Search Theory

Econ 8006

Structural Labor

2020-2021 Term 2

Outline

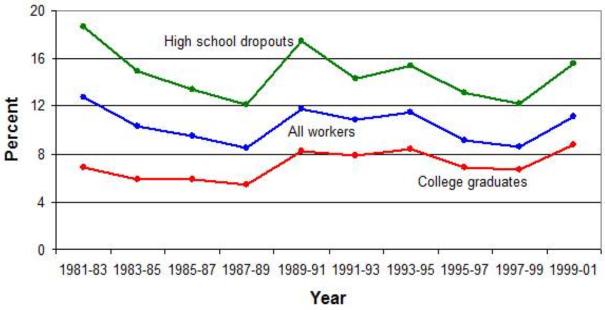
Employee Turnover

Stylized facts

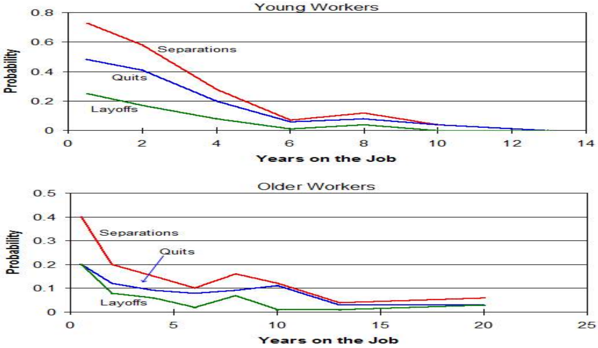
- Newly hired workers tend to leave their jobs within 24 months of being hired, while workers with more seniority rarely leave their jobs.
- The rate of job loss is highest among the least educated workers.
- There is a strong negative correlation between a worker's age and the probability of job separation.
  - This fits with the hypothesis that labor turnover can be an investment in human capital.
  - Young workers are testing the waters?
  - Older workers have a smaller payoff period to recoup the costs associated with job search. Thus, they are less likely to search (or move).

Employee Turnover

The Rate of Job Loss in the United States, 1981-2001



Probability of Job Turnover



Employee Turnover

Job match

- Each particular pairing of a worker and employer has its own unique value.
- Workers and firms might improve their situation by shopping for a better job match.
- Efficient turnover is the mechanism by which workers and firms correct matching errors and obtain a better and more efficient allocation of resources.

Employee Turnover

Specific training and turnover

- A worker's earnings depend on total labor market experience and seniority on the current job.
- When a worker receives specific training, his productivity improves only at the current firm.
- This implies there should be a negative correlation between the probability of job separation and job seniority.

Employee Turnover

Difference between quits and layoffs:

- Young people who quit often experience substantial increases in their wages.
- Workers who are laid off often experience wage cuts.

Employee Turnover

Gender Differences

- Women workers have higher quit rates, and shorter job tenures than men.
- Women have lower wages and shorter careers.
- Higher quit rates in women may reflect lower levels of firm-specific human capital investments.

Employee Turnover

Effects of Employer Size

- Quit rates tend to decline as firm size increases because they offer more opportunities for transfers and promotions.
- Large firms have greater needs for dependable and steady workers because employee shirking could impose great costs - hence they establish internal labor market.

Monthly Quit Rates per 100 Workers by Firm Size, Selected Industries (1977–1981Averages)

	Number of Employees			
Industry	<250	250–499	500–999	1,000 and Over
All manufacturing	3.28	3.12	2.40	1.50
Food and kindred products	3.46	4.11	3.95	2.28
Fabricated metal products	3.33	2.64	2.12	1.20
Electrical machinery	3.81	3.12	2.47	1.60
Transportation equipment	3.90	2.78	2.21	1.41

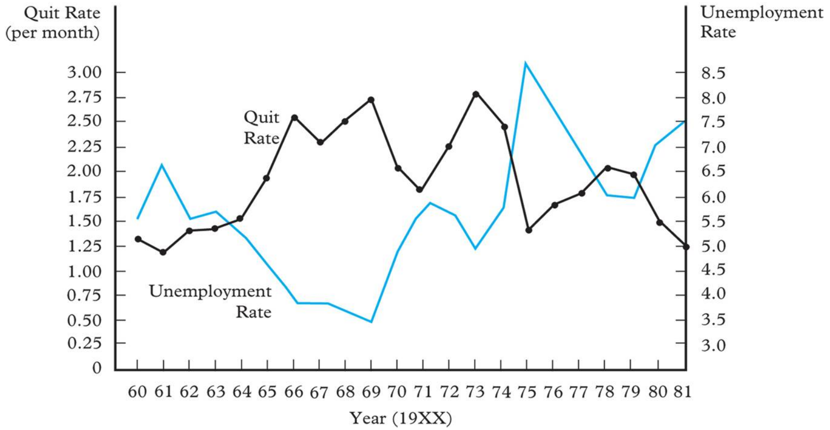
Employee Turnover

Cyclical Effects

- Human capital theory predicts that workers will have a higher probability of quitting a job for another during economic booms.
- Quit rates tend to fall when labor markets are loose and rise when labor markets are tight.
- The unemployment rate is a good measure of labor market conditions - unemployment rate is inversely related to quit rate.

Employee Turnover

The Quit Rate and Labor Market Tightness



The unemployment rate as a good measure of labor market conditions - tightness or looseness - shows the inverse relationship with the quit rate.

A Simple Continuous Time Model

Setup

- Individuals: employed or unemployed, infinitely lived, risk neutral
- Each period the individual receives one job offer which he/she can accept or reject
- No on-the-job search
- Wage offers are drawn from a distribution $F(w)$ with pdf $f(w)$
- Unemployment benefit b and cost of search c

A Simple Continuous Time Model

Continuous time model: take a limit of a discrete time model as the length of search interval h tends to 0.

Setup

- Discount factor is $\beta(h) = e^{-rh}$.
- w, b, c are instantaneous flow rates.
- Number of offers received is Poisson distributed with arrival rate λ. This means that in an interval of length h the expected number of offers is λh and

$$
\begin{aligned}
P(x) &= (\lambda h)^x e^{-\lambda h}/x! \\
P(0 \text{ offer}) &= e^{-\lambda h} \\
P(1 \text{ offer}) &= \lambda h e^{-\lambda h} \\
P(2 \text{ offers}) &= (\lambda h)^2 e^{-\lambda h}/2
\end{aligned}
$$

A Simple Continuous Time Model

Let V denote the value of an unemployed individual. If an individual with a value V is offered a wage with

$$\text{d.p.v. of job} = \frac{w}{1-\beta} > V$$

then he/she should accept it.

A Simple Continuous Time Model

The Bellman equation can be written as

$$\begin{aligned} V =& (b-c)h + e^{-rh}(e^{-\lambda h}V + \lambda h e^{-\lambda h}\int_0^\infty \max(V, \frac{w}{r})f(w)dw) \\ &+ \text{terms of order } h^2 \text{ or higher} \end{aligned}$$

Ignoring the higher order terms we get

$$\begin{aligned} V =& (b-c)h + e^{-(r+\lambda)h}V + e^{-(r+\lambda)h}\lambda h V \\ &+ \lambda h e^{-(r+\lambda)h}\int_0^\infty \max(0, \frac{w}{r} - V)f(w)dw \\ \Rightarrow & V(\frac{1 - e^{-(r+\lambda)h} - e^{-(r+\lambda)h}\lambda h}{h}) = b - c \\ &+ e^{-(r+\lambda)h}\lambda \int_0^\infty \max(0, \frac{w}{r} - V)f(w)dw \end{aligned}$$

A Simple Continuous Time Model

Now take the limit and use the facts that

$$\lim_{h\to 0} \frac{1-e^{-(r+\lambda)h}}{h} = r+\lambda$$
$$\lim_{h\to 0} e^{-(r+\lambda)h}\lambda = \lambda$$

we get

$$rV = b - c + \lambda \int_0^\infty \max(0, \frac{w}{r} - V) f(w) dw$$

A Simple Continuous Time Model

Finally, define $w^* = rV$ as the reservation wage

$$w^* - (b - c) = \frac{\lambda}{r} \int_{w^*}^{\infty} (w - w^*) f(w) dw \tag{1}$$

- The l.h.s is the foregone income from rejecting an offer at $w = w^*$.
- The r.h.s. is the value of waiting one more period and sampling again.

A Simple Continuous Time Model

In this model, each agent has a constant reservation wage w^*, regardless of how long they have been unemployed.
The escape rate from unemployment (or exit hazard) is

$$P(w \geq w^*) = \lambda(1 - F(w^*)) \tag{2}$$

We can differentiate (1) w.r.t b to get

$$\frac{\partial w^*}{\partial b} - 1 = -(\frac{\lambda}{r}\int_{w^*}^{\infty} f(w)dw)\frac{\partial w^*}{\partial b}$$
$$\Rightarrow \frac{\partial w^*}{\partial b} > 0$$

So there is a predicted positive effect of b on w^*.
Using (2), the exit hazard rate from unemployment will be lower when b is raised (or c is lowered).

Solving a Simple Continuous Time Model

This is an equation of the form $w^* = T(w^*)$.

- T is a contraction
- Algorithm for computing w^*
  - Start with a guess w^1 and compute $w^2 = T(w^1)$
  - Keep iterating until converge

A Simple Continuous Time Model

How do we identify the model?

- We do not observe the wage offer distribution $F()$, but we observe the accepted wage distribution $G()$, which is a truncated distribution (at w^*).
- We observe the lowest accepted wage, which identifies w^*.
- If $F()$ is recoverable (i.e., log normal), we can recover $F()$ from $G()$ (Flinn and Heckman, 1982).
- We observe transitions from unemployment to employment, which equals $\lambda(1 - F(w^*))$. U-E transition rates identify λ.

A Continuous Time Model with On-the-job Search

Now assume that people can receive job offers even when working.
Model setup

- b represent the net benefit received by an unemployed person
- c represent the disutility cost of work
- A person with wage w receives a net income flow of $w - c$
- The arrival rate of a new offer in each period follows a Poisson process with parameter λ
- Jobs dissolve following a Poisson process with parameter δ
- Two value functions: a value V if unemployed, a value function $U(w)$ associated with holding a job that pays w
- Discount rate e^{-rh}

A Continuous Time Model with On-the-job Search

Consider someone who enters the period with a job paying w.

- With probability λ they get a new offer $\tilde{w}$, which they will accept if $\tilde{w} > w$ with probability $1 - F(w)$.
- With probability δ the old job disappears and the person ends up at the end of the period with V.

A Continuous Time Model with On-the-job Search

For an employed worker

$$rU(w) = w - c + \delta(V - U(w)) + \lambda \int_{w}^{\infty} (U(\tilde{w}) - U(w)) f(\tilde{w}) d\tilde{w} \tag{3}$$

For an unemployed workers with a reservation wage w^*

$$rV = b + \lambda \int_{w^*}^{\infty} (U(\tilde{w}) - V) f(\tilde{w}) d\tilde{w} \tag{4}$$

A Continuous Time Model with On-the-job Search

Also, we must have $U(w^*) = V$. Look back at the value function (3) for an employed worker $U(w)$, and evaluate at $w = w^*$.

$$rV = rU(w^*) = w^* - c + \delta(V - V) + \lambda \int_{w^*}^{\infty} (U(\tilde{w}) - V) f(\tilde{w}) d\tilde{w}$$

$$\Rightarrow rV = w^* - c + \lambda \int_{w^*}^{\infty} (U(\tilde{w}) - V) f(\tilde{w}) d\tilde{w}$$

Comparing this to (4) we see that $w^* = b + c$.
The reason is that there is no opportunity cost of taking a job: it does not slow down the arrival of offers so you might as well take any job with $w \geq b + c$ while you wait for something better, i.e., λ is the same for unemployed and employed workers.

Solving a Continuous Time Model with On-the-job Search

- With fixed benefit level b and disutility of work c, we showed that $w^* = b + c$.
- Value functions $U(w)$ and $V = U(w^*)$ is a functional equation of the form

$$U(w) = T\{U(w)\}$$

T maps from the space of functions to the space of functions, and we are looking for a fixed point of T.

Solving a Continuous Time Model with On-the-job Search

What does $U(w)$ look like? From equation (3), for higher values of w:

$$rU(w) = w - c + \delta(V - U(w)) + \text{“little bit”}$$

The “little bit” is the option value of a better job coming along. As w rises this option value is smaller and smaller.

For low values of the wage, the option value term can be sizable

$$rU(w) = w - c + \delta(V - U(w)) + \lambda \int_{w}^{\infty} (U(\tilde{w}) - U(w)) f(\tilde{w}) d\tilde{w}$$

Solving a Continuous Time Model with On-the-job Search

If w has a discrete support with a values w^1, w^2, ... w^n and associated probabilities π^1, π^2, ... π^n, it's pretty easy to solve backward from the highest value (which has no option value).

$$
\begin{aligned}
rU(w^n) &= w^n - c + \delta(V - U(w^n)) \\
rU(w^{n-1}) &= w^{n-1} - c + \delta(V - U(w^{n-1})) + \lambda(U(w^n) - U(w^{n-1}))\pi^n \\
&\dots \\
rV &= b + \lambda \sum_{k=1}^{n} (U(w^k) - V)\pi^k
\end{aligned}
$$

Make an initial guess of V, solve $U(w^k)$, for $k \in \{n, ..., 1\}$, and update V using the last equation.

This will converge if the discount rate and rate of job destruction are large enough, and the arrival rate of offers is not too large.

Card-Hyslop Model for SSP

Card and Hyslop, "Estimating the Effects of a Time-Limited Earnings Subsidy for Welfare-Leavers," Econometrica, 2005

What was the SSP program?

- Earnings subsidy offered to randomized treatment group of long-term welfare recipients
- To become entitled: have to leave welfare and begin subsidized job within first year of offer
- Those who fail to establish entitlement - lose all future benefits (revert to no SSP)
- Once entitled: receive subsidy S in any period working (full time only)
- Entitled for 36 months; can go back and forth work-welfare and retain entitlement
- Typical person in experiment:
  $b = \$700/mo, w = \$900/mo, w + S = \$1700$

Card-Hyslop Model for SSP

Model setup

- Welfare pays b
- Full time work pays w and has utility-equivalent $w - c$. Wages drawn from $f(w)$
- Fixed interest rate r, arrival rate λ, job destruction rate δ
- No part-time work

In absence of SSP

- Behavior characterized by $U(w)$ and $V = V^0$
- Constant reservation wage $R = b + c$
- Constant rate of transition from welfare to work $= \lambda(1 - F(R))$
- Constant rate of transition from work to welfare $= \delta$

Card-Hyslop Model for SSP

How to extend the model? Now have 3 more value functions

- $V_i(t)$ = value of being offered SSP, not yet left welfare as of period t
- $U_e(w, d)$ = value of being entitled for SSP, employed at wage w, d months of entitlement left
- $V_e(d)$ = value of being entitled for SSP but currently on welfare, d months of entitlement left
- Allow people to quit a job: some may do when SSP entitlement ends, and also after finding a first job that establishes entitlement.

Card-Hyslop Model for SSP

Relations and facts:

- $V_i(t) = V^0$ for $t \geq 13$ (missed the deadline for entitlement)
- $U_e(w, d) = U(w)$ and $V_e(d) = V^0$ for $d > 36$ (max'ed out entitlement)
- $U_e(w, d) > U(w)$; $V_e(d) > V^0$ for $d \leq 36$
- $V_i(t)$ decreasing in t (less time to establish eligibility)
- $U_e(w, d)$ and $V_e(d)$ decreasing in d
- Reservation wage during the entitlement period $R_e(d)$, satisfies $U_e(R_e(d), d) = V_e(d)$
- $R_e(d) + S = b + c \Rightarrow R_e(d) \equiv R_e < R = b + c$. Once entitled, reservation wage $R_e < R$.

Card-Hyslop Model for SSP

- Prior to establishing entitlement: reservation wage $R_i(t)$ with $V_i(t) = U_e(R_i(t), 1)$
- For $t < 12$: $U_e(R_i(t), 1) = V_i(t) > V_i(t+1) = U_e(R_i(t+1), 1)$ $\Rightarrow U_e(R_i(t), 1) > U_e(R_i(t+1), 1) \quad \Rightarrow \quad R_i(t) > R_i(t+1)$
- $R_i(1) < R_e$ since taking a job when not yet entitled gives "option value" of entitlement plus all other benefits of a job during the entitlement period. Thus

$$R > R_e > R_i(1) > R_i(2) > ... > R_i(12)$$

Card-Hyslop Model for SSP

The eligibility rules created two effects

- An "establishment" incentive to find a job and leave welfare within a year of random assignment
- An "entitlement" incentive to choose work over welfare once eligibility was established

Findings

- The combination of the two incentives explains the time profile of the experimental impacts, which peaked 15 months after random assignment and faded relatively quickly.
- About half of the peak impact of SSP was attributable to the establishment incentive.
- Despite the extra work effort generated by SSP, the program had no lasting impact on wages and little or no long-run effect on welfare participation.

Card-Hyslop Model for SSP

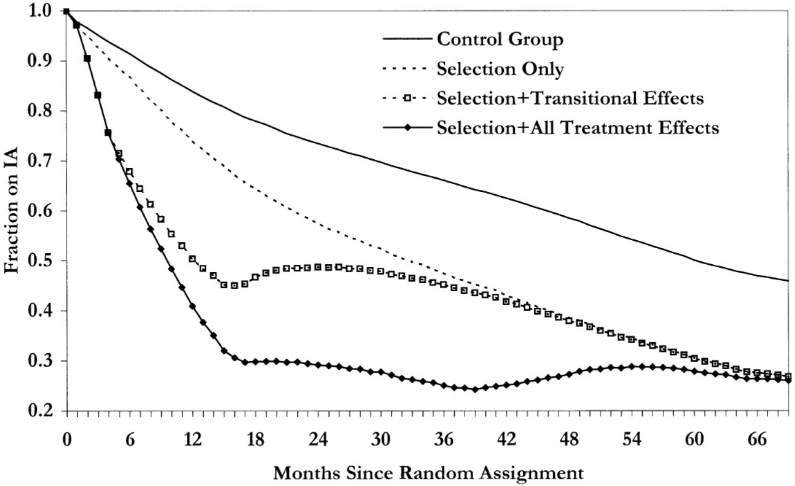


FIGURE 9.—Decomposition of predicted IA of eligible treatments.

Equilibrium Search Model

- Random search model: workers receive wage offers randomly
  - Wage posting model (Burdett & Mortensen (1998)): firms post wage by maximizing their profit
  - Nash bargaining model (Mortensen & Pissarides (1994)): firms and workers bargain over the wage
- Directed search model: workers know all wage offers and choose one to search for

Nash Bargaining Model

Mortensen & Pissarides (1994)

- u: number of unemployed workers; v: number of vacancies
- $M(u, v)$: total number of matches
  - M is continuous, nonnegative, increasing in both arguments and concave, with $M(u, 0) = M(0, v) = 0$.
  - M displays constant returns to scale, that is, $kM(u, v) = M(ku, kv)$.
- θ: market tightness $(v/\ u)$
- $f(\theta) \equiv M(1, \theta) = \frac{M(u,v)}{u}$: employment rate, probability of receiving an offer for workers
- $q(\theta) \equiv M(\frac{1}{\theta}, 1) = \frac{M(u,v)}{v}$: hiring rate, probability of hiring a worker for firms

Nash Bargaining Model

Mortensen & Pissarides (1994)

$$\begin{aligned} J_p(w) &= p - w_p + \beta\{(1-\delta)E_p J_{p'}(w) + \delta E_p V_{p'}\} \\ V_p &= -c_p + \beta\{q(\theta_p)E_p J_{p'}(w) + (1-q(\theta_p))E_p V_{p'}\} \\ U_p &= z + \beta\{f(\theta_p)E_p W_{p'}(w) + (1-f(\theta_p))E_p U_{p'}\} \\ W_p(w) &= w_p + \beta\{(1-\delta)E_p W_{p'}(w) + \delta E_p U_{p'}\} \end{aligned}$$

J: firm's value of a job; V: firm's value of an unfilled vacancy; W: worker's value of having a job; U: worker's value of being unemployed.

- Free entry implies $V_p = 0$ for all p.
- Nash bargaining implies that a worker and a firm split the surplus $S_p = J_p + W_p - U_p$ such that $J_p = (1-\beta)S_p$.

Nash Bargaining Model

Shimer (2005)

- Argues that the textbook search and matching model cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to shocks of a plausible magnitude.
- In the United States, the standard deviation of the vacancy-unemployment ratio is almost 20 times as large as the standard deviation of average labor productivity, while the search model predicts that the two variables should have nearly the same volatility

Nash Bargaining Model

Shimer (2005): Data

TABLE 1—SUMMARY STATISTICS, QUARTERLY U.S. DATA, 1951–2003

		u	v	v/u	f	s	p
Standard deviation		0.190	0.202	0.382	0.118	0.075	0.020
Quarterly autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
	u	1	−0.894	−0.971	−0.949	0.709	−0.408
	v	—	1	0.975	0.897	−0.684	0.364
Correlation matrix	v/u	—	—	1	0.948	−0.715	0.396
	f	—	—	—	1	−0.574	0.396
	s	—	—	—	—	1	−0.524
	p	—	—	—	—	—	1

Notes: Seasonally adjusted unemployment u is constructed by the BLS from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index v is constructed by the Conference Board. The job-finding rate f and separation rate s are constructed from seasonally adjusted employment, unemployment, and mean unemployment duration, all computed by the BLS from the CPS, as explained in equations (1) and (2). u, v, f, and s are quarterly averages of monthly series. Average labor productivity p is seasonally adjusted real average output per person in the non-farm business sector, constructed by the Bureau of Labor Statistics (BLS) from the National Income and Product Accounts and the Current Employment Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

Shimer (2005): Model prediction

TABLE 3—LABOR PRODUCTIVITY SHOCKS

		u	v	v/u	f	p
Standard deviation		0.009	0.027	0.035	0.010	0.020
		(0.001)	(0.004)	(0.005)	(0.001)	(0.003)
Quarterly autocorrelation		0.939	0.835	0.878	0.878	0.878
		(0.018)	(0.045)	(0.035)	(0.035)	(0.035)
Correlation matrix	u	1	−0.927	−0.958	−0.958	−0.958
			(0.020)	(0.012)	(0.012)	(0.012)
	v	—	1	0.996	0.996	0.995
				(0.001)	(0.001)	(0.001)
	v/u	—	—	1	1.000	0.999
					(0.000)	(0.001)
	f	—	—	—	1	0.999
						(0.001)
	p	—	—	—	—	1

Notes: Results from simulating the model with stochastic labor productivity. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5. Bootstrapped standard errors—the standard deviation across 10,000 model simulations—are reported in parentheses. The text provides details on the stochastic process for productivity.

Nash Bargaining Model

Hagedorn and Manovskii (2008)

- Propose a new calibration strategy of the standard model that uses data on the cost of vacancy creation and cyclicality of wages to identify the two key parameters - the value of non-market activity and the bargaining weights.
- Their calibration implies that the model is consistent with the data.

Nash Bargaining Model

Hagedorn and Manovskii (2008)

Table 3: Summary Statistics, quarterly U.S. data, 1951:1 to 2004:4.

		u	v	v/u	p
Standard Deviation		0.125	0.139	0.259	0.013
Quarterly Autocorrelation		0.870	0.904	0.896	0.765
	u	1	-0.919	-0.977	-0.302
	v	—	1	0.982	0.460
Correlation Matrix	v/u	—	—	1	0.393
	p	—	—	—	1

Table 4: Results from the Calibrated Model.

		u	v	v/u	p
Standard Deviation		0.145	0.169	0.292	0.013
Quarterly Autocorrelation		0.830	0.575	0.751	0.765
	u	1	-0.724	-0.916	-0.892
	v	—	1	0.940	0.904
Correlation Matrix	v/u	—	—	1	0.967
	p	—	—	—	1

Reference

- Burdett and Mortensen, "Wage Differentials Employer Size and Unemployment", International Economics Review, 1998
- Card and Hyslop, "Estimating the Effects of a Time-Limited Earnings Subsidy for Welfare-Leavers", Econometrica, 2005
- Hagedorn and Manovskii, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited", American Economic Review, 2008
- Manning, "Monopsony in Motion", Princeton: PU Press, 2003
- Mortensen, "Unemployment Insurance and Job Search Decisions", Industrial and Labor Relations Review, 1977
- Mortensen and Pissarides, "New Developments in Models of Search in the Labor Market", Handbook of Labor Economics, 1999
- Mortensen and Wright, "Competitive pricing and efficiency in search equilibrium", International Economics Review, 2002
- Shimer, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies", American Economic Review, 2005