RBC Models and Its Extensions

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Outline

1. RBC models and its stylized facts
2. MIU and Fisher Quantity Theory
3. CIA and Friedman Rule

1 RBC and Its stylized facts

In this section, we will use a very simple RBC model to simulate the data like output, consumption, investment labor hours and wage etc. Then we calculate the moments of this simulated data. Finally, we will compare the simulated moments to those in the data. The purpose of this section is to see how well the RBC model behaves and how we can improve the model to fit the data well.

1.1 The model

This is a very simple RBC model, so I will not lay out the detailed model. I will simply list the key equations and definitions. The households utility function

$$U\left(c_t, n_t\right) = logc_t + \theta log\left(1 - n_t\right)$$

where $\theta > 0$ is a parameter. The production technology

$$y_t = a_t k_t^{\alpha} n_t^{1-\alpha}$$

where a_t is the technology shock whose log follows AR(1) process. And output are consumed and invested! The capital accumulate follows the standard law of motion. The FOC reads

$$
\begin{aligned}
\frac{1}{c_t} &= \beta E_t \left(\frac{1}{c_{t+1}} \left(R_{t+1} + 1 - \delta \right) \right) && (1) \\
\frac{\theta}{1 - n_t} &= \frac{1}{c_t} w_t && (2) \\
k_{t+1} &= I_t + \left(1 - \delta \right) k_t && (3) \\
y_t &= a_t k_t^{\alpha} n_t^{1-\alpha} && (4) \\
\frac{1}{c_t} &= \beta E_t \left(\frac{1}{c_{t+1}} r_{t+1} \right) && (5) \\
y_t &= I_t + c_t && (6) \\
w_t &= \left(1 - \alpha \right) a_t k_t^{\alpha} n_t^{-\alpha} && (7) \\
R_t &= \alpha a_t k_t^{\alpha-1} n_t^{1-\alpha} && (8) \\
log a_t &= \rho log a_{t-1} + \epsilon_t && (9)
\end{aligned}
$$

We totally have 9 variables and 9 equations: $a_t, c_t, k_t, I_t, n_t, y_t w_t, R_t, r_t$. w_t is the wage rate and R_t is the capital rental rate. And r_{t+1} is the real rate from holding bonds which is predetermined variable. This rate is known at time t and will be paid out at time $t+1$. If we interested in the average labor productivity, we can define a new variable here

$$yn_t = \frac{y_t}{n_t}$$

as the ratio of output to hours worked.

1.2 The code

The Dynare code reads:

```
%This file is written by Xiangyang Li@Aug.,11,2015
%Simulation to find out success and failures of RBC model.
var a n c k i r w R y yn;
varexo ea;
parameters theta alpha delta beta sigma rho sda;
parameters ns cs ks as is rs ws Rs;

alpha = 0.33;
beta  = 0.99;
delta = 0.025;
sigma = 1;
rho = 0.974; %calibrated
sda = 0.009;  %calibrated

as = 1;
ns = 1/3;
```

```
rs = 1/beta;
kn = (alpha/(1/beta - 1 +delta))^(1/(1-alpha));
ks = ns*kn;
Rs= 1/beta - 1 + delta;
ws = (1-alpha)*kn^alpha;
ys =as*kn^alpha*ns;
is=  delta*ks;
cs = ys - is;
theta = (1-ns)/cs*(1-alpha)*as*kn^alpha;

model;
% (1) Euler equation, capital
exp(c)^(-sigma)=beta*exp(c(+1))^(-sigma)*(R(+1)+(1-delta));

% (2) Euler equation, bonds
exp(c)^(-sigma)=beta*exp(r)*exp(c(+1))^(-sigma);

% (3) Labor supply
theta/(1-exp(n))=exp(c)^(-sigma)*exp(w);

% (4) Production func
exp(y)=exp(a)*exp(k(-1))^(alpha)*exp(n)^(1-alpha);

% (5) Capital demand
R=alpha*exp(a)*exp(k(-1))^(alpha-1)*exp(n)^(1-alpha);

% (6) Labor demand
exp(w)=(1-alpha)*exp(a)*exp(k(-1))^(alpha)*exp(n)^(-alpha);

% (7) Resource constraint
exp(y)=exp(c)+exp(i);

% (8) Capital accumulation
exp(k)=exp(i)+(1-delta)*exp(k(-1));

% (9) Productivity shock (TFP)
a=rho*a(-1)+ea;

%(10) labor average productivity
exp(yn) = exp(y)/exp(n);

end;
initval;
k=log(ks);
y=log(ys);
c=log(cs);
```

```
i=log(is);
a=log(as);
r=log(rs);
R=Rs;
w=log(ws);
n=log(ns);
yn = log(ys/ns);
end;

shocks;
var ea = sda^2;
end;
resid(1);
steady;
check;

%Uses HP filter before computing moments.
stoch_simul(order =1, hp_filter =1600,periods=1000);
```

1.3 The output of simulated moments

The following is the results from Dynare output.

MOMENTS OF SIMULATED VARIABLES (HP filter, lambda = 1600)

VARIABLE	MEAN	STD. DEV.	VARIANCE	SKEWNESS	KURTOSIS
a	-0.019705	0.011743	0.000138	-0.026593	-0.273987
n	-1.099827	0.006976	0.000049	-0.053905	-0.223238
c	-0.295289	0.006503	0.000042	0.053327	-0.336704
k	2.204295	0.004354	0.000019	0.189507	-0.152872
i	-1.483129	0.050254	0.002525	-0.043193	-0.244560
r	0.010332	0.000528	0.000000	-0.064945	-0.200056
w	0.670165	0.009682	0.000094	0.011936	-0.321082
R	0.035362	0.000585	0.000000	-0.063627	-0.202855
y	-0.029184	0.016420	0.000270	-0.018192	-0.286889
yn	1.070643	0.009682	0.000094	0.011936	-0.321082

CORRELATION OF SIMULATED VARIABLES (HP filter, lambda = 1600)

VARIABLE	a	n	c	k	i	r	w	R	y	yn
a	1.0000	0.9878	0.9332	0.3178	0.9958	0.9738	0.9827	0.9758	0.9991	0.9827
n	0.9878	1.0000	0.8659	0.1664	0.9979	0.9973	0.9419	0.9980	0.9802	0.9419
c	0.9332	0.8659	1.0000	0.6373	0.8965	0.8270	0.9836	0.8321	0.9478	0.9836
k	0.3178	0.1664	0.6373	1.0000	0.2299	0.0939	0.4880	0.1030	0.3584	0.4880
i	0.9958	0.9979	0.8965	0.2299	1.0000	0.9905	0.9616	0.9917	0.9909	0.9616

Table 1: Standard Deviation and Relative Standard Deviation

	Model		Data	
Variables	Std. Dev.	Relative Std.Dev	Std. dDev.	Relative Std.Dev
output	0.0164	1	0.017	1
Consumption	0.0065	0.44	0.009	0.53
Investment	0.0503	3.13	0.047	2.76
Hours	0.0070	**0.44**	0.019	**1.12**
Average Produ.	0.010	0.63	0.011	0.65
Wage	0.010	0.63	0.009	0.53
Real rate	0.0005	**0.03**	0.004	**0.24**
TFP	0.012	0.75	0.012	0.71

Sources: Data From USA. All Quantity terms are in real, per capita and log term. Population size is the civilian non-institutionalized population aged 16 and above. GDP from BEA accounts. Average Productivity, Hours and Wage are non-farm business sector from BLS. Real interest rate from three months Treasure Bill rate minus the expected inflation as in Fisher Equation. This is ex-post real interest rate since we replace $\pi_{t+1}$with $E_t\pi_{t+1}$ and in general, they are not the same. Investment is fixed private investment plus consumption on durable goods, like cars.

```
r           0.9738  0.9973  0.8270  0.0939  0.9905  1.0000  0.9148  1.0000  0.9631  0.9148
w           0.9827  0.9419  0.9836  0.4880  0.9616  0.9148  1.0000  0.9184  0.9898  1.0000
R           0.9758  0.9980  0.8321  0.1030  0.9917  1.0000  0.9184  1.0000  0.9655  0.9184
y           0.9991  0.9802  0.9478  0.3584  0.9909  0.9631  0.9898  0.9655  1.0000  0.9898
yn          0.9827  0.9419  0.9836  0.4880  0.9616  0.9148  1.0000  0.9184  0.9898  1.0000


AUTOCORRELATION OF SIMULATED VARIABLES (HP filter, lambda = 1600)

VARIABLE    1       2       3       4       5
a           0.7320  0.4834  0.2513  0.0768 -0.0478
n           0.7247  0.4712  0.2362  0.0611 -0.0624
c           0.7880  0.5756  0.3652  0.1932  0.0570
k           0.9560  0.8505  0.7037  0.5359  0.3618
i           0.7263  0.4740  0.2397  0.0648 -0.0588
r           0.7252  0.4720  0.2370  0.0617 -0.0621
w           0.7545  0.5205  0.2972  0.1239 -0.0052
R           0.7250  0.4717  0.2366  0.0614 -0.0624
y           0.7359  0.4899  0.2594  0.0852 -0.0402
yn          0.7545  0.5205  0.2972  0.1239 -0.0052
```

We now summarize the output from Dynare and compare the moments in Data :

Table 2: Correlation and Auto Correlation

	Data		Model	
Variables	Corr(x,GDP)	Auto Corr	Corr(x,output)	Auto Corr
GDP	1.00	0.85	1.00	0.7359
Consumption	0.76	0.79	**0.9487**	0.7880
Investment	0.79	0.87	**0.9909**	0.7263
Hours	0.88	0.90	**0.9802**	0.7247
Average Produ.	**0.42**	0.72	**0.9898**	0.7545
Wage	**0.10**	0.73	**0.9898**	0.7545
Real rate	**0.00**	**0.42**	**0.9631**	**0.7252**
TFP	0.76	0.75	**0.9991**	0.7320
GDP deflator	-0.13	0.91	-	-

Sources: Data From USA. All Quantity terms are in real, per capita and log term. Population size is the civilian non-institutionalized population aged 16 and above. GDP from BEA accounts. Average Productivity, Hours and Wage are non-farm business sector from BLS. Real interest rate from three months Treasure Bill rate minus the expected inflation as in Fisher Equation. This is ex-post real interest rate since we replace $\pi_{t+1}$with $E_t\pi_{t+1}$ and in general, they are not the same. Investment is fixed private investment plus consumption on durable goods, like cars.

1.4 Success and Failures of RBC Model

There are few points that deserve noticed from above two tables. The Model has successfully reproduced most of the moments in the data. I will not detail on this point. Let's move to the points where RBC model has failed. In the first table,

1. The Model fails to reproduce the relative volatility of hours.
2. The Model fails to reproduce the relative volatility of real interest rates.

In the second table,

1. The wage is too procyclical in the model which is slightly to be so in data
2. The real interest rate is too procyclical in the model which is acyclical in data

2 RBC Extensions

2.1 MIU: Money in Utility

1. Money: paper money has no fundamental value, it is issued just by government fiat.

   (a) Medium of exchange, double coincidence of wants,

(b) unit of account, numeraire
(c) store of value

2. budget constraints
   (a) M_{t-1} stock of money you enter t with
   (b) M_t stock you take to $t+1$, you choose at time t and bring it to $t+1$.
   (c) P_t price of goods in term of money
   (d) B_{t-1} nominal stock of bonds you bring into t , what I means nominal here is that we measuring the bonds in term of money, not of real goods.
   (e) i_t interest rate on bonds that pays out in $t+1$, it is a rate, generically, it is unit-less. it is dated when observed not paid out.

3. Flow budget constraints

   $$P_tC_t+P_t\left(K_{t+1}-(1-\delta)K_t\right)+B_t+M_t \leq W_t^pN_t+R_t^pK_t+\left(1+i_{t-1}\right)B_{t-1}+M_{t-1}$$

   W_t^p where superscript p denotes the nominal wage. In real terms, we have

   $$C_t+\left(K_{t+1}-(1-\delta)K_t\right)+\frac{B_t}{P_t}+\frac{M_t-M_{t-1}}{P_t} \leq w_tN_t+R_tK_t+\left(1+i_{t-1}\right)\frac{B_{t-1}}{P_t}$$

4. Firm problem

   $$\max_{N_t,K_t} P_tA_tF\left(K_t,N_t\right)-W_t^pN_t-R_t^pK_t$$

5. FOC for labor and capital:

   $$P_tA_tF_K\left(K_t,N_t\right)=R_t^p$$

   $$P_tA_tF_N\left(K_t,N_t\right)=W_t^p$$

   How many goods you compensating for working one hour?

6. Government

   $$\ln\left(M_t\right)-\ln\left(M_{t-1}\right)=\left(1-\rho_m\right)\pi^*+\rho_m\left(\ln\left(M_{t-1}\right)-\ln\left(M_{t-2}\right)\right)+\epsilon_{M,t}$$

7. Real money balance $m_t=\frac{M_t}{P_t}$:

8. Utility function

   $$u\left(C_t,N_t,M_t\right)=\frac{C_t^{1-\sigma}-1}{1-\sigma}-\theta\frac{N_t^{1+\eta}}{1+\eta}+\psi\frac{\left(\frac{M_t}{P_t}\right)^{1-v}-1}{1-v}$$

9. Lagrangian: Now we have money in the utility function what would be the economics interpretation of Lagrangian multiplier? it is the marginal utility of extra one dollar. Multiplier is always has the meaning of shadow price of constraints. In real budget constraint, the multiplier means the marginal utility of extra one unit of real goods. The more real money balance you have, you more power for you to trade. This is the cheap way to justify this.

10. You write out the Lagrangian. And then find the focs w.r.t. consumption, capital, bond and money holdings.

11. FOC for consumption
$$\lambda_t = \frac{1}{P_t} C_t^{-\sigma}$$
extra one dollar income, how many goods you can buy?

12. for labor
$$\theta N_t^{\eta} = C_t^{-\sigma} w_t$$
where $w_t = \frac{W_t^p}{P_t}$

13. for capital
$$\lambda_t = \beta E_t \left(\lambda_{t+1} \left(R_{t+1} + 1 - \delta \right) \right)$$

14. for bonds
$$\lambda_t = \beta E_t \left(\lambda_{t+1} \left(\frac{1 + i_t}{1 + \pi_{t+1}} \right) \right)$$

15. Fisher relationship
$$1 + r_t = (1 + i_t) \frac{P_t}{P_{t+1}} = \frac{1 + i_t}{1 + \pi_{t+1}}$$
taking logs at both sides, we have
$$r_t \approx i_t - E_t \left(\pi_{t+1} \right)$$
why is it the case? r_t is the real interest rate. If you forgo one unit of consumption or real goods today, how more goods you get tomorrow? it is $1 + r_t$.

16. inflation risk: You get no risk free real interest rate.

17. for the money
$$\psi \left(m_t \right)^{-v} = C_t^{-\sigma} \left(\frac{i_t}{i_t + 1} \right)$$
hence we have the money demand equation
$$m_t = \psi^{\frac{1}{v}} C_t^{\frac{\sigma}{v}} \left(\frac{i_t}{i_t + 1} \right)^{-\frac{1}{v}}$$

then we have

$$\frac{M_t}{P_t} = \psi^{\frac{1}{v}} C_t^{\frac{\sigma}{v}} \left(\frac{i_t}{i_t + 1} \right)^{-\frac{1}{v}}$$

The real money demand is proportional to consumption since $\frac{\partial M_t}{\partial C_t} > 0$. Consumption is proportional to real money balance according the money demand equation. If you have some shocks that driven consumption up, then real money balance is going up too. This mean that the price levels is going down if nominal money stay fixed. $\frac{\partial M_t}{\partial i_t} < 0$, this makes sense since the interest rate is opportunities cost, money does not pay interest rate and bonds pay back the interest rate.

18. Assume $C_t = Y_t$,$v = \sigma = 1$,$\psi = 1$, then the crazy thing is that we have Fisher Quantity theory

$$M_t = P_t Y_t \left(\frac{1 + i_t}{i_t} \right)$$

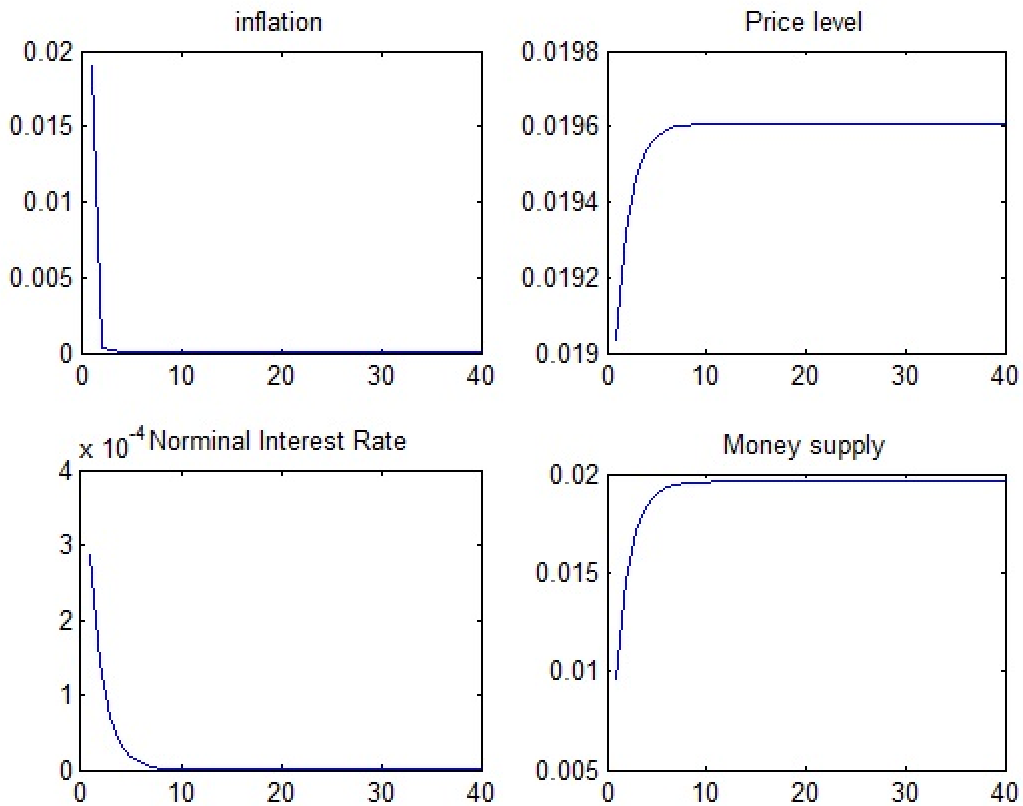
$$M_t V_t = P_t Y_t$$

where the velocity of money is increased function of the interest rates.

$$V_t = \frac{i_t}{i_t + 1}$$

19. In flexible price, money is neutral in both short and long run. In MIU with additionally separable utility, money is super **neutral** (IRF of Real variables are not plotted since they do not respond to the monetary shock, see the figure below). The important thing in MIU model is that money is neutral in the sense that the existence of money firstly does not alter the impulse responses of real variables to real shocks, or in any way shape forms. secondly, changing money supply does not effect real variables.

Figure 1: MIU nominal variables to Monetary supply shock



Since price is not stationary,i.e., do not have a steady state value, then we can not have price in Dynare mod file. What we do here is that we construct IRF for price and nominal money supply after we have IRF for inflation and real money balance. Dynare only plot the IRFs of nominal variables since money is neutral to real variables here in MIU. That is to say that real variables do not response to monetary policy shock.

20. Nominal rigidities (like price rigidity and wage rigidity are two commonly used rigidity in the literature. Sometimes, we interchange rigidity and stickiness later) can make money non-neutral in short run.

21. **Prescott:** It is an established scientific fact that monetary policy has had virtually no effect on output and employment in the U.S. since the formation of the Fed. i.e. nominal quantities had no virtually no effect on real quantities.

2.2 The Dynare Code

```
%MIU, This file is written By xiangyang Li

var c n w R i k a y I dlnm pi m r;
varexo ea em;
parameters theta beta alpha delta psi rhom pistar sigmam sigmaa zeta rhoa;
parameters cs ns ws Rs is ps ks ys Is ms rs;
beta =.99;
alpha =1/3;
delta=.025;
psi = 1;
zeta =1;
rhom=.5;
rhoa = .5;
sigmam =.01;
sigmaa=.01;
pistar = 1.02;
Rs = 1/beta - 1+delta;
kn = (alpha/Rs)^(1/(1-alpha));
ws = (1-alpha)*kn^alpha;
is = pistar/beta;
ns = 1/3;
ks = kn*ns;
Is = delta*ks;
ys = kn^alpha*ns;
cs = ys - Is;
ms = psi^zeta*cs^zeta*(is/(is-1))^zeta;
rs = is/pistar;
theta = ws/cs*(1-ns);

model;
%(1) labor supply equation
theta/(1-exp(n)) = exp(w)/exp(c);

%(2) Euler equation
1/exp(c) = beta*(1/exp(c(+1))* ( exp(R(+1)) + 1-delta) );

%(3) bonds
1/exp(c) = beta*(1/exp(c(+1))* ( exp(i)/exp(pi(+1)) ));

%(4) capital returns
exp(R) = alpha*exp(a)*exp(k(-1))^(alpha -1)*exp(n)^(1-alpha);

%(5) wage
exp(w) = (1-alpha)*exp(a)*exp(k(-1))^alpha*exp(n)^(-alpha);
```

```
%(6) production technology
exp(y) = exp(a)*exp(k(-1))^alpha*exp(n)^(1-alpha);

%(7) capital accumulation
exp(k) = exp(I) + exp(k(-1))*(1 - delta );

%(8) GDP identity
exp(y) = exp(I) + exp(c);

%(9) Fisher relationship
exp(r) = exp(i)/exp(pi(+1));

%(10) real money balance
exp(m) = psi^zeta*exp(c)^zeta*(exp(i)/(exp(i) -1))^zeta;

%(11) monetary shock
dlnm = (1-rhom)*log(pistar) - pi +rhom*(pi(-1)) + rhom*dlnm(-1) +em;

%(12) technology shock
a = rhoa*a(-1) + ea;

%(13) price level and money level
dlnm = m - m(-1);
end;

initval;
c = log(cs);
n = log(ns);
w = log(ws);
R = log(Rs);
i = log(is);
k = log(ks);
a = 0;
y = log(ys);
I = log(Is);
dlnm = 0;
pi = log(pistar);
m = log(ms);
r = log(rs);
end;

resid;
steady;

shocks;
```

```
var ea = sigmaa^2;
var em = sigmam^2;
end;
stoch_simul(order=1);
```

2.3 CIA: Cash in Advance

1. CIA: Money is necessary for consumption essentially, you only add one more constraints and this means that you get more budget constraint or one more multiplier in Lagrangian.

$$M_{t-1} \geq P_t C_t$$

2. the Lagrangian

$$\begin{aligned}\mathcal{L} \quad = \quad & E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \theta \frac{N_t^{1+\eta}}{1+\eta} \right) \\ & -\lambda_t \left(P_t C_t + P_t \left(K_{t+1} - (1-\delta) K_t \right) + B_t + M_t - W_t^p N_t - R_t^p K_t - (1 + i_{t-1}) B_{t-1} - M_{t-1} \right) \\ & +\mu_t \left(M_{t-1} - P_t C_t \right)\end{aligned}$$

For different budget constraints (nominal or real, i.e., divided by price P_t or not), the meaning of the multiplier is different. In this case, the budget constraint is nominal, the multiplier will have the meaning that one extra dollar gives you how much marginal utility, the multiplier has the unit of utility per dollar. Otherwise, it has the meaning that how much marginal utility you have by having one more unit of goods.

3. The foc for consumption and money is now slightly different from the previous one in MIU.

$$C_t^{-\sigma} = P_t \left(\lambda_t + \mu_t \right) \tag{10}$$

$$\lambda_t = \beta E_t \left(\frac{\lambda_{t+1} + \mu_{t+1}}{1 + \pi_{t+1}} \right) \tag{11}$$

μ_t is the shadow price of the CIA constraints, and it means that one extra dollar give you how much utility.

4. For the bonds, still the same FOC

$$\lambda_t = \beta E_t \left(\lambda_{t+1} \frac{(1 + i_t)}{1 + \pi_{t+1}} \right) \tag{12}$$

5. The budget constraint always binds because you have non-satiated preference. By binding means equality hold, i.e., $M_{t-1} = P_t C_t$.

6. Under what scenario will CIA not bind? If not binding, this in turn requires that $\mu_t = 0$. By combining money and bond equations (11,12), we see that zero interest rate $i_t = 0$. What does this means? Holding

bonds will receive nothing since interest rate is zero. In other words, if interest rate is not zero, people will not want to hold extra money and CIA must bind.

7. Why μ_{t+1} in money FOC? If $\mu_t > 0$, this means that it is costly to hold money relative to hold bonds because bonds paying out the interest rates. They never want to hold more money than they need to and hence it could never be non-binding. If the nominal interest rates is zero, holding money is costless relative to hold bonds.

8. And this is actually gives you the **Friedman rule**. It says that it is optimal for the monetary policy to set interest rate to zero. What is the logic of Friedman rule? Friedman basically says that money is costless to produce. Functionally, fiat money is costless to produce. Since the marginal benefit is positive because money reduce the cost of transaction and therefore increase the welfare. But the marginal cost of producing one more unit of money is zero. So you should print sufficient amount of money so to get rid of the marginal cost of holding money from the perspective of households. In other words, you would like bring the social cost of holding money into equality with the marginal cost of producing money which is zero.

9. Basically, Friedman said if $i_t = 0$, you get rid of the constraint. It will be valuable if you get rid of the constraint. You always marginal better off if you get rid of the constraint. You cannot be worse off.

10. What would that imply the level of inflation at steady state if Friedman hold? Inflation has to be negative since real interest rate in steady states has to be positive. steady state real interest rate is the rate of time preference, $\frac{1}{\beta} - 1$, or ρ. So basically, Friedman rule has deflation over time. You have prices falling over time. By nominal price falling, we move the nominal interest rate to zero, and eliminate the opportunity cost of holding money which can only be beneficial. If you have nominal interest rate goes to zero, and this is to say that you have negative steady state inflation rate, price goes down over time and you essentially have real money balance goes to infinity by looking at the money demand equation.

11. Central bankers in practices will not use Friedman rule since they definitely dislike it at Zero-Lower-Bound of money. You need some kind of real effect of monetary policy and some kind of useful rule for endogenous monetary policy to react to economic conditions. If the recession come along, you want to cut interest rate to stimulate demand. You won't do that because you cannot since you have zero interest rate and nominal interest rate can not go negative. That is argument for positive trending inflation. You want lower interest rate, because interest rate dissuade people from holding money and money is useful either because it ease the constraint or because people get utility from it or more generally it solves the double coincidence of want problem.

12. Now in practice, if you want to do some economic stabilization policy, you want interest rate moves up and downs, You do not want be too close to zero or you do not want be too far from zero. The US Fed. functionally has 2% inflation target that you can think the steady state real interest rate is about 1% or 2% means that they targeting at 3% or 4% for nominal interest rate. So there are a little bit above zero, but they might not have about 15% of nominal rate which would give you very high inflation. So this is the gist of Friedman rule.

2.4 The code of CIA

```
%CIA
var c n w R i k a y I dlnm pi m r mu lambda;
varexo ea em;
parameters theta beta alpha delta rhom pistar sigmam sigmaa rhoa;
parameters cs ns ws Rs is ks ys Is ms rs mus lambdas;
beta =.99;
alpha =1/3;
delta=.025;
rhom=.5;
rhoa = .5;
sigmam =.01;
sigmaa=.01;
pistar = 1.02;
Rs = 1/beta - 1+delta;
kn = (alpha/Rs)^(1/(1-alpha));
ws = (1-alpha)*kn^alpha;
is = pistar/beta;
ns = 1/3;
ks = kn*ns;
Is = delta*ks;
ys = kn^alpha*ns;
cs = ys - Is;
ms = cs*pistar;
lambdas = beta/cs/pistar;
mus = 1/cs - lambdas;
rs = is/pistar;
theta = ws*lambdas*(1-ns);

model;
%(1) labor supply equation
theta/(1-exp(n)) = exp(lambda)*exp(w);

%(2) Euler equation
exp(lambda) = beta*(exp(lambda(+1))* ( exp(R(+1)) + 1-delta) );
```

```
%(3) bonds
exp(lambda) = beta*(exp(lambda(+1))* ( exp(i)/exp(pi(+1)) ));

%(4) capital returns
exp(R) = alpha*exp(a)*exp(k(-1))^(alpha -1)*exp(n)^(1-alpha);

%(5) wage
exp(w) = (1-alpha)*exp(a)*exp(k(-1))^alpha*exp(n)^(-alpha);

%(6) production technology
exp(y) = exp(a)*exp(k(-1))^alpha*exp(n)^(1-alpha);

%(7) capital accumulation
exp(k) = exp(I) + exp(k(-1))*(1 - delta );

%(8) GDP identity
exp(y) = exp(I) + exp(c);

%(9) Fisher relationship
exp(r) = exp(i)/exp(pi(+1));

%(10) real money balance
exp(lambda) = beta*(exp(mu(+1)) +exp(lambda(+1)))/exp(pi(+1));

%(11) monetary shock
dlnm = (1-rhom)*pistar - exp(pi) +rhom*exp(pi(-1)) + rhom*dlnm(-1) +em;

%(12) technology shock
a = rhoa*a(-1) + ea;

%(13) price level and money level
dlnm = m - m(-1);

%(14) marginal consumption
1/exp(c) = exp(lambda) + exp(mu);

%(15) CIA
exp(m(-1)) = exp(c)*exp(pi);

end;

initval;
c = log(cs);
n = log(ns);
w = log(ws);
R = log(Rs);
```

```
i = log(is);
k = log(ks);
a = 0;
y = log(ys);
I = log(Is);
dlnm = 0;
pi = log(pistar);
m = log(ms);
r = log(rs);
lambda = log(lambdas);
mu = log(mus);
end;

resid;
steady;

shocks;
var ea = sigmaa^2;
var em = sigmam^2;
end;
stoch_simul(order=1);
```

2.5 IRF of CIA

1. CIA IRFs: Here I show some IRFs of real variables to a unit of positive money supply shocks. Your actually see something weird here. It seems weird relative to NK approach.

   (a) The weird thing here is that output actually decline following the increase of money supply.

   (b) Consumption goes down

   (c) Investment goes up

   (d) Leisure goes down

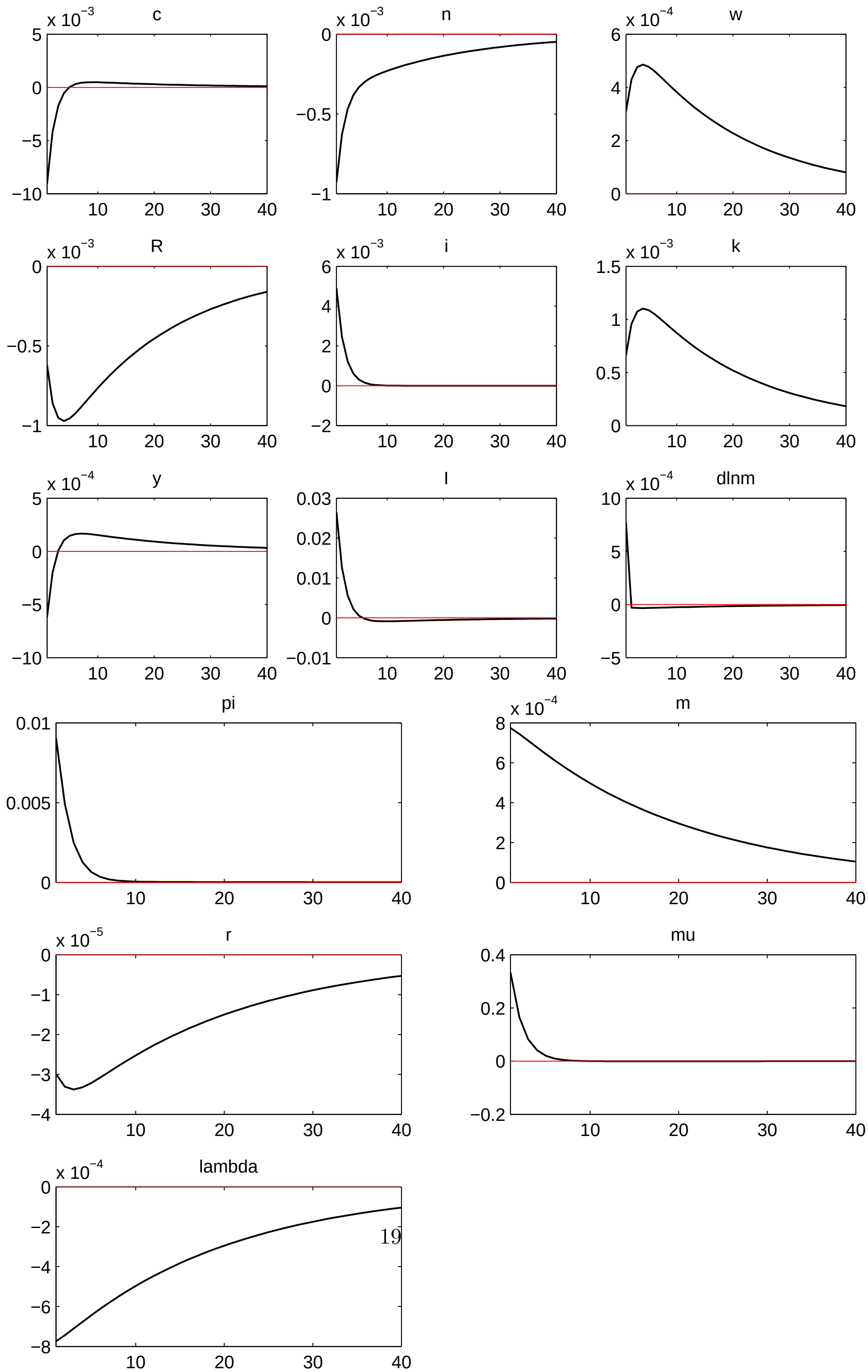
2. Any idea why it might be the case? If you are holding money, inflation sucks. Why? Because your money is worthy less tomorrow if there is inflation. Since real interest rate is kind of fixed, more inflation rises the nominal interest rate which is the opportunities cost of holding money. So If there are persistent increase of money supply that driving up the inflation and nominal interest rate goes up too. If holding money is more costly, it makes sense to give out the things that requires money. So what is that? Consumption. You are going to substitute away from consumption and substitute into which donot require money which are leisure or capital. What you get here is that you consumption goes down, work less and invest

more[1]. After that capital begins to accumulate because investment grows so that cause output goes up. That is kind of counter-intuitive.

3. We typically thinking that You print up more money and we are going to spend up not to reduce it. You can play around with different level of the steady state of inflation which the central bank can choose by setting the mean growth rate of money. The level of output is decreasing with the level of steady state inflation. I get it simpler. The bigger inflation, the bigger nominal interest rate is going to be because $i_t = r_t + \pi_t$ since real rate is relatively fixed. The bigger nominal interest rate, the less consumption, because consumption requires money, the less you going to work. you substitute away from consumption to leisure. And you actually get lower output at higher inflation rate.

[1](Question: If you work less and invest more, why this effect dominates in terms of output: The stock of capital can not move immediately relative bigger to the flow investment, and you actually see output change into positive after few quarters, so capital is remain fixed during the period, so output fall due to the labor goes down.)

Figure 2: CIA,IRF to a money supply shock



4. **CIA, money is not neutral in short and long run both**. This means nominal shocks do have real effects on real variables. You do not get huge effect in the long run unless you have hyperinflation. If you want to think about the hyperinflation in the scenario of Argentina or Germany in the past, it is really bad. If really higher inflation in MIU and it won't do anything since money is neutral. But in CIA constraint model, something like 1000% per year or whatever was, people really do not want to hold money. But people do need money to conduct market transactions and they do not really want to hold money, you will be going to have fewer transactions. So hyperinflation will be costly. When we talk about 2% or 3% percents per years, it wont do too much. Okay, let's call it a day.

5. So to sum up.

   (a) In RBC models, MIU with additively separable utility can be super neutral for money. The crazy stuff is that Under special parameters setting, the FOCs of the MIU model actually produce the Fisher Quantity Identity of Monetary Theory.

   (b) CIA setting actually produces non-neutrality of money albeit it is very small. This means that the nominal shock of money supply has real effect on real variables. The FOCs of the model actually relate to the famous Friedman rule in monetary theory by setting the multiplier of CIA constraint equal to zero.

   (c) At the last, MIU and CIA are just fancy terms for different model settings or structure settings of the RBC or NK models.

6. What do you learn from this class?

   (a) How to program MIU and CIA in DYNARE and know how the two different models setting lead to different implications for the money neutrality.

   (b) You should know why we do not include households budget constraint into equilibrium conditions. Because this condition already implied by the FOCs and equilibrium conditions. (See the notes for Walsh,2010) . For the CIA constraint, the constraint should be included into the model.

3 Appendix

1. Money is a 'good' thing - it relieves exchange frictions. A positive interest rate discourages holding money, however. In addition to relieving exchange rate frictions, one of the things money does is to serve as store of value - you can transfer resources across time with money. But money is 'dominated' by interest-bearing bonds. Hence, the opportunity cost of holding money is the nominal interest rate - the foregone interest rate one

could have earned by holding an interest-bearing asset. Friedman's basic intuition was that it would be costless to engineer a monetary policy of zero interest rates which would raise welfare by eliminating the cost of holding money. This can be formalized in both a money in the utility function and cash in advance function model.

2. **Why nominal interest rates cannot go below zero in theory**.The problem with Friedman rule is that it makes using monetary policy for stabilization difficult because of the fact that nominal rates cannot fall below zero. One may be willing to accept a negative real interest rate if goods are not otherwise storable - a negative real rate of 5 percent, say, would mean loaning 100 goods today in exchange for 95 back tomorrow. That isn't a 'good deal', but if goods aren't storable you may be willing to take it to help better smooth your consumption. Money, in contrast to goods, is storable. A negative nominal interest rate would mean loaning 100 dollars today to get 95 dollars back tomorrow. You would never take that - you'd rather just sit on the 100 dollars. Hence, nominal interest rates cannot go below zero.

3. **ZLB makes the practice of monetary policy difficult.** Standard New Keynesian analysis calls for the central bank to lower nominal interest rates whenever the 'natural interest rate' declines. Just think about the solution of the 3-equation NK model, the output gap and inflation is the positive proportional to the natural interest rate. If natural interest rate goes down (you could have this by a positive technology shock), and by the simple Taylor rule, you should lower the nominal interest rate.