New Keynesian Model

Xiangyang Li

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1 Actors

1. Households
   (a) Firms
      i. competitive final goods firm
      ii. continuum monopolistic competitive intermediates
   (b) Government
      i. monetary and fiscal policy

2 The Model

2.1 Households

1. You write out the utility functions with MIU, households choose consumption,labor,bonds and money. We will talk about MIU more later.

$$\max_{C_t,N_t,B_{t+1},M_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \psi \frac{N_t^{1+\eta}}{1+\eta} + \gamma \log \left(\frac{M_t}{P_t} \right) \right)$$

$$P_t C_t + B_{t+1} + M_t \leq W_t^p N_t + \Pi_t + T_t + (1+i_{t-1}) B_t + M_{t-1}$$

where B_t is the bond that households have when enter the period t which pay out the dividends i_{t-1}. This dividend is known at time $t-1$ and will pay out at time t. M_{t-1} the money the households have when enter the period t. The aggregate supply of money in period M_t is not going to be predetermined but rather set by a central bank. It is easy to show that σ^{-1} is the inter-temporal elasticity of substitution. At the same time, σ is the Relative Risk Aversion, RRA and the curvature-like[1] parameter of

[1] Some facts about curvature. The straight line has zero curvature. and The circle has the constant curvature which is the reciprocal of radius. Roughly speaking, the curvature represents how much the curve deviates from the straight line. Generally, the curvature in mathmatics has a form definiton as $\kappa = |\frac{y''}{(1+(y')^2)^{\frac{3}{2}}}|$ for 2D curve $y = f(x)$.

utility functions. The bigger the σ is, the bigger curvature, and the larger risk averse (the flat the utility function is). Π_t is the dividends receives from firm which assumed to be possessed by households.

2. You write out the Lagrangian and interpret the meaning of multiplier which could change if you use different form of budget constraint: divided by price level or not, whether the constraint in nominal or real term.

3. FOCs for four choices variables

$$\begin{aligned}\mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{N_t^{1+\eta}}{1+\eta} + \gamma \log\left(\frac{M_t}{P_t}\right) \right) \\ &\quad \lambda_t \left(W_t^p N_t + \Pi_t + T_t + (1 + i_{t-1}) B_t + M_{t-1} - P_t C_t - B_{t+1} - M_t \right) \Bigg\}\end{aligned}$$

the focs with respect to (w.r.t.) consumption, labor, bond and money as follows:

$$\begin{aligned} C_t^{-\sigma} &= \lambda_t P_t \\ \psi N_t^{\eta} &= \lambda_t W_t^p \\ \lambda_t &= \beta E_t \lambda_{t+1} (1 + i_t) \\ \gamma \frac{1}{M_t} &= \lambda_t - \beta E_t \lambda_{t+1} \end{aligned}$$

By eliminating the multiplier and above conditions can be re-written as

$$\begin{aligned} \psi N_t^{\eta} &= C_t^{-\sigma} w_t \\ C_t^{-\sigma} &= \beta E_t C_{t+1}^{-\sigma} (1 + i_t) \frac{P_t}{P_{t+1}} \\ \gamma \left(\frac{M_t}{P_t}\right)^{-1} &= \frac{i_t}{1 + i_t} C_t^{-\sigma} \end{aligned}$$

where w_t is real wage.

2.2 Production

2.2.1 Final Good

1. Final Good producer employs a Dixit-Stiglizt aggregator

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

where j is an index for immediate good producer. $\epsilon_p > 1$ is the elasticity of substitution[2] between different immediate good. This means that there

[2]When ϵ_p goes to infinity, this means that the elasticity will goes to infinity. We can easily to see that the production technology will be linear at this time, hence, perfect substitution. When ϵ_p goes to zero, there will be no substitution.

are imperfect substitution between immediate good and this in turn means that immediate good will have some monopolistic power. The profit maximization problem of final goods producer which take the final good price P_t and intermediate good $P_t(j)$ as given:

$$\max_{Y_t(j)} P_t Y_t - \int_0^1 P_t(j) Y_t(j) \, dj$$

$$P_t \frac{\epsilon_p - 1}{\epsilon_p} \left(\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1} - 1} \frac{\epsilon_p}{\epsilon_p - 1} Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p} - 1} = P_t(j) \qquad (1)$$

It easy to see that the right hand side of Eq.(1) is marginal cost and left-hand side is marginal benefit. This will produce a downward sloping demand curve for variety j.

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t$$

This is to say that the relative demand for immediate good j is a function of its relative price, with $\epsilon_p$as the price elasticity of substitution. Zero profit condition for final good producer:

$$P_t Y_t = \int_0^1 P_t(j) Y_t(j) \, dj$$

Replacing the demand for variety j produces the aggregate price level equation:

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_p} dj \right)^{\frac{1}{1-\epsilon_p}}$$

2. The GDP is the aggregation of nominal quantity to derive the Price index evolution

2.2.2 Intermediate good producers

A typical intermediate good producer employs the following constant return to scale technology to produce $Y_t(j)$ while facing a common technology shock A_t

$$Y_t(j) = A_t N_t(j)$$

2.3 Price rigidity

Immediate good producers face a common wage W_t^p. They are assumed to be not freely adjust the price of their good.

1. Each period $1 - \phi$ fixed probability of firms allowed to adjust $P_t(j)$

(a) this adjustment is independent of last adjustment or how far from optimal price

(b) with probability ϕ price you charge in period t is last chosen price;

2. split problem into two parts

   (a) cost minimization of picking $N_t(j)$;

   (b) look at problem of firm getting to adjust, and this produce dynamics of prices

3. cost minimization

$$\min_{N_t(j)} W_t^p N_t(j)$$

   s.t

$$A_t N_t(j) \geq \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$$

4. the Lagrangian

$$\mathcal{L} = -W_t^p N_t(j) + \psi_t(j)\left(A_t N_t(j) - \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t\right)$$

5. FOC

$$W_t^p = \psi_t(j) A_t \equiv \psi_t A_t \equiv mc_t P_t A_t$$

   where $mc_t \equiv \frac{\psi_t}{P_t}$ as the real marginal cost where we drop the subscript j since the marginal cost is common among all intermediate good producers.

Real flow profit

$$\frac{\Pi_t(j)}{P_t} = \frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t^p}{P_t} N_t(j) = \frac{P_t(j)}{P_t} Y_t(j) - mc_t Y_t(j) = P_t(j)^{1-\epsilon_p} P_t^{\epsilon_p - 1} Y_t - mc_t P_t(j)^{-\epsilon_p} P_t^{\epsilon_p} Y_t$$

2.3.1 The 2nd part.

1. Each period, there is a fixed probability $1-\phi$ that an intermediate good firm can adjust their price $P_t(j)$. This means that there is a fixed probability ϕ that the firm can not adjust price. Hence a firm can get stuck at a price for many periods before it can adjust again. The chance for getting stuck for one period is ϕ, and for 2 periods is ϕ^2, and so on. The chance for getting stuck for more and more periods will be smaller and smaller since $0 < \phi < 1$. Think about the problem the firm will be stuck there for ever. The Chance that price chosen today operative s periods from now is

$$\phi^s$$

   The firm will use Stochastic Discount Factor (SDF)

$$M_{t+s} = \beta^s \frac{u'(C_{t+s})}{u'(C_t)}$$

to discount future profit by multiplying $\phi^s M_{t+s}$ and maximize the overall profits as follows:

2. The dynamics

$$\max_{P_t(j)} E_t \sum_{s=0}^{\infty} (\phi\beta)^s \frac{u'(C_{t+s})}{u'(C_t)} \left(\frac{P_t(j)}{P_{t+s}} \left(\frac{P_t(j)}{P_{t+s}} \right)^{-\epsilon_p} Y_{t+s} - mc_{t+s} \left(\frac{P_t(j)}{P_{t+s}} \right)^{-\epsilon_p} Y_{t+s} \right)$$

The first term in brace is the revenue firm j received from its sellings and the second term represents it total cost. Multiplying out, we get,

$$\max_{P_t(j)} E_t \sum_{s=0}^{\infty} (\phi\beta)^s \frac{u'(C_{t+s})}{u'(C_t)} \left(P_t(j)^{1-\epsilon_p} P_{t+s}^{\epsilon_p - 1} Y_{t+s} - mc_{t+s} P_t(j)^{-\epsilon_p} P_{t+s}^{\epsilon_p} Y_{t+s} \right)$$

3. FOC w.r.t. $P_t(j)$

$$P_t^{\star} \equiv P_t(j) = \frac{\epsilon_p}{\epsilon_p - 1} \frac{E_t \sum_{s=0}^{\infty} (\phi\beta)^s u'(C_{t+s}) mc_{t+s} P_{t+s}^{\epsilon_p} Y_{t+s}}{E_t \sum_{s=0}^{\infty} (\phi\beta)^s u'(C_{t+s}) mc_{t+s} P_{t+s}^{\epsilon_p - 1} Y_{t+s}} \equiv \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1t}}{X_{2t}}$$

4. We drop the subscript j and define the optimal price $P_t^{\star}$as above and define two auxiliary variables. Then we write the denominator and numerator recursively so that we can easily put them into Dynare mod block.

$$X_{1t} = u'(C_t) mc_t P_t^{\epsilon_p} Y_t + \phi\beta E_t X_{1t+1}$$

$$X_{2t} = u'(C_t) P_t^{\epsilon_p - 1} Y_t + \phi\beta E_t X_{2t+1}$$

$$P_t^{\star} = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1t}}{X_{2t}}$$

when $\phi = 0$, that means the firm can freely adjust there price. There is no price stickiness anymore. you find out the $X_{1t} = u'(C_t) mc_t P_t Y_t$and $X_{2t} = u'(C_t) P_t^{\epsilon_p - 1} Y_t$ which is exactly the same as the flexible price case, we have $P_t(j) = \frac{\epsilon_p}{\epsilon_p - 1} mc_t P_t$.

2.4 Equilibrium and aggregation

1. Exogenous states A_t and money M_t

$$\Delta \log M_t = (1 - \rho_m)\pi + \rho_m \Delta \log M_{t-1} + \epsilon_t^m$$

The steady states of $\Delta \log M_t = \log M_t - \log M_{t-1}$ is equal to the steady state inflation π.

2. In equilibrium $B_t = 0$ and $T_t = M_t - M_{t-1}$.

3. In equilibrium, the budget constraint becomes using above conditions,

$$P_t C_t = W_t^p N_t + \Pi_t$$

$$C_t = w_t^p N_t + \frac{\Pi_t}{P_t}$$

$$\begin{aligned}
\frac{\Pi_t}{P_t} = \int_0^1 \frac{\Pi_t(j)}{P_t}\mathrm{d}j &= \int_0^1 \left(\frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t^p}{P_t} N_t(j)\right)\mathrm{d}j \\
&= \int_0^1 \left(\frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t^p}{P_t} N_t(j)\right)\mathrm{d}j \\
&= Y_t - w_t^p N_t
\end{aligned}$$

using the two facts:

$$N_t = \int_0^1 N_t(j)\, dj$$

$$P_t^{1-\epsilon_p} = \int_0^1 (P_t(j))^{1-\epsilon_p}\, dj$$

This means that $C_t = Y_t$

4. Aggregation the intermediate producers production functions, you get the price dispersion.

$$Y_t = \frac{A_t N_t}{v_t^p}$$

where

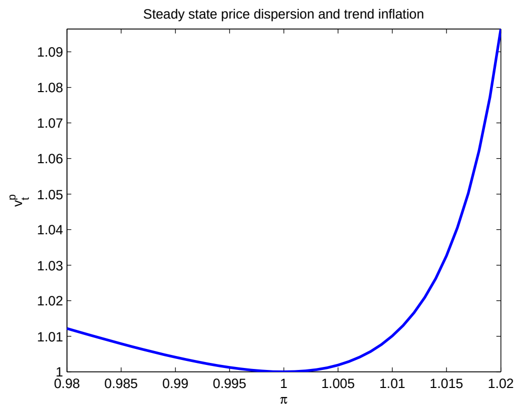
$$v_t^p = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} \mathrm{d}j$$

is the price dispersion. If there is no price rigidity, i.e. $\phi = 0$, then $P_t(j) = P_t^\star$, for any j. then we have $P_t = P_t^\star$,then $v_t^p = 1$. If $\phi = 1$. This is another extreme. That is to say no firms can adjust their price in each period, and there is full price rigidity. In real life, both full flexible and full rigidity price does not seems exist. Hence, in generally, we set $\phi \in (0, 1)$. You can calculate the average time that a firm can adjust its price is $\frac{1}{1-\phi}$.

5. Price dispersion v_t^p is the second order term. In first order it is unity around the zero inflation steady state. You can plot the steady state value of v_t^p against the steady state value of inflation rate. And you will find the steady state value of price dispersion is the convex function of steady state inflation.

6. The full set of equilibrium conditions, 14 equations with 14 variables:

Figure 1: Price Dispersion and Price inflation



(a) three equations for price rigidity;

$$X_{1t} = C_t^{-\sigma} mc_t P_t^{\epsilon_p} Y_t + \phi\beta E_t X_{1t+1}$$
$$X_{2t} = C_t^{-\sigma} P_t^{\epsilon_p - 1} Y_t + \phi\beta E_t X_{2t+1}$$
$$P_t^{\star} = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1t}}{X_{2t}}$$

(b) one for market clear condition $C_t = Y_t$

(c) price level and will finally rewrite in inflation;

$$P_t = \left(\int_0^1 P_t\left(j\right)^{1-\epsilon_p} dj \right)^{\frac{1}{1-\epsilon_p}}$$

(d) aggregation of production with price dispersion;

$$Y_t = \frac{A_t N_t}{v_t^p}$$

(e) definition of price dispersion, finally will rewrite recursively: we can suck whole things out of the integral;

$$v_t^p = \int_0^1 \left(\frac{P_t\left(j\right)}{P_t} \right)^{-\epsilon_p} \mathrm{d}j$$

(f) real marginal cost $mc_t = \frac{w_t}{A_t}$

(g) Euler equations

$$\begin{aligned} \psi N_t^{\eta} &= C_t^{-\sigma} w_t \\ C_t^{-\sigma} &= \beta E_t C_{t+1}^{-\sigma} \left(1 + i_t\right) \frac{P_t}{P_{t+1}} \\ \gamma \left(\frac{M_t}{P_t} \right)^{-1} &= \frac{i_t}{1 + i_t} C_t^{-\sigma} \end{aligned}$$

(h) labor supply

(i) money demand

(j) two for the exogenous states, one is for money supply.

$$log\left(A_t\right) = \rho_a log\left(A_{t-1}\right) + \epsilon_t^a$$

$$\Delta \log M_t = \left(1 - \rho_m\right)\pi + \rho_m \Delta \log M_{t-1} + \epsilon_t^m$$

$$\Delta \log M_t = \log M_t - \log M_{t-1}$$

7. Issues

(a) To get rid of the heterogeneity in price level and price dispersion.

(b) rewrite in stationary terms

i. replace price level with inflation

$$\pi_t \equiv \frac{P_t}{P_{t-1}}$$

$$\pi_t^\star \equiv \frac{P_t^\star}{P_{t-1}}$$

$$x_{1t} \equiv \frac{X_{1t}}{P_t^{\epsilon_p}}$$

$$x_{2t} \equiv \frac{X_{2t}}{P_t^{\epsilon_p - 1}}$$

ii. replace nominal money with real money balance. $m_t = \frac{M_t}{P_t}$

8. How to rewrite v_t^p and price index equation recursively to get rid of heterogeneity? The Calvo assumption will allow us to aggregate out heterogeneity and not worry about keeping track of what is individual firm doing. This is the beauty of Calvo assumption. Since firm who get to update their price are randomly chosen, and because there are a large number of firms, the integral of individual prices over some subset of the unit interval will be proportional to the integral over the entire unit interval, where the proportion is equal to the length of the subset.

$$v_t^p = \int_0^1 \left(\frac{P_t\left(j\right)}{P_t}\right)^{-\epsilon_p} \mathrm{d}j = \int_0^{1-\phi} \left(\frac{P_t^\star}{P_t}\right)^{-\epsilon_p} \mathrm{d}j + \int_{1-\phi}^1 \left(\frac{P_{t-1}\left(j\right)}{P_t}\right)^{-\epsilon_p} \mathrm{d}j$$

$$\int_{1-\phi}^1 \left(\frac{P_{t-1}\left(j\right)}{P_t}\right)^{-\epsilon_p} \mathrm{d}j = \phi \pi_t^{\epsilon_p} \int_0^1 \left(\frac{P_{t-1}\left(j\right)}{P_{t-1}}\right)^{-\epsilon_p} \mathrm{d}j = \phi \pi_t^{\epsilon_p} v_{t-1}^p$$

$$v_t^p = \left(1-\phi\right)\left(\pi_t^*\right)^{-\epsilon_p} \pi_t^{\epsilon_p} + \pi_t^{\epsilon_p} \phi v_{t-1}^p$$

For the price index evolution equation, we can rewrite similarly.

9. The rewritten full set of equilibrium conditions (14 equations):

$$x_{1t} = C_t^{-\sigma} mc_t Y_t + \phi\beta E_t x_{1t+1}\pi_{t+1}^{\epsilon_p} \tag{2}$$

$$x_{2t} = C_t^{-\sigma} Y_t + \phi\beta E_t x_{2t+1}\pi_{t+1}^{\epsilon_p - 1} \tag{3}$$

$$C_t = Y_t \tag{4}$$

$$\pi_t^{\star} = \frac{\epsilon_p}{\epsilon_p - 1}\pi_t \frac{x_{1t}}{x_{2t}} \tag{5}$$

$$\pi_t^{1-\epsilon_p} = (1-\phi)\left(\pi_t^{\star}\right)^{1-\epsilon_p} + \phi \tag{6}$$

$$Y_t = \frac{A_t N_t}{v_t^p} \tag{7}$$

$$v_t^p = (1-\phi)\left(\pi_t^{*}\right)^{-\epsilon_p}\pi_t^{\epsilon_p} + \pi_t^{\epsilon_p}\phi v_{t-1}^p \tag{8}$$

$$mc_t = \frac{w_t}{A_t} \tag{9}$$

$$\begin{aligned} \psi N_t^{\eta} &= C_t^{-\sigma} w_t & (10)\\ C_t^{-\sigma} &= \beta E_t C_{t+1}^{-\sigma}\left(1+i_t\right)\pi_{t+1}^{-1} & (11)\\ \gamma\left(m_t\right)^{-1} &= \frac{i_t}{1+i_t} C_t^{-\sigma} & (12) \end{aligned}$$

$$log\left(A_t\right) = \rho_a log\left(A_{t-1}\right) + \epsilon_t^a \tag{13}$$

$$\Delta \text{log} m_t = (1-\rho_m)\pi - \pi_t + \rho_m \Delta \text{log} m_{t-1} + \rho_m \pi_{t-1} + \epsilon_t^m \tag{14}$$

$$\Delta \text{log} m_t = \text{log} m_t - \text{log} m_{t-1} \tag{15}$$

This is 14 equations with 14 variables $C_t, i_t, \pi_t, N_t, mc_t, w_t, m_t, Y_t, v_t^p, \pi_t^{\star}, x_{1t}, x_{2t}, A_t, \Delta \text{log} m_t$. We will add one more variables $r_t \equiv i_t - E_t\pi_{t+1}$, the real rate.

2.5 Flexible price equilibrium

1. Flexible price equilibrium (noticed by super-script f): this is to say $\phi = 0$.

2. From price evolution equation, we have the $\pi_t^{\star} = \pi_t$.

3. From price rigidity condition, we have $mc_t^f = \frac{\epsilon_p - 1}{\epsilon_p}$, i.e. $P_t = \frac{\epsilon_p - 1}{\epsilon_p}\psi_t$, the real marginal cost is constant.

4. From marginal cost: $w_t^f = \frac{\epsilon_p - 1}{\epsilon_p} A_t$ this means that wage is less than the marginal product of labor. This is distortion. For the planner's problem, planner always set the wage to the marginal product of labor.

5. From labor supply equation, we have $\psi\left(N_t^f\right)^{\eta} = \left(Y_t^f\right)^{-\sigma}\frac{\epsilon_p - 1}{\epsilon_p} A_t$

6. Price dispersion is constant: $v_t^p \equiv 1$. Hence, $Y_t^f = A_t N_t^f$. Then we have $N_t^f = \left(\frac{1}{\psi}\frac{\epsilon_p - 1}{\epsilon_p}\right)^{\frac{1}{\eta+\sigma}} A_t^{\frac{1-\sigma}{\eta+\sigma}}$ and

$$Y_t^f = A_t N_t^f = \left(\frac{1}{\psi}\frac{\epsilon_p - 1}{\epsilon_p}\right)^{\frac{1}{\eta+\sigma}} A_t^{\frac{1+\eta}{\eta+\sigma}}$$

   The flexible output will only respond to real shock, not nominal shock. That is to say that with flexible price, nominal shocks have no real effect.

7. If $\sigma = 1$, i.e., we will have log-utility over consumption, this setting is consistent with KPR(1988)[3]. And this mean that flexible price labor will do not respond to technology shock. What's going on behind? Actually, when set with log utility, the income and substitution effect of consumption from technology will exactly offset with each other.

8. Steady states with price rigidity: Assume $\pi^\star = 0$ in money supply equation this mean that money growth rate is zero. This assumption is kind of unrealistic but it is not far off the truth.

9. Use the flexible price output and define the output gap in Dynare as the log difference.

$$X_t \equiv logY_t - logY_t^f$$

2.6 Efficient allocation

1. Two distortions

   (a) monopolistic competition leads to $y_t^f \neq y_t^e$, where y_t^e is the efficient allocation when no price stickiness and no monopolistic competition and y_t^f is the flexible output when only price is flexible and monopolistic exists. This could be solved by labor income subsidy. see below for more details. Monopolistic competition is kind of long-run distortion.

   (b) price rigidity leads to $y_t \neq y_t^f$ where y_t is actual output when both price and monopolistic competition exist in the model. Price stickiness is kind of short-run distortion.

2. Flexible price allocation

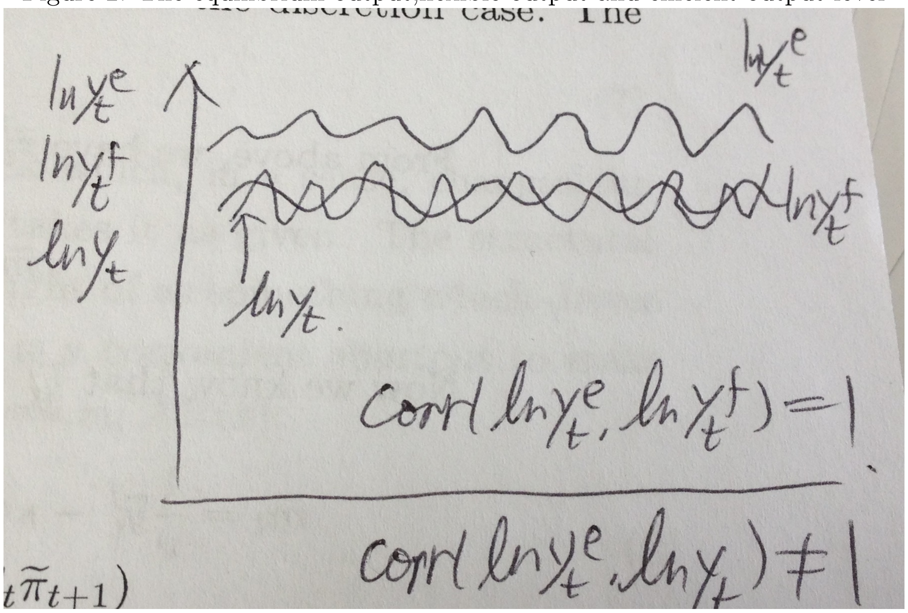
3. labor supply: N^s: $\psi\left(N_t^f\right)^\eta = C_t^{-\sigma} w_t^f$

4. labor demand $N^d : w_t^f = \frac{\epsilon-1}{\epsilon} A_t$

5. efficient output $y_t^e = A_t N_t^e$ and flexible output level $y_t^f = A_t N_t^f$.

[3] King, R. G. and C. I. Plosser, et al. (1988). "Production, growth and business cycles: I. The basic neoclassical model." Journal of Monetary Economics 21 (2–3): 195-232.

Figure 2: The equilibrium output,flexible output and efficient output level



we can see the efficient output level should have perfect correlation with flexible price output level since both are linear functions of technology shock in deviation form. The shape should looks the same since there are determined by the same driving force. The equilibrium output (i.e. the actual output y_t) has the same steady state with the flexible price output level if you steady state price dispersion is unity. Hence, it should has the same mean as the flexible output and could be much noise than flexible price output level.

markup

$\downarrow$

$\mu = \frac{\epsilon}{\epsilon - 1}$

6. efficient wage $w_t^e = A_t$.
7. Then from the labor supply equation and resource constraint (output == cosumption), You will have flexible output level $y_t^f = \left(\frac{1}{\psi}\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\sigma+\eta}} A_t^{\frac{1+\eta}{\sigma+\eta}}$ and efficient output level $y_t^e = \left(\frac{1}{\psi}\right)^{\frac{1}{\sigma+\eta}} A_t^{\frac{1+\eta}{\sigma+\eta}}$, if you take log you will find $y_t^f < y_t^e$ when $\epsilon > 1$. This means that the monopolistic competition leads to flexible price output level less than the efficient level from benevolent social planner problem.

$-\frac{U_n}{U_c} = \frac{W_t}{P_t} = \frac{MPN_t}{\mu}$

$< MPN_t$

8. Assume exist a Pigouvian tax τ, that get $y_t^f = y_t^e$. $\psi N_t^{\eta} = (1-\tau)\, C_t^{-\sigma} w_t = (1-\tau)\, C_t^{-\sigma} \frac{\epsilon-1}{\epsilon} A_t$ which requires that $(1-\tau)\frac{\epsilon-1}{\epsilon} = 1$ this requires that $\tau = -\frac{1}{\epsilon-1} < 0$. Tax rate is negative means that you should give some subsidy.
9. since $X_t = \ln y_t - \ln y_t^f$, we want y_t to be efficient level y_t^e hence we want $X_t > 0$ if no tax. Getting rid of distortion we have $X_t = 0$ it is a good idea.

Table 1: Three allocations under different assumptions

	Actual	Flexible	Efficient
output Y_t	$Y_t = \frac{A_t N_t}{v_t^p}$	$Y_t^f = A_t N_t^f$	$Y_t^e = A_t N_t^e$
	$Y_t < Y_t^f < Y_t^e$		
marginal cost mc_t	$mc_t = \frac{w_t}{A_t}$	$mc_t^f = \frac{\epsilon-1}{\epsilon}$	$mc_t^e = 1$
real wage w_t	$w_t = mc_t A_t$	$w_t^f = \frac{\epsilon-1}{\epsilon} A_t$	$w_t^e = A_t$
labor N_t	$N_t = \left(\frac{1}{\psi}\frac{mc_t}{v_t^p}\right)^{\frac{1}{\eta+\sigma}} A_t^{\frac{1-\sigma}{\eta+\sigma}}$	$N_t^f = \left(\frac{1}{\psi}\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\eta+\sigma}} A_t^{\frac{1-\sigma}{\eta+\sigma}}$	$N_t^e = \left(\frac{1}{\psi}\right)^{\frac{1}{\eta+\sigma}} A_t^{\frac{1-\sigma}{\eta+\sigma}}$
	$N_t < N_t^f < N_t^e$		

(a) **Efficient allocation**: with flexible price and perfect competition

(b) **Flexible allocation**: with flexible price but monopolistic competition

(c) **Actual allocation**: with sticky price and monopolistic competition

But the tax parameter τ do not appears in the log-linearized conditions, it only affect the model's steady state just behaves the same way as the parameter ψ. Hence, in most cases, we can simply assume that there are some subsidy in place to get rid of the distortion caused by monopolistic competition.

10. In the following subsequent analysis, we just assume there are some implicitly Pigouvian tax exists to make efficient level of output the same as the flexible output.

$$\underbrace{y_t^e}_{\text{efficient output}} \overset{\text{monopoly}}{\longrightarrow} \underbrace{y_t^f}_{\text{flexible output}} \overset{\text{sticky price}}{\longrightarrow} \underbrace{y_t}_{\text{actual output}}$$

11. Let's sum the efficient, flexible allocation with actual allocation as follows:

2.7 Steady states

We assume that the model is stationary. Under this assumption, each variables have its steady states. Set steady state of technology $A = 1$. This means that $Y = C$. The exogenous money rule Eq.(14) implies that the real money growth rate $\Delta \log m = 0$. And this in turn means that the real money balances are stationary. The Euler equation implies that

$$i = \frac{\pi}{\beta}$$

i.e., the s.s of gross nominal rate equal to the s.s. of inflation over beta. The reset price inflation

$$\pi^\star = \left(\frac{\pi^{1-\epsilon_p} - \phi}{1-\phi}\right)^{\frac{1}{1-\epsilon_p}}$$

if we set $\pi = 1$, then $\pi^\star = 1$. The price dispersion equation implies

$$v^p = (1-\phi)\left(\pi^\star\right)^{-\epsilon_p}\pi^{\epsilon_p} + \pi^{\epsilon_p}\phi v^p$$

hence,

$$v^p = \frac{(1-\phi)\left(\pi^\star\right)^{-\epsilon_p}\pi^{\epsilon_p}}{1-\pi^{\epsilon_p}\phi}$$

if $\pi = 1$, then $v^p = 1$. The optimal price decision equations together decide the s.s. of marginal cost

$$mc = \frac{1-\phi\beta\pi^{\epsilon_p}}{1-\phi\beta\pi^{\epsilon_p-1}}\frac{\pi^\star}{\pi}\frac{\epsilon_p-1}{\epsilon_p}$$

if $\pi = 1$, then $mc = \frac{\epsilon_p-1}{\epsilon_p}$. The labor demand equation, the resource constraint and the production technology equation together decide the steady state of labor

$$N = \left(\frac{1}{\psi}\left(v^p\right)^\sigma mc\right)^{\frac{1}{\eta+\sigma}}$$

then we can solve the steady state of real money balance

$$m = \gamma\frac{i}{i-1}C^\sigma$$

where $C = Y = \frac{AN}{v^p}$. So we have all the s.s. of all variables.

3 Codes

4 IRF

4.1 Technology shock

There few problems that need discussed:

1. **Why output responds less in price rigidity than in price flexible** (compare the output Y and Y^f in the figure which are indicated by y and yf respectively)? From above figure, we can see that output respond significantly less than the technology shock and flexible price output which is exactly the size of technology shock A_t which is indicated by a in the figure. Hence, the output gap

$$X_t \equiv logY_t - logY_t^f$$

   which falls since Y_t does not respond enough. What is going on behind? Why price rigidity makes the output respond less? At the same time, we see that inflation falls. First, let's see how the level price moves. The IRF of price level P_t (I compute the IRF of price level by recovering from IRF of inflation rate, see the Matlab code below) is roughly the mirror image

Figure 3: IRF to technology shock

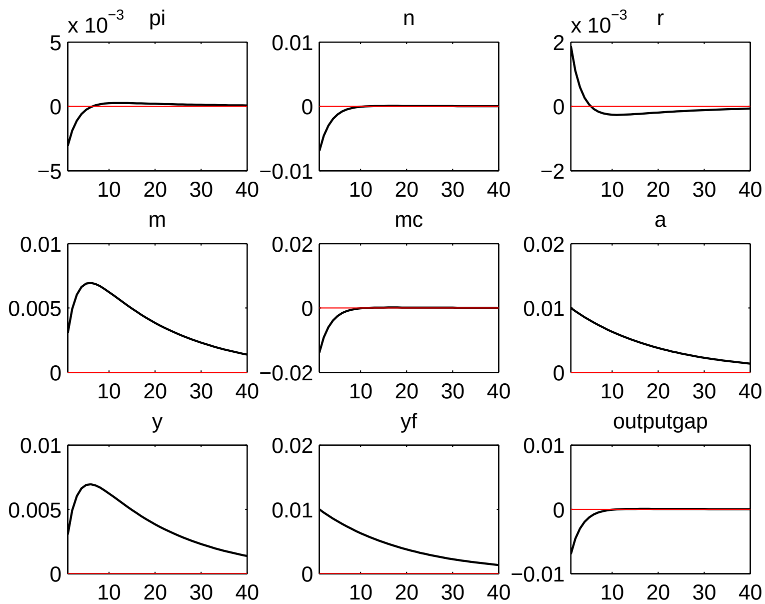
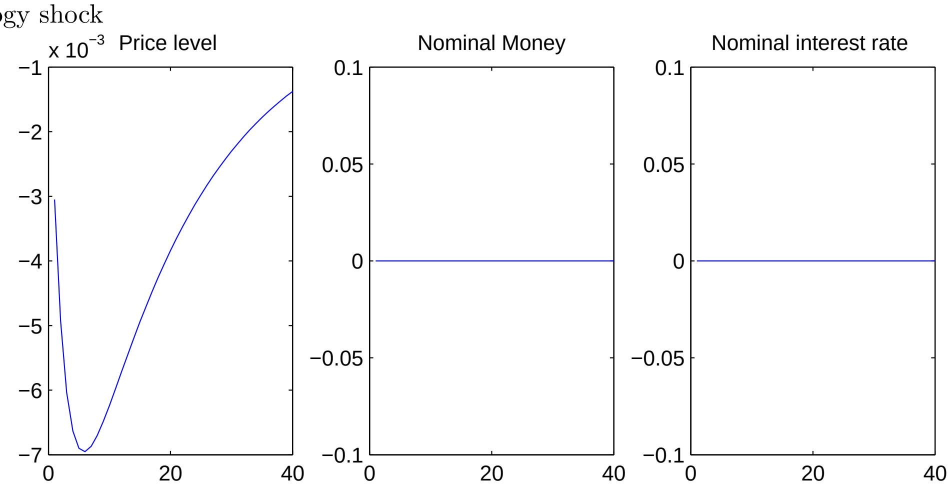


Figure 4: Price level,Nominal Money holdings and nominal interest rate to Technology shock



of output. The nominal interest rate and nominal money holdings do not respond at all the time though the real rate increase. The marginal cost falls, which suggest that real wage w_t rises less than the technology shock A_t (effectively firms charge bigger markups)

```
%% back out the nominal Price level and nominal money to technology shock
% this scripts should be run immediately after running mod file
T = 40;
lnP_em(1) = pi_ea(1);
M_em(1) = m_ea(1)+lnP_em(1);
for ii=2:T
    lnP_em(ii) = lnP_em(ii-1) + pi_ea(ii); %in log, intial price level =1
    M_em(ii) = m_ea(ii)+lnP_em(ii); %in log
end
figure;
subplot(1,3,1);
plot(1:1:T,lnP_em);
title('Price level ');
subplot(1,3,2);
plot(1:1:T,M_em);
axis([0 40 -0.1 0.1])
title('Nominal Money');
subplot(1,3,3);
plot(1:1:T,i_ea);
axis([0 40 -0.1 0.1])
title('Nominal interest rate ');
%% back out the price level and nominal money stock from monetary shock
T = 40;
lnP_em(1) = pi_em(1);
M_em(1) = m_em(1)+lnP_em(1);
for ii=2:T
    lnP_em(ii) = lnP_em(ii-1) + pi_em(ii); %in log, intial price level =1
    M_em(ii) = m_em(ii)+lnP_em(ii); %in log
end
figure;
subplot(2,2,1);
plot(1:1:T,lnP_em);
title('Price level ');
subplot(2,2,2);
plot(1:1:T,M_em);
axis([0 40 -0.1 0.1])
title('Nominal Money');
subplot(2,2,3);
plot(1:1:T,i_em);
axis([0 40 -0.1 0.1])
title('Nominal interest rate ');
```

```
subplot(2,2,4);
plot(1:1:T,yf_em);
axis([0 40 -0.1 0.1])
title('flexible output ');
```

Let's look at the money demand equation, in logs, we have

$$logm_t = log\gamma + logi_t - log\,(i_t - 1) + \sigma logY_t$$

The nominal interest does not move here (see below). Hence, output will move together with real money balance. Since $m_t = \frac{M_t}{P_t}$, this means that output will opposite with the price level. And this is the reason why the IRF of output and price level are almost exactly the opposite to each other. And Prices are sticky and do not move freely, hence the output will not respond that large as the technology shock does.

2. Why nominal interest rate does not respond to technology shock? (Dynare will not plot IRF for the variables whose IRF are all zeroes in the given period)

The foc w.r.t. nominal money holding produces

$$\lambda_t = \gamma \frac{1}{M_t} + \beta E_t \lambda_{t+1}$$

if we solve forward, we can see that

$$\lambda_t = E_t \sum_{i=0}^{\infty} \beta^t \frac{\gamma}{M_{t+i}} \tag{16}$$

If the money does not respond to technology shock, so the multiplier will be the same. From the Euler equation

$$\lambda_t = \beta E_t \lambda_{t+1} (1 + i_t) \tag{17}$$

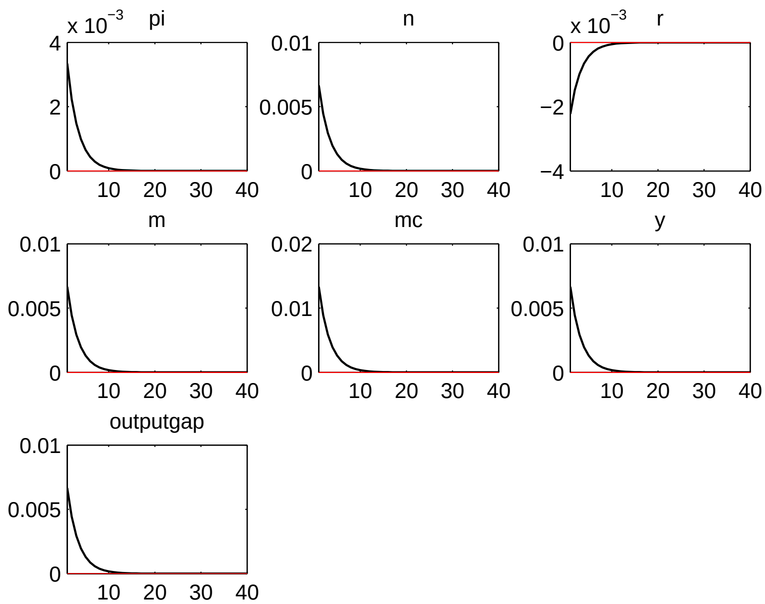
we can see that interest rate does not move either.

4.2 Monetary shock

Let's examine the IRF to monetary shock in the following figure.

1. We see that nominal interest rate, flexible output do not respond to the monetary shock. This seems a little strange because we generally believe that nominal rate will respond to the monetary policy shock.
2. A Permanent Shift in M_t and then stays constant. Nominal money stock follows a random walk since I set $\rho_m = 0$. So the shock will result a permanent shift in M_t as you can see from IRF of money holdings. Once again, we can see from Eq.(16) and (17), a permanent shift in M_t will result the fall in both λ_t and $\lambda_{t+1}$by the same amount since both are infinite sum of money holdings and M_t follows a random walk. This mean that nominal interest rate will not move again by the Euler equation (17).

Figure 5: IRF to money shocks



3. Since nominal interest rate does not respond to shocks, the same logic can apply here for the IRF of output and price level as faced with real shock. That is to say that price level and output will move in opposite direction. Hence, we have the following point.

4. Nominal shock has real effects. That is to say that Output responds to the monetary policy shock.

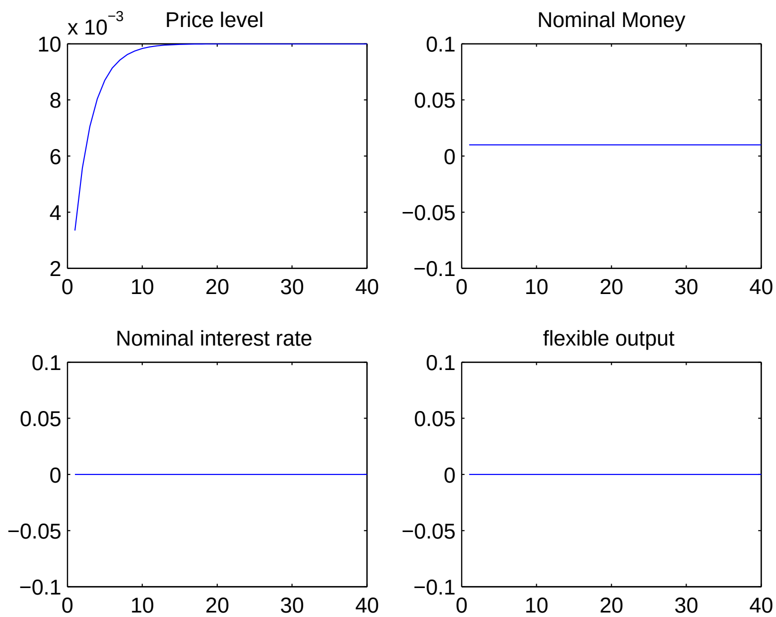
So we can summarize here that with sticky price in the model, output becomes 'demand-determined'. With the exogenous money supply rule here, the price rigidity prevents demand from rising sufficiently when supply increases (technology shock hits, inflation going down, but rigidity in price prevent price going down too much and thus 'limit' the demand in some sense), output will not respond that much as in the flexible price case.

5 How Taylor rule matters?

In this section, we will replace exogenous money supply rule with Taylor rule. And we will see what happens. But we are going to keep money supply in the model because it will be very instructive for us to understand some key points.

But problems will occur with this so-called exogenous Taylor rule, an interest rate rule sometimes. This problem called indeterminacy problem when interest

Figure 6: Price level,Nominal Money holdings and nominal interest rate to monetary shock



rate does not react sufficiently to endogenous variable like inflation and output (typically $\phi_\pi < 1$). We will talk more about it later.

A typically seen Taylor rule will look like as

$$i_t = (1-\rho_i)\, i + \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi\left(\pi_t - \pi\right) + \phi_x\left(logX_t - logX_{t-1}\right)\right) + \epsilon_t^i \quad (18)$$

where X_t is the output gap defined above as the difference between output in sticky price and flexible output. Let's set $\rho_i = 0.8, \phi_\pi = 1.5, \phi_x = 0$.

5.1 The Model

Let's replace the money supply growth rule (the last two equations in above model) with Taylor rule. That is to say we remove the real money growth variable $\Delta logm_t$ from the model and remove the last two equations from the focs. So Now we have 13 variables and 13 equations as follows:

$$x_{1t} = C_t^{-\sigma} mc_t Y_t + \phi\beta E_t x_{1t+1}\pi_{t+1}^{\epsilon_p} \quad (19)$$

$$x_{2t} = C_t^{-\sigma} Y_t + \phi\beta E_t x_{2t+1}\pi_{t+1}^{\epsilon_p - 1} \quad (20)$$

$$C_t = Y_t \quad (21)$$

$$\pi_t^\star = \frac{\epsilon_p}{\epsilon_p - 1}\pi_t \frac{x_{1t}}{x_{2t}} \quad (22)$$

$$\pi_t^{1-\epsilon_p} = (1-\phi)\left(\pi_t^\star\right)^{1-\epsilon_p} + \phi \quad (23)$$

$$Y_t = \frac{A_t N_t}{v_t^p} \quad (24)$$

$$v_t^p = (1-\phi)\left(\pi_t^*\right)^{-\epsilon_p}\pi_t^{\epsilon_p} + \pi_t^{\epsilon_p}\phi v_{t-1}^p \quad (25)$$

$$mc_t = \frac{w_t}{A_t} \quad (26)$$

$$\begin{aligned} \psi N_t^\eta &= C_t^{-\sigma} w_t & (27)\\ C_t^{-\sigma} &= \beta E_t C_{t+1}^{-\sigma}\left(1+i_t\right)\pi_{t+1}^{-1} & (28)\\ \gamma\left(m_t\right)^{-1} &= \frac{i_t}{1+i_t} C_t^{-\sigma} & (29) \end{aligned}$$

$$log\left(A_t\right) = \rho_a log\left(A_{t-1}\right) + \epsilon_t^a \quad (30)$$

$$i_t = (1-\rho_i)\, i + \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi\left(\pi_t - \pi\right) + \phi_x\left(logX_t - logX_{t-1}\right)\right) + \epsilon_t^i \quad (31)$$

5.2 The Code

The mod file (Dynare code):

```
%NK with sticky price, Taylor rule
%we define the output gap as the difference btw output of sticky and
%flexible price 2014-04-11@ND
%we consider zero inflation steady state for a simple life here
%This file is written by Xiangyang Li;

var c i pi r n w mc a y vp pisharp x1 x2 yf outputgap m;
varexo ea ei;
parameters sigma beta psi eta phi epsilon theta phipi phiy rhoa rhoi sigmaa sigmam;
parameters cs is pis ns ws mcs as ys vps pisharps x1s x2s yfs outputgaps ms;

beta =.99;
sigma = 1;
eta = 1;
psi = 1;
epsilon =10;
theta = 1;
phipi = 1.5;
phiy = 0; %0.125/4;
rhoa = .95;
rhoi = 0.8;
sigmaa =.01;
sigmam=.01;
%the stickiness parameter
phi = .75;


%steady state calculation
is = 1/beta;
as = 1;
%zero inflation steady state
pis = 1;
pisharps= ((pis^(1-epsilon) - phi)/(1-phi))^(1/(1-epsilon));
vps = (1-phi)*(pis/pisharps)^epsilon/(1- pis^epsilon*phi);
mcs = (1-phi*beta*pis^epsilon)/(1-phi*beta*pis^(epsilon-1))
           *pis/pisharps*(epsilon-1)/epsilon;
ns = (vps^sigma*mcs/psi)^(1/(eta+sigma));
ys = as*ns/vps;
cs= ys;
yfs =((epsilon-1)/epsilon/psi)^(1/(sigma+eta))* as^((1+eta)/(sigma+eta));
outputgaps = ys/yfs;
ws=  as*mcs;
x1s = cs^(-sigma)*mcs*ys/(1-beta*phi*pis^epsilon);
```

```
x2s =cs^(-sigma)*ys/(1-beta*phi*pis^(epsilon-1));
ms = theta*is/(is -1)*cs^sigma;

model;
%(1) home Euler equation
exp(-sigma*c) = beta * exp(-sigma*c(+1))*exp(i)/exp(pi(+1));

%(2) labor supply
psi*exp(eta*n) =exp(-sigma*c) *exp(w);

%(3) labor demand
exp(mc) = exp(w)/exp(a);

%(4) Taylor interest rate rule
%actual output deviation
%i = (1-rhoi)*log(is) + rhoi*i(-1) + (1-rhoi)*(phipi*(pi - log(pis)) +phiy* (y - log(ys)) )

% output gap
i = (1-rhoi)*log(is) + rhoi*i(-1) + (1-rhoi)*(phipi*(pi - log(pis))
         +phiy* (outputgap ))+ ei;

%(5) accounting identity
exp(c)=  exp(y);

%(6) the production technology
exp(y) = exp(a) *exp(n)/exp(vp);

%(7) the price dispersion
exp(vp) = (1-phi)*exp(-epsilon*pisharp)*exp(epsilon*pi)
               + exp(epsilon*pi) *phi*exp(vp(-1));

%(8) inflation evolution
exp((1-epsilon)*pi) = (1-phi)*exp((1-epsilon)*pisharp) + phi;

%(9)the sticky price equation
exp(pisharp) = epsilon/(epsilon -1 )*exp(pi)*exp(x1)/exp(x2);

%(10) the auxiliary x1
exp(x1) = exp(-sigma*c)*exp(y)*exp(mc) +phi*beta*exp(epsilon*pi(+1))*exp(x1(+1));

%(11) the auxiliary x2
exp(x2) = exp(-sigma*c)*exp(y) +phi*beta*exp((epsilon-1)*pi(+1))*exp(x2(+1));

%(12) technology shock
a = rhoa*a(-1) + ea;
```

```
%(13)flexible output
exp(yf) = ((epsilon-1)/epsilon/psi)^(1/(sigma+eta))* exp(((1+eta)/(sigma+eta))*a);

%(14) output gap
outputgap = y - yf;

%(15) real interest rate
exp(r) = exp(i)/exp(pi(+1));

%(16) the real money balance
exp(m) = theta* exp(i)/(exp(i) -1 )*exp(sigma*c);

end;
initval;
c = log(cs);
i = log(is);
pi = log(pis);
n = log(ns);
w = log(ws);
mc = log(mcs);
a = log(as);
y = log(ys);
vp = log(vps);
pisharp = log(pisharps);
x1 = log(x1s);
x2 = log(x2s);
yf = log(yfs);
outputgap = log(outputgaps);
r =  log(is/pis);
m = log(ms);
end;

shocks;
var ea = .01^2;
var ei =.01^2;
end;

resid(1);
steady;
check;
%when used in loops, noprint and nograph is a good option.
%graph_format = none, only display,no save to disk
%since y and c is the same, we only plot y
stoch_simul(order=1) m i pi n w mc y yf r outputgap a;
```

5.3 The IRF

5.3.1 Technology shock

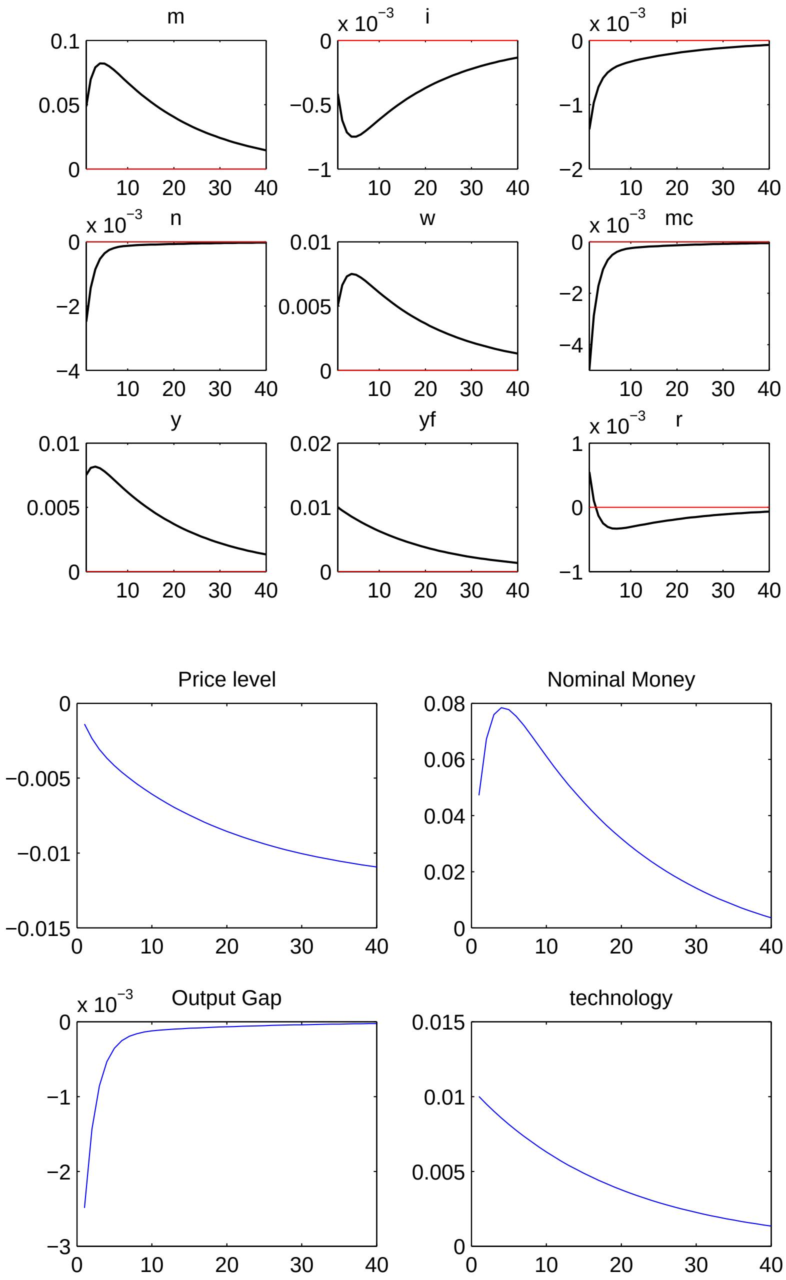
If you compare to the previous section, you will find some substantial differences.

1. The output responds larger in Taylor rule than under exogenous money growth rule. Hence the labor drop less too. And a smaller drop in inflation and real rate.

2. Nominal interest rate moves.

3. Nominal money responds significantly: This means that under Taylor rule, money supply is effectively endogenous, and the central bank reacts to the increased productivity by accommodating it and increasing the money supply.
$$m_t = \frac{M_t}{P_t}$$
This nominal money supply increase will help the real money balance goes up which means that we do not have to rely on the price level goes down to get real money balance goes up. Hence, in this sense, output can expand by more than it would if the money supply were fixed. This endogenous rising of money supply is what allows output to goes up more than it would be under exogenous money rule before.

Figure 7: IRF to technology shock

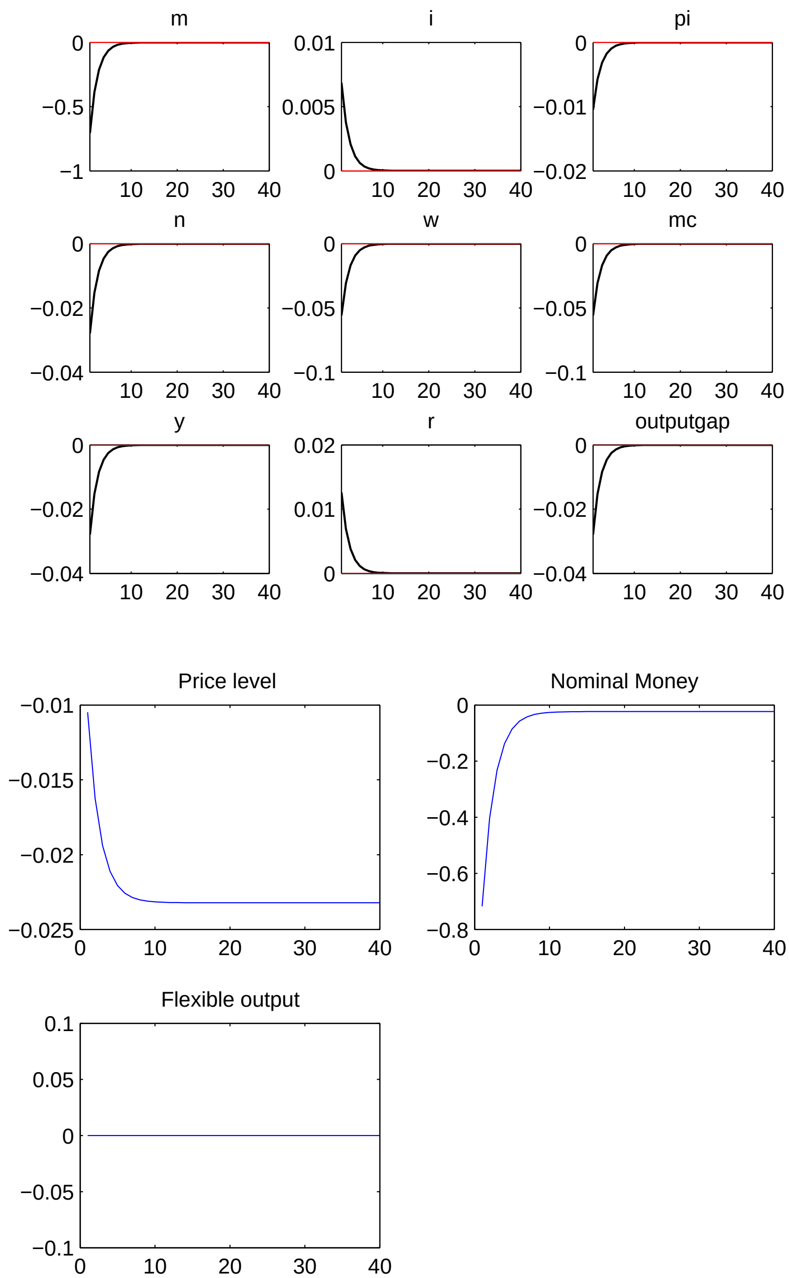


5.3.2 Monetary shock

After a positive shock to interest rate,

1. **The nominal interest rate rises**.: due to the Taylor rule.
2. **The nominal money supply decreases**.
3. **And the real money balance decrease**s: $m_t = \frac{M_t}{P_t}$ since price does not fall that much due to stickiness.
4. **An increase in real rate**: $r_t = i_t - E_t\pi_{t+1}$ since expected inflation does not rise that much as nominal interest rate. And the rise in real rate will to some extent low demand.
5. **And a decline in output**. From the money demand equation listed above, we see that a decline in real money balance will necessitate a decline in output since nominal rate rising is too small.

Figure 8: IRF to one positive unit of monetary shock



6 Summary

There are few important points that need summarized:

1. With exogenous money growth rule and sticky price setting, output can respond significantly low when faced with technology shock. By definition, it suppose to respond proportionally to technology shock. But it do not due to price stickiness.

2. With Taylor rule and stickiness in price, money holdings become effectively endogenous and output respond larger when faced real shocks than when set with exogenous money growth rule. This gives the noticed role of policy (here is monetary policy) to stabilize the output when faced real shocks.

3. Nominal shocks have real effects when set with nominal rigidity in both cases: both the exogenous money growth rule and Taylor rule.