Solution Akcigit Ates JPE 2023

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1 Consumer

Consumer solves

$$V_t = \underset{C_s}{max} \int_t^\infty \exp\left(-\rho(s-t)\right) \ln C_s \, ds \tag{1}$$

The Bellman reads

$$V_t(A_t) = \underset{C_s}{max} \ln C_s + \exp\left(-\rho\right) V(A_{t+1}) \tag{2}$$

Set up the Hamiltonian

2 Final Producer

Final producer solves

$$\underset{y_{jt}}{max}\, P_t Y_t - \int_0^1 p_{jt} y_{jt} \, dj \qquad\qquad \text{subject to} \ln Y_t = \int_0^1 \ln y_{jt} \, dj$$

The marginal profit of the Final producer should equal the marginal cost:

$$\frac{\partial P_t Y_t}{\partial y_{jt}} = p_{jt}$$

$$\Rightarrow P_t \times \exp\left(\int_0^1 \ln y_{jt}\, dj\right) \times \frac{1}{y_{jt}} = p_{jt}$$

$$\Rightarrow P_t Y_t = p_{jt} y_{jt}$$

$$\Rightarrow \ln Y_t = \int_0^1 \ln \frac{P_t Y_t}{p_{jt}}\, dj$$

$$= \ln P_t Y_t - \int_0^1 \ln p_{jt}\, dj$$

$$\Leftrightarrow lnP_t = \int_0^1 \ln p_{jt}\, dj \tag{3}$$

which is the optimal demand.

3 Sectoral intermediate production

On sectoral level, the intermediate production's optimal supply of products solves

$$\underset{y_{ijt}, y_{-ijt}}{max} P_{jt} Y_{jt} - (p_{ijt} y_{ijt} + p_{-ijt} y_{-ijt}) \qquad \text{subject to } Y_{jt} = (y_{ijt}^{\beta} + y_{-ijt}^{\beta})^{1/\beta}$$

Setting marginal cost and product equal yields

$$P_{jt} \frac{\partial Y_{jt}}{\partial y_{ijt}} = p_{ijt}$$

$$P_{jt} \frac{1}{\beta} (y_{ijt}^{\beta} + y_{-ijt}^{\beta})^{\frac{1-\beta}{\beta}} \cdot \beta y_{ijt}^{\beta-1} = p_{ijt}$$

$$y_{ijt} = (\frac{p_{ijt}}{P_{jt}})^{\frac{1}{\beta-1}} \cdot Y_{jt}$$

Symmetry gives

$$y_{-ijt} = (\frac{p_{-ijt}}{P_{jt}})^{\frac{1}{\beta-1}} \cdot Y_{jt}$$

Substituting into