Advanced Tools in Macroeconomics

Occasionally Binding Constraints

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Occasionally binding constraints

- Many interesting economic problem involves various sorts of inequality constraints that "occasionally bind"
  - Borrowing constraints
  - Irreversibility constraints
  - Collateral constraints
  - Implementability constraints
- It is generally perceived as hard to solve such problems
- I disagree

Outline for today

- Start by looking into how one can solve nonlinear models with occasionally binding constraints
  - Focus on two examples
- Look at how Regime switching linear models can accommodate constraints that bind for exogenous reasons (e.g. ZLB).
- Look at how Regime switching linear models can accommodate constraints that bind for enogenous reasons (e.g. irreversible investment). This is supplementary material.
- Exercise: Calculate the fiscal multiplier in a ZLB model.

Occasionally binding constraints

Let's take two examples in a nonlinear world

1. A borrowing constraint
2. Irreversible investment

A borrowing constraint

Consider the following optimisation problem

$$V(b_0, s_0) = \max_{\{c_t(s^t), b_{t+1}(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s^t \in \mathcal{S}^{t+1}} \beta^t u(c_t(s^t)) P(s^t, s_0)$$

$$\text{subject to} \quad c_t(s^t) + b_{t+1}(s^t) = s_t w + (1 - s_t)\mu w + (1 + r) b_t(s^t),$$

$$b_{t+1}(s^t) \geq \underline{b}$$

$$\forall t, \forall s^t \in \mathcal{S}^{t+1} \quad b_0, s_0 \text{ are given}$$

Bellman equation

$$v(b, s) = \max_{b' \geq \underline{b}} \{u(b(1 + r) + w(s) - b') + \beta \sum_{s'=0}^{1} v(b', s') p(s', s)\}$$

A borrowing constraint

$$V(b,s) = \max_{b' \geq \underline{b}} \{u(b(1+r) + w(s) - b') + \beta \sum_{s'=0}^{1} V(b',s')p(s',s)\}$$

- First order condition

$$u'(b(1+r) + w(s) - b') - \mu(b,s) = \beta \sum_{s'=0}^{1} V_{b'}(b',s')p(s',s)$$

where $\mu(b,s)$ is the Lagrange multiplier on the borrowing constraint.

A borrowing constraint

- Suppose we have a guess for $V_{b,n}(b,s)$. Then find $\tilde{b}'$ as

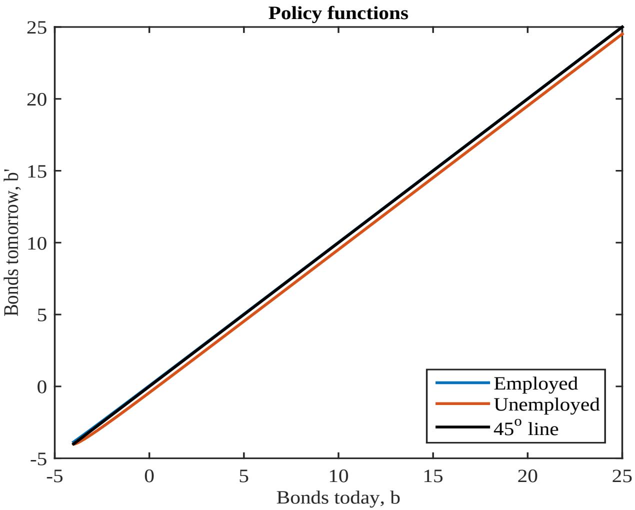
$$u'(b(1+r)+w(s)-\tilde{b}') = \beta \sum_{s'=0}^{1} V_{b'}(\tilde{b}', s')p(s', s), \quad \forall (b,s)$$

- Optimal b' is then $b' = \max\{\tilde{b}', \underline{b}\}$.
- Update the guess

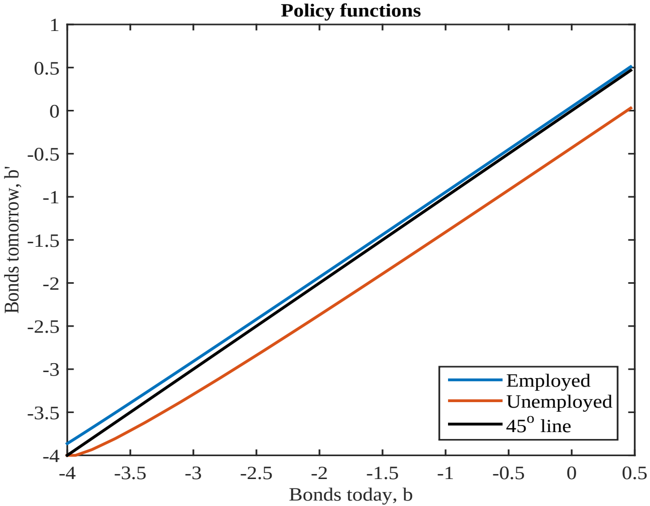
$$V_{b,n+1} = (1+r)u'(b(1+r)+w(s)-b')$$

and repeat until convergence.

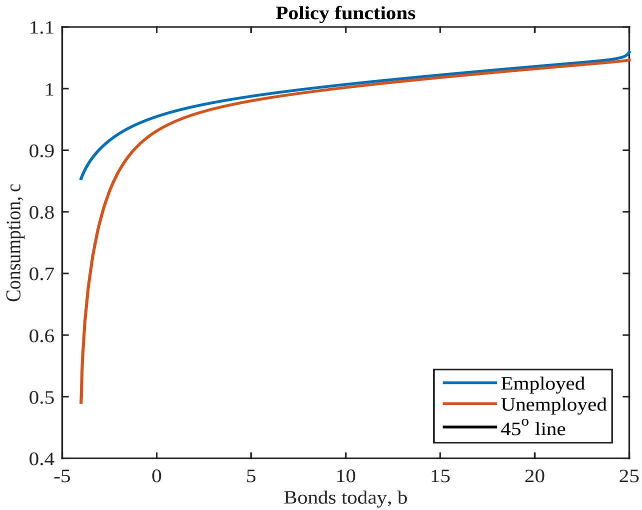
Income fluctuation problem



Income fluctuation problem



Income fluctuation problem



Irreversible investment

Consider the following optimisation problem

$$V(k_0, z_0) = \max_{\{c_t(z^t), k_{t+1}(z^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{z^t \in \mathcal{Z}^{t+1}} \beta^t u(c_t(z^t)) P(z^t, z_0)$$

$$\text{subject to} \quad c_t(z^t) + k_{t+1}(z^t) = z_t f(k_t(z_{t-1})) + (1-\delta) k_t(z^{t-1}),$$

$$k_{t+1}(z^t) \geq (1-\delta) k_t(z^{t-1})$$

$$\forall t, \forall z^t \in \mathcal{Z}^{t+1} \quad k_0, z_0 \text{ are given}$$

Bellman equation

$$v(k, z) = \max_{k' \geq (1-\delta)k} \{u(zf(k) + (1-\delta)k - k') + \beta \sum_{z' \in \mathcal{Z}} v(k', z') p(z', z)\}$$

Irreversible investment

$$v(k,z) = \max_{k' \geq (1-\delta)k} \{u(zf(k) + (1-\delta)k - k') + \beta \sum_{z' \in \mathcal{Z}} v(k',z')p(z',z)\}$$

- First order condition

$$u'(zf(k) + (1-\delta)k - k') - \mu(k,z) = \beta \sum_{z' \in \mathcal{Z}} V_{k'}(k',z')p(z',z)$$

where $\mu(k,z)$ is the Lagrange multiplier on the irreversibility constraint.

Irreversible investment

- Suppose we have a guess $V_{k,n}(k, z)$. Then find $\tilde{k}'$ as

$$u'(zf(k) + (1-\delta)k - \tilde{k}') = \beta \sum_{z' \in \mathcal{Z}} V_{k',n}(\tilde{k}', z')p(z', z)$$

- Optimal k' is then given by $k' = \max\{\tilde{k}', (1-\delta)k\}$

Irreversible investment

- The Lagrange multiplier is

$$\mu_n = u'(zf(k) + (1-\delta)k - k') - \beta \sum_{z' \in \mathcal{Z}} V_{k',n}(k', z') p(z', z)$$

- Lastly, update the guess

$$V_{k,n+1}(k,z) = (1 + zf'(k) - \delta) u'(zf(k) + (1-\delta)k - k') \\ - \mu_n (1-\delta)$$

and iterate until convergence.

Irreversible investment