Notes For Chapter 2, 3,7 in Galí(2008)

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1 Chap2: A Classic Monetary Model

A simple model of a classical monetary economy characterized by perfect competition and fully flexible prices in all markets is developed in this chapter. Although many of the predictions are at odds with empirical evidences, the model does provide us a lot of useful intuitions to understand a lot of facts in more complicated models. For example, there are two predictions that directly at odds with empirical facts:

1. The price level should respond more than one for one with the increase in the money supply, a prediction that contrasts starkly with the sluggish response of the price level observed in empirical estimates of the effects of monetary policy shocks (P23).

2. The model lack of liquidity effect. The nominal interest rate is predicted to go up proportional with the growth of nominal money. While liquidity effect observed indicates that nominal interest rate moves in opposite direction with the growth rate of nominal money supply (P24).

The derivation of real variables are of functions only to real shocks gives us a lot of intuitions. Consumption, output, real interest rate and labor are of functions of technology shock only and independent of monetary shock.

1. Consumption and output are positively related technology.

2. The direction that Labor responds to the technology depends crucially on structural parameter σ, the substitution of elasticity of consumption, which measure the strength of wealth effect of labor supply.

3. The response of the real interest rate depends critically on the time series properties of technology. If technology series keep improving, i.e., $E_t a_{t+1} > a_t$, real rate will move in the same direction with technology shock. If the technology series is transitory, it will will move in opposite direction.

4. If simple Taylor rule is introduced, then inflation is negatively determined by technology. This gives you some intuition that when technology shock impacts, you most often see that inflation goes down.

Nominal variables like inflation and interest rate can not be determined by real shocks or real forces. In order to determine the inflation or price level, exogenous path for nominal interest rate must be introduced. Taylor rule is one of them. Only Taylor rule can uniquely pin down the inflation or price level.

Exogenous rule of quantity of money can also be used as the monetary rule. In turn, it can pin down the interest rate and price level respectively.

In this section, we will code in log form. Note, there is a little bit different from text in notation. In textbook, nominal rate is denoted by i_t. Here we use R_t as the nominal gross rate. They are linked together by $i_t = \log R_t$. Or you can believe $i_t \approx \log(1 + i_t) = \log R_t$.

1.1 Files structure of this sections

1. *chap2_no_money.mod*: simple example without any money quantities involves;

2. *chap2_m_real.mod*: simple example with real money quantities involves;

3. *chap2_m_growth.mod*: simple example with nominal money growth rate involves;

1.2 Equilibrium equations

1. FOC of labor,eq(6), labor supply equation:

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\varphi$$

Here the parameter σ represents the reciprocal of elasticity of the intertemporal substitution (EIS) of consumption. It is easy to see by noticing the definition

$$EIS = \frac{\partial \log \frac{C_{t+1}}{C_t}}{\partial \log \frac{U'(C_t)}{U'(C_{t+1})}}$$

and the assumption that

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}.$$

And the parameter φ represents the reciprocal of Frisch labor supply elasticity (FLSE). In fact, the definition for Frisch labor supply elasticity (under the assumption that constant marginal utility of wealth)

$$FLSE = \frac{\partial N_t}{\partial W_t} \frac{W_t}{N_t}$$

It is easy to find out[1]

$$\frac{\partial N_t}{\partial W_t} = \frac{-\frac{U_N}{W} U_{CC}}{U_{cn}^2 - U_{nn} U_{cc}}$$

Then

$$FLSE = \frac{U_N}{N\left(U_{NN} - \frac{U_{cn}^2}{U_{cc}}\right)}$$

[1]For a simple RBC model, it is easy to get the FOC w.r.t. consumption and labor as

$$U_c = \lambda_t$$

$$-U_N = \lambda_t W_t$$

By noticing that consumption C_t and labor N_t are functions of λ_t, the multiplier of the budget constraint and W_t, then takes derivative at both sides of above two equations, we have

$$U_{cc} \frac{\partial C_t}{\partial W_t} + U_{cN} \frac{\partial N_t}{\partial W_t} = 0$$

$$U_{Nc} \frac{\partial C_t}{\partial W_t} + U_{NN} \frac{\partial N_t}{\partial W_t} = \lambda_t$$

We could easily work out $\frac{\partial N_t}{\partial W_t}$.

2. Euler Equation (7):

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$

where Q_t is the reciprocal of the gross nominal return R_t on one unit of bond, i.e.

$$Q_t = \frac{1}{R_t}$$

Hence, Q_t is the price of the bond that pay one unit of money at maturity.

3. Production function eq.(11)

$$Y_t = A_t N_t^{1-\alpha}$$

4. Wage rate eq.(13): marginal cost = marginal product of labor,labor demand equation,

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

5. The definition of the real rate equal to the nominal minus the expected inflation

$$r_t = R_t / E_t (\pi_{t+1})$$

where all variables denotes the gross ones and in coding, we use the log form version like

$$\log r_t = \log R_t - \log E_t (\pi_{t+1})$$

And in the classic text, we usually see that the Fisher relationship like $\tilde{r}_t = R_t - E_t (\tilde{\pi}_{t+1})$ where the variables denote the log-deviation forms of the gross variables since $\log(1+x) \approx x$ if x is small. And $\tilde{x}$ denotes the log-deviation form, i.e., the percentage deviation form of level variable x.

6. Monetary policy rule eq(22):

$$\phi_\pi \tilde{\pi}_t = E_t (\tilde{\pi}_{t+1}) + \log(r_t) - \rho = \log R_t - \log \beta^{-1}$$

Here we add a monetary policy shock ϵ_t^m in Dynare code to show the neutrality of the money.

7. Market clearing condition eq(15) :

$$Y_t = C_t$$

8. Technology shock, exogenous shock usually be assumed to follow AR(1) process like:

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t^A$$

9. The money equation, postulated or ad hoc equation for money demand is used:

$$m_t - p_t = y_t - \eta \log R_t$$

where small letters denote the log forms of the big letters. But in our programming, we use m_t denotes the log form of real money $\frac{M_t}{P_t}$ as shown *chap2_m_real.mod*. Difference this equation will yield the growth equation for nominal money as shown in the mod file: *chap2_m_growth.mod*.

1.3 Derivation of Eq.(16,17) in Textbook

In order to consistent to the notation in textbook, we use log-level form of variables. First, the resource constraint

$$y_t = c_t$$

and the production technology

$$y_t = a_t + (1-\alpha) n_t$$

then, the labor supply equation (6) and labor demand equation (13) in textbook can be written as the log-level form as in equation(8) and (14) respectively. By eliminating the nominal wage and price level, we could have

$$\sigma c_t + \varphi n_t = a_t - \alpha n_t + \log(1-\alpha)$$

Then this reduces to a system of two equations and two unknowns y_t, n_t while given a_t:

$$\begin{aligned} y_t &= a_t + (1-\alpha) n_t \\ \sigma y_t + \varphi n_t &= a_t - \alpha n_t + \log(1-\alpha) \end{aligned}$$

Here, unknowns mean endogenous variables which is determined by the exogenous variable a_t. This could be solved easily to get equation (17,18) in textbook.

1.4 The Codes

Here is the codes from *chap2_ m_ growth.mod:*

```
/*
 * This mod file implements the baseline Classical Monetary Economy model of Jordi
 * and the Business Cycle, Princeton University Press, Chapter 2
 *
 * Note that this mod-file implements the log-level of non-linear first order cond
 * and that the IRFs show the percentage deviations (not linear deviation) from st
 *
 * It demonstrate the neutrality of money by showing that real variables do not mo
 *
 * This implementation was written by Xiangyang Li. 2016-3@Shanghai
 *
 */

var C           //Consumption
    w      //Real Wage
    pi         //Gross inflation
    A           //AR(1) technology process
    N           //Hours worked
    R           //Gross Nominal Interest Rate
    r //Real Interest Rate
    Y           //Output
    m_growth_ann;//money growth

varexo eps_A    //technology shock
       eps_m;   //monetary policy shock

parameters alpha beta rho sigma phi phi_pi eta;
parameters Cs ws pis As Ns Rs rs Ys m_growth_anns;
%----------------------------------------------------------------
% Follows parametrization of Chapter 3, p. 52
%----------------------------------------------------------------

alpha=0.33; //capital share
beta=0.99;  //discount factor
rho=0.9;     //autocorrelation technology shock
```

```
sigma=1;    //log utility
phi=1;       //unitary Frisch elasticity
phi_pi=1.5;  //inflation feedback Taylor Rule
eta  =0.5;     // semi-elasticity of money demand

As=1;
Rs=1/beta;
pis=1;
rs=Rs;
Ns=(1-alpha)^(1/((1-sigma)*alpha+phi+sigma));
Cs=As*(Ns^(1-alpha));
ws=((1-alpha)*As*Ns^(-alpha));
Ys=Cs;
m_growth_anns=0.14;

%----------------------------------------------------------------
% First Order Conditions
%----------------------------------------------------------------

model;
//1. labor demand, eq. (6) in Chap 2;
exp(w)=exp(sigma*C+phi*N);

//2. Euler equation eq. (7) in Chap 2;
1/exp(R)=beta*(exp(-sigma*(C(+1)- C)) - pi(+1));

//3. Production function eq. (11)
exp(A+(1-alpha)*N)= exp(C);

//4. Wage rate, eq. (13)
exp(w)=(1-alpha)*exp(A-alpha*N);

//5. Definition Real interest rate
exp(r)=exp(R-pi(+1));

//6. Monetary Policy Rule, eq. (22,24)
R=log(1/beta)+phi_pi*(pi)+eps_m;
```

```
//7. Market Clearing, eq. (15)
exp(C)=exp(Y);

//8. Technology Shock
A=rho*A(-1)+eps_A;

//9. Nominal Money growth (derived from eq. (10))
m_growth_ann=m_growth_anns+4*(Y-Y(-1)-eta*(R-R(-1))+pi);

end;

%----------------------------------------------------------------
%  define shock variances
%---------------------------------------------------------------

shocks;
var eps_A; stderr 1;
var eps_m; stderr 1;
end;

%----------------------------------------------------------------
%  Initial Values for steady state
%---------------------------------------------------------------

initval;
A=log(As);
R=log(Rs);
pi=log(pis);
r=log(rs);
N=log(Ns);
C=log(Cs);
w=log(ws);
Y=log(Ys);
m_growth_ann=m_growth_anns;
end;


resid(1);
```

```
steady;
check;

%----------------------------------------------------------------
% generate IRFs to show neutrality of money
%----------------------------------------------------------------

stoch_simul(order=1) Y C pi R r m_growth_ann N;
```

1.5 Experiments

1. Change the value of sigma to see the IRF of labor: $\sigma = 1, 1.5, 0.9$;
2. Show the neutrality of Money: Real variables are not effected by the nominal force, i.e. the monetary policy shock here.

2 Chap3: The Basic NK Model

This equilibrium of this classic model leads to the canonical representation of typical New Keynesian Model in the literature, which includes the New Keynesian Phillips curve, a dynamic IS equation and a description of monetary policy.

2.1 The Equilibrium equations with Taylor rule

In this section, we will code in log-deviation form instead of log form. Read *chap3.mod* for more details.

1. The NPKC eq(21)
$$\tilde{\pi}_t = \beta E_t \left(\tilde{\pi}_{t+1} \right) + \kappa \tilde{y}_t$$
where κ is the function of structural parameters.
2. The output Gap eq(22)
$$\tilde{y}_t = -\frac{1}{\sigma} \left(\tilde{R}_t - E_t \left(\tilde{\pi}_{t+1} \right) - \tilde{r}^n_t \right) + E_t \left(\tilde{y}_{t+1} \right)$$

3. The natural rate eq(23)

$$r_t^n = \rho + \sigma\psi_{ya}^n E_t\left(\Delta a_{t+1}\right)$$

or in log-deviation

$$\tilde{r_t^n} = \sigma\psi_{ya}^n E_t\left(\Delta a_{t+1}\right)$$

4. The interest rate rule eq(25)

$$\log R_t = \rho + \phi_\pi\tilde{\pi}_t + \phi_y\tilde{y}_t + v_t$$

or in log-deviation

$$\tilde{R}_t = \phi_\pi\tilde{\pi}_t + \phi_y\tilde{y}_t + v_t$$

5. The real rate $r_t = R_t/E_t\left(\pi_{t+1}\right)$ in level.

6. Production function $y_t = a_t + (1-\alpha)\, n_t$ in log-level or in log-deviation form $\tilde{y}_t = a_t + (1-\alpha)\,\tilde{n}_t$.

7. The natural output $y_t^n = \psi_{ya}^n a_t + \vartheta_y^n$ in log-level or $\tilde{y}_t^n = \psi_{ya}^n a_t$ in log-deviation form.

8. The monetary policy shock

$$v_t = \rho_v v_{t-1} + \epsilon_t^v$$

9. The output gap $\tilde{y}_t = y_t - y_t^n$

10. Money growth the same as in Chap2.

11. Technology shock process, the same in Chap2.

12. Annualized Terms: All the parameters are calibrated on the basis of quarterly data, hence, the annualized terms are obtained by multiply 4 on their quarterly counterparts.

2.2 The Codes

As you can see from the coding, use the 'shocks' and 'stoch_simul' commands twices:

```
/*
 * This file implements the baseline New Keynesian model of Jordi Gali(2008):
 * Monetary Policy, Inflation,
 * and the Business Cycle, Princeton University Press, Chapter 3
 *
 * Note that all variables are expressed in log-deviations;
 */

var pi          //inflation
    y_gap       //output gap
    y_nat       //natural output
    y           //output
    r_nat       //natural interest rate
    r      //real interest rate
    R           //nominal interst rate
    n           //hours worked
    m_growth_ann//money growth
    nu          //AR(1) monetary policy shock process
    a           //AR(1) technology shock process
    r_ann  //annualized real interest rate
    R_ann       //annualized nominal interest rate
    r_nat_ann   //annualized natural interest rate
    pi_ann;     //annualized inflation rate

varexo eps_a    //technology shock
       eps_nu;  //monetary policy shock

parameters beta sigma psi_n_ya rho_nu rho_a phi;
parameters phi_pi phi_y kappa alpha epsilon eta;

%----------------------------------------------------------------
% Parametrization, p. 52
%----------------------------------------------------------------
sigma = 1;      //log utility
```

```
phi=1;          //unitary Frisch elasticity
phi_pi = 1.5;   //inflation feedback Taylor Rule
phi_y  = .5/4;  //output feedback Taylor Rule
theta=2/3;      //Calvo parameter
//rho_nu = 0.9;   //high persistent monetary policy shock
rho_nu =0.5; // moderately persistent mon. pol. shock
rho_a  = 0.9;   //autocorrelation technology shock
beta = 0.99;    //discount factor
eta  =4;        // semi-elasticity of money demand
alpha=1/3;      //capital share
epsilon=6;      //demand elasticity

//Composite parameters
Omega=(1-alpha)/(1-alpha+alpha*epsilon);  //defined on page 47
psi_n_ya=(1+phi)/(sigma*(1-alpha)+phi+alpha); //defined on page 48
lambda=(1-theta)*(1-beta*theta)/theta*Omega; //defined on page 47
kappa=lambda*(sigma+(phi+alpha)/(1-alpha));  //defined on page 49

%----------------------------------------------------------------
% First Order Conditions
%----------------------------------------------------------------

model(linear);
//1. New Keynesian Phillips Curve eq. (21)
pi=beta*pi(+1)+kappa*y_gap;

//2. Dynamic IS Curve eq. (22)
y_gap=-1/sigma*(R-pi(+1)-r_nat)+y_gap(+1);

//3. Interest Rate Rule eq. (25)
R=phi_pi*pi+phi_y*y_gap+nu;

//4. Definition natural rate of interest eq. (23)
r_nat=sigma*psi_n_ya*(a(+1)-a);

//5. Definition real interest rate
r=R-pi(+1);
```

```
//6. Definition of natural output
y_nat = psi_n_ya*a;

//7. Definition output gap
y_gap=y-y_nat;

//8. Monetary policy shock
nu=rho_nu*nu(-1)+eps_nu;

//9. TFP shock
a=rho_a*a(-1)+eps_a;

//10. Production function (eq. 13)
y=a+(1-alpha)*n;

//11. Nominal Money growth (derived from eq. (4))
m_growth_ann=4*(y-y(-1)-eta*(R-R(-1))+pi);

//12. Annualized nominal interest rate
R_ann=4*R;

//13. Annualized real interest rate
r_ann=4*r;

//14. Annualized natural interest rate
r_nat_ann=4*r_nat;

//15. Annualized inflation
pi_ann=4*pi;
end;

%----------------------------------------------------------------
%  define shock variances
%---------------------------------------------------------------

shocks;
var eps_nu = 0.25^2; //1 standard deviation shock of 25 basis points
end;
```

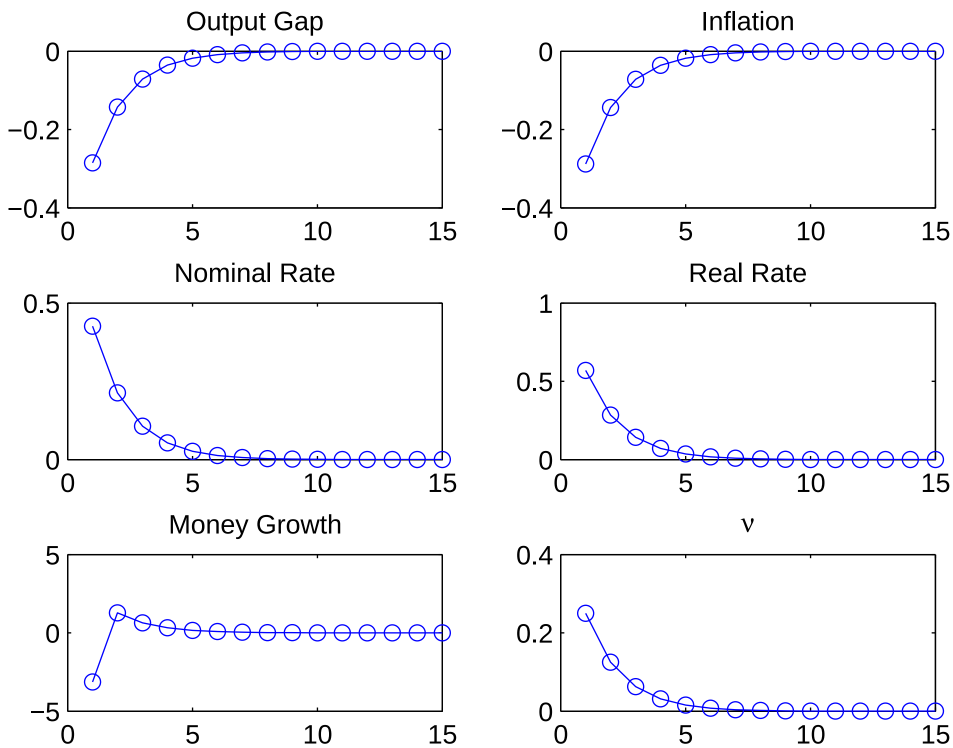
```
%----------------------------------------------------------------
%  steady states: all 0 due to linear model, no initval block anymore
%----------------------------------------------------------------
resid(1);
steady;
check;

%----------------------------------------------------------------
% generate IRFs, replicates Figures 3.1, p. 53
%----------------------------------------------------------------
stoch_simul(order = 1,irf=15) y_gap pi_ann R_ann r_ann m_growth_ann nu;


shocks;
var eps_nu = 0;   //shut off monetary policy shock
var eps_a  = 1^2; //unit shock to technology
end;

%----------------------------------------------------------------
% generate IRFs, replicates Figures 3.2, p. 55
%----------------------------------------------------------------
stoch_simul(order = 1,irf=15) y_gap pi_ann y n R_ann r_ann m_growth_ann a ;
```


2.3 Some Explanations

The IRF of the two shocks: technology shock and the monetary policy shock. In a way consistent with the empirical results it is seen that the policy shock generates an increase in the real rate, and a decrease in inflation and output. There are liquidity effects to the monetary policy shock (i.e., the liquidity contraction by the central bank will have negative effect on nominal interest rate). In order to observed the interest rate responses (downward slope which is induced by the declines in output and inflation as in the Taylor rule), the central bank must engineer a reduction in money supply.

Note also that the nominal rate goes up less than the real rate as the first period as a result of a decrease of inflation. You can carry out a simple example to illustrate this point:

Figure 1: The IRF of Monetary Policy Shock



```
>> r_ann_eps_nu(1)
ans =
    0.5698
>> R_ann_eps_nu(1)
ans =
    0.4260
>> pi_ann_eps_nu(1)
ans =
   -0.2877
>> pi_ann_eps_nu(2)
ans =
   -0.1439
>> pi_ann_eps_nu(2) + r_ann_eps_nu(1)
ans =
    0.4260
```

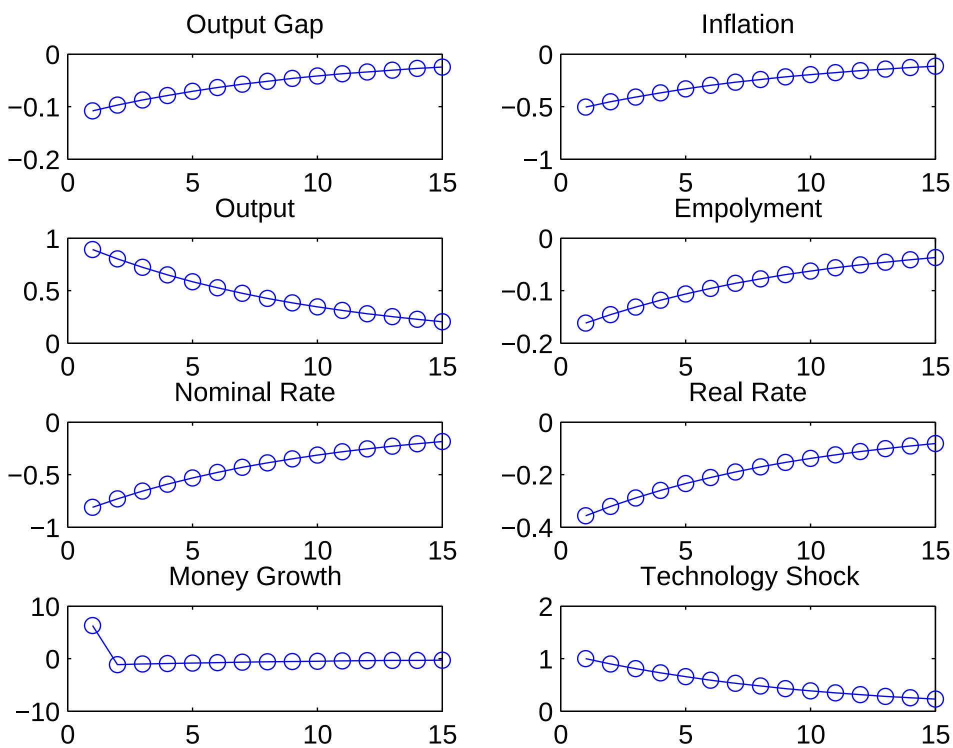
The IRF to a favorable technology shock shows the responses of a number of variables in Fig.2. Notice that the improvement in technology is partly accommodated by the central bank, which lowers nominal and real rates, while increasing the quantity of money in circulation. That policy, however, is not sufficient to close a negative output gap, which is responsible for the decline in inflation[2].

2.4 The codes for Plotting

```
%this m script should be run after runing chap3.mod
%% IRF plotting: monetary shock
%-----------------------------------------------------------------
% generate IRFs, replicates Figures 3.1, p. 53
%-----------------------------------------------------------------
close all
T=1:1:15;
```

[2] Due to the sticky price in the model which generates some inefficiency and as a result, the actual level of output is lower contrast to the flexible price output equilibrium which is proportional to technology, output gap falls to be negative as a result. From the NKPC, the inflation will be lower if we holding the expected inflation as fixed. Hence, nominal rate will low due to the Taylor rule. By input 'phi_pi*pi_ann_eps_a(1) + phi_y*4*y_gap_eps_a(1)' in command window you will get -0.8112 which is exactly the same as the 1st period IRF of R_ann_eps_a(1).

Figure 2: The IRF of Technology Shock



```
figure(1)
subplot(3,2,1)
plot(T,y_gap_eps_nu,'-o');
title('Output Gap');

subplot(3,2,2)
plot(T,pi_ann_eps_nu,'-o');
title('Inflation');

subplot(3,2,3)
plot(T,R_ann_eps_nu,'-o');
title('Nominal Rate');

subplot(3,2,4)
plot(T,r_ann_eps_nu,'-o');
title('Real Rate');

subplot(3,2,5)
plot(T,m_growth_ann_eps_nu,'-o');
title('Money Growth');

subplot(3,2,6)
plot(T,nu_eps_nu,'-o');
title('\nu');

%% IRF plotting: technology shock
%----------------------------------------------------------------
% generate IRFs, replicates Figures 3.2, p. 55
%----------------------------------------------------------------
close all
T=1:1:15;
figure(1)
subplot(4,2,1)
plot(T,y_gap_eps_a,'-o');
title('Output Gap');

subplot(4,2,2)
plot(T,pi_ann_eps_a,'-o');
```

```
title('Inflation');

subplot(4,2,3)
plot(T,y_eps_a,'-o');
title('Output');

subplot(4,2,4)
plot(T,n_eps_a,'-o');
title('Empolyment');

subplot(4,2,5)
plot(T,R_ann_eps_a,'-o');
title('Nominal Rate');

subplot(4,2,6)
plot(T,r_ann_eps_a,'-o');
title('Real Rate');

subplot(4,2,7)
plot(T,m_growth_ann_eps_a,'-o');
title('Money Growth');

subplot(4,2,8)
plot(T,a_eps_a,'-o');
title('Technology Shock');
```


2.5 Some Derivation

2.5.1 Derivation for Eq.(6) in Textbook

The aggregate price dynamics

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{1}$$

We approximate the above equation at the zero inflation steady state, i.e., $\Pi = 1$ and $P_t^* = P_{t-1} = P_t$ for all t. If we define

$$\pi_t = \tilde{\Pi}_t$$

$$\pi_t^* = \left(\frac{\tilde{P_t^*}}{P_{t-1}}\right) = \log P_t^* - \log P_{t-1} = p_t^* - p_{t-1}$$

Using the definition of log-linearization, $X_t \approx X \times (1 + \tilde{x}_t)$ where $\tilde{x} = \log \frac{X_t}{X}$ and X is the steady state of X_t, we have

$$\left(\Pi \left(1 + \pi_t\right)\right)^{1-\epsilon} = \theta + (1-\theta)\left(1 + \pi_t^*\right)^{1-\epsilon}$$

Once again using the formula $(1+x)^\alpha \approx 1 + \alpha x$ when x is not far away from zero,

$$\pi_t = (1-\theta)\,\pi_t^* = p_t^* - p_{t-1} \tag{2}$$

There is another way around. Taking logarithm at both side of Eq(1),

$$(1-\epsilon)\log \Pi_t = \log\left(\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}\right)$$

Differentiate at both sides

$$(1-\epsilon)\,d\log \Pi_t = d\log\left(\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}\right)$$

$$(1-\epsilon)\frac{d\Pi_t}{\Pi} = \frac{d\left(\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}\right)}{\theta + (1-\theta)\left(\frac{\bar{P}_t^*}{P_{t-1}}\right)^{1-\epsilon}}$$

we have by using the fact that $\frac{dX_t}{X} \approx \tilde{x}_t$ and $dX_t^\alpha = \alpha X^\alpha \frac{dX_t}{X} = \alpha X^\alpha \tilde{x}_t$.

2.5.2 Derivation for Eq.(15) in Textbook

Many beginners are strange to this equation and questions always asked about how this equation come from. It does need some algebra. Let's recopy two equations in P47 of the textbook before detailed derivation.

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\hat{mc}_{t+k|t} + p_{t+k} - p_{t-1}\right\} \tag{3}$$

$$mc_{t+k|t} = mc_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k}) \tag{4}$$

where small letters represent the logarithm of correspondent big letters. For example, $p_t = logP_t$. And hat ˆ denotes percentage deviation from its steady state value. For example $\hat{mc}_{t+k|t} = log(MC_{t+k|t}/\overline{MC})$.

From above two equations, we can obtain the following recursive formula for $p_t^* - p_{t-1}$ which is the eq(15) in textbook:

$$p_t^* - p_{t-1} = \beta\theta E_t \left\{p_{t+1}^* - p_t\right\} + (1-\beta\theta)\Theta\hat{mc}_t + \pi_t \tag{5}$$

where $\Theta \equiv (1-\alpha)/(1-\alpha+\alpha\varepsilon)$. The following will give the derivation of equation (5).

First, Eq.(4) can be rewritten as follows:

$$\hat{mc}_{t+k|t} = \hat{mc}_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k}) \tag{6}$$

Substituting Eq.(6) into Eq.(3), we have

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\hat{mc}_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k}) + p_{t+k} - p_{t-1}\right\} \tag{7}$$

A step forward producing

$$p_{t+1}^* - p_t = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_{t+1}\left\{\hat{mc}_{t+k+1} - \frac{\alpha\varepsilon}{1-\alpha}(p_{t+1}^* - p_{t+k+1}) + p_{t+k+1} - p_t\right\} \tag{8}$$

Hence,

$$\begin{aligned}
E_t(p_{t+1}^* - p_t) &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t \\
&\quad \left\{E_{t+1}\left[\hat{mc}_{t+k+1} - \frac{\alpha\varepsilon}{1-\alpha}(p_{t+1}^* - p_{t+k+1}) + p_{t+k+1} - p_t\right]\right\} &(9)\\
&= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \\
&\quad E_t\left[\hat{mc}_{t+k+1} - \frac{\alpha\varepsilon}{1-\alpha}(p_{t+1}^* - p_{t+k+1}) + p_{t+k+1} - p_t\right] &(10)\\
&= (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^{k-1}
\end{aligned}$$

$$
\begin{aligned}
& E_t\left[\hat{mc}_{t+k}-\frac{\alpha\varepsilon}{1-\alpha}(p^*_{t+1}-p_{t+k})+p_{t+k}-p_t\right] && (11)\\
= & \frac{(1-\beta\theta)}{\beta\theta}\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\hat{mc}_{t+k}-\frac{\alpha\varepsilon}{1-\alpha}(p^*_{t+1}-p_{t+k})+p_{t+k}-p_t\right] \\
& -\frac{(1-\beta\theta)}{\beta\theta}\left[\hat{mc}_t-\frac{\alpha\varepsilon}{1-\alpha}\left(p^*_{t+1}-p_t\right)\right] && (12)\\
= & \frac{(1-\beta\theta)}{\beta\theta}\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\hat{mc}_{t+k}-\frac{\alpha\varepsilon}{1-\alpha}(p^*_{t}-p_{t+k})+p_{t+k}-p_{t-1}\right] \\
& -\frac{(1-\beta\theta)}{\beta\theta}\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[(p_t-p_{t-1})-\frac{\alpha\varepsilon}{1-\alpha}(p^*_t-p^*_{t+1})\right] \\
& -\frac{(1-\beta\theta)}{\beta\theta}\left[\hat{mc}_t-\frac{\alpha\varepsilon}{1-\alpha}\left(p^*_{t+1}-p_t\right)\right] && (13)\\
= & \frac{1}{\beta\theta}\left[(p^*_t-p_{t-1})-(p_t-p_{t-1})+\frac{\alpha\varepsilon}{1-\alpha}E_t\left(p^*_t-p^*_{t+1}\right)\right] \\
& -\frac{(1-\beta\theta)}{\beta\theta}\left[\hat{mc}_t-\frac{\alpha\varepsilon}{1-\alpha}\left(p^*_{t+1}-p_t\right)\right] && (14)\\
= & \frac{1}{\beta\theta}\left[(p^*_t-p_{t-1})-\pi_t+\frac{\alpha\varepsilon}{1-\alpha}E_t\left((p^*_t-p_{t-1})-(p^*_{t+1}-p_t)-\pi_t\right)\right] \\
& -\frac{(1-\beta\theta)}{\beta\theta}\left[\hat{mc}_t-\frac{\alpha\varepsilon}{1-\alpha}\left(p^*_{t+1}-p_t\right)\right] && (15)\\
= & \frac{1}{\beta\theta}\left(1+\frac{\alpha\varepsilon}{1-\alpha}\right)\left[(p^*_t-p_{t-1})-\pi_t\right]-\frac{\alpha\varepsilon}{1-\alpha}E_t\left(p^*_{t+1}-p_t\right) \\
& -\frac{(1-\beta\theta)}{\beta\theta}\hat{mc}_t && (16)
\end{aligned}
$$

Hence, Eq.(5) holds.

3 Chap7: Monetary Policy and the Open Economy

I used the equilibrium conditions in the textbook, but I could not replicate the IRFs. In some cases, I could not run the Dynare mod file that follows the textbook. So I have to find the working paper of this RES published paper. By adding some equilibrium equations for foreign countries found in that working paper[3], I could manage to run the mod file and get some hints of the IRF similar to those in the textbook. But I could not fully replicate most results in the textbook. The mod file I provide here could only give you some intuitions.

3.1 Introduction

This paper has developed a small open economy model in which

1. Exchange rate, terms of trade, export, import and international financial market are explicitly introduced.
2. CPI inflation π_t versus domestically produced goods π_t^H;
3. The openness of goods market which measured by a simple parameter α
$$C_t = \left((1-\alpha)^{\frac{1}{\eta}} \left(C_t^H \right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_t^F \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$
4. Nest a closed version of economy when $\alpha \to 0$
5. Consider what the role played by the exchange rate in the design of optimal monetary policy and what is the measure of inflation (i.e., the CPI inflation or the inflation solely concerns with the domestically produced goods) that the central bank should seek to stabilize.
6. It provides the simple the way to assess the welfare implication of simple rules.

There are other issues that this model:

1. Large versus Small: Small means that no repercussions effect from the rest of the world is considered for the home country. Technically speaking, small economy will greatly simplify the analysis.
2. Openness of financial market: complete market (full international risk sharing) or financial autarky.
3. PTM or Law of One Price: Law of one price setting means that there are no price discrimination.

[3] Gali & Monacelli, NBER w.p. 8905, April 2002.

4. Other issues not incorporated into this model
    (a) allowance or not of nontradeable goods
    (b) Trading cost
    (c) international policy coordination

The paper by Clarida, Gali and Gertler(2002)[4], Pappa(2004)[5] and Benigno and Benigno(2006)[6] depart from the assumption of a small open economy and analyze the consequences of alternative monetary policy arrangement in a two-country framework with staggered price setting as in Calvo(1983) and with a focus on the gains from cooperation.

3.2 The Variables interested

There are totally 23 endogenous variables that we interested in:

$$\pi_t, \pi_{H,t}, \pi_t^*, s_t, p_t, p_{H,t}, e_t, q_t, y_t, y_t^*, \tilde{y}_t y_t^n, \tilde{R}_t, \tilde{r}_t^n, \tilde{mc}_t, nx_t, a_t, r_t^*, mc_t^*, w_t, n_t, a_t^*, v_t$$

There is one slightly difference in notation with the textbook. We use R_t to denote the nominal rate instead of i_t in the textbook. All the other will be the same. The description of the variables will be given in the next section. The complete description of basic structural parameters will be also listed below.

3.3 The Equilibrium Conditions

1. The Home CPI inflation $\pi_t = \pi_{H,t} + \alpha\,(s_t - s_{t-1})$, where s_t denotes the log-level of the effective term of trade.

2. The definition of the home inflation $\pi_{H,t} = p_{H,t} - p_{H,t-1}$

3. The definition of CPI level $\pi_t = p_t - p_{t-1}$

[4] Clarida R., Gal J., Gertler M. A Simple Framework for International Monetary Policy Analysis[J]. Journal of Monetary Economics, 2002(5):879-904.

[5] Pappa E. Do the ECB and the Fed Really Need to Cooperate? Optimal Monetary Policy in a Two-Country World[J]. Journal of Monetary Economics, 2004(4):753-779.

[6] Benigno G., Benigno P. Designing Targeting Rules for International Monetary Policy Cooperation[J]. Journal of Monetary Economics, 2006(3):473-506.

4. The log effective real exchange rate $q_t = (1-\alpha)\, s_t$
5. Difference version of eq(16): $s_t = e_t + p_t^* - p_{H,t}$
6. Market clearing condition eq(29): $y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t$, where $\sigma_\alpha \equiv \frac{\sigma}{1+\alpha(\omega-1)}, \omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$
7. Definition of output gap : $\tilde{y}_t = y_t - y_t^n$
8. Home NKPC, eq(37): $\pi_{H,t} = \beta E_t (\pi_{H,t+1}) + \kappa_\alpha \tilde{y}_t, \kappa_\alpha = \lambda (\sigma_\alpha + \varphi), \lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$
9. Home IS curve, eq(38): $\tilde{y}_t = E_t (\tilde{y}_{t+1}) - \frac{1}{\sigma_\alpha} \left(\tilde{R}_t - E_t (\pi_{H,t+1}) - \tilde{r}_t^n \right)$
10. Natural level of output eq(36): $y_t^n = \Gamma_0 + \Gamma_a a_t + \Gamma_* y_t^*$,where $\Gamma_0 = \frac{\nu-\mu}{\sigma_a+\varphi}, \Gamma_a = \frac{1+\varphi}{\sigma_a+\varphi}, \Gamma_* = -\frac{\alpha\Theta\sigma_a}{\sigma_a+\varphi}$
11. The definition of natural rate, eq(39):

    $$r_t^n = \rho - \sigma_\alpha \Gamma_a (1-\rho_a)\, a_t + \frac{\alpha\Theta\sigma_\alpha\varphi}{\sigma_\alpha+\varphi} E_t \left(\Delta y_{t+1}^* \right)$$

    or

    $$\tilde{r}_t^n = -\sigma_\alpha \Gamma_a (1-\rho_a)\, a_t + \frac{\alpha\Theta\sigma_\alpha\varphi}{\sigma_\alpha+\varphi} E_t \left(\Delta y_{t+1}^* \right)$$

    where $\Theta = \omega - 1$
12. The net export $nx_t = \alpha \left(\frac{\omega}{\sigma} - 1 \right) s_t$
13. The home marginal cost $\tilde{mc}_t = (\sigma_\alpha + \varphi)\, \tilde{y}_t$
14. Home's monetary policy shock:
    (a) Strict CPI inflation target: $\pi_t = 0$ for all t (SCIT);
    (b) CPI Inflation Target $R_t = \rho + \phi_\pi \pi_t$ or $\tilde{R}_t = \phi_\pi \pi_t$ (CIT) where $\phi_\pi > 1$;
    (c) Exchange rate peg: $e_t = 0$ for all t (PEG);
    (d) Strict Domestic inflation target: $\pi_{H,t} = 0$ for all t (SDIT);

(e) Optimal policy: $R_t = r_t^n + \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t$,where ϕ_π, ϕ_y are non-negative coefficients chosen by the monetary authority and in general $\phi_\pi > 1$. This rule is equivalent to SDIT in equilibrium. The reason why we called it optimal is that in equilibrium $\pi_{H,t} = 0$ and $\tilde{y}_t = 0$[7]. This will achieve the minimum welfare loss zero and full stabilization of domestic inflation[8]. This is because nominal rate R_t responds to natural rate r_t^n one to one. And That will make the equilibrium system independent of the real force, i.e., the technology shock here since the natural rate is a function of technology shock and foreign output growth rate. For the technical reason why it is the case, see the appendix below.

(f) Simple Taylor rule: $R_t = r_t^n + \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t + v_t$; This policy rule will no longer optimal since the output gap and domestic inflation have to adjust according to the exogenous shock in equilibrium.

(g) Domestic inflation target: $R_t = \rho + \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t$ or $\tilde{R}_t = \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t$(DIT)[9];

(h) Simple Taylor rule: $R_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$ or $\tilde{R}_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$where v_t is an exogenous component;

15. Technology shock: $a_t = \rho_a a_{t-1} + \epsilon_t^a + a_{corr} \epsilon_t^{a^*}$; if $a_{corr} = 0$, there are no correlation of two technology shocks. You could study the correlation effects on home output by the foreign technology shock.

16. Foreign Euler condition in working paper, eq(22): $y_t^* = E_t\left(y_{t+1}^*\right) - \frac{1}{\sigma}\left(r_t^* - E_t\left(\pi_{t+1}^*\right)\right)$ where y_t^*, r_t^* and π_t^* in log-deviation form. But you can use the AR(1) process for the foreign output in textbook: P174 as equilibrium condition for Dynare mod file.

17. Foreign NKPC in working paper, eq(31): $\pi_t^* = \beta E_t\left(\pi_{t+1}^*\right) + \lambda mc_t^*$ where mc_t^* are in log-deviation form.

[7] In Dynare output, you can see that both the mean and variance of this two endogenous variables are zeros.

[8] The criteria of optimality is the welfare loss function developed in the textbook, or more specifically, the expected period welfare loss of eq(45) in P173 in textbook.

[9] Since nominal rate R_t does not respond to natural rate r_t^n one to one, hence the equilibrium system will be dependent on real force. In other words, in equilibrium, domestic inflation and output gap will depend on the real force, the technology shock here for example. Generally speaking, a Taylor rule will be an optimal policy as long as it responds to natural rate one to one in this model.

18. Foreign marginal cost,in working paper, eq(32): $mc_t^* = (\sigma + \varphi)\, y_t^* - (1 + \varphi)\, a_t^*$ which is similar to eq.(35) in textbook
19. Foreign's Taylor rule eq(51) and see also note.19: $r_t^* = \phi_\pi^* \pi_{t+1}^* + \phi_a^* a_t^*$;
20. Foreign technology shock: $a_t^* = \rho_{a^*} a_{t-1}^* + \epsilon_t^{a^*}$;
21. Home labor in P154, eq(8), log-linearized: $w_t = \sigma y_t + \varphi n_t$
22. Home resource constraint in P162, eq(32): $y_t = a_t + n_t$
23. Exogenous Monetary policy rule: $v_t = \rho_v v_{t-1} + \epsilon_t$;

3.4 The Structural Parameters

1. $\alpha = 0.4$, the openness of the economy;
2. $\sigma = 1$, log utility;
3. $\beta = 0.99$, discount factor;
4. $\gamma = 1$, the substitutability between goods produced in different foreign countries;
5. $\eta = 1$, the substitutability between domestic and foreign goods from the viewpoint of the domestic consumer;
6. $\varphi = 3$, the reciprocal of the elasticity of Frisch labor supply;
7. $\epsilon = 6$, the elasticity of substitution between varieties produced within any given country;
8. $\theta = .75$, the price stickiness parameter;
9. $\phi_\pi = 1.5$, the coefficient of (CPI) inflation;
10. $\phi_y = 0$, the coefficient of output gap.
11. $\phi_\pi^* = 1.01$, the coefficient on π_t^* in foreign interest rate rule
12. $\rho_a = 0.66$, the persistence of domestic technology shock, see P174;
13. $\rho_{y^*} = 0.86$, the persistence of foreign output process, see P174;

14. $\rho_{a^*} = 0.9$, the persistence of foreign technology shock;

15. $a_{corr} = 0.3$, the correlation of the two technology shocks,see P174;

For parameters not listed here, please refer to the mod file: chap7_book.mod.

3.5 The Dynare code

The following mod file is the chap7_book.mod that you can find at the chap7 directory:

```
/*
 * This file tries to implements the Open Economy model of Jordi Gali(2008):
 * Monetary Policy, Inflation,
 * and the Business Cycle, Princeton University Press, Chapter 7
 *
 * This file is written by Xiangyang Li@Guangzhou, 2016-3-18
 * I have tried to use the model equilibrium conditions in textbook
 * but it seems that there are colinearity or indeterminacy problem.
 * Thus, I borrow some equlibrium conditions from
 * Gali & Monacelli, NBER w.p. 8905, April 2002
 * Note that all model variables are expressed in log-deviations;
 */
var y_star  //overseas output
  a   // domestic technology shock;
 ygap //domestic output gap
rnat  //natrual rate
R      //nominal interest rate;
y      //domestic output
ynat // natural level of output
pi    // CPI inflation
pi_h //domestic home infaltion
pi_star // foreign inflation
s   //effective term of trade
q //  effective real exchange rate
e //  effective nominal exchange rate
p_h  //domestic price level
cpi_level // CPI price level
mc // marginal cost
```

```
nx // net export
r_star //foreign nominal interest rate
mc_star // foreign marginal cost
a_star // foreign technology shock;
n   //domestic labor
w  //domestic real wage
v  // monetary policy shock
;
varexo  eps_a eps_y_star eps_a_star eps_v;

parameters beta sigma alpha eta epsilon phi theta rho_a rho_y_star
      phi_pi phi_y phi_pi_star phi_a_star rho_a_star a_shock_correl
      kappa_a omega sigma_a lambda  BigGamma_a BigGamma_star BigTheta
      rho_v;


// Calibrations as per p.174
beta = 0.99; // Pure temporal discount factor
sigma = 1; // Intertemporal consumption elastiticy
alpha = 0.4; // Degree of 'openness' in the Home economy
eta = 1; // Elast. of sub. between Home and Foreign goods
epsilon = 6; // Dixit-Stiglitz parameter for within-sector consumption
phi = 3; // Labour disutility parameter
theta = 0.75; // Calvo probability
gamma =1;
lambda = (1-(beta*theta))*(1-theta)/theta; // Coefficient on marginal cost in the
omega = sigma*gamma+(1-alpha)*(sigma*eta-1);
sigma_a = sigma/(1+alpha*(omega-1));
kappa_a = lambda*(phi +sigma_a); // Real rigidity; see eq(37) in textbook, hencefo
BigTheta = omega - 1;
BigGamma_a = (1+phi)/(sigma_a + phi);  //See eq(36);
BigGamma_star = -alpha*BigTheta*sigma_a/(sigma_a + phi); //See eq(36);

// Parameters of the productivity shocks (p.174 in textbook)
rho_a = 0.66;
rho_y_star = 0.86;
rho_a_star =0.9; //not present in textbook
rho_v  = 0.9;      //not present in textbook
```

```
phi_pi = 1.5;
phi_y = 0.5;

//// See (51) in the working paper and note 19;
phi_a_star = -(sigma*(1+phi)*(1-rho_a_star)) / (phi+sigma);
phi_pi_star = 1.01;
a_shock_correl = 0.3;

model(linear);
//(1) Home CPI inflation eq(15), P155 in textbook, the same afterward
pi = pi_h + alpha*(s - s(-1));

//(2) An identity to pin down the relative price of home goods,P155
p_h = p_h(-1) + pi_h;

//(3)An identity to pin down the consumer price level
cpi_level = cpi_level(-1) + pi;

//(4)Real exchange rate P156 in  textbook;
q = (1-alpha)*s;

//(5) term of trade, eq(16), differenced version
s - s(-1) = e - e(-1) + pi_star - pi_h;

//(6)Market clearing eq(29)
y = y_star +s/sigma_a;

//(7) Definition of Home output (p. 164)
y = ynat + ygap;

//(8)Home's Phillips curve, eq(37)
pi_h = beta*pi_h(+1) + kappa_a*ygap;

//(9)Home's IS curve, eq(38)
ygap = ygap(+1) - (1/sigma_a)*(R - pi_h(+1) - rnat);

//(10) Home's natural level output (39)
ynat = BigGamma_a*a + BigGamma_star*y_star;
```

```
//(11)The definition of Home's Wicksellian interest rate, eq(39):
rnat = -sigma_a*(1-rho_a)*BigGamma_a*a
 - phi*BigGamma_star*(y_star(+1) - y_star);

//(12)The net export
nx = alpha*(omega/sigma-1)*s;

//(13)The home marginal cost
mc = (sigma_a + phi)*ygap;

//(14)The home monetary policy
//Home's monetary policy;
//pi = 0; // Strict inflation targeting (SCIT)
//e = 0; // Exchange rate peg (PEG)
//pi_h = 0; // Strict Domestic inflation targeting (SDIT)
// R = 0.5*R(-1) + phi_pi*pi + phi_y*ygap; // Simple Taylor rule
//R = rnat; // indeterminacy problem arises, can not run;
//R = rnat +phi_pi*pi_h + phi_y*ygap;  //optimal policy, equi. to SDIT
R = rnat +phi_pi*pi_h + phi_y*ygap + v; //nolonger optimal due to shock presence
//R =  phi_pi*pi; //Domestic inflation Taylor rule (DIT)

//(15)Home technology shock
// you can turn off the correlation by setting a_shock_correl = 0;
a = rho_a*a(-1) + eps_a +a_shock_correl*eps_a_star;

//(16)Foreign Euler condition,in working paper, eq(22):
y_star = y_star(+1) - (r_star - pi_star(+1))/sigma;

//AR(1) for y_star in textbook(P174) like this also works
//y_star = rho_y_star*y_star(-1) + eps_y_star;

//(17) Foreign's marginal cost, ,in working paper, eq(32):
mc_star = (sigma + phi)*y_star - (1+phi)*a_star;

// (18)Foreign's Phillips curve,in working paper, eq(31):
pi_star = beta*pi_star(+1) + lambda*mc_star;
```

```
//(19)foreign interest rate rule, Taylor rule in working paper eq(51)
r_star = phi_pi_star*pi_star(+1) + phi_a_star*a_star;

// (20)Foreign's technology process
a_star = rho_a_star*a_star(-1) + eps_a_star;

//(21) domestic real wage,in P154, eq(8), log-linearized:
w = sigma*y +phi*n;

//(22) domestic production technology,in P162, eq(32):
y = a+n;

//(23) monetary policy shock
v = rho_v*v(-1) + eps_v;
end;

//you can turn on all the shocks; I only turn on the technology shock here;
//using the standard deviation in P174, it is unrealistic to have
// that sizes of IRFs in the textbook.
shocks;
var eps_a; stderr 1;
var eps_y_star; stderr 0;
var eps_a_star; stderr 0;
var eps_v; stderr 0.1;
end;

stoch_simul(irf=16)  ygap pi_h R pi e q p_h cpi_level;
```


3.6 Appendix

The policy rule: $R_t = \zeta\rho + (1-\zeta)\, r_t^n + \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t$, $\zeta \in [0,1]$ where r_t^n is a function of technology shock. Under this so-called Taylor rule, the canonical representation of the equilibrium system can be reduced to[10]

$$\begin{pmatrix} \tilde{y}_t \\ \pi_{H,t} \end{pmatrix} = A_\alpha \begin{pmatrix} E_t\left(\tilde{y}_{t+1}\right) \\ E_t\left(\pi_{H,t+1}\right) \end{pmatrix} + B_\alpha \left(\zeta \tilde{r}_t^n - v_t\right)$$

where A_α, B_α are coefficient matrices. I will not list them since they are of no interests here. But you can derive them by paper and pencil by yourself if you want.

If we set $\zeta = 0$ and shut off the exogenous monetary policy shock, This will make the terms $\tilde{r}_t^n, v_t$ disappear from the equilibrium system. That is equivalent to say that $B_\alpha = 0$ all the time. Hence the system reduces to

$$\begin{pmatrix} \tilde{y}_t \\ \pi_{H,t} \end{pmatrix} = A_\alpha \begin{pmatrix} E_t\left(\tilde{y}_{t+1}\right) \\ E_t\left(\pi_{H,t+1}\right) \end{pmatrix}$$

Hence, the equilibrium solution for domestic inflation and output gap must be the case: $\pi_{H,t} = 0$ and $\tilde{y}_t = 0$ if $\|A_\alpha\| < 1$[11]. If exogenous shock presents or $\zeta \neq 0$, this will generally make the system depends on and acts to real force and hence sub-optimal policy on the welfare loss ground.

[10]The canonical representation of the equilibrium system can be described by three equations: NPKC, DIS and Taylor rule. After substituting out the nominal rate in the first two equations using Taylor rule, we only have NPKC and DIS equations left here. You can refer to Eq(41) in P165 in the textbook for more information.

[11]For both eigenvalues of $A_\alpha$lying within the unit circle.This condition could be satisfied under reasonable and proper parameterization.