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INTERNATIONAL TECHNOLOGY DIFFUSION: THEORY AND MEASUREMENT*

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We model the invention of new technologies and their diffusion across countries. In our model all countries grow at the same steady-state rate, with each country's productivity ranking determined by how rapidly it adopts ideas. Research effort is determined by how much ideas earn at home and abroad. Patents affect the return to ideas. We relate the decision to patent an invention internationally to the cost of patenting in a country and to the expected value of patent protection in that country. We can thus infer the direction and magnitude of the international diffusion of technology from data on international patenting, productivity, and research. We fit the model to data from the five leading research economies. A rough summary of our findings is that the world lies about two-thirds of the way from an extreme of technological autarky to an extreme of free trade in ideas. Research performed abroad is about two-thirds as potent as domestic research. Together the United States and Japan drive at least two-thirds of the growth in each of the countries in our sample.

1. INTRODUCTION

What is the geographic scope of technical progress? One camp holds that by its very nature technology is freely available everywhere. A questionable implication is that countries enjoy no relative advantage from being innovative.[2] At the other extreme, the new growth theory typically relates a country's technical advances to only its own innovations. A troubling implication here is that innovative countries leave everyone else behind.[3] In contrast to either polar position, economic historians describe world growth in terms of the gradual diffusion of advances from a small set of innovators.

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[2] A corollary is that differences in worker productivity across countries must result from differences in capital per worker, where capital may be construed very broadly to include human capital. Mankiw, Romer, and Weil (1992) take this stand.

[3] See, for example, Romer (1990) and Agrion and Howitt (1992).

Innovative countries are the most productive, but their innovations also drive growth elsewhere.[4]

We develop, estimate, and simulate a model of world growth driven by innovation. In the spirit of new growth theory, innovation is endogenous.[5] However, in the spirit of historical accounts, we allow for much richer interdependence among different countries' inventive activity and growth. Our microfoundation of innovation leads naturally to equations for productivity dynamics among countries that are simultaneously inventing and adopting each others' inventions.[6] In steady-state countries all grow at the same rate, with countries that are quicker to adopt innovations taking the lead.[7] Together, the ingredients of our model provide an explanation of why research occurs in different countries, how this research gives rise to innovations, and where these innovations generate increases in productivity.

Reflecting two strands of the new growth literature, we build two versions of the model. In one, the world growth rate is fully endogenous, reacting to research incentives and the extent of diffusion.[8] In the second, the growth rate is only "semiendogenous" and is not influenced by factors affecting research and diffusion.[9]

A fundamental problem in trying to quantify a model with richer and more realistic diffusion patterns is identifying who is getting what from whom. We exploit international patenting to overcome this problem. The international patent system requires inventors to patent in each country where they seek protection from imitators.[10] Our model incorporates an inventor's choice of where to patent an idea, relating the decision to seek patent protection in a particular country in part to the likelihood that it will be used there. This feature of the model allows us to use data on international

[4] Gerschenkron (1962) interprets the comparative experiences of different European countries and Japan during their industrial revolutions in terms of the diffusion of technology. Rosenberg (1982) argues for both the historical importance and practical difficulty of diffusion. "... the transfer of technology has never been easy. Typically high levels of skill and competence are needed in the recipient country" (p. 247). He attributes Britain's decline in relative productivity over the past hundred years to its slowness in "exploiting the new industries that began to emerge in the late nineteenth and earlier twentieth centuries, and that are sometimes referred to collectively as the 'second industrial revolution' " (p. 256). On this same point Lewis (1957) argues that "Britain would have done well enough if she merely imitated German and American innovations. Japan, Belgium, and Swizerland owe more of their success as exporters of manufactures to imitation than they do to innovation."

[5] We build on existing theoretical models of research and economic growth. The last decade saw a spurt of activity on this topic, while Phelps (1966) and Shell (1966) provide earlier contributions.

[6] In particular, we incorporate Kortum's (1997) search model of how research generates innovation.

[7] Several theoretical models of economic growth incorporate diffusion. Nelson and Phelps (1966) provide an early contribution and Parente and Prescott (1994) a recent one. Fagerberg (1994), Grossman and Helpman (1995), and Jovanovic (1996) survey subsequent contributions. These models have been too stylyzed to provide a framework for quantitative analysis in a multicountry setting. In particular, they typically assume a single innovating country. Extending them to a world in which a group of innovators draws on each others' ideas is not straightforward.

[8] Grossman and Helpman (1991a), for example, develop several versions of multicountry growth models in this spirit.

[9] Jones (1995) develops a single-economy version of such a model and finds that it explains the postwar U.S. experience better than a model with fully endogenous growth.

[10] Penrose (1951) provides the classic discussion of the international patenting system. Griliches (1990) surveys the literature on using patents as indicators of inventive activity.

patenting as evidence of who is coming up with ideas and where those ideas are more likely to be used.[11]

Where previous studies provide compelling evidence, we use them to fix key parameters of the model. We estimate the rest by fitting the steady state of our model to research employment, productivity levels, and international patenting among the five leading research economies—the United States, Japan, Germany, the United Kingdom, and France—in the late 1980s.[12] Employment of researchers provides a measure of innovative effort, while productivity reflects countries' use of these innovations. International patenting reflects the link between the sources and uses of innovations.[13]

Inventors patent much more at home than abroad. This propensity could reflect technological immobility, but it also could mean that patents provide much less protection abroad. The patent data themselves cannot sort this out, but these competing explanations have different implications for the distribution of productivity. Since our analysis also takes into account data on productivity and research effort, we can distinguish between the two. We infer that much of the home bias derives from lesser incremental protection provided by foreign patents. Otherwise, the model could not explain the tight distribution of productivity levels relative to the skewness of research effort.

Indeed, our estimates do suggest substantial, but not perfect, sharing of ideas. Relative to the adoption of their own potentially useful ideas, countries generally adopt from one half to three-fourths of those generated abroad. Another way to quantify the extent of diffusion is to decompose each country's growth into the contribution made by its own and others' innovation. We find that the United States and Japan together contribute two-thirds or more to growth in each of the five countries, and only the United States derives most of its growth from its own innovation.

Do inventors' earnings reflect the pervasiveness of diffusion? Not quite. We also decompose earnings from innovation in each country into what emanates from each of the five. We find that inventors in every country earn more from their inventions at home than in any other single country. The U.S. market is the source of about one-fourth of the returns to invention in each of the other four.[14]

[11] Coe and Helpman (1995) confront this issue by equating the direction of technology diffusion to exports. Keller (1998b), however, raises serious doubts about the connection between trade and diffusion. We think that patenting abroad is a much more direct, albeit imperfect, indicator of where ideas are going.

[12] Why do we fit our model only to this one snapshot of the data? By the late 1980s these five countries appear to have converged to roughly similar growth rates. To explain the much faster growth of Japan, Germany, and France in the earlier decades would require us to appeal to the much more unwieldy out-of-steady-state predictions of the model. Eaton and Kortum (1997a) take a first step in that direction.

[13] Others have looked at the determinants of subsets of these variables in partial equilibrium. For example, Coe and Helpman (1995), Park (1995), Keller (1998a), and Eaton and Kortum (1996) attempt to link productivity to research in different countries, taking research as given. These analyses cannot be used to infer the effect of changes in policies or parameters on research effort and productivity, as we do here.

[14] While we find that inventions earn most at home, our analysis implies that only the best and hence most valuable inventions are patented in many countries. This result is consistent with the findings of Harhoff, Scherer, and Vopel (1997) and Putnam (1997).

Putting our general equilibrium approach to work, we simulate the effects of various counterfactuals and policies. While the two versions (fully and semiendogenous growth) of the model produce very similar estimation results, they have quite different implications for simulation.

Using the model and parameter estimates, we can imagine a world in which diffusion patterns are more extreme. We find, for example, that cutting off the United States from the rest of the world would cause its productivity to fall far behind the other four. Going in the opposite direction, removing the impediments to diffusion created by national borders would not only shrink productivity differences, it also would raise productivity overall. With growth fully endogenous, world growth rises by 1 percentage point, whereas, in the case of semiendogenous growth, U.S. productivity rises by 40 percent. The overall level of research effort also rises substantially.

What about the role of patent policy? Our estimates imply that eliminating protection would reduce steady-state growth, when it is fully endogenous, by about 0.1 percentage point and, when it is only semiendogenous, by about 6 percent across the board. In either case there are substantial reductions in research effort in every country. Going to the other extreme, eliminating any imitation of patented ideas would raise growth, when it is fully endogenous, by 0.7 percentage point. With growth semiendogenous, productivity rises by about 40 percent. In both cases research rises by an order of magnitude.

We proceed as follows: In Section 2 we describe our data. Section 3 sets forth our model, while Section 4 characterizes its steady state. In Section 5 we describe the estimation procedure. Section 6 explores the implications of our estimates and their sensitivity to different assumptions. Section 7 simulates several counterfactuals, and Section 8 offers some concluding remarks. The Appendix provides a list of symbols and derivations of results.

2. FEATURES OF THE DATA

Our interest is in the generation and sharing of technology among technological leaders. Hence we restrict ourselves to the five leading research nations in the OECD: the United States, Japan, Germany, the United Kingdom, and France. In the late 1980s these countries employed over 80 percent of the OECD's research scientists and engineers (OECD, 1991). Furthermore, each of these five countries obtained between 70 and 80 percent of its foreign patent applications from one of the other four (WIPO, 1990). About 60 percent of the world's gross domestic product (GDP) was produced in these countries (Summers and Heston, 1991). Hence our five countries account for most of the world's inventive activity and a majority of the market for ideas.

Our concern is with the relationships among research and productivity in these countries, and we use patent data as an indicator of the extent to which these countries make use of each others' ideas. Table 1 presents the data. Our task in the model we present below is to capture the basic features of these numbers. We discuss each in turn.

TABLE 1
DATA

	Germany	France	U.K.	Japan	U.S.
Adjusted research employment (thousands)	97	41	74	289	477
Workforce (millions)	29	25	28	61	120
Relative output per hour in manufacturing	0.86	0.91	0.66	0.78	1
Patent applications seeking protection in			By inventors from		
Germany	43342	5130	4739	14553	18849
France	13428	15365	4389	10859	17627
U.K.	13029	4961	24176	12984	19720
Japan	7487	2685	2811	65128	17505
U.S.	13159	5178	6418	32829	83333
Adjusted GDP ($ billions)	1751	1299	921	3662	5876
Application costs ($) to patent in			Paid by inventors from		
Germany	1066	1066	1066	3066	1066
France	992	992	992	3042	992
U.K.	1200	1200	1200	4020	1200
Japan	4772	4772	4772	9590	4772
U.S.	3390	3440	1390	4210	1390

SOURCES: R&D RSE's employed in the business sector in 1988, adjusted by the fraction of business sector R&D financed either by the business sector or from abroad, are from OECD (1991). Workforces are from Summers and Heston (1991). Labor productivity levels are manufacturing value added per hour relative to the United States, in 1990 (van Ark, 1996). Patent applications (1988–1990 average) are from WIPO. Domestic applications in Japan are scaled down by a factor of 4.9 (see text). Adjusted GDP is from IMF (1994) with R&D expenditure from OECD (1991) subtracted. Costs of filing a patent application (including translation and agents fees) are from Helfgott (1993). The cost for a Japanese inventor filing an application in Japan is scaled up by a factor of 4.9 (see text).

2.1. *Research.* The first row of Table 1 shows private-sector employment of research and development (R&D) scientists and engineers in 1988 (in thousands).[15] A striking feature is the skewness of the location of research. Nearly half these researchers work in the United States, and Japan employs more than half the remainder. This skewness suggests significant concentration in the generation of ideas in these two countries, but, of course, with technological diffusion, the adoption of these ideas could be much more equal.

Much of the skewness in absolute research activity reflects the scale of the countries in question. Dividing the number of researchers in each country by its workforce, reported in row 2, greatly reduces the range of variation. The two largest countries also have the largest fraction of their workforce engaged in research, but research intensity in the third largest, Germany, is not far behind. Hence, while we do observe skewness in research activity, we do not observe the extreme specialization predicted by some models of international innovation. Finally, in any country, the incentive to

[15] To eliminate researchers working on defense, we multiply these employment numbers by the fraction of business sector R&D financed by either the business sector or from abroad. All data on research are from OECD (1991).

undertake research at the margin seems to die out at a very low level of activity; no more than 0.5 percent of the workforce in any of these countries is engaged as researchers.[16]

2.2. *Productivity.* The third row in Table 1 reports productivity as measured by manufacturing value added per hour in 1990 relative to the United States. We use manufacturing productivity because this represents the major component of what Griliches (1994) terms the "measurable" sector of the economy. In addition, since our interest is in productivity rather than in living standards, we follow Heston and Summer's (1996) recommendation and use van Ark's (1996) data for this purpose.[17]

Regardless of which productivity measure one uses, a striking feature is that differences in productivity levels are nowhere near differences in research effort. Moreover, national growth rates do not appear to be related to research effort. Various measures of productivity growth for our five countries show growth rates bunching together during the postwar period, independent of research effort.[18]

2.3. *Patents.* The next five rows of Table 1 report patent applications, averaged over 1988 through 1990, by country of application and residence of inventor.[19] Note that applications from inventors abroad comprise a large share of the applications in each country. France obtains more applications from the United States than from its own inventors. However, domestic inventors are the single most important source of applications in each of the others.[20]

Foreign patent applications roughly reflect the scale of research activity in the source country (as opposed, for example, to the intensity of research effort there). The United States is the dominant source of foreign patents in the other four countries, followed either by Japan or (in Europe) by Germany.

[16] Of course, research effort involves more than scientists and engineers. Including technical and clerical workers augments these figures by a factor of about 2.3 (based on OECD numbers from all these countries except the United States, for which they are not available).

[17] Summers and Heston's (1991) data on GDP per worker put the United States much further ahead and Japan in the rear.

[18] This tendency appears in van Ark's (1996) data on worker productivity as tabulated in Eaton and Kortum (1997a). It also appears in data on total factor productivity in manufacturing, as shown in Gordon (1996).

[19] Data are from WIPO (1990). We report applications rather than grants because they are much more comparable across countries. Patent applications in Japan by Japanese inventors have been scaled down by a factor of 4.9. The Japanese apply for over 300,000 patents domestically each year, nearly four times the domestic U.S. number. Using data on the number of claims of invention, Okada (1992) finds that Japanese patents granted to foreigners contain on average 4.9 times as many inventive claims as those granted to Japanese inventors. This extremely low level of claims per patent seems particular to Japanese domestic patents. Tong and Frame (1994) show that U.S. patents granted to U.S. inventors, Japanese inventors, and others all have a similar average number of claims per patent.

[20] Putnam (1997) finds that of inventions that are patented in at least one country, 72 percent are patented only there, while 18 percent are patented in three or more countries. Jaffe and Trajtenberg (1998) report that there is also a strong home bias in patent citations; i.e., inventors disproportionately cite other patents originating in the same country. Interestingly, this home bias tends to disappear (or even reverse) when the cited patent has been around for 20 years or more.

Scale, as reflected in GDP (also reported in Table 1), plays a more mixed role in the choice of foreign destinations for patenting.[21] The United States is usually the largest foreign destination for the others. Japan, however, is the least popular destination. Patent costs, reported in the last five rows of Table 1, provide some explanation. Japan is by far the most expensive destination for foreign inventors.[22]

3. THE MODEL

Our task now is to construct a model that captures the key features of these data and that will enable us to use them to measure the role of innovation and international technology diffusion in growth. Since we seek to capture the relationships, within and between countries, among technology, research, patenting, and productivity, our model is necessarily intricate. Hence we review its components before describing each in detail. Production of output, described in Section 3.1, combines a continuum of inputs of varying quality, which are themselves produced by labor and capital. An alternative activity for workers is doing research to come up with ideas for better inputs. Section 3.2 describes how ideas are produced and how they disseminate. A key variable describing a country at any moment is the stock of ideas that have reached it up to that point. Our assumptions about production and diffusion imply a relationship between this stock of knowledge and the distribution of input qualities (technologies) in the country. Section 3.3 derives this distribution and its dynamics. In Section 3.4 we show how productivity relates to the stock of ideas through the implied distribution of technologies.

We make assumptions about market structure, with implications for pricing and firm profit, which we discuss in Section 3.5. These assumptions also have implications, which we turn to in Section 3.6, about the value of having an innovation adopted in a country, depending on the imitation rate. We can thus infer the return to patenting in a country and relate the decision to patent an idea to the quality of the idea, the speed of diffusion, the cost of patenting, and market size. This we do in Section 3.7. Putting these things together, we then calculate, in Section 3.8, the expected value of an idea, incorporating the optimal patenting decision in each country.

The value of ideas determines the return to doing research, whereas labor productivity determines the opportunity cost of this activity. In Sections 3.9 and 3.10 we relate the two to solve for the equilibrium amount of research effort and the wage.

[21] These GDP figures are constructed to be consistent with the patent fees discussed below. They are GDP for 1992 in local currencies from the IMF (1994), translated into U.S. dollars at the 1992 fourth quarter exchange rates, also from the IMF (1994). The share of GDP spent on R&D, from OECD (1991), is subtracted.

[22] Patenting costs are based on country-specific filing fees, agents fees, and translation costs taken from Helfgott (1993). We scale up the cost of an application for a Japanese inventor in Japan by the factor of 4.9 so that the costs are compatible with the way that we count Japanese domestic patents. We ignore the more complicated fee structure applying to patents through the European Patent Office and complications introduced by patent renewal fees. Helfgott collected the cost of application data from a survey in 1992 and translated all the figures into U.S. dollars using the exchange rate in effect "near the end of 1992." Since we ignore patent renewal fees and the possible cost of disclosure of information in taking out a patent, our measure of the cost of patenting is a lower bound on the true costs.

Throughout, lengthy derivations are moved to the Appendix. The Appendix also contains a list of symbols.

3.1. *Production.* We consider a world consisting of $n = 1, \ldots, N$ countries. Output in country n at time t (Y_{nt}) is produced by combining intermediate inputs subject to a constant-returns-to-scale Cobb-Douglas production function:

$$\ln(Y_{nt}/J) = J^{-1}\int_0^J \ln[Z_{nt}(j)X_{nt}(j)]dj$$

where $X_{nt}(j)$ is the quantity of intermediate input j produced at time t in country n and $Z_{nt}(j)$ is the quality of that input. The range of inputs is fixed over time and the same across countries.[23] Output is homogeneous and tradable across countries, whereas inputs are nontraded.

Each input j is produced by a Cobb-Douglas combination of capital $K(j)$ and labor $L(j)$:

$$X(j) = K(j)^{\phi}L(j)^{1-\phi}$$

where $\phi \in [0, 1]$ is the capital elasticity. We treat this production relationship as the same across inputs, countries, and time. Productivity grows over time as research provides ideas for higher-quality inputs. Imperfect diffusion of these ideas leads to cross-country differences in productivity.

3.2. *Ideas.* An idea, our basic unit of research output, is the result of research effort. While all workers in each country are equally productive making intermediates, they differ in their talent for research. We assume that workers are compensated in proportion to their marginal productivity either as production or as research workers. Hence those who are the most productive at doing research will become researchers.

Consider country i at time t with L_{it} workers. If a fraction s_{it} of these workers are doing research, they create ideas at rate $\alpha_{it}s_{it}^{\beta}L_{it}$. The parameter α_{it} reflects the overall productivity of research effort in country i at time t, whereas β reflects the rate at which research productivity declines as less talented workers become researchers.[24] We consider alternative specifications of α_{it} in our analysis of the steady state of the model.

There are three dimensions to an idea: its quality, its use, and how long it takes to diffuse. For simplicity, we treat each dimension as independent of the other two.

[23] This production structure slightly generalizes that in Grossman and Helpman (1991b), in which our J is equal to 1. A larger value of J means that the market for a given intermediate is smaller.

[24] This specification arises, for example, if a worker's talent as a researcher is drawn from a Pareto distribution (see Phelps, 1966). Since large countries have more to draw from, this specification naturally introduces "scale effects." Given its research intensity s, a larger country invents more. Through the process of diffusion, small countries nevertheless can grow at the same rate as everyone else. In the absence of diffusion, size bias can be eliminated by assuming that inventive output depends only on research intensity. Dinopoulos and Thompson (1996) develop a model along these lines. We find the strong scale effects in patenting compelling evidence for our approach.

An idea's quality Q is a random variable drawn from the cumulative distribution function $F(q)$. The quality of an idea is common to all countries to which it diffuses. We assume a Pareto distribution of qualities, $F(q) = 1 - q^{-\theta}$.[25]

An idea applies to only one out of the continuum of intermediates. The intermediate j to which the idea applies is drawn from the uniform distribution on $[0, J]$.[26]

Ideas do not diffuse immediately. If an idea is discovered at time t in country i, then it diffuses to country n at time $t + \tau_{ni}$, for $n = 1, 2, \ldots, N$. We assume that the diffusion lag to country n, τ_{ni}, has an exponential distribution with parameter ϵ_{ni}, $\Pr[\tau_{ni} \leq x] = 1 - e^{-\epsilon_{ni}x}$. Thus ϵ_{ni} is the speed of diffusion from country i to country n, and ϵ_{ni}^{-1} is the mean diffusion lag.[27]

We distinguish between the concepts of diffusion and adoption. An idea has *diffused* to a country when people there know about it and can in principle make use of it. It is *adopted*, however, only if it is actually used. Hence adoption implies diffusion, but not the opposite. While every idea eventually will diffuse to every other country (if the ϵ_{ni} values are strictly positive), many ideas will never be adopted because they are not as good as ideas already being used.

3.3. *The Technological Frontier.* In equilibrium, only the best available idea for each input in each country is actually used. For each country n, $Z_{nt}(j)$ represents the highest-quality idea yet adopted for input j in country n by time t, i.e., the state of the art. The state of the art across all inputs forms the technological frontier. Consider an idea of quality q for input j discovered somewhere at time t. If the idea diffuses to country n with a lag of τ, then it will be adopted if and only if $q > Z_{nt+\tau}(j)$.

To derive the dynamics of the technological frontier in a given country, we need to know the rate at which ideas were discovered in all countries over all history. Researchers in country i produce a flow of new ideas $\alpha_{it} s_{it}^{\beta} L_{it}$. Let $\dot{\mu}_{nt}$ be the stochastic rate at which ideas diffuse to country n (normalized by the measure of inputs J) from all the research that has been done throughout the world. (The corresponding stock is $\mu_{nt} = \int_{-\infty}^{t} \dot{\mu}_{ns} ds$.) An idea may be the result of domestic research or may arrive from

[25] Bental and Peled (1996) and Kortum (1997) also use the Pareto distribution to characterize the pool of undiscovered techniques from which researchers draw. The Pareto distribution has the convenient feature that if we truncate the distribution of Q at some level z, then the random variable Q/z (≥ 1) inherits the Pareto distribution. Thus, if a new idea is better than current best practice (say z), then the distribution of the inventive step (Q/z) does not depend on the level of the best practice that is surpassed.

[26] The continuum allows us to abstract from randomness in aggregate outcomes. To simplify further, we ignore the possibility that research could be aimed at improving the quality of a specific input.

[27] Although we can allow any correlations among the diffusion lags to different countries, we do assume that these lags are independent of the quality of the idea that is diffusing. While there is greater incentive to learn about a more important idea, learning about it is likely to prove more difficult. Since our data are unlikely to allow us to identify the direction and magnitude of any correlation between an idea's quality and the time it takes to diffuse, we take the simplest course and treat the two as independent. More disaggregated data might allow the identification of any correlation. In Eaton and Kortum (1996) we relate diffusion parameters to country characteristics, finding that diffusion from country i to country n is related negatively to the distance between the countries, positively to the level of human capital (schooling) in country n, and positively to imports of country n from country i.

some other country. It may be the outcome of research performed recently or years before. Integrating over the appropriately weighted past research done in country i and summing over source countries, the flow of ideas diffusing to country n is

$$\dot{\mu}_{nt} = J^{-1} \sum_{i=1}^{N} \epsilon_{ni} \int_{-\infty}^{t} e^{-\epsilon_{ni}(t-s)} \alpha_{is} s_{is}^{\beta} L_{is} ds \tag{1}$$

To summarize the technological frontier, we employ the cumulative distribution function $H_n(z;t)$. It represents, for country n at time t, the fraction of inputs whose state of the art is below z. In the Appendix, we show that

$$H_n(z;t) = e^{-\mu_{nt} z^{-\theta}} \tag{2}$$

Note that the technological frontier depends only on the total stock of ideas μ_{nt}, regardless of when these ideas were adopted for production or where they came from. This feature of the distribution simplifies the analysis drastically.

3.4. *Productivity.* Output is maximized when factors are evenly divided among production of the individual inputs. In this case, total output is

$$A_{nt} K_{nt}^{\phi} [L_{nt}(1 - s_{nt})]^{1-\phi}$$

where total factor productivity, $A_{nt} = \exp\{\int_1^{\infty} \ln(z) dH_n(z;t)\}$, is the geometric mean of the technological frontier. We show in the Appendix that as μ_{nt} becomes large,

$$A_{nt} = e^{\psi/\theta} \mu_{nt}^{1/\theta} \tag{3}$$

where $\psi \approx 0.5772$ is Euler's constant. Thus productivity growth in a country is proportional to the growth in the stock of ideas that have diffused to that country.

The market structure that we assume does not, in fact, imply an even allocation of production workers among inputs, since the markup of price over cost differs across intermediates. Total factor productivity is proportional to A_{nt}, however, as can be seen from equation (11) below. We now discuss the market structure that gives rise to this result.

3.5. *Market Structure.* All producers of intermediates in a country face the same wage w and cost of capital (interest rate plus depreciation) r'. Hence they incur the same unit cost of production $c = (r'/\phi)^{\phi}[w/(1-\phi)]^{1-\phi}$. They will not all set the same price, however.

The reason is that the owner of the state-of-the-art idea for an input in a country competes against the previous state of the art for that input, i.e., the state of the art prevailing at the time of invention. Competition is Bertrand, so the owner of the idea charges the highest price at which the previous state of the art is unprofitable.[28] If

[28] Grossman and Helpman (1991b) make similar assumptions. The production technology implies a unit elastic demand for an individual input given the prices of all other inputs. Hence, to maximize profit, the owner of the invention charges the highest price at which it remains the only seller of that input.

the previous state of the art was z, an owner of an idea of quality $q > z$ will charge $p = (q/z)c$. Since q/z differs across inputs, so does the price.

At the time of invention, the inventor is the owner of the idea in any country to which it might diffuse. Ownership in any country might be stolen, in which case it passes to a local, monopolistic imitator.[29]

3.6. *The Value of an Idea.* Total purchases of the leading-edge version of intermediate j with price $p(j)$ are $Y/[Jp(j)]$, where we use final output as numeraire. Given the pricing equilibrium, the profit to the owner of the right to use a technology of quality q improving on an existing input of quality z is $\pi = [1 - (z/q)]Y/J$ if $q > z$ and zero otherwise.

The owner can earn a profit only after his or her idea has been adopted and only before it has been surpassed by a more advanced technology. Consider, then, the expected profit in country n at time s from an idea of quality q invented at time $t < s$ in country i. The probability of its having diffused there by then is $(1 - e^{-\epsilon_{ni}(s-t)})$. The probability of its not having become obsolete by then is $e^{-(\mu_{ns}-\mu_{nt})q^{-\theta}}$. Finally, we assume that ideas face a hazard ι of imitation, so the probability of its not having been copied by time s is $e^{-\iota(s-t)}$. The expected discounted value of the right to use an idea from country i of quality q in country n, given that the previous state of the art for the relevant input in country n was z, is therefore

$$V_{nit}(z, q) = \int_0^{\infty} \pi_{nt+s}(z, q) e^{-(r+\iota)s}(1 - e^{-\epsilon_{ni}s}) e^{-(\mu_{nt+s}-\mu_{nt})q^{-\theta}} ds \tag{4}$$

if $q \geq z$. Otherwise, the value is zero.

We assume that at the time of invention the researcher knows the quality of his or her idea but not the quality of the competing input in any country. The expected value of an idea of quality q from country i in country n,

$$V_{nit}(q) = \int_1^{q} V_{nit}(z, q) dh_n(z; t) \tag{5}$$

is consequently the basis for the patenting decision.

3.7. *The Decision to Patent.* Our modeling of patenting is guided by findings, from surveys of researchers and analyses of patent renewal data, that imitation rates are extremely high and that patenting reduces, but only modestly, the hazard of imitation.[30] Patents need not provide perfect protection from imitation, nor is imitation necessarily immediate if the inventor fails to patent. While we assume that

[29] We assume that imitation does not lead to a lower price. The more natural and appealing assumption, that competition between imitator and inventor drives the markup to zero, substantially complicates the analysis. To keep the analysis as simple as possible, we also assume that the owner of the right to use an idea (whether inventor or imitator) competes with the state of the art at the time of invention. The owner is thus protected from any inferior invention that surpasses the initial state of the art.

[30] For survey evidence, see Mansfield and Romeo (1980), Mansfield et al. (1981), and Levin et al. (1987). Schankerman and Pakes (1986) find that while the total value of patent rights is substantial, it is small relative to total R&D investment. Hence researchers must be appropriating a substantial return to their inventive activity through other means than patents.

patenting does not affect the speed of diffusion, more rapid diffusion increases the incentive to patent because the rewards will be achieved sooner.

Hence we relate the hazard of imitation to whether the inventor has a patent in that country. We denote the hazard of imitation if the idea was patented as ι^{pat} and if it was not patented as ι^{not}.[31] For a patent to have any value requires, of course, that $\iota^{pat} < \iota^{not}$. If patents provide perfect protection, then $\iota^{pat} = 0$. However, if trade secrets are impossible to keep, then $\iota^{not} = \infty$.

We can distinguish between the expected value of a patented idea $V_{nit}^{pat}(q)$ and an unpatented idea $V_{nit}^{not}(q)$ by evaluating (4) and (5) at imitation rates ι^{pat} and ι^{not}, respectively. A patent is therefore worth $V_{nit}^{pat}(q) - V_{nit}^{not}(q)$. Although the inventor does not know how long it will take his or her idea to diffuse, all patenting decisions must be made up front.[32]

Denoting by f_{nit} the total fees and other costs to an inventor in country i of patenting in country n, the inventor will seek patent protection in that country if $V_{nit}^{pat}(q) - V_{nit}^{not}(q)$ exceeds f_{nit} and not otherwise. The return to patenting rises with the quality of the idea q. Hence the condition

$$V_{nit}^{pat}(q) - V_{nit}^{not}(q) = f_{nit} \tag{6}$$

determines a threshold quality level $\bar{q}_{nit}$ such that ideas of higher quality are patented, while those of lower quality are not.[33] We assume that if an idea is potentially useful in a country, then a patent is granted automatically to the inventor seeking protection.

Since researchers in country i produce ideas at rate $\alpha_{it} s_{it}^{\beta} L_{it}$, the rate at which they patent in country n, P_{nit}, is

$$P_{nit} = \alpha_{it} s_{it}^{\beta} L_{it} (\bar{q}_{nit})^{-\theta} \tag{7}$$

3.8. *The Return to R&D.* The value of an idea of quality q from country i in country n is the maximum of $V_{nit}^{pat}(q) - f_{nit}$ and $V_{nit}^{not}(q)$. The expected value of an idea in that country before its quality is known is therefore the expectation of this amount across all possible values of q, which is

$$V_{nit} = \int_{1}^{\bar{q}_{nit}} V_{nit}^{not}(q)dF(q) + \int_{\bar{q}_{nit}}^{\infty} V_{nit}^{pat}(q)dF(q) - f_{nit}(\bar{q}_{nit})^{-\theta} \tag{8}$$

where $F(q)$ is the Pareto distribution. The expected return to an idea of unknown quality in country i at time t is therefore the sum of its expected returns across

[31] We treat the imitation lag, like the diffusion lag, as independent of the quality of the idea in question. While there is greater incentive to try to imitate a bigger idea, imitating it is probably harder, and the inventor has more reason to prevent imitation. Again, our data are unlikely to allow us to identify any correlation between the importance of an idea and its hazard of imitation. Hence, in the absence of any presumption one way or the other, we treat the two as independent.

[32] Specifically, we do not allow the inventor to wait until the idea is adopted in a country to apply for a patent there. This assumption reflects the requirement of most patent systems that patents be taken out in additional countries within 1 year of the first, or priority, application. We assume that inventors do not delay seeking a priority application.

[33] A possibility, of course, is that the cost of patenting would exceed the benefit for any idea regardless of its quality, in which case patenting would be zero and $\bar{q}_{nit}$ infinite. At the other extreme, if $f_{nit} = 0$, then $\bar{q}_{nit} = 1$, so any idea would be patented.

countries, or

$$V_{it} = \sum_{n=1}^{N} V_{nit} \tag{9}$$

This amount is the expected value of an idea.

3.9. *Equilibrium R&D.* The marginal researcher in country i is indifferent between doing research and making intermediates. Equilibrium in the labor market thus implies that the fraction s_{it} of workers doing research in country i will solve

$$\alpha_{it} \beta V_{it} s_{it}^{\beta-1} = w_{it} \tag{10}$$

where w_{it} is what a researcher would earn making intermediates.

3.10. *Technology, Wages, and Income.* Bertrand competition in intermediates implies that the markup over unit cost, $M(j) \equiv p(j)/c$, is low where the currently adopted input is only marginally better than the input it replaced, while $M(j)$ is large where the current input is a substantial improvement over its predecessor. Since expenditure on each input is the same, more factors are allocated to the production of inputs with low markups. Consequently, productivity is lower than it would be if production workers were equally allocated among inputs. As we show in the Appendix, the markup has a time-invariant distribution depending only on the parameter θ.

Even though producers of different inputs in a country set different markups over unit cost, since they face the same wage and interest rate, they employ the same capital-labor ratio k. Taking into account the distribution of the markup, aggregate output in country i at time t is

$$Y_{it} = \frac{\kappa_1(\theta)}{\kappa_2(\theta)} A_{it} K_{it}^{\phi} [L_{it}(1 - s_{it})]^{1-\phi} = \frac{\kappa_1(\theta)}{\kappa_2(\theta)} A_{it} k_{it}^{\phi} L_{it}(1 - s_{it}) \tag{11}$$

where $k_{it} \equiv K_{it}/[L_{it}(1 - s_{it})]$ is the ratio of capital to production workers, and the constants $\kappa_1(\theta)$ and $\kappa_2(\theta)$ are derived in the Appendix. The wage is

$$w_{it} = (1 - \phi)\kappa_1(\theta) A_{it} k_{it}^{\phi} \tag{12}$$

Since our Cobb-Douglas assumption implies that

$$k_{it} = \frac{\phi}{1 - \phi} \frac{w_{it}}{r'}$$

we obtain

$$Y_{it} = \frac{1}{\kappa_2(\theta)} [\kappa_1(\theta) A_{it} (\phi/r')^{\phi}]^{1/(1-\phi)} L_{it}(1 - s_{it}) \tag{13}$$

and

$$w_{it} = (1 - \phi)[\kappa_1(\theta) A_{it} (\phi/r')^{\phi}]^{1/(1-\phi)} \tag{14}$$

We have now fully specified our model. Given the paths of the workforces L_{it}, patenting costs f_{nit}, and research productivities α_{it}, the state of the world economy at any moment t can be described in terms of the N technology state variables μ_{nt}. Three equations describe the dynamic equilibrium. At each moment individuals decide where to work according to (10) and what ideas to patent according to (6). The labor allocation decisions in turn govern the evolution of the technology state variables according to (1). The economy is in equilibrium when patenting and labor allocation decisions are individually optimal at any moment, given the paths of technology that these decisions will generate. We now turn to the steady state of this three-equation system.

4. THE STEADY STATE

The economy is in steady state when the state variables μ_{nt} grow at a constant common rate that we denote g. In order to obtain a steady-state outcome, we make the following assumptions:

1. Workforces in each country grow at a constant rate $g_L \geq 0$.
2. The relative productivity of researchers in a country is proportional to the relative level of technology there. Research productivity in any country also depends on the world stock of ideas. Specifically, we assume that

$$\alpha_{it} = \alpha\left(\frac{\mu_{it}}{\overline{\mu}_t}\right)\overline{\mu}_t^{\gamma}$$

   where α is a constant term, $\overline{\mu}_t = \sum_{i=1}^{N} \mu_{it}$, and $\gamma \leq 1$. To ensure a steady state with growth, we restrict $\gamma = 1$ for $g_L = 0$ and $\gamma < 1$ for $g_L > 0$.
3. Patenting costs are a constant proportion of output, i.e., $f_{nit} = f_{ni} Y_{nt}$.[34]
4. The interest rate r and cost of capital r' are constant across countries and over time.[35]

Features of a steady state are:

1. A constant fraction s_i of workers in each country works as researchers.
2. Patenting is constant.
3. Total factor productivity grows at rate g/θ, the wage grows at rate $g/[\theta(1-\phi)]$, and total output grows at rate $g_Y = g/[\theta(1-\phi)] + g_L$ in all countries.

[34] If patenting costs did not grow with market size, eventually all new ideas, no matter how bad, would be patented. Since we observe a rate of domestic patenting that is not growing over time (with the exception of Japan and a recent jump in the United States), we find it reasonable to assume that patenting costs have not been falling relative to market size.

[35] For most of the analysis we treat r as a parameter even though in general equilibrium the interest rate is determined by intertemporal preferences as they interact with the growth rate. In the standard case of isoelastic marginal utility, in steady state $r = \rho + \sigma g_Y$, where ρ is the discount factor and σ the elasticity of marginal utility, with these parameters assumed common across countries. For purposes of estimation (and simulation if g_Y is unaffected), r summarizes all we need to know about the parameteres ρ and σ.

4. Conditional on an idea eventually being adopted, the mean lag between invention in country i and adoption in n is $1/(\epsilon_{ni} + g)$.[36]
5. Of the ideas generated by country i that would have been useful to n had they diffused immediately after invention in i, country n eventually adopts a fraction $\epsilon_{ni}/(\epsilon_{ni} + g)$.[37]

If $g_L = 0$ and $\gamma = 1$, our model exhibits (fully) endogenous growth as in the R&D-based growth models of Romer (1990) and Grossman and Helpman (1991a, 1991b). In these models, the existing stock of knowledge raises research productivity to the extent that constant research effort can generate perpetual growth of knowledge. If $\gamma < 1$, then, as the stock of knowledge increases, innovations become harder to find. Perpetual growth of knowledge requires ever increasing research effort. In steady state population growth thus dictates the rate of productivity growth, as in the single-country semiendogenous growth model of Jones (1995).[38]

We begin by describing the steady-state dynamics of the model given how much research is done. We then turn to steady-state patenting. We finish by solving for how much research will be done.

4.1. *Steady-State Relative Productivities and Growth.* Given a constant research intensity s_i in each country, (1) implies that the stock of ideas in each country n grows at a common rate g. These equations can be written in terms of time-invariant variables as

$$g = \frac{\dot{\mu}_n}{\mu_n} = \frac{\alpha}{J} \sum_{i=1}^{N} \frac{\epsilon_{ni}}{\epsilon_{ni} + g} \frac{\mu_i}{\mu_n} s_i^{\beta} \widetilde{L}_i \qquad n = 1, \ldots, N \tag{15}$$

where $\widetilde{L}_i \equiv L_{it} \overline{\mu}_t^{\gamma-1}$ is country i's workforce, scaled by the drag on research productivity imposed by a rising stock of world knowledge in the case of semiendogenous growth ($\gamma < 1$). The $\widetilde{L}_i$ values are constant in either the fully or semiendogenous growth case. With fully endogenous growth, $\gamma = 1$ and L_i is constant, whereas with semiendogenous growth, $g = g_L/(1 - \gamma)$.

Equation (15) decomposes growth in any country into the contribution made by each country, including itself. We can further break down the contribution of any single country into the product of three parts: (1) the generation of ideas in source

[36] This lag differs from the mean diffusion lag $1/\epsilon_{ni}$ because, with growth, ideas that diffuse rapidly are more likely to be adopted, since they are less likely to be obsolete when they arrive.

[37] A larger growth rate relative to the diffusion rate implies that more ideas that might have been useful at the time of invention are obsolete when they finally arrive.

[38] How does the number of existing ideas affect researchers' productivity in developing new ones? On one hand, a larger existing stock of ideas may provide researchers more stimulus for thinking up new ones. On the other, it could mean greater depletion of the stock of possible ideas so that coming up with new ones is harder. In an international context, an additional issue is identifying the relevant country-specific stocks of ideas that researchers have to build on and to compete against. Our specification here implies that for a given world stock of ideas, a country's own stock enhances the productivity of its researchers in proportion. For $\gamma < 1$, however, a larger world pool makes coming up with original ideas harder. While there are other plausible specifications of the effect of knowledge stocks on research productivity, the one we adopt here allows us to nest conveniently the two major approaches appearing in the single-economy literature on R&D and growth.

country i, $\alpha\mu_{it}s_i^{\beta}\widetilde{L}_i$, (2) the fraction of those ideas which are potentially useful in country n at the time of invention, μ_{nt}^{-1} [as shown in (22) of the Appendix], and (3) the fraction of those potentially useful ideas which are eventually adopted, $\epsilon_{ni}/(\epsilon_{ni}+g)$. This last magnitude captures the extent to which research findings are shared between countries and serves as our central measure of diffusion links.

We describe in the Appendix how we solve this system of equations. The solution gives, as a function of the steady-state research intensities s_i across countries, $N-1$ relative levels of total factor productivity:

$$\frac{A_{nt}}{A_{Nt}} = \left(\frac{\mu_{nt}}{\mu_{Nt}}\right)^{1/\theta} \qquad n = 1, \ldots, N-1 \tag{16}$$

In the case of fully endogenous growth, the solution also gives the world rate of total factor productivity growth $\dot{A}/A = g/\theta$. In the case of semiendogenous growth, $\dot{A}/A = g_L/[(1-\gamma)\theta]$ and the solution gives the steady-state level of $\overline{\mu}_t$ relative to any L_{it}, which has implications for the world *levels* of productivity rather their rate of growth.

4.2. *Steady-State Patenting.* Of the ideas diffusing to country n from country i, equation (6) determines the quality threshold for patenting $\bar{q}_{nit}$. A fraction μ_{nt}^{-1} is adopted. In steady state, the ratio of patented ideas to adopted ideas is a constant given by[39]

$$\bar{b}_{ni} = \mu_{nt}(\bar{q}_{nit})^{-\theta} \tag{17}$$

Substituting this expression into equation (7), patenting by inventors from country i in country n is

$$P_{ni} = \alpha s_i^{\beta}\widetilde{L}_i \frac{\mu_i}{\mu_n}\bar{b}_{ni} \tag{18}$$

4.3. *Steady-State Labor Market Equilibrium.* Substituting (13) and (14), the condition for steady-state labor market equilibrium in each country, from (10), becomes

$$\alpha\beta s_i^{\beta-1}\sum_{n=1}^{N} v_{ni}\widetilde{L}_n(1-s_n)\left(\frac{\mu_n}{\mu_i}\right)^{\frac{1-\theta+\phi\theta}{\theta(1-\phi)}} = (1-\phi)\kappa_2(\theta) \tag{19}$$

where $v_{ni} \equiv \mu_{nt}V_{nit}/Y_{nt}$ is constant in steady state, as shown by (25) in the Appendix.[40]

To summarize, the $N(N+2)$ equations (15), (17), and (19) determine $N-1$ relative technology levels μ, N^2 patenting thresholds $\bar{b}$, and N levels of R&D intensity s, and g (with growth fully endogenous) or the level of $\overline{\mu}_t^{\gamma-1}$ relative to each L_{it} (with growth semiendogenous).

[39] Equation 24 in the Appendix allows us to solve directly for each of the constants $\bar{b}_{ni}$ as a function of the parameters of the model.

[40] Since v_{ni} is independent of technology levels, an implication of (19) is that the (partial) elasticity of R&D with respect to the relative level of technology has the sign of $\theta(1-\phi)-1$ (holding research effort elsewhere constant). Researchers in countries with more advanced technologies, as measured by μ, are proportionately more productive as researchers, but their opportunity cost of doing research is also greater in proportion to $\mu^{1/[\theta(1-\phi)]}$. The net effect is more research in advanced countries if $\theta(1-\phi)$ exceeds 1 and less otherwise.

5. FITTING THE MODEL

To bring the model to data, we rewrite it as

$$Y = G(\Theta^*, X) \tag{20}$$

where Y is a vector of endogenous variables (productivity growth, relative productivity, research, and patents), X is a vector of exogenous variables (workforces and patenting costs relative to GDP), and Θ^* is a vector of parameters. The function $G(\cdot)$ represents the simultaneous solution of (15), (17), and (19), as well as the productivity and patenting equations, (16) and (18), respectively.

5.1. *Parameter Restrictions.* In full generality, the parameters of the model, $\theta, \beta, J, \alpha, \iota_{ni}^{pat}, \iota_{ni}^{not}, \epsilon_{ni}, r, g_L, \gamma, \phi$, are too numerous to identify with our equations. To give the model empirical content, we tie things down as follows:

Imitation rates. We assume that all patent systems are the same.[41] We allow for the possibility, however, that they treat nationals differently from foreigners. Hence we define ι_D^{pat} as the imitation rate when an idea is patented domestically and ι_F^{pat} as the imitation rate when an idea is patented abroad. The corresponding hazards for unpatented ideas are ι_D^{not} and ι_F^{not}. We set three of these four rates on the basis of survey evidence. The rate at which foreigners imitate nonpatented ideas is set to $\iota_F^{not} = 0.25$ based on Mansfield and Romeo (1980).[42] The imitation rates of domestic patented and unpatented ideas are set to $\iota_D^{pat} = 0.23$ and $\iota_D^{not} = 0.415$, respectively, based on Mansfield, Schwartz, and Wagner (1981).[43]

Diffusion rates. We restrict the rate of diffusion from country i to country n to be the product of a parameter governing the speed at which county n adopts new

[41] Rapp and Rozek (1990) classify countries into five categories according to the strength of protection provided by their intellectual property regime. Of our countries, all but Japan are in the highest category (Japan is in the next-to-highest category).

[42] Following the transfer of technology from a U.S. parent company to a foreign subsidiary, Mansfield and Romeo report that it took 4 years, on average, for a non-U.S. competitor to obtain the technology. This average delay implies an imitation rate of 0.25 if the imitation lag is exponentially distributed. Mansfield and Romeo continue: "In fact, the observed distribution is not very different from such an exponential distribution, but the sample is too small to carry out a goodness-of-fit test" (fn. 8, p. 740). We apply this imitation rate to nonpatented ideas, but we could just as well have applied it to patented ideas, since our estimates below imply that patenting ideas abroad has only a small effect on the imitation rate.

[43] "Within 4 years of their introduction, 60 percent of the patented successful innovations in our sample were imitated" (p. 913). Assuming an exponential distribution, these figures imply an imitation rate for domestic patented ideas of 0.23. One interpretation of our model is that there are two types of imitators working independently, one type imitating only unpatented domestic ideas and the other imitating any domestic idea. This specification implies that patenting an idea domestically will do no good with probability $\iota_D^{pat}/\iota_D^{not}$. With probability $[(\iota_D^{not} - \iota_D^{pat})/\iota_D^{not}]e^{-\iota_D^{pat} x}$ patenting an idea will increase the time until imitation by at least x years. Mansfield, Schwartz, and Wagner go on to report that "although patent protection seems to have only a limited effect on entry in about half of the cases, it seems to have a very important effect in a minority of them. For about 15 percent of the innovations, patent protection was estimated to have delayed the time when the first imitator entered the market by 4 years or more." Given $\iota_D^{pat} = 0.23$, the first fact suggests $\iota_D^{not} = 0.46$, while the second suggests $\iota_D^{not} = 0.37$. We average these values and set $\iota_D^{not} = 0.415$.

ideas, a parameter governing the speed at which ideas from country i are ready for adoption, and a parameter governing the percentage increase in adoption speed for domestic ideas. Formally, $\epsilon_{ni} = \epsilon_{n.}\epsilon_{.i}\epsilon_D$, where we normalize $\epsilon_{N.} = 0.1$ and $\epsilon_D = 1$ if $n \neq i$. Thus we require 10 parameters to account for the 25 diffusion rates between and within our 5 countries. Evidence reported by Pakes and Schankerman (1984) suggests that of U.S. inventions that are eventually adopted in the United States, the mean lag to adoption is 1.2 to 2.5 years. Taking the upper end of this range, we restrict $1/(\epsilon_{USUS} + g) = 2.5$. We use this restriction to determine ϵ_D in terms of the other parameters. The effect of this restriction is to anchor the absolute levels of diffusion to outside evidence.

Other parameter restrictions. We set the interest rate to $r = 0.07$, the long-run real return on the U.S. stock market. We fix the capital elasticity $\phi = 0.3$ based on evidence reported in Lysko (1995). We choose α so that, given all the other parameters, the model predicts $g_A = 0.018$, the mean of total factor productivity growth in the U.S., German, and French manufacturing sectors averaged over 1979–1990, from Lysko (1995). For our baseline, we assume the value $g_L = 0$ and $\gamma = 1$ to obtain fully endogenous growth. To explore the semiendogenous growth case, we set γ to solve $g_A = 0.018 = g_L/[(1-\gamma)\theta]$ and $g_L = 0.02$, representing a compromise between population growth and growth of research employment.

5.2. *Estimation.* To get at the parameters that have not been measured before, we turn to our data on relative productivity, research, and patenting for our set of five countries. We assume that by the late 1980s their situation can be described by the steady state of our model.

Incorporating our restrictions, and introducing multiplicative measurement error, we rewrite (20) in logarithms as

$$y = g(\Theta, X) + u \tag{21}$$

where $\Theta = (\theta, \beta, J, \iota_F^{pat}, \epsilon_{.1}, \ldots, \epsilon_{.5}, \epsilon_{1.}, \ldots, \epsilon_{4.})'$ and

$$y \equiv \Big[\ln(A_1/A_5), \ldots, \ln(A_4/A_5), \ln R_1, \ldots, \ln R_5,$$

$$\ln(P_{11}+1), \ldots, \ln(P_{ni}+1), \ldots, \ln(P_{55}+1)\Big]'$$

(which no longer includes productivity growth, since we choose α to match it exactly).[44] The productivity ratios are relative value added per hour in manufacturing, as shown in Table 1, converted to a measure of relative total factor productivity assuming a common interest rate across countries and a capital share of 0.3.[45]

One final adjustment takes into account the employment of clerical and technical workers in research. We assume that employing one research scientist or

[44] Note that in order to allow the model to predict zero patenting, we compare the model's prediction for the number of patents plus one with the actual number of patents plus one. The large amount of patenting that we observe among our five countries makes it reasonable to ignore, for estimation purposes, the count nature of the patent data.

[45] Hence, if X_i/X_N is value added per hour in country i relative to country N, then $A_i/A_N = (X_i/X_N)^{0.7}$ is total factor productivity in country i relative to country N.

engineer requires the additional employment of 1.3 staff, earning the production wage.[46]

Our estimate $\hat{\Theta}$ of the parameter vector minimizes

$$[y - g(\hat{\Theta}, X)]'\Omega^{-1}[y - g(\hat{\Theta}, X)]$$

where the matrix Ω reflects our weighting of productivity, research, and patents in the objective function.[47] This procedure would constitute standard nonlinear least squares if the function g had an analytic solution.[48]

6. RESULTS

Table 2 reports our parameter estimates for the baseline case, with the first column reporting the case of fully endogenous growth ($g_L = 0$) and the second that of semiendogenous growth ($g_L = 0.02$).[49] First note the remarkable similarity of the parameter estimates in the two cases.

Table 3 reports how well we fit the data for the fully endogenous growth case.[50] The fit with semiendogenous growth (not shown) is nearly identical.

[46] This is the average ratio of R&D clerical and technical workers to R&D scientists and engineers across the countries of our sample for which data were available (all but the United States) (OECD, 1991). Equation 19 becomes

$$\alpha\beta s_i^{\beta-1}\sum_{n=1}^{N} v_{ni}\widetilde{L}_n(1-2.3s_n)\left(\frac{\mu_n}{\mu_i}\right)^{\frac{1-\theta+\phi\theta}{\theta(1-\phi)}} = 2.3(1-\phi)\kappa_2(\theta)$$

[47] Denoting our weights on productivity and patents, relative to research, as ω_A and ω_P, respectively,

$$\Omega = \begin{bmatrix} \Omega_A & 0 & 0 \\ 0 & I_5 & 0 \\ 0 & 0 & (1/\omega_P)I_{25} \end{bmatrix}$$

where $\Omega_A = (1/\omega_A)[I_4 - \iota_4\iota_4']$, I_N is an $N \times N$ identity matrix, and ι_N is an N-vector of ones. This specification assumes that measurement error is independent across research, patents, and *levels* of productivity. The last induces correlations in relative levels accounted for in Ω_A. The weights reflect the importance that we attach to fitting the different types of data. Given the small number of observations on relative productivity and researchers, we do not attempt to base this weighting on the estimated variances. In our baseline we set $\omega_P = 1/5$ and $\omega_A = 30$, reflecting our desire to take the productivity data more seriously than the patent data. We also examine the implications of alternative weights on productivity.

[48] We have written a GAUSS program to calculate the function g. It begins with a parameter vector and data for all the exogenous variables. Next, it finds the set of b_{ni} that solves (17). Then it iterates between (15) and (19) until it finds technology levels and research employments that are consistent with each other. Finally, (3) and (18) are used to infer productivity and patenting. The entire process takes about 1 to 5 minutes (depending on the parameter values) on a Pentium-120 PC. We nest this calculation in a standard minimization routine to find $\hat{\Theta}$.

[49] Table 2 also reports standard errors, although these have limited meaning. They are correct in the very special case in which Ω reflects the variance matrix of the measurement error (up to a scalar multiple). We use the delta method to calculate standard errors of transformations of the estimated parameters appearing in subsequent tables.

[50] Since we placed so much weight on fitting productivity relative to the other endogenous variables, we explain it nearly perfectly, with a root mean square error (RMSE) of 0.01. The RMSE for research is 0.10, while for patenting it is 0.36. (RMSE is calculated from the differences of the logarithms of the actual and predicted values from Table 3.)

TABLE 2
PARAMETER VALUES

Definition	Symbol	Parameter Value: Endogenous	Parameter Value: Semiendogenous
Population growth	g_L	0.00	0.02
Parameter of research spillover	γ	1	0.40
Parameter of search distribution	θ	1.87 (1.06)	1.85 (1.05)
Parameter of talent distribution	β	0.18 (0.02)	0.16 (0.02)
Number of inputs (millions)	J	1.11 (0.62)	1.12 (0.62)
Imitation rates			
If not patented at home	ι_D^{not}	0.415	0.415
If not patented abroad	ι_F^{not}	0.250	0.250
If patented at home	ι_D^{pat}	0.230	0.230
If patented abroad	ι_F^{pat}	0.244 (0.002)	0.245 (0.002)
Diffusion factor from			
Germany	$\epsilon_{.1}$	0.93 (0.67)	0.93 (0.67)
France	$\epsilon_{.2}$	0.28 (0.23)	0.28 (0.22)
U.K.	$\epsilon_{.3}$	0.58 (0.56)	0.56 (0.54)
Japan	$\epsilon_{.4}$	1.17 (1.10)	1.20 (1.14)
U.S.	$\epsilon_{.5}$	0.21 (0.20)	0.21 (0.20)
Diffusion factor to			
Germany	$\epsilon_{1.}$	0.19 (0.13)	0.20 (0.13)
France	$\epsilon_{2.}$	0.22 (0.14)	0.22 (0.14)
U.K.	$\epsilon_{3.}$	0.07 (0.05)	0.07 (0.05)
Japan	$\epsilon_{4.}$	0.11 (0.08)	0.12 (0.08)
U.S.	$\epsilon_{5.}$	0.10	0.10
Diffusion factor domestic	ϵ_D	17.7	17.8

NOTE: Numbers in parentheses are approximate standard errors. Parameters without standard errors have been calibrated based on outside sources of information (described in the text). We parameterize the diffusion rate to destination n from source i as $\epsilon_{ni} = \epsilon_{n.}\epsilon_{.i}\epsilon_D$, where ϵ_D takes on the value 1 if $n \neq i$ and the value in the last row of the table if $n = i$. The value of $\epsilon_{5.} = 0.1$ is simply a normalization.

TABLE 3
MODEL FIT

	Germany		France		U.K.		Japan		U.S.	
Researchers (thousands)	97	(110)	41	(40)	74	(68)	289	(329)	477	(425)
Productivity (relative to U.S.)	0.90	(0.91)	0.94	(0.94)	0.75	(0.75)	0.84	(0.85)	1	
Patents (thousands) for protection in					By inventors from					
Germany	43	(27)	5	(6)	5	(6)	15	(15)	19	(36)
France	13	(10)	15	(19)	4	(6)	11	(13)	18	(32)
U.K.	13	(9)	5	(4)	24	(20)	13	(9)	20	(17)
Japan	7	(8)	3	(4)	3	(4)	65	(45)	18	(20)
U.S.	13	(8)	5	(4)	6	(6)	33	(16)	83	(119)

NOTE: Actual values with predicted values in parentheses. Here and in subsequent tables productivity measures reflect total factor productivity assuming a constant capital-output ratio and a capital share of 0.3.

Since the two cases are so similar, we pursue for now only the fully endogenous growth case. We resurrect semiendogenous growth when we turn to counterfactual experiments.

Our estimate of the research elasticity β is precise at a level significantly below the value of one assumed in much theoretical work. We think that this estimate reflects the small but rather even fraction of the workforce engaged in research across our countries. A value closer to 1 implies more international specialization. To explain the near-diagonal dominance of the patenting matrix, our estimates imply that foreign patents provide rather limited protection, reducing the hazard of imitation by about half a percentage point.

Turning to diffusion, we find that ideas from Japan and Germany diffuse most rapidly, while France and Germany are the quickest to exploit ideas. Ideas diffuse much faster within than between countries.

6.1. *Diffusion Lags and Adoption.* The diffusion parameters are not precisely estimated and, on their own, are hard to interpret. Three more meaningful and digestible concepts are (1) the mean lag between the invention of an idea in country i and its arrival in country n, regardless of whether or not it is adopted when it arrives $(1/\epsilon_{ni})$, (2) the mean lag between invention and arrival conditional on adoption $[1/(\epsilon_{ni}+g)]$, and (3) the fraction of ideas that are ever adopted of those which were potentially useful when they were invented $[\epsilon_{ni}/(\epsilon_{ni}+g)]$.[51] The second concept corresponds to the adoption lags measured in microeconomic studies (which consider only ideas that are actually adopted). The third concept reflects how much of the research output of one country makes it to another, as one can see in (15). The simple and conditional diffusion lags are estimated about as precisely as the diffusion parameters themselves. However, our central measures of diffusion, the fractions of potentially useful ideas that are adopted, are estimated much more precisely.

As for the unconditional lag, its mean is just over 1 year for domestic ideas and about 21 years for ideas from abroad. The second figure hides considerable variation across individual country pairs. Looking at ideas that eventually will be adopted, the figure for domestic ideas remains about 1 year, while for foreign inventions it decreases to about 11 years. This last figure falls in between the survey results of Mansfield and Romeo (1980) on the mean lags between U.S. adoption and adoption abroad by U.S. subsidiaries (5.8 years for developed countries) and nonsubsidiaries (13.1 years for all countries).[52]

Table 4 presents concept (3), our estimates of ideas that are ever adopted as a fraction of those which would have been adopted if they had diffused instantaneously, along with standard errors (in parentheses). The matrix is highly diagonal-dominant. Over 95 percent of potentially useful domestic ideas are adopted eventually, in contrast with an average of about 60 percent for foreign ideas.

[51] Recall that $g=\theta g_A$, where we set g_A to match average growth in total factor productivity of 0.018 and θ is estimated.

[52] Jovanovic and Lach (1997) also find evidence of slow international diffusion.

TABLE 4
ADOPTION PERCENTAGES

Fraction of Potentially Useful Ideas that Are Ever Adopted in	Originating from Research Performed in				
	Germany	France	U.K.	Japan	U.S.
Germany	0.99 (0.01)	0.62 (0.14)	0.77 (0.07)	0.67 (0.07)	0.54 (0.16)
France	0.86 (0.07)	0.97 (0.03)	0.79 (0.07)	0.88 (0.06)	0.57 (0.15)
U.K.	0.65 (0.18)	0.37 (0.18)	0.95 (0.05)	0.70 (0.14)	0.30 (0.17)
Japan	0.76 (0.13)	0.49 (0.17)	0.66 (0.10)	0.99 (0.02)	0.41 (0.17)
U.S.	0.73 (0.14)	0.46 (0.19)	0.63 (0.17)	0.78 (0.14)	0.92 (0.05)

NOTE: The element in row n and column i is $\epsilon_{ni}/(\epsilon_{ni}+g)$, which in steady state has the interpretation given in the table. Numbers in parentheses are approximate standard errors.

TABLE 5
GROWTH DECOMPOSITION

Fraction of Productivity Growth in	Due to Research Performed in				
	Germany	France	U.K.	Japan	U.S.
Germany	0.16 (0.02)	0.08 (0.01)	0.07 (0.01)	0.27 (0.02)	0.42 (0.04)
France	0.13 (0.01)	0.11 (0.02)	0.07 (0.01)	0.26 (0.02)	0.42 (0.04)
U.K.	0.15 (0.02)	0.07 (0.01)	0.13 (0.02)	0.32 (0.04)	0.33 (0.06)
Japan	0.14 (0.02)	0.07 (0.01)	0.07 (0.01)	0.35 (0.05)	0.36 (0.05)
U.S.	0.10 (0.01)	0.05 (0.02)	0.05 (0.01)	0.20 (0.03)	0.60 (0.06)

NOTE: Rows may not sum to 1 due to rounding. Numbers in parentheses are approximate standard errors.

6.2. *The Sources of Growth.* To what extent do countries depend on each other for their growth? Imbedding the figures in Table 4 into (15) provides a means of decomposing growth into its sources by country, taking into account our estimates of the production of potentially useful ideas by each country. Table 5 quantifies this decomposition.

The table portrays a world in which diffusion is pervasive. Differences in the scale of research output swamp the diagonal dominance of the adoption-rate figures in Table 4. The United States, Japan, and Germany, in that order, are the leading contributors to growth in every country. Together, the United States and Japan account for over 65 percent of the growth of each of the five countries.[53]

6.3. *The Rewards to Research.* Do the rewards to inventive activity reflect this breadth of diffusion? While the growth decomposition in Table 5 looks at foreign countries as sources of new technology, Table 6 looks at foreign countries as markets for new technology. We use our estimates to calculate the fraction of the average value of an idea arising from markets in each of the five countries.

[53] In Eaton and Kortum (1996) we perform a similar exercise for a broader sample of countries using a quite different model that does not endogenize research effort. Despite the differences in sample and methodology, the results are remarkably similar. The one difference worth noting is that in this other study Germany and Japan switch places in their rankings as sources of growth for the other European countries.

TABLE 6
INVENTION VALUE DECOMPOSITION

Fraction of Invention Value from Markets in	For Ideas Originating in				
	Germany	France	U.K.	Japan	U.S.
Germany	0.33 (0.08)	0.11 (0.02)	0.11 (0.02)	0.10 (0.01)	0.04 (0.02)
France	0.11 (0.02)	0.48 (0.13)	0.11 (0.02)	0.09 (0.01)	0.04 (0.02)
U.K.	0.07 (0.01)	0.05 (0.01)	0.35 (0.13)	0.06 (0.01)	0.02 (0.01)
Japan	0.19 (0.03)	0.15 (0.02)	0.17 (0.02)	0.50 (0.12)	0.06 (0.02)
U.S.	0.30 (0.10)	0.22 (0.12)	0.26 (0.13)	0.25 (0.11)	0.84 (0.05)

NOTE: Columns may not sum to 1 due to rounding. Numbers in parentheses are approximate standard errors.

The return to ideas appears to be much more local than the benefit from ideas. The largest single source of returns is always from the domestic market. Nevertheless, for all but the United States, foreign markets taken as a whole provide at least half the returns. The United States is a valuable market for ideas from all countries and provides over 80 percent of the market for its own ideas.

6.4. *Robustness.* The results that we have presented so far rely on a number of specific assumptions. We now examine the implications of changing a few of the potentially more controversial ones. Specifically, we reestimate the model after either (1) lowering the interest rate from 0.07 to 0.03 (closer to the return on bonds rather than on stocks), (2) doubling the number of research scientists and engineers (to take account of less formal research activity that might escape measurement), (3) increasing the cost of patenting abroad by \$10,000 (to proxy for the potential hassle of dealing with foreign legal systems),[54] or (4) reducing the weight placed on productivity in our objective function (ω_A) from 30 to 10.

The one notable effect of the lower interest rate is a lower elasticity of research output with respect to research employment β of 0.14. Raising the level of research effort has the opposite impact, raising β to 0.35, with no other effect worth mentioning.[55] Higher costs of foreign patenting are, quite logically, manifested entirely in a lower imitation rate of foreign patents, with $\iota_F^{pat} = 0.208$. These changes affect the fit hardly at all, except for the higher fee, which hurts it substantially.

Downweighting productivity in our objective function has broader effects. The major change is that Japan becomes a source of world growth on a par with the United States.[56] However, in trying to fit the patent data better, the model flounders on relative productivity, predicting only tiny differences among countries and putting the

[54] Harvard Business School (1991) documents the frustration vented by U.S. patent seekers in Japan.

[55] Less discounting raises the return to R&D. To explain observed levels of research, our model offsets this effect by reducing the productivity of R&D through a lower β. By this same logic, when confronted with larger numbers of researchers, our model concludes that research effort is a more productive undertaking.

[56] As shown in Table 3, our baseline, in striving to explain the U.S. productivity edge over Japan, tends to overpredict patenting by U.S. researchers and to underpredict patenting by the Japanese.

United States behind all but the United Kingdom.[57] The reason for this difference is the mixed messages sent by the productivity and patent data about the positions of Japan versus the United States. As shown in Table 1, Japan is 22 percent behind in value added per hour, but given the much smaller scale of its economy, the Japanese patent performance is formidable. While the alternative weighting scheme highlights this intriguing feature of the data, we pay more attention to the message sent by the productivity data, since trying to understand them is our primary objective.

7. COUNTERFACTUALS

Exploiting the general equilibrium nature of our model, we finish off by using it to observe hypothetical worlds with different patterns of diffusion and rigor of patent protection. We run two types of experiments, assuming first that growth is fully endogenous and then that it is semiendogenous (using the parameter values in the first and second columns, respectively, of Table 2). In the first case, the counterfactual worlds we explore have different steady-state growth rates, while in the second, levels of productivity differ, but the long-run growth rate is the same.[58]

7.1. *Alternative Patterns of Diffusion.* We can get another perspective on the role of international technology diffusion by simulating the effects of technological autarky. In particular, we calculate what would happen if the single largest economy, the United States, were severed from the others. The results appear in Table 7. (For comparison, we repeat the predictions of the model from Table 3 under the column "baseline.") Our first experiment, "technological isolation," reduces to 0.0001 the rate of diffusion between the United States and the block of four other countries.

When growth is fully endogenous, it falls by about 0.7 of a percentage point. Since the block of four other countries grows faster on its own than does the United States, the U.S. level of productivity must fall relative to the others before the resulting technology gap supports the new steady-state growth rate.[59] Note that research effort falls everywhere. Hence the direct effect of technological isolation—lowering growth by cutting off access to foreign ideas—is exacerbated by a reduction in research effort.

With growth semiendogenous, the U.S. productivity growth path in the new steady state is about two-thirds of what it otherwise would have been. Since the United States has fewer ideas, researchers find it easier to come up with new ones. As a

[57] The RMSE for productivity jumps to 0.19, while for research it falls to 0.08 and for patents to 0.22.

[58] In the simulations reported we fix the interest rate at its baseline value of 0.07. With semiendogenous growth coupled with isoelastic marginal utility, this assumption is innocuous, since the steady-state growth rate is unaffected. In the case of fully endogenous growth coupled with a nonzero elasticity of marginal utility σ, however, the change in the growth rate would feed back into r, with subsequent effects on research activity and growth. We also simulated the fully endogenous growth model endogenizing r and setting $\sigma = 2$ (with the discount rate ρ chosen to match the baseline). The interest rate obviously responded to the change in the growth rate, but the subsequent feedback onto research activity and growth was negligible. Hence the results for this case are very close to those reported in Tables 7 and 8.

[59] Thus in the new steady state the United States is not completely isolated because it obtains many innovations from abroad, albeit years later.

TABLE 7
EXPERIMENTS WITH THE RATE OF DIFFUSION

	Baseline		Technological Isolation		Borderless Diffusion	
	Endog.	Semi	Endog.	Semi	Endog.	Semi
Productivity growth:	0.018	0.018	0.011	0.018	0.028	0.018
U.S. productivity level:	1.00	1.00	1.00	0.67	1.00	1.41
Productivity (per U.S.)						
Germany	0.91	0.92	3.20	2.09	1.02	1.01
France	0.94	0.94	3.25	2.15	1.02	1.02
U.K.	0.75	0.75	2.94	1.87	0.98	0.99
Japan	0.85	0.85	3.13	2.05	1.01	1.00
Research intensity						
Germany	0.0037	0.0037	0.0025	0.0040	0.0115	0.0073
France	0.0016	0.0016	0.0012	0.0020	0.0081	0.0053
U.K.	0.0025	0.0025	0.0018	0.0029	0.0101	0.0065
Japan	0.0054	0.0054	0.0039	0.0063	0.0119	0.0075
U.S.	0.0035	0.0036	0.0030	0.0049	0.0071	0.0047

NOTE: We do each experiment for both fully endogenous (using the parameters in the first column of Table 2) and semiendogenous growth (using the parameters in the second column of Table 2). In "Baseline" we display the prediction of the model (for fully endogenous growth those predictions are from Table 3). In "Technological Isolation" we set the diffusion rates between the United States and the other four countries equal to 0.0001. Since these diffusion rates are not zero, the United States still grows at the same rate as the other four countries in steady state. In "Borderless Diffusion" we set $\epsilon_D = 17.7$ (in the fully endogenous growth case) or $\epsilon_D = 17.8$ (in the semiendogenous growth case) even for $n \neq i$.

consequence, more research is performed everywhere. For countries other than the United States this effect dominates to the extent that productivity is somewhat higher than in the base case.

Our second experiment, "borderless diffusion," eliminates the effect of country borders on diffusion rates. In particular, we set $\epsilon_D = 17.7$ (or 17.8 in the semiendogenous growth case) even if $n \neq i$. Since ideas now spread more rapidly and evenly across countries, productivity levels become tightly clustered. With growth fully endogenous, the growth rate rises by a percentage point. In the semiendogenous growth case the steady-state level of U.S. productivity rises by 41 percent. In either case technological integration stimulates research, particularly in the smaller countries (Germany, France, and the United Kingdom), which now have a much larger effective market for their ideas.

7.2. *The Strength of Patent Protection.* We conclude with some counterfactual experiments on the strength of patent protection. Table 8 reports the results. In the first experiment we eliminate all forms of patent protection by setting $\iota_D^{pat} = \iota_D^{not} = 0.415$ and $\iota_F^{pat} = \iota_F^{not} = 0.250$ (i.e., making the hazard of imitation of patented ideas as great as for unpatented ones). In the fully endogenous growth case, productivity growth falls by one tenth of a percentage point. In the case of semiendogenous growth, the level of productivity in all countries falls by about 6 percent. In either case, relative productivity levels remain about the same, while research intensity falls.

In the second experiment we make patent protection perfect by setting $\iota_D^{pat} = \iota_F^{pat} = 0$ so that there is no hazard of imitation if an idea is patented. As before, relative

TABLE 8
EXPERIMENTS WITH THE STRENGTH OF PATENT PROTECTION

	Baseline		No IPP		Perfect IPP	
	Endog.	Semi	Endog.	Semi	Endog.	Semi
Productivity growth:	0.0180	0.0180	0.0168	0.0180	0.0248	0.0180
U.S. productivity level:	1.00	1.00	1.00	0.94	1.00	1.41
Productivity (per U.S.)						
Germany	0.91	0.92	0.93	0.92	0.89	0.92
France	0.94	0.94	0.95	0.95	0.92	0.94
U.K.	0.75	0.75	0.77	0.76	0.71	0.75
Japan	0.85	0.85	0.87	0.86	0.82	0.85
Research intensity						
Germany	0.0037	0.0037	0.0030	0.0032	0.0399	0.0443
France	0.0016	0.0016	0.0011	0.0012	0.0207	0.0254
U.K.	0.0025	0.0025	0.0019	0.0021	0.0298	0.0348
Japan	0.0054	0.0054	0.0039	0.0042	0.0462	0.0491
U.S.	0.0035	0.0036	0.0017	0.0018	0.0340	0.0373

NOTE: We do each experiment for both fully endogenous (using the parameters in the first column of Table 2) and semiendogenous growth (using the parameters in the second column of Table 2). In "Baseline" we display the prediction of the model (for fully endogenous growth those predictions are from Table 3). In "No IPP" we set $\iota_D^{pat} = \iota_D^{not} = 0.415$ and $\iota_F^{pat} = \iota_F^{not} = 0.250$. In "Perfect IPP" we set $\iota_D^{pat} = \iota_F^{pat} = 0$.

productivities do not change much, but productivity growth rises by over a half a percentage point in the fully endogenous growth case while long-run productivity tables rise by almost 40 percent in the semiendogenous growth case. In either case, research effort rises dramatically.

To get a crude sense of the net benefits of patent protection, we compare the steady-state gain in productivity with the cost of diverting labor from production. (In the case of fully endogenous growth, we consider the effect of a permanent percentage increase in productivity with the same present value as the increase in the growth rate.) Moving from no protection to our baseline level raises steady-state output by the equivalent of 2.3 percent (growth fully endogenous) or 6.3 percent (growth semiendogenous), while pulling only about half a percent of the workforce into research away from production. Moving from the baseline to perfect protection generates productivity gains of 15 percent (fully) or 41 percent (semi), at a cost of employing about 8 percent more of the workforce in research.[60]

8. CONCLUSION

We have provided a quantitative explanation of research effort, the growth of productivity, and the spread of technology across countries. A rough summary of our findings is that countries lie two-thirds of the way from technological autarky toward free trade in ideas; i.e., research performed abroad is about two-thirds as potent as domestic research. Furthermore, the United States and Japan together drive at least two-thirds of the growth in each of our five countries. Our results jive with a

[60] These calculations, by ignoring the transit to the new steady state, overstate the benefits of patent protection, especially in the semiendogenous growth case.

view of economic history that relates a country's productivity to its ability to adopt ideas.[61]

While these results might suggest that the barriers to the spread of technology are minor, they nevertheless can account for the substantial productivity differences among our five countries. Eliminating international barriers to diffusion would not only bring productivity levels very close together but also would raise productivity substantially everywhere.

Our purpose has been to develop a basic methodology for analyzing the determinants of research and productivity in a multicountry world. While the focus here has been on economic aggregates, the methodology can be extended to accommodate additional dimensions of the data. For example, giving the analysis a sectoral dimension would provide insight into why countries specialize in research as they do and how this specialization shapes comparative advantage in production.[62]

We have taken only a first look at policies toward research (by examining the effects of eliminating or perfecting patent protection). Many more issues of patent protection deserve attention. For example, to what extent is coordination required to obtain an optimal patent system? Moreover, governments pursue a wide range of other policies, including tax incentives, research grants, and government labs, that affect innovation. Our framework is a natural one for evaluating the payoffs from these efforts.[63]

APPENDIX

A.1. *Symbols Used in the Model.*

Y_{nt}	Output in country n at time t
J	Range of inputs, $j \in [0, J]$
$X_{nt}(j)$	Quantity of input j in country n at time t
$Z_{nt}(j)$	Quality of input j in country n at time t
L_{nt}	Workforce in country n at time t
K_{nt}	Capital stock in country n at time t
s_{it}	Fraction of workers doing research in country i at time t
α_{it}	Productivity of researchers in country i at time t
β	Parameter of the distribution of research talent

[61] Unlike economic historians, however, we have only taken a snapshot of the world at a given moment. In Eaton and Kortum (1997a) we used the model (parameterized with our estimates here) to explain the postwar growth of these five economies as convergence toward the steady state from initial productivity levels. While endogenizing R&D in steady state, as we have done here, requires very intricate numerical integration, outside of steady state the problem is an order of magnitude more difficult. Hence in this out-of-steady-state exercise we conditioned on the actual paths of research. We found that the semiendogenous growth version of the model captures broad movements in productivity quite successfully, while the fully endogenous growth version predicted much more rapid convergence than actually occurred.

[62] In Eaton and Kortum (1999) we introduce international trade into a static version of the model to investigate the extent to which trade, as opposed to the diffusion of knowledge, spreads the benefits of innovation.

[63] Eaton, Gutierrez, and Kortum (1998) use a variant of the framework here to analyze European research policies.

Q	Random variable representing the quality of an idea
$F(q)$	Distribution from which the quality of an idea is drawn
θ	Parameter of the quality distribution, $F(q) = 1 - q^{-\theta}$
τ	Random diffusion lag
ϵ_{ni}	Rate of diffusion from country i to n
μ_{nt}	Stock of ideas that have diffused to country n by time t
$H_n(z; t)$	Cumulative distribution of the technological frontier in country n at time t
A_{nt}	Level of productivity in country n at time t
ψ	Euler's constant (≈ 0.5772)
c	Unit cost of production
r	Rate of interest
r'	Cost of capital (rate of interest plus depreciation rate)
w_{nt}	Wage (in units of output) of production workers in country n at time t
ϕ	Capital elasticity
$p(j)$	Price (in units of output) of input j
$\pi_{nt}(z, q)$	Profit from marketing input of quality q in country n at time t if the next best input has quality z
$V_{nit}(z, q)$	Expected discounted value as of time t in country n of an idea from country i where the idea has quality q and replaces an input of quality z
$V^k_{nit}(q)$	Expected value of an idea before its use (hence z) is known The index $k = pat, not$ specifies if it is patented
ι^{pat}_n	Rate of imitation in country n if the idea is patented
ι^{not}_n	Rate of imitation in country n if the idea is not patented
f_{nit}	Cost of seeking protection in country n from country i at time t
$\bar{q}_{nit}$	Cut-off quality to patent in country n from country i on an idea invented at time t
P_{nit}	Number of ideas from country i seeking protection in n at time t
b_{nit}	Defined as $\mu_{nt}(\bar{q}_{nit})^{-\theta}$ (constant in steady state)
V_{nit}	Expected value of an idea from country i in country n at time t (quality unknown but optimal patenting assumed)
V_{it}	Expected value of an idea from country i at time t
g	Steady-state growth rate of μ, the stock of diffused ideas
g_L	Growth rate of the labor force
g_A	Growth rate of total factor productivity
v_{ni}	Normalized value of an idea from country i in country n defined as $V_{nit}\mu_{nt}/Y_{nt}$ (constant in steady state)
$\kappa_1(\theta)$	Constant relating productivity index to the wage
$\kappa_2(\theta)$	Average value of the inverse of the price markup

A.2. *Mathematical Appendix.*

A.2.1. *The distribution of the technological frontier.* We derive the distribution of the technological frontier by considering the distribution of the best quality idea at time t for a given input in country n. Consider an input that is currently of quality z. New ideas will be adopted for this input at a stochastic rate of $\dot{\mu}_{nt}z^{-\theta}$. The probability that no idea is adopted in the time interval $[t, t+dt]$ is thus $e^{-\dot{\mu}_{nt}z^{-\theta}dt}$. Therefore,

$$H_n(z; t+dt) = H_n(z; t)e^{-\dot{\mu}_{nt}z^{-\theta}dt}$$

or

$$\frac{\partial \ln H_n(z; t)}{\partial t} = -\dot{\mu}_{nt}z^{-\theta}$$

Solving this differential equation, with the two initial conditions, (1) $\lim_{s\to-\infty} H_n(z; s) = 1$ for all $z \geq 1$ and (2) $\lim_{s\to-\infty} \mu_{ns} = 0$, yields the cumulative distribution function for the technological frontier, equation 2.

A.2.2. *The productivity equation*. The natural log of the geometric mean of the technological frontier is

$$\ln A_{nt} = \int_1^{\infty} \ln(z) dh_n(z; t)$$

Changing the variable of integration to $x = \mu_{nt}z^{-\theta}$,

$$\ln A_{nt} = \theta^{-1} \int_0^{\mu_{nt}} \ln(\mu_{nt}/x)e^{-x}dx = \theta^{-1} \ln(\mu_{nt})(1 - e^{-\mu_{nt}}) - \theta^{-1} \int_0^{\mu_{nt}} \ln(x)e^{-x}dx$$

For large μ_{nt} we have an arbitrarily good approximation:

$$\ln A_{nt} = \theta^{-1} \ln(\mu_{nt}) - \theta^{-1} \int_0^{\infty} \ln(x)e^{-x}dx$$

The Laplace transform of $-\psi - \ln t$ is $s^{-1}\ln s$, where ψ is Euler's constant. Evaluating the Laplace transform at $s = 1$ implies

$$\int_0^{\infty} \ln(x)e^{-x}dx = -\psi$$

This gives us the desired result that

$$\ln A_{nt} = \theta^{-1} \ln(\mu_{nt}) + \psi/\theta$$

As we discussed in the introduction, our model implies that an idea is more likely to be adopted in a country with a relatively low level of productivity. The probability that an idea of quality q will prove useful is simply $H_n(q; t)$. Integrating this probability over the Pareto density of Q, and noting that μ_{nt} becomes arbitrarily large over time, we get

$$(22) \qquad \int_1^{\infty} H_n(q; t)dF(q) = \int_1^{\infty} \theta q^{-(\theta+1)}e^{-\mu_{nt}q^{-\theta}}dq = \mu_{nt}^{-1}$$

A.2.3. *Wages, productivity, and the distribution of the markup.* From Proposition 3.3 in Kortum (1997), the time invariant distribution of the markup is given by

$$G(m) \equiv 1 - \frac{\theta \ln(m)}{1 - m^{-\theta}} m^{-\theta}$$

This distribution is used to derive our equations for the wage and labor productivity conditional on technology, (12) and (11).

To derive an expression for the wage, start with the result that the quantity purchased of input j is

$$X(j) = \frac{Y}{Jp(j)} = \frac{Y}{JcM(j)}$$

where $M(j)$ is the markup for input j. Plugging this into the production function and rearranging,

$$\ln c = \ln(A) - J^{-1} \int_0^J \ln M(j) dj = \ln(A) - \int_1^\infty \ln(m) dG(m)$$

Hence $c_{it} = \kappa_1(\theta) A_{it}$, where $\kappa_1(\theta) \equiv \exp\{-\int_1^\infty \ln(m) dG(m)\}$. The corresponding wage is $w_{it} = (1-\phi)\kappa_1(\theta) A_{it} k_{it}^{\phi}$ in terms of the capital-labor ratio and $w_{it} = (1-\phi)[\kappa_1(\theta) A_{it} (\phi/r')^{\phi}]^{1/(1-\phi)}$ in terms of the cost of capital r'.

Output is the sum of profit and factor income. Total profit across all inputs is

$$\int_0^J (1 - M(j)^{-1}) \frac{Y_{it}}{J} dj = Y_{it}[1 - \int_1^\infty m^{-1} dG(m)]$$

Output of intermediates is $k^{\phi} L(1-s)$, while each intermediate costs c in terms of factor payments to produce. Summing factor payments and profits and rearranging, we obtain as an expression for total income

$$Y_{it} = \frac{c_{it}}{\kappa_2(\theta)} k_{it}^{\phi} L_{it}(1 - s_{it}) = \frac{\kappa_1(\theta)}{\kappa_2(\theta)} A_{it} k_{it}^{\phi} L_{it}(1 - s_{it})$$

in terms of k_{it} and

$$Y_{it} = \frac{1}{\kappa_2(\theta)} [\kappa_1(\theta) A_{it} (\phi/r')^{\phi}]^{1/(1-\phi)} L_{it}(1 - s_{it})$$

in terms of the cost of capital r'. Here $\kappa_2(\theta) \equiv \int_1^\infty m^{-1} dG(m)$.

A.2.4. *Steady-state relative productivities and growth.* We can write the system of equations (15) in matrix form as

$$\mu g = \Delta(g) \mu \tag{23}$$

where $\mu \equiv (\mu_{1t}/\mu_{Nt}, \ldots, \mu_{N-1t}/\mu_{Nt}, 1)'$ and

$$\Delta(g) = \begin{bmatrix} \delta_{11} & \ldots & \delta_{1N} \\ \vdots & \ddots & \vdots \\ \delta_{N1} & \ldots & \delta_{NN} \end{bmatrix}$$

where

$$\delta_{ni} \equiv \frac{\epsilon_{ni}}{\epsilon_{ni} + g} \frac{\alpha}{J} s_i^{\beta} \widetilde{L}_i$$

In the case of fully endogenous growth, we solve the system iteratively. Based on an initial g_0, we set $\Delta_0 = \Delta(g_0)$. The system of equations (23) then constitutes a standard system of linear differential equations. The Frobenius root of this system determines a new growth rate g_1. We iterate accordingly until $g_n \approx g_{n-1}$. We then set $g = g_n$. The vector μ is given by the eigenvector corresponding to the Frobenius root g. In the case of semiendogenous growth, $g = g_L/(1-\gamma)$. Thus we iterate on the level of $\overline{\mu}_t^{\gamma-1}$ relative to the L'_{it}s rather than on g.

A.2.5. *Steady-state patenting thresholds*. In steady state, Y_{nt} grows at a constant rate g_Y, while μ_{nt} grows at a constant rate g. In order to obtain an expression for the thresholds in terms of time-invariant variables, we define $b \equiv \mu_{nt} q^{-\theta}$. Imposing these steady-state relationships and integrating equation (4) over the distribution of the technological frontier, we get

$$V_{nit}^{l}(q) = \widetilde{V}_{nit}^{l}(b) = Y_{nt} \frac{\Upsilon(b, \theta)}{J} \int_0^{\infty} e^{-(r+\iota_{ni}^{l}-g_Y)s}(1 - e^{-\epsilon_{ni}s}) e^{-be^{gs}} ds$$

where $l = pat, not$ depending on whether or not the idea is patented, $\Upsilon(b, \theta) \equiv 1 - e^b b^{1/\theta} \Gamma[(\theta-1)/\theta, b]$, where $\Gamma(a, b) \equiv \int_b^{\infty} e^{-x} x^{a-1} dx$ is the incomplete gamma function. The value of an idea relative to market size therefore does not depend on time:

$$\widetilde{V}_{nit}^{l}(b)/Y_{nt} = v_{ni}^{l}(b) = (Jg)^{-1} \Upsilon(b, \theta) \{\Psi(\widehat{r}_{ni}^{l}/g, b) - \Psi[(\widehat{r}_{ni}^{l} + \epsilon_{ni})/g, b]\}$$

where

$$\widehat{r}_{ni}^{\,l} \equiv r + \iota_{ni}^{l} - g_Y \quad l = pat, not$$

and $\Psi(a/g, b) \equiv g \int_0^{\infty} e^{-as} e^{-be^{gs}} ds$.[64]

Equation (6) determining the patent threshold then becomes

$$v_{ni}^{pat}(\overline{b}) - v_{ni}^{not}(\overline{b}) = f_{ni} \tag{24}$$

Given g, the world growth rate of μ, the only country characteristics that directly affect the patenting threshold are the adoption lag ϵ_{ni}, the strength of patent protection, as reflected by ι_{ni}^{pat} and ι_{ni}^{not}, and the cost of patenting f_{ni}.

[64] In order to compute this integral, we rely on the result that for $b > 0$,

$$\Psi(a/g, b) = b^{a/g} \Gamma(-a/g, b)$$

As a consequence, we also have

$$\Upsilon(b, \theta) = 1 - be^b \Psi\left(\frac{1-\theta}{\theta}, b\right)$$

There is a continued fraction representation for the incomplete gamma function (that admits $d < 0$) leading to a speedy numerical algorithm (from Press et al., 1989, pp. 160–163).

A.2.6. *The steady-state value of an idea*. In the text we define

$$v_{ni} \equiv V_{nit}\mu_{nt}/Y_{nt}$$

Substituting the definitions from the previous section into equation (8), it follows that

$$v_{ni} = \int_0^{\bar{b}_{ni}} v_{ni}^{pat}(b)db + \int_{\bar{b}_{ni}}^{\infty} v_{ni}^{not}(b)db - f_{ni}\bar{b}_{ni} \tag{25}$$

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