Perturbation and Dynare

IRFs and Simulation

Tools for Macroeconomists: The essentials

Petr Sedláček

Impulse response functions

Effect of a one-time, one-standard-deviation, shock

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Effect of a one-time, one-standard-deviation, shock

Consider a “baseline”

Effect of a one-time, one-standard-deviation, shock

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- take as given k_0, z_0 and a series of exogenous shocks $\{\epsilon_t\}_{t=1}^{T}$
- corresponding solution (associated time series) for capital $\{k_t\}_{t=1}^{T}$

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$IRF_j^k(\sigma) = k_{\tau+j}^* - k_{\tau+j}$ for $j \geq 0$

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- state of the economy, i.e. particular realizations of $\{\epsilon_t\}_{t=1}^{T}$
  - size of shocks
- therefore, you're free to pick the most convenient point

Doing IRFs yourself: Dynare provides you with decision rules, e.g.

$$k_t = \overline{k} + a_{kk}(k_{t-1} - \overline{k}) + a_{kz}(z_{t-1} - \bar{z}) + a_{k\epsilon}\epsilon_t$$

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- can consider other interesting starting points(e.g. booms/recessions)
- can simulate economy and compute IRF at each point (IRF will be a band)

Simulation

Simulation and relating to data

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Same procedure for higher-order perturbation solutions, but may be explode

- look into the extensions for pruning

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        - can repeat multiple times (e.g. $1,000$), compute st. errors of statistics

Taking stock

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Impulse response functions

- response of model to a one-time, one-standard deviation, shock
- relative to a baseline scenario (can be steady state for linear solutions)
- higher-order approximations – range of IRFs

Simulation

- same principle as IRFs, just with a sequence of shocks
- higher-order approximations – careful with explosive behavior


That's all Folks!