Solution Akcigit Ates JPE 2023

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1 Consumer

Consumer solves

$$V_t = \max_{C_s} \int_t^{\infty} \exp\left(-\rho(s-t)\right) \ln C_s \, ds \tag{1}$$

subject to $\dot{A}_t = w_t L_t + r_t A_t + G_t - P_t C_t$.

The Bellman reads

$$\rho V(A_t) = \max_{C_t} \ln C_t + V'(A_t)\dot{A}_t \tag{2}$$

Take derivate w.r.t. C_t gives

$$0 = \frac{1}{C_t} - V'(A_t)P_t$$

So we have

$$C_t = P_t^{-1} V'(A_t)^{-1}$$

And the Envelope theorem

$$\begin{aligned}
(\rho - r_t)V'(A_t) &= V''(A_t)\dot{A}_t \\
(\rho - r_t)(P_t C_t)^{-1} &= -P_t^{-1} C_t^{-2} \frac{dC_t}{dA_t} \frac{dA_t}{dt} \\
\frac{\dot{C}_t}{C_t} &= r_t - \rho \\
r_t &= \rho + g_t
\end{aligned}$$

where $g_t = \frac{\dot{C}_t}{C_t}$ and $V''(A_t) = -P_t^{-1} C_t^{-2} C_t'$.

2 Final Producer

Final producer solves

$$\underset{y_{jt}}{max}\, P_t Y_t - \int_0^1 p_{jt} y_{jt}\, dj \qquad\qquad \text{subject to} \ln Y_t = \int_0^1 \ln y_{jt}\, dj$$

The marginal profit of the Final producer should equal the marginal cost:

$$\begin{aligned}
\frac{\partial P_t Y_t}{\partial y_{jt}} &= p_{jt} \\
\Rightarrow P_t \times \exp\left(\int_0^1 \ln y_{jt}\, dj\right) \times \frac{1}{y_{jt}} &= p_{jt} \\
\Rightarrow P_t Y_t &= p_{jt} y_{jt} \\
\Rightarrow \ln Y_t &= \int_0^1 \ln \frac{P_t Y_t}{p_{jt}}\, dj \\
&= \ln P_t Y_t - \int_0^1 \ln p_{jt}\, dj
\end{aligned}$$

$$\Leftrightarrow lnP_t = \int_0^1 \ln p_{jt}\, dj \tag{3}$$

which is the optimal demand.

3 Sectoral intermediate production

On sectoral level, the intermediate production's optimal supply of products solves

$$\underset{y_{ijt}, y_{-ijt}}{max} P_{jt}Y_{jt} - (p_{ijt}y_{ijt} + p_{-ijt}y_{-ijt}) \qquad \text{subject to } Y_{jt} = (y_{ijt}^{\beta} + y_{-ijt}^{\beta})^{1/\beta}$$

Setting marginal cost and product equal yields

$$P_{jt}\frac{\partial Y_{jt}}{\partial y_{ijt}} = p_{ijt}$$

$$P_{jt}\frac{1}{\beta}(y_{ijt}^{\beta} + y_{-ijt}^{\beta})^{\frac{1-\beta}{\beta}} \cdot \beta y_{ijt}^{\beta-1} = p_{ijt}$$

$$y_{ijt} = (\frac{p_{ijt}}{P_{jt}})^{\frac{1}{\beta-1}} \cdot Y_{jt}$$

Symmetry gives

$$y_{-ijt} = (\frac{p_{-ijt}}{P_{jt}})^{\frac{1}{\beta-1}} \cdot Y_{jt}$$

Substituting into sectoral constraint gives

$$Y_{jt} = ((\frac{p_{ijt}}{P_{jt}})^{\frac{\beta}{\beta-1}} + (\frac{p_{-ijt}}{P_{jt}})^{\frac{\beta}{\beta-1}})^{1/\beta} Y_{jt}$$

This gives the expression of P_{jt}

$$P_{jt} = (p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}})^{\frac{\beta-1}{\beta}} \tag{4}$$

And we combine with the firm level production and the fact that $P_tY_t = p_{jt}y_{jt}$

$$y_{ijt} = \frac{p_{ijt}^{\frac{1}{\beta-1}}}{p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}}} Y_t \tag{5}$$

The market share is given by

$$z_{ijt} = \frac{p_{ijt}y_{ijt}}{Y_t} = \frac{p_{ijt}^{\frac{\beta}{\beta-1}}}{p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}}} \tag{6}$$

4 Firm Production

Firm maximizes its profit by choosing price and production

$$\pi_{ijt} = \max_{y_{ijt}, p_{ijt}} (p_{ijt} - mc_{ijt})y_{ijt} \tag{7}$$

Taking derivative of p_{ijt}

$$y_{ijt} + (p_{ijt} - mc_{ijt})\frac{dy_{ijt}}{dp_{ijt}} = 0$$
$$\frac{p_{ijt}^{\frac{1}{\beta-1}}}{p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}}}Y_t + (p_{ijt} - mc_{ijt})\frac{\frac{1}{\beta-1}p_{ijt}^{\frac{2-\beta}{\beta-1}} - p_{ijt}^{\frac{1}{\beta-1}}\frac{\beta}{\beta-1}p_{ijt}^{\frac{1}{\beta-1}}}{(p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}})^2}Y_t = 0$$

Making use of the definition of z_{ijt} gives

$$(\beta - 1)p_{ijt} + (p_{ijt} - mc_{ijt})(1 - \beta z_{ijt}) = 0$$

We yield

$$p_{ijt} = \frac{1 - \beta z_{ijt}}{\beta(1 - z_{ijt})} mc_{ijt} \tag{8}$$

We derive

$$\begin{aligned}
\frac{p_{-ijt}}{p_{ijt}} &= \frac{1 - \beta z_{-ijt}}{1 - \beta z_{ijt}} \frac{\beta(1 - z_{ijt})}{\beta(1 - z_{-ijt})} \frac{q_{ijt}}{q_{-ijt}} \\
&= \Xi(z_{ijt})\lambda^{\mathbb{F}(m_{ijt})} \\
&= \Xi(z_{ijt})\lambda_{ijt}^{m}
\end{aligned}$$

Suppressing j and we have the labour demand

$$\begin{aligned}
l(m_{it}) &= \frac{y_{it}}{q_{it}} \\
&= q_{it}^{-1}\frac{z_{it}}{p_{it}}Y_t \\
&= \frac{z_{it}}{w_{it}}\frac{\beta(1 - z_{it})}{1 - \beta z_{it}}Y_t \\
&= \omega_t^{-1} z_i t \frac{\beta(1 - z_{it})}{1 - \beta z_{it}}
\end{aligned}$$

where $\omega_t = \frac{Y_t}{w_t}$ is the proportion of labor wage in the economy.

The profit

$$\begin{aligned}\pi(m_{it}) &= (p_{it} - mc_{it})y_{it} \\ &= \frac{(1-\beta)z_{it}}{1-\beta z_{it}}Y_t\end{aligned}$$

And markup

$$\begin{aligned}mu(m_{it}) &= \frac{p_{it}}{mc_{it}} - 1 \\ &= \frac{1-\beta}{\beta(1-z_{it})}\end{aligned}$$

And finally the elasticity of price

$$\begin{aligned}\varepsilon_{ijt} &= \frac{\partial \ln y_{ijt}}{\partial \ln p_{ijt}} \\ &= \frac{\partial}{\partial \ln p_{ijt}}\left(\frac{1}{\beta-1}\ln p_{ijt} - \ln(p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}}) + \ln Y_t\right) \\ &= \frac{1-\beta z_{ijt}}{\beta-1}\end{aligned}$$