1 True model

Suppose the true model has the following equation

$$x_t = \mathbb{E}_t\left[y_{t+1}\right]\mathbb{E}_t\left[z_{t+1}\right]. \tag{1}$$

2 Dynare

How would you incorporate this into the Dynare model block? This is a bit tricky since Dynare equations do not include expectation operators. You may be tempted to program this equation as follows:

$$\texttt{x = y(+1)*z(+1).} \tag{2}$$

This would be wrong! This Dynare equation would be correct for the alternative model given by

$$x_t = \mathbb{E}_t\left[y_{t+1}z_{t+1}\right]. \tag{3}$$

This is a different equation unless y_t and z_t are independent.

The correct way to incorporate the original equation into Dynare would be

$$\texttt{yexp} = \texttt{y}(+1), \tag{4}$$

$$\texttt{zexp} = \texttt{z}(+1), \tag{5}$$

$$\texttt{x} = \texttt{yexp} * \texttt{zexp}. \tag{6}$$

Why do you need the extra two equations? Recall from the slides (on the derivation of perturbation solutions) that we add *one* forecast error to each equation. But the original equation does not have one forecast error. It has two.

3 First-order?

Although equation (1) can be solved with Dynare you do need second-order. The reason is that the interaction between y_t and z_t is key and that would be lost in a linear approximation (both for the original and the alternative model).

4 Example

Suppose the complete model is the following:

$$x_t = \mathbb{E}_t\left[y_{t+1}\right]\mathbb{E}_t\left[z_{t+1}\right], \tag{7}$$

$$y_t = \rho_y y_{t-1} + \varepsilon_t, \tag{8}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t. \tag{9}$$

The alternative model is given by

$$
\begin{aligned}
x_t &= \mathbb{E}_t\left[y_{t+1}z_{t+1}\right], && (10)\\
y_t &= \rho_y y_{t-1} + \varepsilon_t, && (11)\\
z_t &= \rho_z z_{t-1} + \varepsilon_t. && (12)
\end{aligned}
$$

Note that y_t and z_t are driven by the same innovation. Such perfect correlation is only done to simplify the model. Recall that these two models are different as long as y_{t+1} and z_{t+1} are not independent.

True solution. The true solution for the original model is (obviously)

$$x_t = \rho_y \rho_z y_t z_t \tag{13}$$

and the solution of the alternative model is given by

$$x_t = \rho_y \rho_z y_t z_t + \sigma^2, \tag{14}$$

where σ is the standard deviation of ε_t.

First-order perturbation. The first-order perturbation soluton for x_t is given by

$$\texttt{x = 0} \tag{15}$$

for both models. As mentioned above, the cross product is key and first-order perturbation does not give us that.

Second-order perturbation. The Dynare program for the original model would be

$$
\begin{aligned}
&\texttt{y = rho\_y*y(-1)+e,} && (16)\\
&\texttt{z = rho\_z*z(-1)+e,} && (17)\\
\texttt{yexp} &= \texttt{y}(+\texttt{1}), && (18)\\
\texttt{zexp} &= \texttt{z}(+\texttt{1}), && (19)\\
\texttt{x} &= \texttt{yexp} * \texttt{zexp}. && (20)
\end{aligned}
$$

This Dynare program gives the correct solution for the original model when second-order perturbation is used.

If we use the following dynare model

$$
\begin{aligned}
&\texttt{y = rho\_y*y(-1)+e,} && (21)\\
&\texttt{z = rho\_z*z(-1)+e,} && (22)\\
\texttt{x} &= \texttt{y}(+\texttt{1}) * \texttt{zexp}(+\texttt{1}), && (23)
\end{aligned}
$$

then we get a *different* solution,namely the one corresponding to the alternative model.