# Solving Heterogeneous-Agent Models: Bopart, Krusell &Mitman MIT shock approach

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# BKM is part of the Reiter Family

- Generic Reiter: Use first-order perturbation to deal with aggregate fluctuations
  (and projection to deal with individual problem)

- Bopart, Krusell, and Mitman (2018) or BKM: Use MIT shock approach *plus linearity assumption* to deal with aggregate fluctuations

# Bopart, Krusell, Mitman

# Overview Bopart, Krusell, and Mitman

- As in Reiter, individual problem solved with projection method
- As in Reiter, linear solution with respect to aggregate random variables
    - Linearity is assumed, *not* formally derived with perturbation method
    - MIT shock used to get solution for one particular aggregate shock
    - Linearity assumption does the rest

# What is an MIT disturbance?

An MIT shock is a one-time shock that

1. Was believed to never ever occur and
2. Never ever to happen again after it occurs

# MIT shock in rep. agent RBC

- Start with $k_0 = k_{ss}$ and $z_1 = +\sigma + z_{ss}$
- MIT shock $\Rightarrow$ perfect foresight
- System of equations

$$c_t^{-1} = \beta c_{t+1}^{-1} z_{t+1} \alpha k_{t+1}^{\alpha-1}$$
$$c_t + k_{t+1} = z_t k_t^{\alpha}$$
$$\ln z_{t+1} = \rho \ln z_t$$

# MIT shock in rep. agent RBC

- Substitute out consumption

$$
\begin{aligned}
\frac{1}{z_t k_t^{\alpha} - k_{t+1}} &= \beta \frac{z_{t+1} \alpha k_{t+1}^{\alpha-1}}{z_{t+1} k_{t+1}^{\alpha} - k_{t+2}} \\
k_0 & \quad \text{given}
\end{aligned}
$$

- $T$ equations in $T$ unknowns for given final value for $k_t$
- Only one value for $k_1$ such that $k_t$ converges to $k_{ss}$. (See Blanchard-Kahn/Sunspots slides)

# Combining MIT shocks

- Q: What is time path for $k_t$ if $z_1 = \lambda\sigma$?
  A: solution for $z_1 = \sigma$ scaled by $\lambda$
- Q: What is time path for $k_t$ if $z_5 = \sigma$?
  A: same, just shifted five periods
- Q: What is time path for $k_t$ if $z_1 = \sigma$ *and* $z_5 = \sigma$?
  A: the sum of the outcomes for the two individual events

**All answers above are correct under linearity assumption**

Linearity also makes extension to model with more than one type of disturbance

# MIT shock in het. agent model

- Solve for no-aggregate-uncertainty steady state
- Consider a one-time never-again aggregate schock
- Policy rules are now time dependent, assume $k_{i,T+1}$ is the no-aggregate-risk policy rule
- Also assume $K_{T+1} = K_{ss}$
- Solve for transition time path
  - Guess a solution for aggregate capital time path $\{K_t\}_{t=1}^{T}$
  - Solve for individual $k_{i,t}$ policy rules for $t = 1, \ldots, T$ backwards
  - Update guess for timepath for $K_t$

# Sequence-Space Jacobian

- Title of BKM: " *Exploiting MIT shocks in heterogeneous-agent economies: the impulse response as a numerical derivative*.
- But "derivative" is calculated numerically for a shock of certain magnitude.
- SSJ calculates those derivates explicitly. Big advantage: SPEED
- See Auclert, Bardoczy, Rognlie, and Straub (2021)

# References


- Auclert, A. B. Bardoczy, M. Rognlie, and L. Straub, 2021, Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models, Econometrica 89, 2375-2408.
- Bopart, T., Krusell, P., and K. Mitman, 2018, Exploiting MIT shocks in heterogeneous-agent economies: the impulse response as a numerical derivative, Journal of Economic Dynamics and Control 89, 68-92.
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# References


- Reiter, M., 2009, Solving heterogeneous-agent models by projection and perturbation, *Journal of Economic Dynamics and Control*, 32, 1120-1155.
- Winberry, T., 2016, A toolbox for solving and estimating heterogeneous agent macro models. Available at http://faculty.chicagobooth.edu/thomas.winberry/research/winber