Perturbation and Dynare

Simple DSGE Model

Tools for Macroeconomists: The essentials

Petr Sedláček

Neoclassical Growth Model

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- production subject to exogenous fluctuations in productivity

Production

$$y_t = Z_t k_t^{\alpha}$$

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$$Z_t = 1 - \rho + \rho Z_{t-1} + \epsilon_t$$

$$\mathbb{E}\epsilon_t = 0$$

$$\mathbb{E}\epsilon_t^2 = \sigma_z^2$$

Household decision

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$$Z_0 \quad \text{given}$$

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- different $k_0 \rightarrow$ optimal sequences different!
- different realizations of $Z_t \rightarrow$ optimal sequences different!

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  - realizations of exogenous variables, pre-determined variables (e.g. capital stock)
  - not always easy to know what the state variables are!

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$$u_c(c_t) = \beta \mathbb{E}_t u_c(c_{t+1}) \left(\alpha Z_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta \right)$$

$$c_t + k_{t+1} = y_t + (1 - \delta) k_t$$

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$$Z_{+1} = 1 - \rho + \rho Z + \epsilon$$

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$$c_t = (1 - \alpha\beta) Z_t k_t^{\alpha}$$

Neoclassical Growth Model

Taking stock

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- workhorse DSGE model which we'll encounter throughout the course
- solution consists of policy functions
- computational tools necessary to approximate such policy functions


That's all Folks!