

Solution Akcigit Ates JPE 2023

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1 Consumer

Consumer solves

$$V_t = \max_{C_s} \int_t^{\infty} \exp(-\rho(s-t)) \ln C_s ds \quad (1)$$

subject to $\dot{A}_t = w_t L_t + r_t A_t + G_t - P_t C_t$.

The Bellman reads

$$\rho V(A_t) = \max_{C_t} \ln C_t + V'(A_t) \dot{A}_t \quad (2)$$

Take derivate w.r.t. C_t gives

$$0 = \frac{1}{C_t} - V'(A_t) P_t$$

So we have

$$C_t = P_t^{-1} V'(A_t)^{-1}$$

And the Envelope theorem

$$\begin{aligned}
(\rho - r_t)V'(A_t) &= V''(A_t)\dot{A}_t \\
(\rho - r_t)(P_t C_t)^{-1} &= -P_t^{-1}C_t^{-2}\frac{dC_t}{dA_t}\frac{dA_t}{dt} \\
\frac{\dot{C}_t}{C_t} &= r_t - \rho \\
r_t &= \rho + g_t
\end{aligned}$$

where $g_t = \frac{\dot{C}_t}{C_t}$ and $V''(A_t) = -P_t^{-1}C_t^{-2}C'_t$.

2 Final Producer

Final producer solves

$$\max_{y_{jt}} P_t Y_t - \int_0^1 p_{jt} y_{jt} dj \quad \text{subject to } \ln Y_t = \int_0^1 \ln y_{jt} dj$$

The marginal profit of the Final producer should equal the marginal cost:

$$\begin{aligned}
\frac{\partial P_t Y_t}{\partial y_{jt}} &= p_{jt} \\
\Rightarrow P_t \times \exp\left(\int_0^1 \ln y_{jt} dj\right) \times \frac{1}{y_{jt}} &= p_{jt} \\
\Rightarrow P_t Y_t &= p_{jt} y_{jt} \\
\Rightarrow \ln Y_t &= \int_0^1 \ln \frac{P_t Y_t}{p_{jt}} dj \\
&= \ln P_t Y_t - \int_0^1 \ln p_{jt} dj \\
\Leftrightarrow \ln P_t &= \int_0^1 \ln p_{jt} dj
\end{aligned} \tag{3}$$

which is the optimal demand.

3 Sectoral intermediate production

On sectoral level, the intermediate production's optimal supply of products solves

$$\max_{y_{ijt}, y_{-ijt}} P_{jt} Y_{jt} - (p_{ijt} y_{ijt} + p_{-ijt} y_{-ijt}) \quad \text{subject to } Y_{jt} = (y_{ijt}^\beta + y_{-ijt}^\beta)^{1/\beta}$$

Setting marginal cost and product equal yields

$$\begin{aligned} P_{jt} \frac{\partial Y_{jt}}{\partial y_{ijt}} &= p_{ijt} \\ P_{jt} \frac{1}{\beta} (y_{ijt}^\beta + y_{-ijt}^\beta)^{\frac{1-\beta}{\beta}} \cdot \beta y_{ijt}^{\beta-1} &= p_{ijt} \\ y_{ijt} &= \left(\frac{p_{ijt}}{P_{jt}} \right)^{\frac{1}{\beta-1}} \cdot Y_{jt} \end{aligned}$$

Symmetry gives

$$y_{-ijt} = \left(\frac{p_{-ijt}}{P_{jt}} \right)^{\frac{1}{\beta-1}} \cdot Y_{jt}$$

Substituting into sectoral constraint gives

$$Y_{jt} = \left(\left(\frac{p_{ijt}}{P_{jt}} \right)^{\frac{\beta}{\beta-1}} + \left(\frac{p_{-ijt}}{P_{jt}} \right)^{\frac{\beta}{\beta-1}} \right)^{1/\beta} Y_{jt}$$

This gives the expression of P_{jt}

$$P_{jt} = (p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}})^{\frac{\beta-1}{\beta}} \quad (4)$$

And we combine with the firm level production and the fact that $P_t Y_t = p_{jt} y_{jt}$

$$y_{ijt} = \frac{p_{ijt}^{\frac{1}{\beta-1}}}{p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}}} Y_t \quad (5)$$

The market share is given by

$$z_{ijt} = \frac{p_{ijt} y_{ijt}}{Y_t} = \frac{p_{ijt}^{\frac{\beta}{\beta-1}}}{p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}}} \quad (6)$$

4 Firm Production

Firm maximizes its profit by choosing price and production

$$\pi_{ijt} = \max_{y_{ijt}, p_{ijt}} (p_{ijt} - mc_{ijt}) y_{ijt} \quad (7)$$

Taking derivative of p_{ijt}

$$\begin{aligned} y_{ijt} + (p_{ijt} - mc_{ijt}) \frac{dy_{ijt}}{dp_{ijt}} &= 0 \\ \frac{p_{ijt}^{\frac{1}{\beta-1}}}{p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}}} Y_t + (p_{ijt} - mc_{ijt}) \frac{\frac{1}{\beta-1} p_{ijt}^{\frac{2-\beta}{\beta-1}} - p_{ijt}^{\frac{1}{\beta-1}} \frac{\beta}{\beta-1} p_{ijt}^{\frac{1}{\beta-1}}}{(p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}})^2} Y_t &= 0 \end{aligned}$$

Making use of the definition of z_{ijt} gives

$$(\beta - 1)p_{ijt} + (p_{ijt} - mc_{ijt})(1 - \beta z_{ijt}) = 0$$

We yield

$$p_{ijt} = \frac{1 - \beta z_{ijt}}{\beta(1 - z_{ijt})} mc_{ijt} \quad (8)$$

We derive

$$\begin{aligned} \frac{p_{-ijt}}{p_{ijt}} &= \frac{1 - \beta z_{-ijt}}{1 - \beta z_{ijt}} \frac{\beta(1 - z_{ijt})}{\beta(1 - z_{-ijt})} \frac{q_{ijt}}{q_{-ijt}} \\ &= \Xi(z_{ijt}) \lambda^{\mathbb{F}(m_{ijt})} \\ &= \Xi(z_{ijt}) \lambda_{ijt}^m \end{aligned}$$

Suppressing j and we have the labour demand

$$\begin{aligned} l(m_{it}) &= \frac{y_{it}}{q_{it}} \\ &= q_{it}^{-1} \frac{z_{it}}{p_{it}} Y_t \\ &= \frac{z_{it}}{w_{it}} \frac{\beta(1 - z_{it})}{1 - \beta z_{it}} Y_t \\ &= \omega_t^{-1} z_{it} \frac{\beta(1 - z_{it})}{1 - \beta z_{it}} \end{aligned}$$

where $\omega_t = \frac{Y_t}{w_t}$ is the proportion of labor wage in the economy.

The profit

$$\begin{aligned}\pi(m_{it}) &= (p_{it} - mc_{it})y_{it} \\ &= \frac{(1 - \beta)z_{it}}{1 - \beta z_{it}} Y_t\end{aligned}$$

And markup

$$\begin{aligned}mu(m_{it}) &= \frac{p_{it}}{mc_{it}} - 1 \\ &= \frac{1 - \beta}{\beta(1 - z_{it})}\end{aligned}$$

And finally the elasticity of price

$$\begin{aligned}\varepsilon_{ijt} &= \frac{\partial \ln y_{ijt}}{\partial \ln p_{ijt}} \\ &= \frac{\partial}{\partial \ln p_{ijt}} \left(\frac{1}{\beta - 1} \ln p_{ijt} - \ln(p_{ijt}^{\frac{\beta}{\beta-1}} + p_{-ijt}^{\frac{\beta}{\beta-1}}) + \ln Y_t \right) \\ &= \frac{1 - \beta z_{ijt}}{\beta - 1}\end{aligned}$$