Maximum Likelihood Estimation

Introduction

Tools for Macroeconomists: The essentials

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Overview for today

- looking at the big picture
  - what you've done in the past days
  - what we'll do today and Friday
- intro into estimation
  - Kalman filter (based on Hamilton)
  - Maximum Likelihood estimation
- estimating DSGE models
  - DSGE and time-series models
- extensions

Introduction

What tools are needed in modern macro?
(according to Victor Rios-Rull)

1. theoretical tools
   - use models to look at data
2. computational tools
   - characterize model outcomes
   - assign “right” parameter values
3. empirical tools
   - analyze statistical properties of data and model
4. common sense

Introduction

What you've already done

What you've already learned

What does it mean to "solve" a model?

- solving for policy rules
- forward-looking behavior makes this tough

Which (general) tools did you learn towards this end?

- function approximation
- numerical integration

What ways of constructing model solutions did you cover?

- perturbation
- projection

Introduction

What we'll cover next

What will we learn today?

How to parameterize models

- alternative methods
  - calibration
  - matching moments
  - estimation
    - Maximum Likelihood
    - Bayesian methods

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"The goal of computing is insight, not numbers"

Richard Hamming

Mathematician and
computer scientist

Alternative Parameterization Methods

Alternative methods of parametrizing models

- calibration
- matching moments
  - general methods of moments
  - simulated method of moments
  - indirect inference
- estimation
  - Maximum Likelihood
  - Bayesian estimation

Alternative Parameterization Methods

Calibration

Calibration

- wide-spread methodology
  - at least since Kydland and Prescott (1982)
- although calibration is also an empirical exercise
- it lacks the probabilistic interpretation
- constrained by (a priori identified) features in the data

Kydland and Prescott (1996):
It is important to emphasize that the parameter values selected are not the ones that provide the best fit in some statistical sense.

Calibration

Main idea:

- model parameters pinned down by *selected* real-world features
- "evaluate" model based on a *different* set of features

Criticism of calibration

- no formal rules on selecting targets
- no formal rules on selecting dimensions of model fit
- no formal rules on comparing alternatives

We'll go through an example to give you an idea

Alternative Parameterization Methods

Matching moments

Matching moments (GMM, SMM, II)

- idea similar to calibration:
  - parametrize by a set of moments (features) of the data
  - judge model performance by a different set of moments
- matching moments adds statistical rigor
  - estimation (limited information)
  - hypothesis testing

Generalized method of moments (Hansen, 1982)

Main idea:

- select a set of moments (orthogonality conditions)

$$\mathbb{E}[f(x_t, \Psi)] = 0$$

  - x_t is a vector of variables, Ψ are model parameters

- choose Ψ s.t. sample analogs $g(X, \Psi) = 1/T \sum_t f(x_t, \Psi)$ hold
  - exactly identified case: # of params. = # of conditions
  - over-identified case: # of params. $<$ # of conditions

Simulated method of moments

- sometimes orthog. conditions cannot be assessed analytically
- moment-matching estimation based on simulations
  - retains asymptotic properties of GMM

Indirect inference

Main idea:

- parameters pinned down by selected reduced-form estimates
  - choose parameters s.t. simulated data from structural model
  - replicates reduced-form results in the data
- judge model performance by a different set of moments

Alternative Parameterization Methods

Estimation

Estimation

Full information method

- need to specify entire distributions (of driving forces)
  - model fit (and parameters) based on implied likelihood function
  - and prior information in case of Bayesian estimation

We'll talk about estimation in detail!


That's all Folks!