Mandelman and Waddle (2020)

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1 Setting

- Based on Ghironi and Melits (2005) and Holmes et al. (2015)
- two-country, differ in productivity, tech capital production ability, protection of intellectual property rights
- Home more developed, two representative agents: Entrepreneurs produce tech capital, HH supply labor and own firms.
- Foreign less developed, only HH supply labor and own two firms: licensed and appropriating firms. only licensed firms can do trade.
- Licensed entrants buy tech capital from entrepreneurs and pay royalty fee as sunk-cost to use it.
- Appropriating firms replicate goods with higher cost (identical to lower quality proved by Holmes et al. (2015))
- Policy instruments: tariffs and intellectual property protection

2 Consumption

- HH utility: $C_t = (\int_{\omega \in \Omega_t} c_t(\omega)^{\frac{\theta-1}{\theta}} d\omega)^{\frac{\theta}{\theta-1}}$
- $\theta > 1$: elasticity of substitution between goods, ω: variety of goods, one firm one good, Ω_t: continuum of goods at time t
- CPI index: $P_t = (\int_{\omega \in \Omega_t} p_t(\omega)^{1-\theta} d\omega)^{\frac{1}{\theta-1}}$
- $p_t(\omega)$: price of good ω at time t
- Aggregate consumption: $C_t = C_{h,t} + C_{e,t}$

3 Entrepreneurs

- $C_{e,t} = \mathbb{E}_t \sum\limits_{s=t}^{\infty} \beta^{s-t} \frac{C_{e,s}^{1-\gamma}}{1-\gamma}$
- γ: elasticity of inter-temporal substitution
- $L_{e,t} \equiv L_t \equiv 1$: labor supply and do not receive wage
- technology capital: $M_t = X_t + (1 - \delta_M) M_{t-1}$
- X_t: investment in new technology, δ_M: depreciation rate

- royalty fee: $R_t M_{t-1}$

3.1 Deployment of technology in Foreign

- appropriated tech capital by foreign: $h(q_t) = \Theta_t[q_t \exp(-\eta(1-q_t))]$
- Θ_t: exogenous changes in policy, q_t: intensity of tech capital deployed, or fraction of capital rented
- foreign tech capital law of motion $M_t^* = X_t + (1-\delta_M)(1-h(q_t))M_{t-1}^*$

3.2 Optimality

Entrepreneur maximization problem:

$$\max_{C_{e,t},X_t,q_t} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_{e,s}^{1-\gamma}}{1-\gamma}$$

s.t.

$$\begin{aligned} C_{e,t} + X_t &= R_t M_{t-1} N_{E,t} + \mathbb{Q}_t R_t^* q_t M_{t-1}^* N_{E,t}^* + \Pi_{e,t} \\ M_t^* &= X_t + (1-\delta_M)(1-h(q_t))M_{t-1}^* \\ M_t &= X_t + (1-\delta_M)M_{t-1} \end{aligned}$$

where $\mathbb{Q}_t = \varepsilon_t P_t^*/P_t$ is the real exchange rate, $N_{E,t}$ is the number of prospective entrants in Home, $N_{E,t}^*$ is the number of prospective entrants in Foreign, $\Pi_{e,t}$ is the lump-sum transfer of tariffs.

Lagrangian reads:

$$\begin{aligned} &L = \mathbb{E}_t \sum_{s=t}^{\infty} \beta_{s-t} (C_{e,s}^{1-\gamma}/(1-\gamma) \\ &+ \lambda_s^* \left(X_s + (1-\delta_m)(1-hq_t)M_{s-1}^* - M_s^*\right) \\ &+ \lambda_s \left(X_s + (1-\delta_m) M_{s-1} - M_s\right) \\ &+ \mu_s \left(C_{e,s} + X_s - R_s M_{s-1} N_{E,s} - Q_s R_t^* q_s M_{s-1}^* N_{E,s}^* - \Pi_{e,s}\right)) \end{aligned} \tag{1}$$

FOCs read:

$$\begin{aligned} &\frac{\partial}{\partial C_{e,t}} = C_{e,t}^{-\gamma} + \mu_t = 0 \\ &\frac{\partial}{\partial x_t} = \lambda_t^* + \lambda_t + \mu_t = 0 \\ &\frac{\partial}{\partial M_t} = -\lambda_t + \beta \mathbb{E}_t \left(\lambda_{t+1}(1-\delta_m) - \mu_{t+1} R_{t+1} N_{E,t+1}\right) = 0 \\ &\frac{\partial}{\partial M_t^*} = -\lambda_t^* + \beta_t \mathbb{E} \left(\lambda_{t+1}^* (1-\delta_M)(1-h(q_{t+1})) - \mu_{t+1} \mathbb{Q}_{t+1} R_{t+1}^* q_{t+1} N_{E,t+1}^*\right) = 0 \\ &\frac{\partial}{\partial q_t} = \lambda_t^* (1-\delta_m) h'(q_t) M_{t-1}^* - \mu_t \mathbb{Q}_t R_t^* M_{t-1}^* N_{E,t}^* = 0 \end{aligned} \tag{2}$$

Gives equilibrium conditions:

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\begin{aligned}
&C_{e,t}^{-\gamma} = -\left(\lambda_t^{*} + \lambda_t\right) \\
&\lambda_t = \rho \mathbb{E}_t \left(C_{e,t+1}^{-\nu} R_{t+1} N_{t,t+1} + \lambda_{t+1} \left(1 - \delta_m\right)\right. \\
&\lambda \hat{i} = \beta E_t \left(C_{,t+1}^{-\nu} Q_{t+1} R_{t+1}^{*} q_{t+1} N_{E,t+1}^{*} + \lambda_{t+1}^{*} \left(1 - \partial_m\right) \left(1 - h\left(q_{t+1}\right)\right)\right) \\
&Ce_{e,t}^{-\nu} Q_t R_t^{*} M_{t-1}^{*} N_{E,t}^{+} = -\nu_t^{t} \left(1 - J_m\right) \otimes h'\left(q_t\right) \phi M_{t-1}^{+}. \\
&h'\left(q_t\right) = \left(x_t \exp\left(-\eta\left(1 - q_t\right)\right) + \left(q_{+} q_t \exp\left(-\eta\left(1 - q_t\right)\right) \cdot \eta\right.\right. \\
&\;= \theta_t \left(1 + \eta q_t\right) \exp\left(-\eta\left(1 - q_t\right)\right.
\end{aligned}
\tag{3}
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Bibliography

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