Macroeconomics Summer School
Part II: Advanced Tools
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Thursday Assignment
Solving and simulating the Diamon-Mortensen-Pissarides model

For this problem set, you are asked to solve the Diamon-Mortensen-Pissarides model in continuous time with risk-averse firm owners. For simplicity, I will assume that real wages are sticky, such that profits, π_t, are exogenously given. In Δ-units of time, the model is given by the equations

$$J_t = \Delta\pi_t + (1-\Delta\rho)\frac{u'(c_{t+\Delta})}{u'(c_t)}J_{t+\Delta}(1-\Delta\delta), \tag{1}$$

$$\Delta\kappa = \Delta h(\theta_t)J_t, \tag{2}$$

$$\Delta c_t = \Delta(n_t - \kappa\theta_t(1-n_t)), \tag{3}$$

$$n_{t+\Delta} = (1-n_t)\Delta f(\theta_t) + (1-\Delta\delta)n_t, \tag{4}$$

where $f(\cdot)$ and $h(\cdot)$ refer to the job-finding rate and job-filling rate, respectively.

Part A. Equations (2) and (3) are trivial as the Δ cancel out. Equation (4) is straightforward. Rearrange such that

$$\frac{n_{t+\Delta} - n_t}{\Delta} = (1-n_t)f(\theta_t) - \delta n_t \tag{5}$$

Taking the limit gives

$$\dot{n}_t = (1-n_t)f(\theta_t) - \delta n_t. \tag{6}$$

For equation (1) use the approximations $u'(c_{t+\Delta}) \approx u'(c_t) + u''(c_t)\dot{c}_t\Delta$, and $J_{t+\Delta} \approx J_t + \dot{J}_t\Delta$, and substitute in

$$J_t = \Delta\pi_t + (1-\Delta\rho)\frac{u'(c_t) + u''(c_t)\dot{c}_t\Delta}{u'(c_t)}(J_t + \dot{J}_t\Delta)(1-\Delta\delta), \tag{7}$$

or

$$J_t = \Delta\pi_t + (1-\Delta\rho)\left(1 - \gamma\frac{\dot{c}_t}{c_t}\Delta\right)(J_t + \dot{J}_t\Delta)(1-\Delta\delta). \tag{8}$$

Expand

$$J_t = \Delta\pi_t + \left(1 - \gamma\frac{\dot{c}_t}{c_t}\Delta - \Delta\rho + \Delta^2\rho\gamma\frac{\dot{c}_t}{c_t}\right)(J_t + \dot{J}_t\Delta - \Delta\delta J_t - \Delta^2\delta\dot{J}_t). \tag{9}$$

Drop all Δ^2 terms as they will anyway vanish when we divide by Δ and take limits. Thus,

$$J_t = \Delta\pi_t + \left(1 - \gamma\frac{\dot{c}_t}{c_t}\Delta - \Delta\rho\right)(J_t + \dot{J}_t\Delta - \Delta\delta J_t). \tag{10}$$

Expand again and drop all Δ^2 terms

$$J_t = \Delta\pi_t + J_t + \dot{J}_t\Delta - \delta J_t\Delta - \gamma\frac{\dot{c}_t}{c_t}\Delta J_t - \Delta\rho J_t.$$

Subtract J_t from both sides and divide by Δ

$$0 = \pi_t + \dot{J}_t - \delta J_t - \gamma\frac{\dot{c}_t}{c_t}J_t - \rho J_t.$$

Rearrange

$$(\rho + \delta)J_t = \pi_t + \dot{J}_t - \gamma\frac{\dot{c}_t}{c_t}J_t.$$

Given that $\dot{J}_t = J'(n_t)\dot{n}_t$ and $\dot{c}_t = c'(n_t)\dot{n}_t$, we finally get the model in continuous time

$$(\rho + \delta)J_t = \pi_t - J_t\gamma\frac{c'(n_t)\dot{n}_t}{c_t} + J'(n_t)\dot{n}_t \tag{11}$$

$$\kappa = h(\theta_t)J_t, \tag{12}$$

$$c_t = n_t - \kappa\theta_t(1 - n_t), \tag{13}$$

$$\dot{n}_t = (1 - n_t)f(\theta_t) - \delta n_t, \tag{14}$$