005
Welf	-51.1572	0.2105	0.0443
Util	-0.5116	0.0229	0.0005

On the section 'POLICY AND TRANSITION FUNCTIONS', we just focus on the multipliers we have defined by ourselves. All the other part is the same as above.

As expected, λ_{1t} is zero all the time since its coefficients are all zeroes. Though the coefficients are not all zeros, λ_{2t} is zero all the time if start from its steady state which is zero. The output also shows the coefficients on the third multiplier λ_{3t} which is not visible in the first case.

This part of output will change if we change the value of v, for example if we set $v = 0$ in the model file, this will change the steady state of λ_{2t} and in turn it will lead to λ_{2t} not equal to zero.

2.5 Time Inconsistency

Consider the following model parameter values:

$$\beta = 0.99, \epsilon = 5, \phi = 100, \rho = 0.9, v = 0, \chi = 1$$

Note that we have set $v = 0$, that is to say that there is no subsidy on monopolistic competition and distortion will remain there.

The model solution after linearizing the equilibrium conditions about steady states:

$$\begin{aligned}
h_t &= 0.8944 + 2.1128 \times (\lambda_{2,t-1} + 0.025) \\
R_t &= 0.0101 - 0.7056 \times (\lambda_{2,t-1} + 0.025) - 0.1010 \times (A_t - 1) \\
C_t &= 0.8944 + 2.1128 \times (\lambda_{2,t-1} + 0.025) + 0.8944 \times (A_t - 1) \\
\pi_t &= 1 + 0.4550 \times (\lambda_{2,t-1} + 0.025) \\
\lambda_{2,t} &= -0.025 + 0.5905 \times (\lambda_{2,t-1} + 0.025)
\end{aligned}$$

Suppose the system is in a Ramsey steady state, with $A_t = 1$ for all t. Then

- with no deviation from the steady states, we have

$$C_t = h_t = 0.8944, R_t = 0.0101, \pi_t = 1, \lambda_{2t} = -0.025$$

- suppose the monetary authority deviates in period t by restaring the Ramsey program by setting $\lambda_{2,-1} = 0$. The values of the variables in periods t:

$$C_t = h_t = 0.8944 + 2.1127 \times 0.025 = 0.9472 \longrightarrow h_t = 1$$

$$\pi_t = 1 + 0.4550 \times 0.025 = 1.0114$$

$$R_t = 0.0101 - 0.7056 \times 0.025 = -0.0075$$

- The deviation pushes consumption, hours worked and inflation up, and the interest rate down. This pushes employment in the direction of the first best: $h_t = 1$ all the time. And also push the consumption in the direction of first best $C_t = 1$, hence the output also in the direction of the first best.

- The nature of the temptation to deviate under this parameterization corresponds to the famous 'inflation bias' studied by Kydland-Prescott(1977) and Barro-Gordon(1983) and many others. Kydland-Prescott(1977) points that there will be time inconsistency problem in optimal policy for discretionary. Hence there are credibility issues for the monetary policy. The monetary authority will choose the positive inflation to stimulate the economy and improve employment given the short term Philips curve exists. This positive inflation means that there are deviation from steady state inflation. This deviation is called inflation bias.

- in effect, the 'surprise inflation deviation' is the only way monetary policy can address the monopoly power problem. The problem is that if there is a deviation, then credibility breaks down and the Ramsey optimal plan is not implemented. What occurs instead will be very bad.

2.6 Conclusion

- When labor market is treated optimally, policy achieves first best (i.e., it solves both the monopoly power and inflation problems) and there is no time inconsistency problem.

- When labor market not treated optimally, Ramsey optimal policy solves the inflation problem (stabilizing inflation, hence there is no adjustment cost, so that all resources contribute to the welfare and no loss of price adjustment), but does not touch the monopoly power problem. There is a time consistency problem. One interpretation is that surprise deviations from Ramsey policy are the only way to address the monopoly problem.

3 Exogenous Monetary Policy Shock

If we consider an exogenous monetary policy shock, like the following form

$$R_t = \frac{\pi^*}{\beta} - 1 + \alpha(\pi_t - \pi^*)$$

where π^* is the target inflation rate. We set $\pi^* = 1$ in the mod file.

In this section, we write a mod file which a standard RBC model with a simple Taylor rule. This file is located in the 'Exogenous_Monetary_rule' directory named 'Exogenous_Monetary_rule.mod'. In this model

4 Comparison of Two Equilibriums

We compare the two equilibriums: one is the Ramsey equilibrium, the other one is equilibrium with exogenous monetary policy. I have placed the codes in 'IRF' directory. There are three main files in this directory:

1. **IRF_Ramsey_Exogenous_equilibrium.m**: the main file that invokes the following two mod files. In this m file, I will run the two mod files iterating on one parameter: $v = 0$ and $v = \frac{1}{\epsilon - 1}$. I will dynamically save and load the results in the iteration.

2. **Exogenous_Monetary_rule.mod**: mod file with exogenous monetary policy rule.

3. ROTEMBERG_RAMSEY_POLICY.mod: mod file with Ramsey equilibrium.

Here is the m code in IRF_RAMSEY_EXOGENOUS_EQUILIBRIUM.M:

```
%this file will plot the RBC model's IRF from exogenous monetary policy
%rule and Ramsey equilibrium.

clear all;
close all;
clc;

%we iterate on parameter nu
epsilon = 5;
nu_arr = [0, 1/(epsilon-1)];
%run the dynare mod file and store the results;
for ii=1:length(nu_arr)
    nu = nu_arr(ii);
    save parameterfile_irf nu;
    dynare exogenous_monetary_rule noclearall;
    str=['save exogenous_mon_rule',int2str(ii)];
    eval(str);

    dynare Rotemberg_ramsey_policy noclearall;
    str=['save ramsey_policy',int2str(ii)];
    eval(str);
end
%% plot the IRF together
close all;
clear all;

epsilon = 5;
nu_arr = [0, 1/(epsilon-1)];

irfn=40;
T=1:1:irfn;
netinflation=zeros(length(nu_arr),irfn);
hours = zeros(length(nu_arr),irfn);
net_nom_interest =zeros(length(nu_arr),irfn);
```

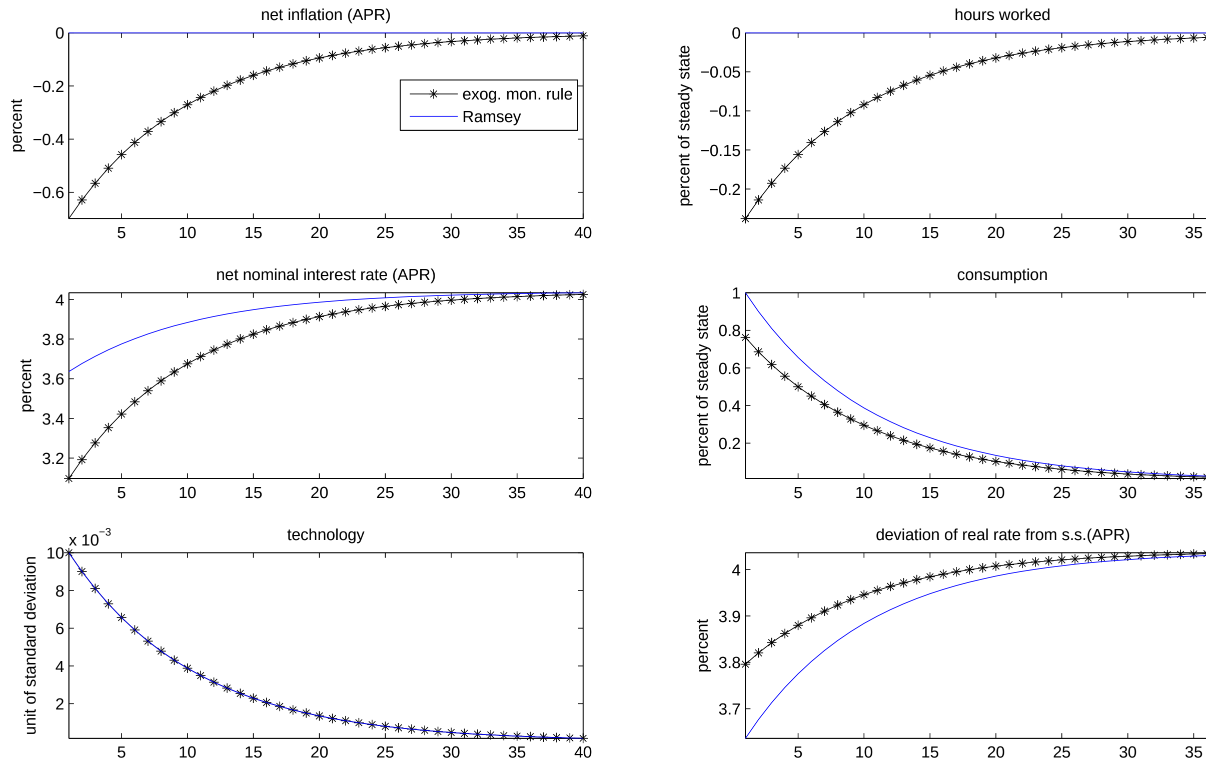
```
consumption = zeros(length(nu_arr),irfn);
technology = zeros(length(nu_arr),irfn);
real_rate =zeros(length(nu_arr),irfn);


for ii=1:length(nu_arr)
    str=['load exogenous_mon_rule',int2str(ii)];
    eval(str);
    netinflation(1,:)=400*(pie_eps_A + pie_SS-1);
    hours(1,:) = 100*h_eps_A/h_SS;
    net_nom_interest(1,:) = 400*(R_eps_A +R_SS);
    consumption(1,:) = 100*C_eps_A/C_SS;
    technology(1,:) = A_eps_A;
    real_rate(1,:) = 400*(R_eps_A +R_SS- pie_eps_A-pie_SS+1);

    str=['load ramsey_policy',int2str(ii)];
    eval(str);
    netinflation(2,:)=400*(pie_eps_A + pie_SS-1);
    hours(2,:) = 100*h_eps_A/h_SS;
    net_nom_interest(2,:) = 400*(R_eps_A +R_SS);
    consumption(2,:) = 100*C_eps_A/C_SS;
    technology(2,:) = A_eps_A;
    real_rate(2,:) = 400*(R_eps_A+R_SS- pie_eps_A -pie_SS +1);

    figure;
    subplot(321);
    plot(T,netinflation(1,:),'k*-',T,netinflation(2,:));
    legend('exog. mon. rule','Ramsey')
    ylabel('percent')
    title('net inflation (APR)')
    axis tight

    subplot(322);
    plot(T,hours(1,:),'k*-',T,hours(2,:));
    ylabel('percent of steady state')
    title('hours worked')
    axis tight
```

```
    subplot(323);
    plot(T,net_nom_interest(1,:),'k*-',T,net_nom_interest(2,:));
    ylabel('percent')
    title('net nominal interest rate (APR)')
    axis tight

    subplot(324);
    plot(T,consumption(1,:),'k*-',T,consumption(2,:));
    ylabel('percent of steady state')
    title('consumption')
    axis tight

    subplot(325);
    plot(T,technology(1,:),'k*-',T,technology(2,:));
    ylabel('unit of standard deviation')
    title('technology')
    axis tight

    subplot(326);
    plot(T,real_rate(1,:),'k*-',T,real_rate(2,:));
    ylabel('percent')
    title('deviation of real rate from s.s.(APR)')
    axis tight
end
```

and here is the IRF under technology shock when $v = 0$[10]:

[10] The IRF of two cases does not differ too much. I only show the first one.

Figure 1: IRF under technology shock of the two equilibriums



5 The Model: Clarida-Gali-Gertler Model

Let's look at a simple New Keynesian Model without Capital[11].This model is a NK model with price stickiness. Hence the model has final and intermediate good firms. The households maximize the lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(C_t\right) - \exp\left(\tau_t\right) \frac{N_t^{1+\psi}}{1+\psi} \right)$$

[11]Clarida, R. and J. Galí, et al. (1999). "The Science of Monetary Policy: A New Keynesian Perspective." Journal of Economic Literature 37 (4): 1661-1707.

which s.t.

$$P_t C_t + B_t \leq R_{t-1} B_{t-1} + W_t N_t + \Pi_t$$

where

$$\tau_t = \lambda \tau_{t-1} + \epsilon_t^\tau, \epsilon_t^\tau \sim \text{i.i.d.}$$

We are not going to layout the model and equilibrium conditions in details. Please refer to original paper for details.

The are some notations that need defined first. Let's define

1. $\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t}$ is relatively price which is believed to be stationary where $\tilde{P}_t$ is the optimal price set by the intermediate good producers;

2. $\pi_t = \frac{P_t}{P_{t-1}}$ is the overall CPI inflation rate.

3. The price distortion $p_t^\star = \left(\frac{P_t^\star}{P_t}\right)^\epsilon$ which is believed to be stationary as the ratio of two price levels, where $P_t^\star \equiv \left(\int_0^1 P_{it}^{-\epsilon} di\right)^{-\frac{1}{\epsilon}}$ and P_{it} is the price index for intermediate goods Y_{it}. And in general, prices levels are believed to be non-stationary.

5.1 The Equilibrium Conditions

We have six equations.

- K_t , F_t

5.1.1 Price Setting

We introduce two auxiliary variables: K_t, F_t which could be written in recursively form:

$$K_t = (1-\nu) \frac{\epsilon}{\epsilon - 1} \frac{\exp(\tau_t) N_t^\psi C_t}{\exp(a_t)} + \beta\theta E_t \pi_{t+1}^\epsilon K_{t+1} \tag{12}$$

$$F_t = 1 + \beta\theta E_t \pi_{t+1}^{\epsilon-1} F_{t+1} \tag{13}$$

5.1.2 Intermediate Goods Firm's Price Adjustment Equation

$$\frac{K_t}{F_t} = \tilde{p}_t = \left(\frac{1-\theta\pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{1-\epsilon}} \tag{14}$$

5.1.3 Price Distortion

$$\frac{1}{p_t^\star} = \left((1-\theta)\left(\frac{1-\theta\pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{1-\epsilon}} + \frac{\theta\pi_t^\epsilon}{p_{t-1}^\star}\right) \tag{15}$$

5.1.4 Household Euler Equation

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\pi_{t+1}} \tag{16}$$

where R_t is the gross nominal interest rate.

5.1.5 Aggregate Resource Constraint

$$C_t = Y_t = p_t^\star \exp(a_t) N_t \tag{17}$$

where aggregate employment $N_t \equiv \int_0^1 N_{it} di$. Hence we have 6 equations and 7 unknowns $p_t^\star, C_t, N_t, \pi_t, F_t, K_t, R_t$.

5.2 The Solution

5.2.1 Equilibrium Conditions

The system is fundamentally under-determined since we do not have the monetary policy rule included. We have 7 variables $p_t^\star, C_t, N_t, \pi_t, F_t, K_t, R_t$, but we only have six equations as mentioned above. There are 2 ways to pin down the variables R_t. One way is to set it in a monetary policy. This other way is to compute the optimal policy. Here we are going to compute the Ramsey optimal policy.

The Ramsey optimal policy is an allocation where the 7 variables will maximize the objective, i.e., the lifetime utility function, which is believed to be the social welfare function. In some cases, objective maybe the preferences of policymakers if we the discount factor β is the policymakers'. There are many configurations that satisfy the six equations. We are looking for the best one (Ramsey optimal) where the value of tax subsidy ν and nominal rate R stand for the optimal policy.

As in the first example, Finding the Ramsey optimal setting of eight variables involves solving a simple Lagrangian optimization problem:

$$\max_{p_t^\star, C_t, N_t, \pi_t, F_t, K_t, R_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t) - \exp(\tau_t) \frac{N_t^{1+\psi}}{1+\psi} \right.$$

$$+\lambda_{1t} \left[\frac{1}{C_t} - E_t \frac{\beta}{C_{t+1}} \frac{R_t}{\pi_{t+1}} \right]$$

$$+\lambda_{2t} \left[\frac{1}{p_t^\star} - \left((1-\theta) \left(\frac{1-\theta(\pi_t)^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon}{\epsilon-1}} + \frac{\theta \pi_t^\epsilon}{p_{t-1}^\star} \right) \right]$$

$$+\lambda_{3t} \left[1 + E_t \beta\theta (\pi_{t+1})^{\epsilon-1} F_{t+1} - F_t \right]$$

$$+\lambda_{4t} \left[(1-\nu) \frac{\epsilon}{\epsilon-1} \frac{C_t \exp(\tau_t) N_t^\psi}{\exp(a_t)} + E_t \beta\theta (\pi_{t+1})^\epsilon K_{t+1} - K_t \right]$$

$$+\lambda_{5t} \left[F_t \left(\frac{1-\theta(\pi_t)^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{\epsilon-1}} - K_t \right]$$

$$\left. +\lambda_{6t} \left[C_t - p_t^\star \exp(a_t) N_t \right] \right\}$$

First substitute out C_t, the consumption everywhere in above Lagrangian problem[12], we have

$$\max_{\nu, p_t^\star, N_t, \pi_t, F_t, K_t, R_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(N_t) + \log(p_t^\star) - \exp(\tau_t) \frac{N_t^{1+\psi}}{1+\psi} \right.$$

$$+\lambda_{1t} \left[\frac{1}{p_t^\star N_t} - E_t \frac{\exp(a_t)\beta}{p_{t+1}^\star \exp(a_{t+1}) N_{t+1}} \frac{R_t}{\pi_{t+1}} \right]$$

$$+\lambda_{2t} \left[\frac{1}{p_t^\star} - \left((1-\theta) \left(\frac{1-\theta(\pi_t)^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon}{\epsilon-1}} + \frac{\theta \pi_t^\epsilon}{p_{t-1}^\star} \right) \right]$$

$$+\lambda_{3t} \left[1 + E_t \beta\theta (\pi_{t+1})^{\epsilon-1} F_{t+1} - F_t \right]$$

$$+\lambda_{4t} \left[(1-\nu) \frac{\epsilon}{\epsilon-1} p_t^\star \exp(\tau_t) N_t^{1+\psi} + E_t \beta\theta (\pi_{t+1})^\epsilon K_{t+1} - K_t \right]$$

$$\left. +\lambda_{5t} \left[F_t \left(\frac{1-\theta(\pi_t)^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{\epsilon-1}} - K_t \right] \right\}$$

[12]Eliminating any linear constraints.

The Kuhn-Tacker Theorem may give us some hint. It easy to show that λ_{1t} must be zero. The nominal rate R_t does not show anywhere except in this constraint. Hence the change of nominal rate is not relevant at all. The intuition here is that if the constraint is binding, and this means that $p_t^\star, N_t$ must be restricted in some way. This will be stupid. Since the change of the nominal rate is irrelevant why we need restrict $p_t^\star, N_t$. Restriction means that You Are Limited and in turn this means that maximization may not achieve because maximization will always achieve in overall space instead of a restricted space.

Let's continue. Before we solve the problem, we make a conjecture:

- Restriction 1,3,4,5 are all nonbinding, i.e., $\lambda_{1t} = \lambda_{3t} = \lambda_{4t} = \lambda_{5t} = 0$; This conjecture is intuitively supported by argument we listed above.
- Optimize w.r.t $p_t^\star, N_t, \pi_t$, while ignoring restrictions 1,3,4,5;
- Solve for ν, F_t, K_t, R_t to satisfy restrictions 1,3,4,5;
- If this can be done, then we verified our conjecture.

Second, Let's look at the simplified version of the problem by imposing above assumption: $\lambda_{1t} = \lambda_{3t} = \lambda_{4t} = \lambda_{5t} = 0$[13],

$$\max_{p_t^\star, N_t, \pi_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(N_t) + \log(p_t^\star) - \exp(\tau_t) \frac{N_t^{1+\psi}}{1+\psi} \right.$$
$$\left. +\lambda_{2t} \left[\frac{1}{p_t^\star} - \left((1-\theta) \left(\frac{1-\theta(\pi_t)^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon}{\epsilon-1}} + \frac{\theta \pi_t^\epsilon}{p_{t-1}^\star} \right) \right] \right\}$$

First order conditions w.r.t $p_t^\star, N_t, \pi_t$, we have

$$p_t^\star + \beta \lambda_{2t+1} \theta \pi_{t+1}^\epsilon = \lambda_{2t}$$

$$N_t = \exp\left(-\frac{\tau_t}{\psi+1} \right) \tag{18}$$

$N_t^{1+\psi} = e^{-\tau_t}$

$$\pi_t = \left(\frac{(p_{t-1}^\star)^{\epsilon-1}}{1-\theta+\theta(p_{t-1}^\star)^{\epsilon-1}} \right)^{\frac{1}{\epsilon-1}} = \left(\theta + (1-\theta)(p_{t-1}^\star)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \tag{19}$$

[13] The planner's discount factor here is set to be the same as agent's one.

Substituting the solution for inflation π_t into the price distortion equation (15), we have price distortion motion:

$$p_t^\star = \left(1 - \theta + \theta\left(p_{t-1}^\star\right)^{\epsilon-1}\right)^{\frac{1}{\epsilon-1}} \tag{20}$$

and Combining Eq.(19) and (20), we have the CPI inflation rate related to the price distortion:

$$\pi_t = \frac{p_{t-1}^\star}{p_t^\star} \tag{21}$$

$= \frac{P_t}{P_{t-1}}$

Now we are going to find solution for ν, R_t [14] to satisfy restrictions 1,3,4,5. Since nominal interest rate is of indifference in utility, we choose it to satisfy restriction 1:

$$\frac{1}{p_t^\star N_t} - E_t \frac{\exp(a_t)\beta}{p_{t+1}^\star \exp(a_{t+1}) N_{t+1}} \frac{R_t}{\pi_{t+1}} = 0$$

$$R_t = \frac{\frac{1}{p_t^\star N_t}}{E_t \frac{\exp(a_t)\beta}{p_{t+1}^\star \exp(a_{t+1}) N_{t+1}\pi_{t+1}}}$$

Then let's examine three price setting equations:

$$\frac{K_t}{F_t} = \tilde{p}_t = \left(\frac{1 - \theta\pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{1-\epsilon}} \tag{22}$$

$$K_t = (1-\nu)\frac{\epsilon}{\epsilon-1} p_t^\star \exp(\tau_t) N_t^{1+\psi} + E_t\beta\theta(\pi_{t+1})^\epsilon K_{t+1} \tag{23}$$

$$F_t = 1 + \beta\theta E_t \pi_{t+1}^{\epsilon-1} F_{t+1} \tag{24}$$

F_{t+1} $\frac{p^\star_{t+1}}{p^\star_t} = \pi^{-1}_{t+1}$

Combining Eq.(19) and (21), Eq.(22) reduces to

$$K_t = F_t p_t^\star \tag{25}$$

Imposing Eq.(25), divide Eq.(23) by $p_t^\star$, and by noticing Eq.(22), and $\exp(\tau_t) N_t^{1+\psi} = 1$ in optimal allocation, We have:

$$F_t = (1-\nu)\frac{\epsilon}{\epsilon-1}\exp(\tau_t) N_t^{1+\psi} + E_t\beta\theta(\pi_{t+1})^{\epsilon-1} F_{t+1}$$

[14] since F_t, K_t do not have too much economic meanings, we do not stress too much.

Imposing Eq(18) and (24), we have

$$1 - \nu = \frac{\epsilon - 1}{\epsilon}$$

Let's summarize the optimal allocation as follows:

$$p_t^\star = \left(1 - \theta + \theta \left(p_{t-1}^\star\right)^{\epsilon-1}\right)^{\frac{1}{\epsilon-1}}$$

$$\pi_t = \frac{p_{t-1}^\star}{p_t^\star}$$

$$1 - \nu = \frac{\epsilon - 1}{\epsilon}$$

$$N_t = \exp\left(-\frac{\tau_t}{\psi + 1}\right)$$

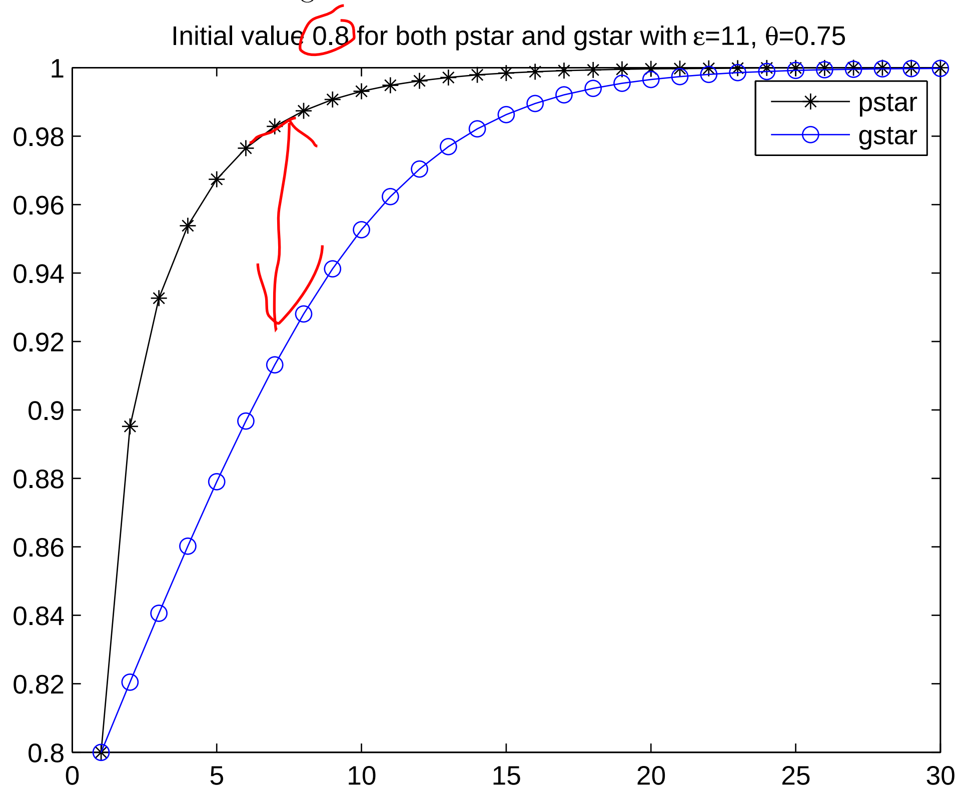
$$C_t = p_t^\star \exp\left(a_t\right) N_t$$

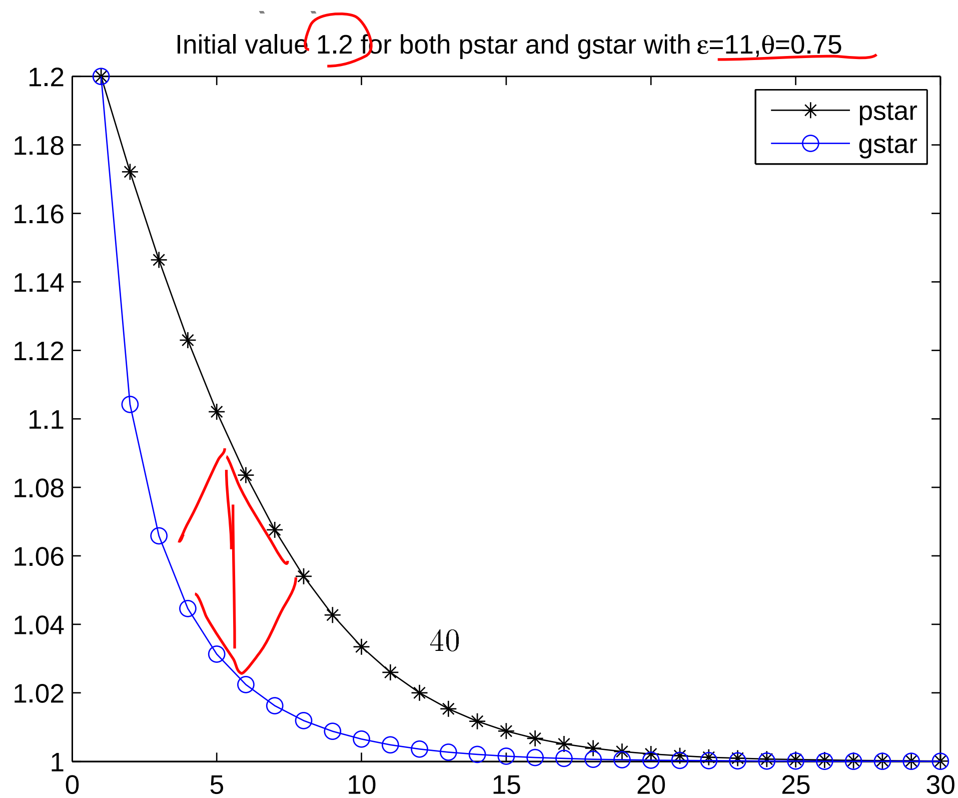
The Ramsey allocations are eventually the best allocations in the economy without price frictions (i.e., 'first best allocations').

5.2.2 Price Evolution Across Time

Let's study one more issue. we plot $p_t^\star$ and $g_t^\star = \left(1 - \theta + \theta \left(g_{t-1}^\star\right)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$ together as in Fig. 2:

Figure 2: Price distortion





We could see that p_t^* converges much faster than g_t^*. This show that eventually, price distortion will be gone since $p_t^\star \rightarrowtail 1$ as time flies. This will be correspondent to flexible price equilibrium where there is the efficient allocation in the economy.

Here is the codes for plotting the two figures:

```
%we show how the price distortion evolution
clear all;
H=30;
pstar = zeros(2,H);
gstar = zeros(2,H);


epsilon=11;
theta=0.75;
pstar(1,1)=0.8;
pstar(2,1)=1.2;
gstar(1,1)=0.8;
gstar(2,1)=1.2;


for ii=2:H
    pstar(1,ii)=(1-theta+theta*pstar(1,ii-1)^(epsilon-1))^(1/((epsilon-1)));
    pstar(2,ii)=(1-theta+theta*pstar(2,ii-1)^(epsilon-1))^(1/((epsilon-1)));
    gstar(1,ii)=(1-theta+theta*gstar(1,ii-1)^(1-epsilon))^(1/((1-epsilon)));
    gstar(2,ii)=(1-theta+theta*gstar(2,ii-1)^(1-epsilon))^(1/((1-epsilon)));
end
T=1:1:H;
close all;
figure;
plot(T,pstar(1,:),'k*-',T,gstar(1,:),'bo-');
title('Initial value 0.8 for both pstar and gstar with \epsilon=11, \theta=0.75');
legend('pstar' ,'gstar')
figure;
plot(T,pstar(2,:),'k*-',T,gstar(2,:),'bo-');
legend('pstar' ,'gstar')
title('Initial value 1.2 for both pstar and gstar with \epsilon=11,\theta=0.75');
```

5.3 Any Codes For CGG model?

The section will left for exercise. You should refer to the abve section to see how to program in Dynare.

6 Appendix: How to Use Andy Levin's Code

6.1 What does Andy Levin's Codes do?

Andy Levin's Codes will help you translate your Dynare mod file into a new one which contains all the Ramsey optimal conditions as well as the equilibrium conditions. All necessary Lagrangian multipliers will be declared automatically. This will great relief your work if you want to study the Ramsey optimal policy of your model. But it will not help you to find the steady states of the declared multipliers. For a not-so-small model, finding the steady states is a not-so-easy job. But for small models, it is a piece of cake[15].

6.2 How Can I use it?

I have prepared a mod file named 'modelin.mod' in the Andy Levin's Codes directory. This modelin.mod file should place at the Andy Levin's code directory. This mod file is the usually file which contains a monetary policy. Read the 'readme.m' or readme.doc for detailed instruction to write your own modelin.mod file. Then Use the following command in Command window to create a new mod file named 'modelout.mod'. This newly-created mod file will contain all equilibrium conditions and FOCs from Lagrangian problem with multipliers.

```
>> infilename = 'modelin';
>> outfilename='modelout';
>> get_ramsey
// Monetary Policy Rule
Note: 1 policy rules declared in model file
```

[15]Dynare has a command that does similar issue. This command is ramsey_model. See Dynare reference manual for more details.