# Medium Scale DSGE Model

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1. Today,Medium Scale DSGE
   (a) Features of the model
      i. capital accumulates
      ii. sticky prices
      iii. sticky wage
      iv. adjustment costs of investment
      v. variable utilization for capital stock
      vi. habit formation
      vii. no price and wage indexation
   (b) shocks
      i. neutral technology
      ii. investment shock
      iii. monetary policy
      iv. government shock
2. Index the firm by $j$ and households by $l$.

## 1 Production

1. Write down the intermediate goods $j$'s demand curve and Final CPI equation.
2. Write down the households $l$'s labor demand curve and Final Wage Index;

*Comments are welcomed!

## 1.1 Final Good

1. Final Good producer employs a Dixit-Stiglizt aggregator

$$Y_t = \left( \int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} \, dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

where $j$ is an index for immediate good producer. $\epsilon_p > 1$ is the elasticity of substitution[1] between different immediate good. This means that there are imperfect substitution between immediate good and this in turn means that immediate good will have some monopolistic power. The profit maximization problem of final goods producer which take the final good price $P_t$ and intermediate good $P_t(j)$ as given:

$$\max_{Y_t(j)} \; P_t Y_t - \int_0^1 P_t(j) \, Y_t(j) \, dj$$

$$P_t \frac{\epsilon_p - 1}{\epsilon_p} \left( \int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} \, dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1} - 1} \frac{\epsilon_p}{\epsilon_p - 1} Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p} - 1} = P_t(j) \tag{1}$$

It easy to see that the right hand side of Eq.(1) is marginal cost and left-hand side is marginal benefit. This will produce a downward sloping demand curve for variety $j$.

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t$$

This is to say that the relative demand for immediate good $j$ is a function of its relative price, with $\epsilon_p$as the price elasticity of substitution. Zero profit condition for final good producer:

$$P_t Y_t = \int_0^1 P_t(j) \, Y_t(j) \, dj$$

Replacing the demand for variety $j$ produces the aggregate price level equation:

$$P_t = \left( \int_0^1 P_t(j)^{1-\epsilon_p} \, dj \right)^{\frac{1}{1-\epsilon_p}}$$

## 1.2 Intermediate goods producer

1. You write down the intermediate goods production technology

$$Y_t(j) = A_t \bar{K}_t(j)^{\alpha} \, N_t(j)^{1-\alpha}$$

[1]When $\epsilon_p$ goes to infinity, this means that the elasticity will goes to infinity. We can easily to see that the production technology will be linear at this time, hence, perfect substitution. When $\epsilon_p$ goes to zero, there will be no substitution.

while taking the factor prices (the nominal rental rate and nominal wage) as given and where capital service is defined as

$$\bar{K}_t(j) = u_t K_t(j)$$

Firms are not freely adjust their price at each period. So they will choose input to minimize their cost while given the input factor prices and subject to the constraint that it produce enough to meet demand by final goods producers.

$$\min_{\bar{K}_t(j), N_t(j)} W_t^p N_t(j) + R_t^p \bar{K}_t(j)$$

s.t.

$$A_t \bar{K}_t(j)^\alpha N_t(j)^{1-\alpha} \geq \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t$$

The Lagrangian is:

$$\mathcal{L} = -W_t^p N_t(j) - R_t^p \bar{K}_t(j) + \psi_t(j) \left(A_t \bar{K}_t(j)^\alpha N_t(j)^{1-\alpha} - \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t\right)$$

The FOCs w.r.t. labor and capital stock are:

$$R_t^p = \psi_t(j)\,\alpha A_t \bar{K}_t(j)^{\alpha-1} N_t(j)^{1-\alpha}$$

$$W_t^p = \psi_t(j)(1-\alpha) A_t \bar{K}_t(j)^\alpha N_t(j)^{-\alpha}$$

We can combine these two equations to eliminate the multiplier, and we get:

$$\frac{W_t^p}{R_t^p} = \frac{1-\alpha}{\alpha}\frac{\bar{K}_t(j)}{N_t(j)}$$

Define the aggregate labor demand and capital as :

$$N_t^d \equiv \int_0^1 N_t(j)\,dj$$

and

$$\bar{K}_t \equiv \int_0^1 \bar{K}_t(j)\,dj$$

hence we have

$$\frac{w_t}{R_t} = \frac{W_t^p}{R_t^p} = \frac{1-\alpha}{\alpha}\frac{\bar{K}_t}{N_t^d}$$

where $w_t, R_t$ are real wage and real rental rate respectively. Cost minimization problem of intermediate goods firms produce the same capital and labor ratio, i.e., the individual firm capital and labor is independent of subscript $j$. So does the marginal cost. Firms will hire labor and capital in the same rate and in turn equal to aggregate ratio.

2. The households make the capital accumulation and utilization decisions, and rent capital services to firms at nominal rate $R_t^p$. This seems to be a little bit unreasonable to assume that households make utilization decisions, but it simplifies the overall analysis. So we just assume that for simplification purpose. Labor is paid by unit nominal wage $W_t^p$. The first order condition w.r.t. labor or capital implicitly define the marginal cost

$$w_t = mc_t \left(1-\alpha\right) A_t \left(\frac{\bar{K}_t}{N_t^d}\right)^{\alpha}$$

or

$$R_t = mc_t \alpha A_t \left(\frac{\bar{K}_t}{N_t^d}\right)^{\alpha-1}$$

The real flow profit for intermediate firm

$$\frac{\Pi_t^p\left(j\right)}{P_t} = \frac{P_t\left(j\right)}{P_t} Y_t\left(j\right) - mc_t Y_t\left(j\right)$$

Just as what we have done before, firms will have a fixed probability $1-\phi_p$ to optimally adjust their prices. By the same logic, the optimal price satisfies the following conditions:

$$X_{1t} = \lambda_t mc_t P_t^{\epsilon_p} Y_t + \phi\beta E_t X_{1t+1}$$

$$X_{2t} = \lambda_t P_t^{\epsilon_p-1} Y_t + \phi\beta E_t X_{2t+1}$$

$$P_t^{\star} = \frac{\epsilon_p}{\epsilon_p-1}\frac{X_{1t}}{X_{2t}}$$

3. Why we need utilization of capital not in terms of labor? For variable factor utilization to make any sense, you need there to be some dimension along which an input is fixed in the short run. Otherwise you could just hire more or less of the input. With capital, the stock of capital is fixed within period. When shocks hit which raise the marginal product of capital, we would like to have more capital but are constrained by the inherited stock. With utilization we can effectively get more capital out of it. In the case of labor, in our standard model there is no aspect of labor that makes it a state variable, though one could modify the model in sensible directions to make that so. Hence, if a shock hits that makes you want more labor, you can simply hire more labor. There is no sense in utilizing labor more or less if you can just vary the actual quantity of labor in response to shocks.

4. Explain why all firms face the same utilization rate, wage rate and rental rate for the capital.

## 2 Households

Households supply differentiated labor input and are index by $l \in (0,1)$. Household labor is packed into a bundled labor that is sold to firms (which we may also called as final labor). Since household labor is imperfectly substituable (this means that elasticity of substitution between different household lab is not zero, generally, it is larger than unity) , there is a downward-sloping demand for each variety of labor, which gives the household some wage-setting power.

We first consider about the final labor production, which generates a downward-sloping demand curve for labor and implies a wage index. This is same logic as we derive the downward-sloping demand curve for immediate goods and which implies a CPI equation. Then we consider the problem of the household's decision on consumption, bond-holdings, wages, labor supply and capital accumulation, and capital utilization etc.

### 2.1 Final labor

1. The final labor demand

$$N_t = \left( \int_0^1 N_t(j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

where $\epsilon_w > 1$ is the elasticity of substitution between differentiated labor inputs which populate the unit interval. The labor packer maximize its profit while taking the wages given

$$\underset{N_t(l)}{max} W_t^p \left( \int_0^1 N_t(j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} - \int_0^1 W_t(l) N_t(l) \, dl$$

the first order condition with labor is the downward sloping demand curve

$$N_t(l) = \left( \frac{W_t(l)}{W_t} \right)^{-\epsilon_w} N_t$$

The zero profit for final labor packer produce the wage Index

$$W_t = \left( \int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}}$$

### 2.2 Household

1. There are two costs. One is investment adjustment cost. This cost arise if your current investment deviates too much from last period. That is to say that too much or too less investment will cost a lot. This is adjustment cost. We model it in capital accumulation equation. Of course, we can model it in resource constraint. The form of adjustment cost of investment take the form of CEE(2005) as the second equation as follows:

$$K_{t+1} = z_t I_t + (1-\delta) K_t$$

$$K_{t+1} = z_t \left(1 - \frac{\tau}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t + (1-\delta) K_t$$

The first equation above only set an investment shock. The adjustment cost is showed up in the capital accumulation equation and not in the resource constraint. This will produce the hump-shape of investment better than the Hayashi style.

2. Hayashi style

$$K_{t+1} = z_t \left(1 - \frac{\tau}{2}\left(\frac{I_t}{K_t} - \delta\right)^2\right) I_t + (1-\delta) K_t$$

3. The other one is capital utilization cost as showed below. This capital cost will show up in the resource constraint. Resource cost of utilization as follows

$$RC_t = \frac{K_t}{z_t}\left(\chi_1 (u_t - 1) + \chi_2 (u_t - 1)^2\right)$$

why we need the divide by $z_t$. We use consumption as the numeraire. This utilization cost is showed up in resource constraint. The $\chi_1$ turns out to be a constrained parameter and $\chi_2$ is not. We can freely calibrated this parameters but with cautiousness. Bigger value for $\chi_2$ will mute the amplification of shocks.

4. State contingent securities insure wage risk.

5. The budget constraint in real terms

$$C_t + I_t + \frac{B_{t+1}}{P_t} \leq w_t(l) N_t(l) + R_t u_t K_t + \frac{\Pi_t}{P_t} + T_t - RC_t + (1 + i_{t-1}) \frac{B_t}{P_t}$$

6. The flow utility function

$$U(C_t, N_t) = \ln(C_t - bC_{t-1}) - \psi \frac{N_t(l)^{1+\eta}}{1+\eta}$$

7. The Lagrangian function. You have two multipliers, one for budget constraint $\lambda_t$ and one for the capital accumulation $\mu_t$. $\lambda_t$ means that the marginal utility from extra one unit of income and $\mu_t$ means that the marginal utility of extra unit of capital accumulation. If $\tau = 0$, then we have $\lambda_t = z_t \mu_t$ and we define $q_t = \frac{\mu_t}{\lambda_t} = \frac{1}{z_t}$

$$\begin{aligned} \mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(C_t - bC_{t-1}) - \psi \frac{N_t(l)^{1+\eta}}{1+\eta}\right) + \mu_t \left(z_t \left(1 - \frac{\tau}{2}\left(\frac{I_t}{K_t} - \delta\right)^2\right) I_t + (1-\delta) K_t - K_{t+1}\right) \\ &\quad + \lambda_t \left(w_t(l) N_t(l) + R_t u_t K_t + \frac{\Pi_t}{P_t} + T_t - RC_t + (1+i_{t-1}) \frac{B_t}{P_t} - C_t - I_t - \frac{B_{t+1}}{P_t}\right) \end{aligned}$$

The FOCs w.r.t. consumption, utilization rate, bond,investment and capital as follows:

$$\lambda_t = \frac{1}{C_t - bC_{t-1}} - \beta b E_t \frac{1}{C_{t+1} - bC_t}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left(1 + i_t\right) \pi_{t+1}^{-1}$$

$$R_t = \frac{1}{z_t} \left(\chi_1 + \chi_2 \left(u_t - 1\right)\right)$$

$$\lambda_t = \mu_t z_t \left( \left(1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) - \tau \left(\frac{I_t}{I_{t-1}} - 1\right) \frac{I_t}{I_{t-1}} \right) + \beta E_t \mu_{t+1} z_{t+1} \tau \left(\frac{I_{t+1}}{I_t} - 1\right) \left(\frac{I_{t+1}}{I_t}\right)^2$$

$$\mu_t = \beta E_t \left( \lambda_{t+1} \left( R_{t+1} u_{t+1} - \frac{1}{z_{t+1}} \left( \chi_1 \left(u_{t+1} - 1\right) + \chi_2 \left(u_{t+1} - 1\right)^2 \right) \right) + \mu_{t+1} \left(1 - \delta\right) \right)$$

8. You derive the FOC for the wage decision of Household. Allows for the wage stickiness. We still follow the assumption in Calvo(1983) for the household's wage decision. Each period there is a fixed probability $1 - \phi_w$ that they can adjust their wage, or there is a fixed proportion $1 - \phi_w$ that household can optimally adjust their wage. For simplicity, we do not allow wage indexation. The household will choose the optimal wage $w_t^\star(w_t(l))$ to maximize their utility (or minimize the dis-utility of labor) subject the labor demand curve labor final labor packer. The Lagrangian is

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \left(\beta \phi_w\right)^s \left( -\psi \frac{N_{t+s}\left(l\right)^{1+\eta}}{1+\eta} + \lambda_{t+s} \left( \frac{W_{t+s}\left(l\right)}{P_{t+s}} N_{t+s}\left(l\right) \right) \right)$$

where

$$N_t\left(l\right) \leq \left(\frac{W_t\left(l\right)}{W_t}\right)^{-\epsilon_w} N_t$$

The FOC w.r.t. wage $w_t\left(l\right)$ produces the following condition

$$\left(w_t^\star\right)^{1+\epsilon_w \eta} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{E_t \sum_{s=0}^{\infty} \left(\beta \phi_w\right)^s \psi w_{t+s}^{\epsilon_w (1+\eta)} \Pi_{t,t+s}^{\epsilon_w (1+\eta)} N_{t+s}^{1+\eta}}{E_t \sum_{s=0}^{\infty} \left(\beta \phi_w\right)^s \lambda_{t+s} w_{t+s}^{\epsilon_w} \Pi_{t,t+s}^{\epsilon_w - 1} N_{t+s}^{1+\eta}}$$

where $\Pi_{t,t+s} = \frac{P_{t+s}}{P_t}$ is the cumulative gross price inflation between period $t$ and $t+s$. We replace $w_t\left(l\right)$ with $w_t^\star$ since all household who optimally adjust their wages will choose the same wage. We define

$$H_{1t} = E_t \sum_{s=0}^{\infty} \left(\beta \phi_w\right)^s \psi w_{t+s}^{\epsilon_w (1+\eta)} \Pi_{t,t+s}^{\epsilon_w (1+\eta)} N_{t+s}^{1+\eta}$$

and

$$H_{2t} = E_t \sum_{s=0}^{\infty} \left(\beta \phi_w\right)^s \lambda_{t+s} w_{t+s}^{\epsilon_w} \Pi_{t,t+s}^{\epsilon_w - 1} N_{t+s}^{1+\eta}$$

then we rewrite the optimal wage focs as follows:

$$\left(w_t^\star\right)^{1+\epsilon_w\eta} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{H_{1t}}{H_{2t}}$$

and

$$H_{1t} = \psi w_t^{\epsilon_w(1+\eta)} N_t^{1+\eta} + \phi_w \beta E_t \left(\pi_{t+1}\right)^{\epsilon_w(1+\eta)} H_{1t+1}$$

$$H_{2t} = \lambda_t w_t^{\epsilon_w} N_t + \phi_w \beta E_t \left(\pi_{t+1}\right)^{\epsilon_w - 1} H_{2t+1}$$

But there is a problem when we solve this model. For a reasonable parameterizations, the exponent $1+\epsilon_w\eta$ could be very large which could lead to some numerical problem in Dynare. So the focs need further consideration before we can input into the mod file. We define two new auxiliary variables

$$\bar{H}_{1t} = \frac{H_{1t}}{\left(w_t^\star\right)^{\epsilon_w(1+\eta)}}$$

and

$$\bar{H}_{2t} = \frac{H_{2t}}{\left(w_t^\star\right)^{\epsilon_w}}$$

Then focs can be rewrite as

$$w_t^\star = \frac{\epsilon_w}{\epsilon_w - 1}\frac{\bar{H}_{1t}}{\bar{H}_{2t}}$$

$$\bar{H}_{1t} = \psi \left(\frac{w_t}{w_t^\star}\right)^{\epsilon_w(1+\eta)} N_t^{1+\eta} + \phi_w \beta E_t \left(\pi_{t+1}\right)^{\epsilon_w(1+\eta)} \left(\frac{w_{t+1}^\star}{w_t^\star}\right)^{\epsilon_w(1+\eta)} \bar{H}_{1t+1}$$

$$\bar{H}_{2t} = \lambda_t \left(\frac{w_t}{w_t^\star}\right)^{\epsilon_w} N_t + \phi_w \beta E_t \left(\pi_{t+1}\right)^{\epsilon_w - 1} \left(\frac{w_{t+1}^\star}{w_t^\star}\right)^{\epsilon_w} \bar{H}_{2t+1}$$

### 2.3 Aggregation

With the intermediate goods demand curve and production function we have

$$A_t \bar{K}_t\left(j\right)^\alpha N_t\left(j\right)^{1-\alpha} = \left(\frac{P_t\left(j\right)}{P_t}\right)^{-\epsilon_p} Y_t$$

this aggregation produces

$$Y_t = \frac{A_t \bar{K}_t^\alpha N_t^{1-\alpha}}{v_t^p}$$

where the price dispersion

$$v_t^p = \int_0^1 \left(\frac{P_t\left(j\right)}{P_t}\right)^{-\epsilon_p} \mathrm{d}j$$

is defined as before which by Calvo assumption, can be written as

$$v_t^p = (1-\phi_p)(\pi_t^*)^{-\epsilon_p}\pi_t^{\epsilon_p} + \pi_t^{\epsilon_p}\phi_p v_{t-1}^p$$

the aggregate prices and optimal prices focs can be re-written as the follows:

$$x_{1t} = C_t^{-\sigma} mc_t Y_t + \phi_p \beta E_t x_{1t+1}\pi_{t+1}^{\epsilon_p}$$

$$x_{2t} = C_t^{-\sigma} Y_t + \phi_p \beta E_t x_{2t+1}\pi_{t+1}^{\epsilon_p - 1}$$

$$\pi_t^{1-\epsilon_p} = (1-\phi_p)(\pi_t^\star)^{1-\epsilon_p} + \phi_p$$

$$\pi_t^\star = \frac{\epsilon_p}{\epsilon_p - 1}\pi_t \frac{x_{1t}}{x_{2t}}$$

where

$$x_{1t} \equiv \frac{X_{1t}}{P_t^{\epsilon_p}}$$

$$x_{2t} \equiv \frac{X_{2t}}{P_t^{\epsilon_p - 1}}$$

The Government consumption is just time-variant share of output.

$$G_t = \omega_t^g Y_t$$

where

$$\omega_t^g = (1-\rho_g)\omega^g + \rho_g \omega_{t-1}^g + \epsilon_t^g$$

I assume that government consumption balances each period and financed by the lump sum taxes.

$$G_t = T_t$$

If there are capital in the model, it is almost impossible to have closed form solution for the flexible output and hence, it is impossible to have the flexible output in Taylor rule. We could use another way that out. We close the model by set the Taylor rule as the monetary policy. We could model interest rate respond to the growth rate of output.

$$i_t = (1-\rho_i)\, i + \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi(\pi_t - \pi) + \phi_y(logY_t - logY_{t-1})\right) + \epsilon_t^i$$

## 2.4 Equilibrium and Calibration

I am not going to estimate the parameters. I will simply calibrate most of the parameters used the values typically used in the literature.

1. Normalization that we have to impose. The steady states of utilization rate $u = 1$. The steady states of multipliers $\mu = \lambda$ since $z = 1$. This turn out the parameter $\chi_1$ is not a free parameter. It easy to see that $\chi_1 = R = \frac{1}{\beta} - (1-\delta)$. Why we need a small number of $\chi_2 = 0.01$ for example. If we set it to infinity, we literally set $u_t = 1$ because moving around will be costly.

2. What does the parameter $\chi_2$ does here? If $\chi_2 = 100$, this will mute the response of output to shock. The bigger is $\chi_2$,the more costly it is to engage in capital utilization. Hence, larger values of $\chi_2$ will lead to smaller movements in $u_t$ and hence less amplification of shocks.

3. We set $\beta = 0.99$, $\alpha = 1/3, \omega^g = 0.2$. $\epsilon_w = \epsilon_p = 10$ means that the markup is about 10%.

4. the habit formation parameter $b = 0.65$;$\tau = 2$; $\delta = 0.025$; $\eta = 1$; the Calvo parameters $\phi_w = \phi_p = 0.75$ which means the frequency to adjust is about 4 quarters, or one year.

5. The parameters in monetary $\phi_\pi = 1.5$ and $\phi_y = 0.5$ for a standard calibration.

6. The persistence parameters $\rho_i, \rho_a, \rho_z, \rho_g$ are all set to 0.9 for simplicity. And the standard deviation for the for shocks are all set to 0.01.

7. In equilibrium, the labor supply $N_t$ and labor demand $N_t^d$ must be equal.

8. The steady states: First, the shock variables $A_t, z_t, \omega_t^g$. For the first two, the steady states are set to unity. and the steady state of $\omega_t^g$ is calibrated to 0.2. We set steady state value for final labor $N = \frac{1}{3}$. From home Euler equation, we have the steady state for the gross nominal rate $i = \frac{\pi}{\beta}$, where $\pi = 1$ this means a zero inflation rate (we assume zero inflation rate for a simple calculation). Zero inflation means that $\pi^\star = 1$ and $v^p = 1$. The investment decision equation says the two multipliers are equal to each other in steady states. Hence Hayashi Q has the steady state unity. The capital decision implies the steady state of capital rental rate $R = \frac{1}{\beta} - (1-\delta)$. The optimal price equations implies that marginal cost $mc = \frac{\epsilon_p - 1}{\epsilon_p}$. The capital-labor ratio equation and the marginal cost of labor equation together decide the steady state of real wage $\omega = (1-\alpha)\left(\alpha^\alpha \frac{mc}{R^\alpha}\right)^{\frac{1}{1-\alpha}}$. Hence, the capital labor ratio equation will decide the steady state of capital stock $K = \frac{w}{R}\frac{\alpha}{1-\alpha}$. The capital accumulation equation decides $I = \delta K$. The production technology decides the steady state output $Y = \frac{AK^\alpha N^{1-\alpha}}{v^p}$. The resource constraint will decides the steady consumption $C = Y - G - I$. The optimal wage equations together decides the parameter $\psi = \frac{\epsilon_p - 1}{\epsilon_p}\frac{w}{\lambda}N$ and $\lambda = \frac{1-\beta b}{1-b}C$. Under the calibration of the parameters, we have $\psi = 8.4428$. Since $\lambda$ is known, we also have $\mu$.

9. The full set of equilibrium conditions as follows: there are 26 equations with 26 variables:
$\lambda_t, C_t, R_t, z_t, i_t, u_t, \mu_t, I_t, w_t^\star, w_t, x_{1t}, x_{2t}, Y_t, A_t, N_t, v_t^p$,$\pi_t, \pi_t^\star, \bar{H_{1t}}, \bar{H}_{2t}, mc_t, K_t, \bar{K}_t, G_t, q_t, \omega_t^g$.

$$\lambda_t = \frac{1}{C_t - bC_{t-1}} - \beta b E_t \frac{1}{C_{t+1} - bC_t} \tag{2}$$

$$\lambda_t = \beta E_t \lambda_{t+1} (1 + i_t) \pi_{t+1}^{-1} \tag{3}$$

$$R_t = \frac{1}{z_t} (\chi_1 + \chi_2 (u_t - 1)) \tag{4}$$

$$\lambda_t = \mu_t z_t \left( \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) - \tau \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right) + \beta E_t \mu_{t+1} z_{t+1} \tau \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \tag{5}$$

$$\mu_t = \beta E_t \left( \lambda_{t+1} \left( R_{t+1} u_{t+1} - \frac{1}{z_{t+1}} \left( \chi_1 (u_{t+1} - 1) + \chi_2 (u_{t+1} - 1)^2 \right) \right) + \mu_{t+1} (1 - \delta) \right) \tag{6}$$

$$w_t^\star = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\bar{H}_{1t}}{\bar{H}_{2t}} \tag{7}$$

$$\bar{H}_{1t} = \psi \left( \frac{w_t}{w_t^\star} \right)^{\epsilon_w (1+\eta)} N_t^{1+\eta} + \phi_w \beta E_t (\pi_{t+1})^{\epsilon_w (1+\eta)} \left( \frac{w_{t+1}^\star}{w_t^\star} \right)^{\epsilon_w (1+\eta)} \bar{H}_{1t+1} \tag{8}$$

$$\bar{H}_{2t} = \lambda_t \left( \frac{w_t}{w_t^\star} \right)^{\epsilon_w} N_t + \phi_w \beta E_t (\pi_{t+1})^{\epsilon_w - 1} \left( \frac{w_{t+1}^\star}{w_t^\star} \right)^{\epsilon_w} \bar{H}_{2t+1} \tag{9}$$

$$w_t^{1-\epsilon_w} = (1 - \phi_w) (w_t^\star)^{1-\epsilon_w} + \pi_t^{\epsilon_w - 1} \phi_w w_{t-1}^{1-\epsilon_w} \tag{10}$$

$$Y_t = \frac{A_t \bar{K}_t^\alpha N_t^{1-\alpha}}{v_t^p} \tag{11}$$

$$v_t^p = (1 - \phi_p) (\pi_t^*)^{-\epsilon_p} \pi_t^{\epsilon_p} + \pi_t^{\epsilon_p} \phi_p v_{t-1}^p \tag{12}$$

$$x_{1t} = C_t^{-\sigma} mc_t Y_t + \phi_p \beta E_t x_{1t+1} \pi_{t+1}^{\epsilon_p} \tag{13}$$

$$x_{2t} = C_t^{-\sigma} Y_t + \phi_p \beta E_t x_{2t+1} \pi_{t+1}^{\epsilon_p - 1} \tag{14}$$

$$\pi_t^{1-\epsilon_p} = (1 - \phi_p) (\pi_t^\star)^{1-\epsilon_p} + \phi_p \tag{15}$$

$$\pi_t^\star = \frac{\epsilon_p}{\epsilon_p - 1} \pi_t \frac{x_{1t}}{x_{2t}} \tag{16}$$

$$w_t = mc_t (1 - \alpha) A_t \left( \frac{\bar{K}_t}{N_t} \right)^\alpha \tag{17}$$

$$\frac{w_t}{R_t} = \frac{1 - \alpha}{\alpha} \frac{\bar{K}_t}{N_t} \tag{18}$$

$$Y_t = C_t + I_t + G_t + \frac{K_t}{z_t} \left( \chi_1 (u_t - 1) + \chi_2 (u_t - 1)^2 \right) \tag{19}$$

$$G_t = \omega_t^g Y_t \tag{20}$$

$$\omega_t^g = (1 - \rho_g) \omega^g + \rho_g \omega_{t-1}^g + \epsilon_t^g \tag{21}$$

$$i_t = (1 - \rho_i) i + \rho_i i_{t-1} + (1 - \rho_i) (\phi_\pi (\pi_t - \pi) + \phi_y (log Y_t - log Y_{t-1})) + \epsilon_t^i \tag{22}$$

$$K_{t+1} = z_t \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t + (1-\delta) K_t \tag{23}$$

$$\bar{K}_t = u_t K_t \tag{24}$$

$$logA_t = \rho_a logA_{t-1} + \epsilon_t^a \tag{25}$$

$$logz_t = \rho_z logz_{t-1} + \epsilon_t^z \tag{26}$$

$$q_t = \frac{\mu_t}{\lambda_t} \tag{27}$$

# 3 The Dynare Mod file

The m file (main file)

```
clear all
close all
beta = 0.99;
alpha =1/3;
delta = 0.025;
b = 0.65;
tau = 2;
eta = 1;
omegags = 0.2;

epsw = 10;
epsp = 10;
phiw =0.75;
phip=0.75;
phipi = 1.5;
phiy = 0.5;
chi1 = 1/beta - (1-delta);
chi2 = 0.01;

rhog = 0.9;
rhoa =0.9;
rhoi =0.9;
rhoz =0.9;
sdg =0.01;
sda =0.01;
sdi =0.01;
sdz =0.01;


vps = 1;
us  =1;
```

```
zs =1;
As =1;
pis = 1; %zeros inflation steady state
pistars =1;
is = pis/beta; %nominal interest rate

%the following algorithm is only valid under the assumption of zero
%inflation steady states. For a non-zeros inflation s.s, you need
%re-calculate, this is left for homework.

%index.m +mediumdsge.mod will lay out a framework within which to
%recursively run mod file for sensitivity test, or different parameter values.

%xs means the steady state value of x;
Rs= 1/beta - (1-delta); %capital rate
mcs = (epsw-1)/epsw;
ws = (1-alpha)*(mcs*alpha^alpha/Rs^alpha)^(1/(1-alpha));
kn= ws/Rs*alpha/(1-alpha);
yn = kn^alpha;
yk = yn/kn;
iy = delta/yk;
cy = 1 - iy -omegags;
Ns = 1/3;
c1 = (1-beta*b)/(1-b);
Ks = kn*Ns;
Ys = Ks*yk;
invs = delta*Ks;
Gs = Ys*omegags;
Cs = Ys - invs - Gs;
psi = ws*(epsw-1)/epsw/Cs/Ns^eta*c1;

Kbars = Ks;
lams = c1/Cs;
mus = lams;
qs = mus/lams;
wstars = ws;
x1s = lams*mcs*Ys/(1-phip*beta);
x2s = lams*Ys/(1-phip*beta);
h1s = psi*Ns^(1+eta)/(1-phiw*beta);
h2s = lams*Ns/(1-phiw*beta);

%save all the variables into mat file and then load in mod file
save mediumdsge;

%run the dynare mod file or the compiled mod m file
dynare mediumdsge
```

and the mod file (mediumdsge.mod)

```
%this file is written by Xiangyang Li @2015.8.8
%
var lam C R z i u mu inv wstar w x1 x2 Y A N vp pi pistar h1 h2 mc K Kbar G q omegag;
varexo eg ei ea ez;
parameters beta alpha delta b tau eta omegags psi epsw epsp phiw phip chi1 chi2;
parameters rhog rhoi rhoa rhoz sdg sdi sda sdz phipi phiy;
parameters lams Cs Rs zs is us mus invs wstars ws x1s x2s Ys As Ns;
parameters vps pis pistars h1s h2s mcs Ks Kbars Gs qs;

load mediumdsge
set_param_value('beta',beta);
set_param_value('alpha',alpha);
set_param_value('delta',delta);
set_param_value('b',b);
set_param_value('tau',tau);
set_param_value('eta',eta);
set_param_value('omegags',omegags);
set_param_value('psi',psi);
set_param_value('epsw',epsw);
set_param_value('epsp',epsp);
set_param_value('phip',phip);
set_param_value('phiw',phiw);
set_param_value('chi1',chi1);
set_param_value('chi2',chi2);
set_param_value('rhog',rhog);
set_param_value('rhoa',rhoa);
set_param_value('rhoz',rhoz);
set_param_value('rhoi',rhoi);
set_param_value('sdg',sdg);
set_param_value('sda',sda);
set_param_value('sdz',sdz);
set_param_value('sdi',sdi);
set_param_value('phipi',phipi);
set_param_value('phiy',phiy);
set_param_value('lams',lams);
set_param_value('Cs',Cs);
set_param_value('Rs',Rs);
set_param_value('zs',zs);
set_param_value('is',is);
set_param_value('us',us);
set_param_value('mus',mus);
set_param_value('invs',invs);
set_param_value('wstars',wstars);
set_param_value('ws',ws);
```

```
set_param_value('x1s',x1s);
set_param_value('x2s',x2s);
set_param_value('Ys',Ys);
set_param_value('As',As);
set_param_value('Ns',Ns);
set_param_value('vps',vps);
set_param_value('pis',pis);
set_param_value('pistars',pistars);
set_param_value('h1s',h1s);
set_param_value('h2s',h2s);
set_param_value('mcs',mcs);
set_param_value('Ks',Ks);
set_param_value('Kbars',Kbars);
set_param_value('Gs',Gs);
set_param_value('qs',qs);


model;
# invt  = exp(inv)/exp(inv(-1)) -1;
# invtp= exp(inv(+1))/exp(inv) -1;
# invr  =exp(inv)/exp(inv(-1));
# invrp =exp(inv(+1))/exp(inv);
# rc     = chi1*(exp(u)-1) + chi2/2*(exp(u)-1)^2;
# rcp     = chi1*(exp(u(+1))-1) + chi2/2*(exp(u(+1))-1)^2;

%(1) home Euler equation 1
exp(lam) = 1/(exp(C) - b*exp(C(-1))) - beta*b/(exp(C(+1)) - b*exp(C));

%(2) utilization cost, already in ratio
R= (chi1 +chi2*(exp(u)-1))/exp(z);

%(3) home Euler equation 2
exp(lam) = beta*exp(lam(+1))*exp(i)/exp(pi(+1));

%(4)investment decision
exp(lam) = exp(mu)*exp(z)*(1-tau/2*invt^2 - tau*invt*invr)
                +beta*exp(mu(+1))*exp(z(+1))*tau*invtp*invrp^2;

%(5)capital stock desicion
exp(mu) = beta*(exp(lam(+1)) *(R(+1)*exp(u(+1))- rcp/exp(z(+1)))
                +exp(mu(+1))*(1-delta));

%(6) optimal wage decision
exp(wstar) = epsw/(epsw-1) * exp(h1 - h2);

%(7) auxiliary h1
```

```
exp(h1) = psi*exp(epsw*(1+eta)*(w-wstar))*exp((1+eta)*N)
            +phiw*beta*exp(epsw*(1+eta)*pi(+1))*exp(epsw*(1+eta)*(wstar(+1)-wstar))*exp(

%(8) auxiliary h2
exp(h2) = exp(lam)*exp(epsw*(w-wstar))*exp(N)
            +phiw*beta*exp((epsw-1)*pi(+1))*exp(epsw*(wstar(+1)-wstar))*exp(h2(+1));

%(9) wage index
exp((1-epsw)*w) = (1-phiw)*exp((1-epsw)*wstar)+exp((epsw-1)*pi)*phiw*exp((1-epsw)*w(-1));

%(10) production technology
exp(Y) = exp(A)*exp(alpha*Kbar)*exp((1-alpha)*N)/exp(vp);

%(11) the price dispersion
exp(vp) = exp(epsw*pi)*((1-phip)*exp(-epsw*pistar)+phip*exp(vp(-1)));

%(12) the CPI
exp((1-epsp)*pi) = (1-phip)*exp((1-epsp)*pistar) +phip;

%(13) optimal price decision
exp(pistar) = epsp/(epsp-1)*exp(pi)*exp(x1-x2);

%(14) auxiliary variable x1
exp(x1) = exp(lam+mc+Y)+phip*beta*exp(epsp*pi(+1))*exp(x1(+1));

%(15) auxiliary variable x2
exp(x2) = exp(lam+Y)+phip*beta*exp((epsp-1)*pi(+1))*exp(x2(+1));

%(16)capital-labor ratio
exp(w)/R=(1-alpha)/alpha*exp(Kbar-N);

%(17)wage decision
exp(w) = exp(mc+A)*(1-alpha)*exp(alpha*(Kbar-N));

%(18)resource constraint
exp(Y) = exp(C) + exp(inv) + exp(G) + rc*exp(K(-1))/exp(z);

%(19)capital evolution
exp(K) = exp(z)*(1-tau/2*invt^2)*exp(inv)+(1-delta)*exp(K(-1));

%(20)capital service
exp(Kbar) = exp(u)*exp(K(-1));

%(21) government spending
exp(G) = omegag*exp(Y);
```

```
%(22) law of government spending share,omegag could be zero or negative in simulation
omegag = (1-rhog)*omegags + rhog*omegag(-1) + eg;

%(23)technology shock
A = rhoa*A(-1) + ea;

%(24)investment-specific shock
z = rhoz*z(-1) + ez;

%(25)Hayashi Q
exp(q) = exp(mu - lam);

%(26) Taylor rule
i = (1-rhoi)*log(is) + rhoi*i(-1) + (1-rhoi)*(phipi*(pi - log(pis))+phiy*(Y-log(Ys)))+ei;

end;

initval;
lam = log(lams);
C = log(Cs);
R =Rs;
z  =log(zs);
i   = log(is);
u  = log(us);
mu =log(mus);
inv  =log(invs);
wstar =log(wstars);
w =log(ws);
x1 =log(x1s);
x2 =log(x2s);
Y  =log(Ys);
A =log(As);
N =log(Ns);
vp =log(vps);
pi =log(pis);
pistar =log(pistars);
h1 =log(h1s);
h2 =log(h2s);
mc =log(mcs);
K   =log(Ks);
Kbar =log(Kbars);
G =log(Gs);
q =log(qs);
omegag = omegags;
end;
```

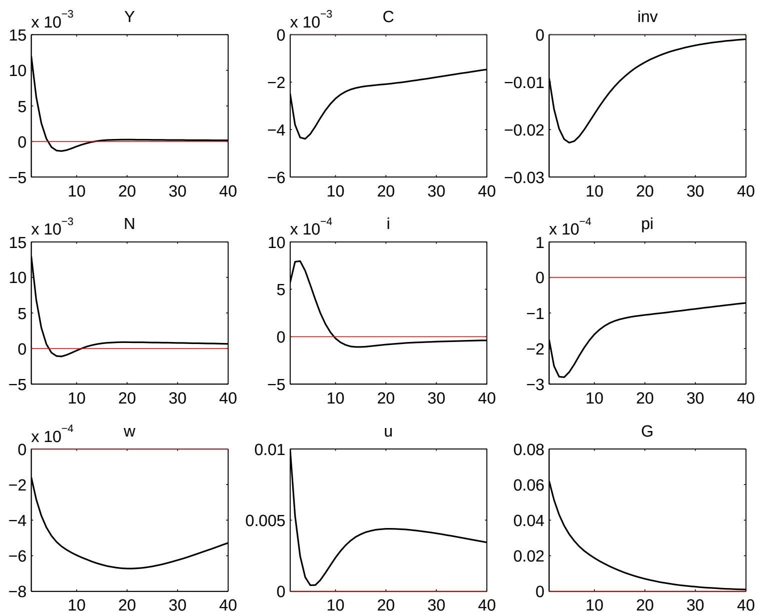
```
shocks;
var eg = sdg^2;
var ea = sda^2;
var ei  = sdi^2;
var ez = sdz^2;
end;

resid(1);
steady;
check;

stoch_simul(order=1)  Y C inv N i pi w u z A G;
```

## 4 The IRF

Figure 1: IRF to government spending shock




The government spending shock, this leads to a rise in output, hours but crowds out private expenditure, with consumption and investment declines which is typically found in the literature. The output multiplier is about 0.9648 which suggest that output rise less than government spending. Furthermore, utilization rate increases at the same time and this means that cost rises too. Hence, consumption and investment are crowded out seriously.