

# Interest Rate Peg and Zero Low Bound

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## 1 Introduction

In the standard New Keynesian model, monetary policy is often described by an interest rate rule (e.g. a Taylor rule) that moves the nominal interest rate in response to inflation and output gap. Nominal interest rates are bound from below by 0 in theory since money is storable, one would never accept a negative nominal return. How does the behavior of the NK model change when interest rates hit zero and cannot freely adjust in response to changing economic conditions?

To answer this question, we consider the implications of an interest rate peg in the model. This isn't literally what happens at the zero lower bound, but what matters in the model is not that the interest rate is zero *per se*, but rather that it becomes unresponsive to economic conditions in the peg period. In the experiments we consider, the nominal interest rate is pegged at a fixed value for a finite (and deterministic) period of time (peg period). Right after the peg, monetary policy obeys a simple Taylor rule as usual. It turns out to be relatively straightforward to modify a Dynare code to take this into account. I include a government spending shock and net export in the model so that we can analyze the effects of demand shocks at the zero lower bound, which has been a topic of much recent interest.

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\*Some of the notes are borrowed from Prof. Sims's Notes. I especially thank him for his generosity in sharing his note and code with me. His inspiring teaching has benefited me tremendously.

The interest rate peg ends up exacerbating the effects of price stickiness which has already low the response of output to a supply shock (productivity or technology shock). In particular, output responds even less to a positive supply shock and more to “demand” shocks (government spending shock or net export shock) than under a standard Taylor rule. This operates through an inflation channel and the Fisher relationship: positive supply shocks lower inflation, which raises real interest rates if nominal rates are unresponsive, with the reverse holding for a demand shock like government spending.

## 2 Model

We use a simple “open” New Keynesian model with sticky price to illustrate our idea. We call it an “open” economy model by simply introducing net export in the total absorption as displayed in Eq.(13). The model is quite standard. We begin this section by briefly describing the model.

### 2.1 A simple NK model

#### 2.1.1 Households

We assume that the economy has a representative households who has the period utility function:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{N_t^{1+\eta}}{1+\eta}, \quad \sigma, \eta > 0 \quad (1)$$

The parameter  $\sigma$  is the inverse elasticity of inter-temporal substitution of consumption. And  $\eta$  is the inverse of elasticity of Frisch labor supply.  $\psi$  is a only scaling parameter which only affect the steady states rather than the dynamics of the model. The households maximize its lifetime utility using the discount factor  $\beta$ , with  $0 < \beta < 1$ :

$$\max_{C_t, N_t} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad (2)$$

The households consumes, supplies labor to firms and earns wage, accumulates bonds, holds shares in firms and hence can get dividends from firms every periods. The nominal flow budget constraint of households as follows:

$$P_t C_t + B_{t+1} \leq W_t N_t + \Pi_t + (1 + i_{t-1}) B_t$$

Here  $P_t$  is the nominal price of goods,  $\Pi_t$  is the nominal dividend or profit from firms.  $B_t$  is the stock of nominal bonds a households decides to have when entering into period  $t$  and payout of this bonds which is known at period  $t - 1$  is the nominal interest rate  $i_{t-1}$  and will be paid in period  $t$ . The first order conditions of households utility maximization problems w.r.t consumption, labor and bonds are standard:

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} (1 + i_t) \frac{P_t}{P_{t+1}} \quad (3)$$

$$\psi N_t^\eta = C_t^{-\sigma} \frac{W_t}{P_t} \quad (4)$$

Equation(3) and (4) are Euler equation and labor supply equation respectively.

### 2.1.2 Production

As in the standard sticky model setting, we have both immediate and final goods producers in the economy. Let begin with the final goods producer.

Assuming there is one representative final goods producer who takes the prices of final goods it produces and the input prices of intermediate goods as given, uses the Dixit-Stiglitz aggregator to produce the final goods:

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \varepsilon > 1$$

The parameter  $\varepsilon$  is the elasticity of substitution between different immediate goods  $Y_t(i)$ . We set  $\varepsilon > 1$  to allows that the immediate goods are substitutes. The profit maximization problem of final goods producer gives the downward-slope demand

curve for immediate goods  $i$ ,

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \quad (5)$$

where  $P_t(i)$ ,  $P_t$  are prices index for the immediate goods  $i$  and final goods respectively. The zero profit conditions of final goods firm gives the CPI index:

$$P_t = \left( \int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \quad (6)$$

We assume immediate goods producer use the following simple production technology:

$$Y_t(i) = A_t N_t(i) \quad (7)$$

where  $A_t$  is technology shock and  $N_t(i)$  is the labor demanded by immediate goods producer  $i$ . We assumes that technology shock follows a standard AR(1) process:

$$\log(A_t) = \rho_A \log(A_t) + \varepsilon_t^A \quad (8)$$

The decision problem of immediate goods firms can be divided into two stages. In the first stage, the firms minimizes its employment cost while taking the nominal wage  $W_t$  and the demand curve (5) as given. And this gives the labor demand equation:

$$mc_t = \frac{w_t}{A_t} \quad (9)$$

where  $mc_t$  is the marginal cost and  $w_t = \frac{W_t}{P_t}$  is the real wage. In the second stage, we introduce the price stickiness. The imperfect substitute between different immediate goods gives rise to some monopolistic power for firms to price its goods. As it is common in the literature that we assume that the firms subject to Calvo(1983) price setting mechanism, i.e., they are not free to adjust its immediate goods prices. In each period, the firm has the probability  $1 - \phi$  to optimal adjust its price and the probability  $\phi$  to fix its price.

The real profit of the firm  $i$  is

$$\frac{\Pi_t(i)}{P_t} = \frac{P_t(i)}{P_t} Y_t(i) - w_t N_t(i)$$

Considering the firm which is able to adjust its price at period  $t$ . The firm adjusts its price will expect that it could be stuck in the future for some periods. The probability that the price optimally chosen in period  $t$  still operative in period  $s$  from now is that  $\phi^s$ . Hence the profit maximization of immediate goods firms is dynamic.

$$\max_{P_t(i)} E_t \sum_{s=0}^{\infty} \phi^s \lambda_{t+s} \frac{\Pi_{t+s}(i)}{P_{t+s}}$$

where  $\lambda_{t+s} = \beta^s \frac{U'(C_{t+s})}{U'(C_t)}$  is the stochastic discount factor of the firm. We consider symmetric solution and the optimal price chosen is independent of the specific firm  $i$ . We use  $P_t^*$  to replace  $P_t(i)$  as the optimally chosen price and the FOC is

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{X_{1t}}{X_{2t}} \quad (10)$$

where the two auxiliary variables are defined as:

$$X_{1t} = U'(C_t) m c_t P_t^\varepsilon Y_t + \phi \beta E_t X_{1t+1} \quad (11)$$

$$X_{2t} = U'(C_t) P_t^{\varepsilon-1} Y_t + \phi \beta E_t X_{2t+1} \quad (12)$$

$$Y_t = C_t + G_t + X_t \quad (13)$$

### 2.1.3 Interest Rate Peg and Demand Shocks

We incorporate a simple “interest rate peg” into our model to see what happens in our economy when faced with demand shocks. Let  $t$  be the current period, and

we fix the short nominal interest rate to be fixed for  $H$  periods:

$$i_{t+h} = i_{t-1}, \quad h = 0, 1, \dots, H-1$$

and after that the interest rate is assumed to follow the simple and implementable Taylor rule.

$$i_{t+s} = i^* + \phi_\pi (\pi_{t+s} - \pi^*) \quad (14)$$

when  $s \geq H$ . In our model and simulation, we only consider the case where  $H$  is fixed instead of a random number. Hence here the peg is the deterministic interest rate peg. We can set  $H$  to different values to see how it effects the IRFs of endogenous variables upon the demand shocks. The determinacy properties of the policy rule do not depend on the length of the peg, so long as it is nite, but rather only how interest rates are set after the peg ends.

It is reasonably straightforward to introduce this into Dynare. To do so, you simply need to create some auxiliary state variables. Suppose I want a four period peg,  $H = 4$ . I would introduce four new state variables, call them  $S1$ ,  $S2$ ,  $S3$ , and  $S4$ . I would write my code:

$$\begin{aligned} i_t &= S1(t-1) \\ S1(t) &= S2(t-1) \\ S2(t) &= S3(t-1) \\ S3(t) &= S4(t-1) \\ S4(t) &= i^* + \phi_\pi (\pi_{t+4} - \pi^*) \end{aligned}$$

$S4(t)$  describes how the interest rate will be set in the fourth period from now (period  $t + 4$ , where we take period  $t$  to be the present. In period  $t + 1$ ,  $S3$  will equal this; in period  $t + 2$ ,  $S2$  will equal this; in period  $t + 3$ ,  $S1$  will equal this. In period  $t + 4$  and forward,  $i_{t+4} = i^* + \phi_\pi (\pi_{t+4} - \pi^*)$  which is the standard Taylor rule. This will setup a mechanism for interest rate fixed for a period time when faced exogenous shock. That is to say that it will take shocks 4 periods time to pass onto the interest rate. Before that it will remain on steady state value<sup>1</sup> (**LET'S**

<sup>1</sup>This could also be verified by simulation. By specifying periods option in `stoch_simul` command, you could see that nominal interest rate will remain fixed at steady states level for  $H$  periods and then fluctuates. And the auxiliary variable  $S1$  will fixed for 3 periods,  $S2$  for 2 periods,  $S3$  for 1 period.

**SHOW IT MATLAB).**

We simply assume that both government spending  $G_t$  and net export  $X_t$  to be AR(1) processes:

$$G_t = (1 - \rho_g) \bar{G} + \rho_g G_{t-1} + \varepsilon_t^g \quad (15)$$

$$X_t = (1 - \rho_x) \bar{X} + \rho_x X_{t-1} + \varepsilon_t^x \quad (16)$$

where  $\varepsilon_t^g, \varepsilon_t^x$  are assumed to be followed i.i.d normal distributions.  $\bar{G}$  and  $\bar{X}$  are steady states or long-run values for government spending and net export.

#### 2.1.4 Parametrization

Before we solve the model, we parametrize the structural parameters in the model. Most parameters values are calibrated according to the literature and some of them will be calibrated using Chinese macro-economy data.

We use the annual data of government spending (1993-2012) and net export (1994-2013) to calibrate the persistence of the AR(1) process and volatility of the i.i.d shocks<sup>2</sup>.

#### 2.1.5 Equilibrium and Aggregation

The final output will be absorbed by consumption, government spending and exports. Hence we have the following resource constraint as given in Eq.(13). Define the real interest rate  $r_t$  through the Fisher relationship,

$$r_t = i_t - E_t(\pi_{t+1}) \quad (17)$$

where the inflation  $\pi_t \equiv \frac{P_t}{P_{t-1}}$ . Define the aggregate labor demand as

$$N_t^d = \int_0^1 N_t(i) di$$

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<sup>2</sup>The data is from CEIC database and is available upon request.

Table 1: Parametrization

parameter	interpretation	value
$\beta$	objective discount factor	.99
$\eta$	the inverse of elasticity of Frisch labor supply	1
$\psi$	labor dis-utility scaling parameter	1
$\sigma$	the inverse of elasticity of inter-temporal sub.	1
$\varepsilon$	intertemporal elasticity of intermediate goods	10
$\phi$	the price stickiness parameter	.75
$\phi\pi$	coefficient of inflation in monetary policy	1.5
$\rho_g$	persistence of government spending	.9658
$\rho_x$	persistence of net export	.8257
$\sigma_g$	volatility of government spending shock	.1459
$\sigma_x$	volatility of net export shock	.5010
$\frac{G}{Y}$	government spending share	.1413
$\frac{X}{Y}$	net export share	.0459

The labor market clear requires that labor supply equals the labor demand

$$N_t = N_t^d$$

The bonds market in equilibrium requires  $B_t = 0$  for all  $t$ . The monopolistic power of immediate goods firms will create some distortion and inefficiency in the economy. That is the output level will be lower than the one competitive economy. Aggregating demand curve Eq.(5) and production function Eq.(7), we have

$$Y_t = \frac{A_t N_t}{v_t^p} \quad (18)$$

where  $v_t^p$  is the price dispersion and defined as

$$v_t^p = \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} di$$

It could easily show that the price dispersion is bounded below by unity as in ?. And this price dispersion is one source of inefficiency. And it can be written in



recursive form to get rid of the heterogeneity:

$$v_t^p = (1 - \phi) (\pi_t^*)^{-\varepsilon} \pi_t^\varepsilon + \pi_t^\varepsilon \phi v_{t-1}^p \quad (19)$$

where  $\pi_t^* \equiv \frac{P_t^*}{P_{t-1}}$ . Under the sticky price setting, the CPI index can be rewritten as

$$\pi_t^{1-\varepsilon} = (1 - \phi) (\pi_t^*)^{-\varepsilon} + \phi \quad (20)$$

A competitive equilibrium of this model is that a set of prices  $\{i_t, r_t, w_t\}$  and allocations  $\{Y_t, C_t, N_t, \pi_t, \pi_t^*, v_t^p, X_{1,t}, X_{2,t}, mc_t\}$ , taking the TFP  $A_t$ , government spending shock  $G_t$  and net export shock  $X_t$  as given, such that all markets are clear and all agents behaves optimally. The equilibrium system is characterized by equations (13,3,4,8,9,10,11,12,14,15,16,18,19,20). Totally, there 15 endogenous variables and 15 equations.

#### 2.1.6 ZLB

The zero lower bound refers to the fact that nominal interest rates cannot be negative (whereas real rates can). The argument for why this is the case is fairly straightforward, though without money explicitly in the model is not terribly transparent. The nominal interest rate tells you the dollar return on foregoing one dollar's worth of current consumption, whereas the real interest rate tells you the consumption return on forgoing one unit of current consumption. If consumption goods are not storable, then you may be willing to accept a negative real return - giving up 1 unit of fruit today for 0.9 fruits tomorrow isn't a great deal, but if your outside option is zero fruit tomorrow, you may be willing to take this. But since money is a store of value, one would never take a negative return on money - you could simply hold your wealth in money between periods and have as many dollars tomorrow as you saved today. Hence, no one would ever take a negative nominal interest rate.

What is relevant in this model is not whether the nominal interest rate is zero or positive per se, but rather whether the nominal interest rate is responsive to

changing economic conditions.

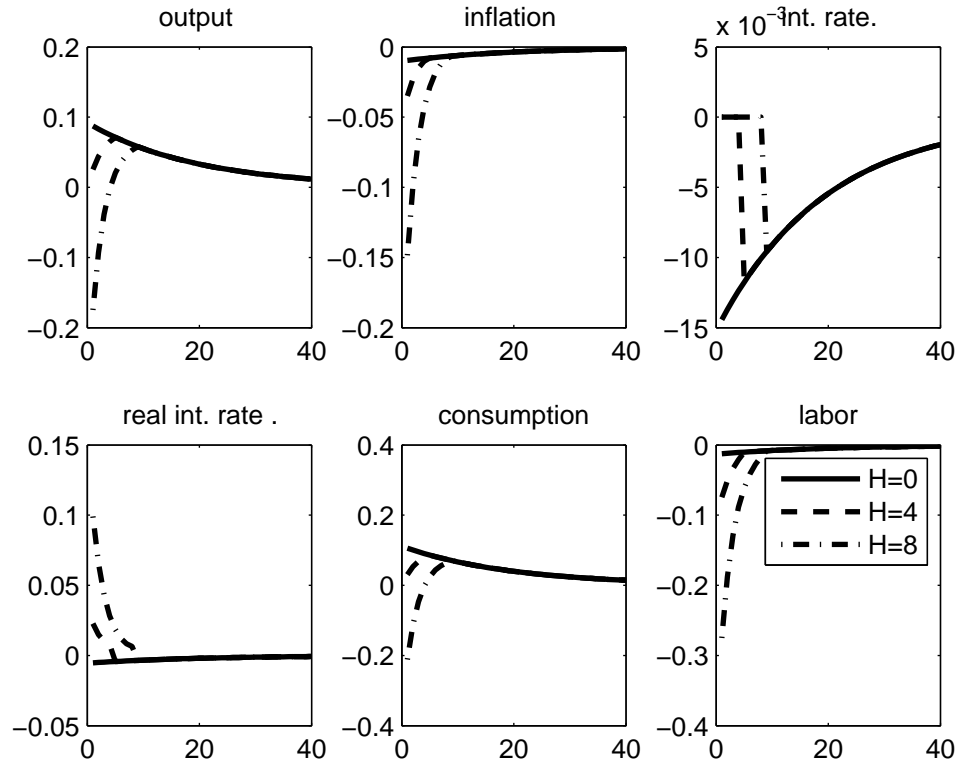
### 2.1.7 IRF Analysis

The following two figures are the IRFs of few endogenous variables to one unit of the technology shock and government spending shock. we set the nominal interest rate fixed for  $H = 0, 4, 8$  periods which are showed by solid, dashed and dotted lines respectively.<sup>3</sup>. From the figures, we can see that interest rates are indeed fixed for 4 or 8 periods in the IRFs of interest rate after shocks. The two demand shocks behaves differently.

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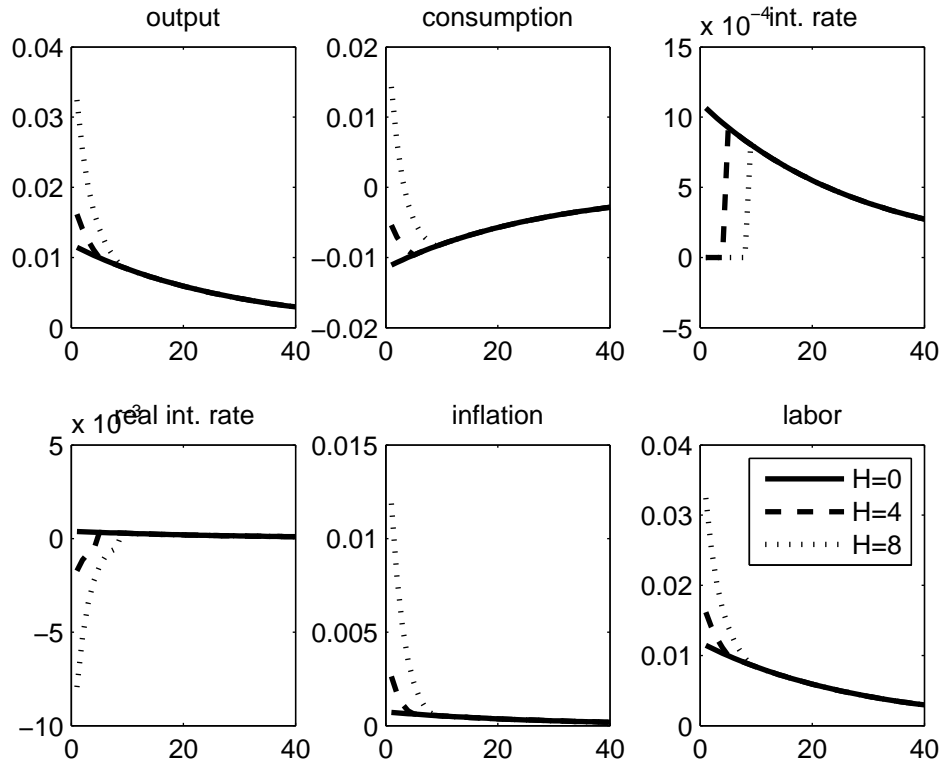
<sup>3</sup>One standard deviation of exogenous shocks in the government spending and technology AR(1) equation.

Figure 1: The IRFs of technology shock



Notes: vertical axis represents how much the variables deviate from the its steady states in respond to one unit or one standard deviation of shock to technology shock. For interest rate, inflation, the vertical axis denotes the absolute deviation.

Figure 2: The IRFs to government shock



We see something pretty interesting here. In particular, the interest rate peg exacerbates the effects of price stickiness. For the case of the productivity shock, under a standard Taylor rule with sticky prices, output rises by less than it would if prices were flexible. The longer the interest rate is pegged, the less output rises. We clearly see how the nominal interest rate is fixed for the specified number of periods. After the peg is over, the IRFs lie on top of one another - this occurs because there is no endogenous state variable in the model (for example, capital stock), so once the peg is over it is irrelevant that the peg was ever in place in the first place.

What's going on here is fairly simple and operates through the real interest rate in conjunction with the Fisher relationship. The positive productivity shock

is a “supply shock” that results in lower inflation. When the central bank follows a Taylor rule, it reacts to lower inflation by lowering the nominal interest rate, which prevents the real interest rate from rising by much (indeed, in the baseline scenario here under a Taylor rule it falls). But when the nominal interest rate is fixed, falling inflation means a higher real interest rate. This higher real interest rate works to “choke off” demand. The longer the interest rate is pegged, the more inflation initially falls. And with a fixed nominal rate, this translates into a bigger increase in the real interest rate and an even more contractionary effect on output. This means that a negative response of real interest rate actually helps improve the consumption.

Under one positive demand shock (either government spending or net export shock), output will rise upon impact. As shown in the figure, the more periods the nominal interest rate is fixed, the more the output will respond. With the positive responses of output, inflation rises too. Consumption is dropping at first if we do not fix the interest rate and actually increases when we fix the interest rate. The more periods interest rate is fixed, the more the consumption rises. With the households increasing its consumption, it also increases its labor supply.

What is going on behind? Taylor rule says if inflation is rising, nominal interest rate will rise more and hence the real interest rate is rising too by the classical Fisher relationship. But if the nominal interest rate is fixed, then rising inflation actually means that falling real interest rate. Hence, interest rate peg just reverses the direction that real interest rate and inflation move. And this helps explain why consumption rises when facing positive demand shock. If we log-linearize the Euler equation (3),  $c_t = E_t c_{t+1} - \frac{1}{\sigma} r_t$  where the small letter denotes the deviation form. If we solve forward, we could get

$$c_t = -\frac{1}{\sigma} \sum_{j=0}^{\infty} r_{t+j}$$

This means that a negative response of real interest rate actually helps improve the consumption.

### 3 The Codes

In this section, I will list out the codes that implements our ideas. Please run the files in Dynare v.4.4.3. There are four files involved and most of them are easy to understand:

1. **MAIN\_ZLB.M**: this is the index file that invoke all of the three mod files. And then collecting all needed data and plot the IRFs. There are few cells insides. You should run cells in order.
2. **ZLB.MOD**: there is no Interest rate peg;
3. **ZLB\_FOUR.MOD**: Interest rate peg for four periods;
4. **ZLB\_EIGHT.MOD**:Interest rate peg for eight periods;

Here is the main\_ZLB.m

```
%% main file for the ZLB and interest rate peg
%please use dynare v4.4.3, version like 4.2.0 will not work;
clear all
dynare ZLB noclearall nolog
save oo_zlb_baseline ;

clear all;
dynare ZLB_four noclearall nolog;
save oo_zlb_four ;

clear all;
dynare ZLB_eight noclearall nolog;
save oo_zlb_eight ;

%% pluck out data
clear all
%1st dimension: number of IRF simulated;
%number of cases: H=0,4,8, totally 3 cases;
%number of shocks: technology,government spending
%net export shock, totally 3 shocks;
output = zeros(40,3,3);
inflation = zeros(40,3,3);
```

```

int          = zeros(40,3,3);
realint      = zeros(40,3,3);
cons        = zeros(40,3,3);
labor= zeros(40,3,3);
wage = zeros(40,3,3);

```

```

load oo_zlb_baseline;
output(:,1,1) = y_ea;
output(:,1,2) =y_eg;
output(:,1,3) =y_ex;
inflation(:,1,1) = pi_ea;
inflation(:,1,2) = pi_eg;
inflation(:,1,3) = pi_ex;
int(:,1,1) = i_ea ;
int(:,1,2) =i_eg;
int(:,1,3) =i_ex;
realint(:,1,1)= r_ea;
realint(:,1,2) =r_eg;
realint(:,1,3) =r_ex;
cons(:,1,1)=c_ea;
cons(:,1,2)=c_eg;
cons(:,1,3)=c_ex;

```

```

labor(:,1,1)=n_ea;
labor(:,1,2)=n_eg;
labor(:,1,3)=n_ex;
wage(:,1,2)=w_eg;
wage(:,1,3)=w_ex;

```

```

load oo_zlb_four;
output(:,2,1) = y_ea;
output(:,2,2) =y_eg;
output(:,2,3) =y_ex;
inflation(:,2,1) = pi_ea;
inflation(:,2,2) = pi_eg;
inflation(:,2,3) = pi_ex;
int(:,2,1) = i_ea ;
int(:,2,2) =i_eg;

```

```

int(:,2,3) =i_ex;
realint(:,2,1)= r_ea;
realint(:,2,2) =r_eg;
realint(:,2,3) =r_ex;
cons(:,2,1)=c_ea;
cons(:,2,2)=c_eg;
cons(:,2,3)=c_ex;
labor(:,2,1)=n_ea;
labor(:,2,2)=n_eg;
labor(:,2,3)=n_ex;
wage(:,2,2)=w_eg;
wage(:,2,3)=w_ex;

load oo_zlb_eight;
output(:,3,1) = y_ea;
output(:,3,2) =y_eg;
output(:,3,3) =y_ex;
inflation(:,3,1) = pi_ea;
inflation(:,3,2) = pi_eg;
inflation(:,3,3) = pi_ex;
int(:,3,1) = i_ea ;
int(:,3,2) =i_eg;
int(:,3,3) =i_ex;
realint(:,3,1)= r_ea;
realint(:,3,2) =r_eg;
realint(:,3,3) =r_ex;
cons(:,3,1)=c_ea;
cons(:,3,2)=c_eg;
cons(:,3,3)=c_ex;
labor(:,3,1)=n_ea;
labor(:,3,2)=n_eg;
labor(:,3,3)=n_ex;
wage(:,3,2)=w_eg;
wage(:,3,3)=w_ex;

%% technology shock
set(0,'DefaultAxesColorOrder',[0 0 0],...
    'DefaultAxesLineStyleOrder','-|---|-.|:')
```



```

T= 1:1:40;
subplot(2,3,1);
plot(T,output(:, :, 1), 'LineWidth', 2);
title('output ');

subplot(2,3,2);
plot(T,inflation(:, :, 1), 'LineWidth', 2);
title('inflation ');

subplot(2,3,3);
plot(T,int(:, :, 1), 'LineWidth', 2);
title('int. rate. ');

subplot(2,3,4);
plot(T,realint(:, :, 1), 'LineWidth', 2);
title('real int. rate ');

subplot(2,3,5);
plot(T,cons(:, :, 1), 'LineWidth', 2);
title('consumption');

subplot(2,3,6);
plot(T,labor(:, :, 1), 'LineWidth', 2);
title('labor');

legend('H=0', 'H=4', 'H=8');
%% government shock
set(0, 'DefaultAxesColorOrder', [0 0 0], ...
      'DefaultAxesLineStyleOrder', '-|--|:|-.');
figure;
subplot(2,3,1);
plot(1:1:40,output(:, :, 2), 'LineWidth', 2);
title('output');

subplot(2,3,2);
plot(1:1:40,cons(:, :, 2), 'LineWidth', 2);
title('consumption');

```

```

subplot(2,3,3);
plot(1:1:40,int(:, :, 2), 'LineWidth', 2);
title('int. rate');

subplot(2,3,4);
plot(1:1:40,realint(:, :, 2), 'LineWidth', 2);
title('real int. rate');

subplot(2,3,5);
plot(1:1:40,inflation(:, :, 2), 'LineWidth', 2);
title('inflation');

subplot(2,3,6);
plot(1:1:40,labor(:, :, 2), 'LineWidth', 2);
title('labor');

legend('H=0', 'H=4', 'H=8');

%% net export shock
set(0, 'DefaultAxesColorOrder', [0 0 0], ...
      'DefaultAxesLineStyleOrder', '-|--|:|-.');
figure;
subplot(3,2,1);
plot(1:1:40,output(:, :, 3), 'LineWidth', 2);
title('output');

subplot(3,2,2);
plot(1:1:40,cons(:, :, 3), 'LineWidth', 2);
title('consumption');

subplot(3,2,3);
plot(1:1:40,int(:, :, 3), 'LineWidth', 2);
title('nominal interest rate');

subplot(3,2,4);
plot(1:1:40,realint(:, :, 3), 'LineWidth', 2);
title('real interest rate');
legend('H=0', 'H=4', 'H=8');

```

```
subplot(3,2,5)
plot(1:1:40,inflation(:,:,3),'LineWidth',2);
title('inflation');
```

```
subplot(3,2,6)
plot(1:1:40,labor(:,:,3),'LineWidth',2);
title('labor');
```

and here is the ZLB\_four.mod:

```
/*
    %Zero lower bound, interest rate peg,NK with sticky price, Taylor rule
    %1st version 2014-04-11@ND
    %2nd version 2016-3-26@Shanghai
    %we consider zero inflation steady state for a simple life here
    %This file is written by Xiangyang Li
*/
var c i p i r n w m c a y yf v p pisharp x1 x2 g nex s1 s2 s3 s4;
varexo ea ei eg ex;
parameters sigma beta psi eta phi epsilon phipi phiy etag etax;
parameters rhog rhox rhoa rhoi sigmaa sigmai sigmax sigmag;
parameters cs is pis ns ws mcs as ys yfs vps pisharps x1s x2s gs nexs;

beta = .99;
sigma = 1;
eta = 1;
psi = 1;
epsilon = 10;
phipi = 1.5;
phiy = 0;
etag = .1413; % 1993-2012, annual data, CEIC
etax = .0320; % 2005-2012 annual data, 0.0459, CEIC; 1994-2013, 0.0320
rhoa = .95;
rhoi = .0;
rhog = .9658;
rhox = .8257;
```

```

sigmaa = .1;%shut off
sigmai=.1; %shut off
sigmax = .5010;
sigmag = .1459;

%the stickiness parameter
phi = .75;
%zero inflation steady state
pis = 1;
%steady state calculation
is = 1/beta*pis;
as = 1;

pisharps= ((pis^(1-epsilon) - phi)/(1-phi))^(1/(1-epsilon));
vps = (1-phi)*(pis/pisharps)^epsilon/(1- pis^epsilon*phi);
mcs = (1-phi*beta*pis^epsilon)/(1-phi*beta*pis^(epsilon-1))*pis/pisharps*(epsilon-1);
ns = (vps^sigma*mcs/psi/(1-etax-etag)^(sigma))^(1/(eta+sigma));
ys = as*ns/vps;
yfs = ((epsilon-1)/epsilon/psi/(1-etag - etax)^sigma)^(1/(sigma + eta))*as^((1+eta));
gs = ys*etag;
nexs = ys*etax;
cs= ys*(1-etag - etax);
ws= as*mcs;
x1s = cs^(-sigma)*mcs*ys/(1-beta*phi*pis^epsilon);
x2s =cs^(-sigma)*ys/(1-beta*phi*pis^(epsilon-1));

model;
%(1) home Euler equation
exp(-sigma*c) = beta * exp(-sigma*c(+1))*exp(i)/exp(pi(+1));

%(2) labor supply
psi*exp(eta*n) =exp(-sigma*c) *exp(w);

%(3) labor demand
exp(mc) = exp(w)/exp(a);

%(4) accounting identity
exp(c) +exp(g) + exp(nex) = exp(y);

```

```

%(5) the production technology
exp(y) = exp(a) *exp(n)/exp(vp);

%(6) the price dispersion
exp(vp) = (1-phi)*exp(-epsilon*pisharp)*exp(epsilon*pi) + exp(epsilon*pi) *phi*exp

%(7) inflation evolution
exp((1-epsilon)*pi) = (1-phi)*exp((1-epsilon)*pisharp) + phi;

%(8)the sticky price equation
exp(pisharp) = epsilon/(epsilon -1 )*exp(pi)*exp(x1)/exp(x2);

%(9) the auxiliary x1
exp(x1) = exp(-sigma*c)*exp(y)*exp(mc) +phi*beta*exp(epsilon*pi(+1))*exp(x1(+1));

%(10) the auxiliary x2
exp(x2) = exp(-sigma*c)*exp(y) +phi*beta*exp((epsilon-1)*pi(+1))*exp(x2(+1));

%(11) technology shock
a = rhoa*a(-1) + ea;

%(12)government spending shock
g = (1-rhog)*log(gs) + rhog*g(-1) + eg;

%(13) net export shock
nex = (1-rhox)*log(nexs) + nex(-1)*rhox + ex;

%(14) real interest rate
exp(r) = exp(i)/exp(pi(+1));

%(15) the flexible price output
exp(yf) = ((epsilon-1)/epsilon/psi/(1-etag - etax)^sigma)^(1/(sigma + eta))*exp(a*

%(16 -19) for interest rate peg model
i = s1(-1);
s1 = s2(-1);
s2 = s3(-1);

```

```

s3 = s4(-1);
s4 = (1-rhoi)*log(is) + rhoi*s4(-1) + (1-rhoi)*(phipi*(pi(+4) - log(pis)) +phiy*(y

end;
initval;
c = log(cs);
i = log(is);
pi = log(pis);
n = log(ns);
w = log(ws);
mc = log(mcs);
a = log(as);
y = log(ys);
yf = log(yfs);
vp = log(vps);
pisharp = log(pisharps);
x1 = log(x1s);
x2 = log(x2s);
g = log(gs);
r = log(is/pis);
nex = log(nexs);
s1 = log(is);
s2 = log(is);
s3 = log(is);
s4 = log(is);
end;

shocks;
var ea = sigmaa^2;
var ei = sigmai^2;
var eg = sigmag^2;
var ex = sigmax^2;
end;

resid(1);
steady;
check;
%when used in loops, noprint and nograph is a good option.

```

```
%graph_format = none, only display, no save to disk
%since y and c is the same, we only plot y
stoch_simul(order=1, nograph, noprint) i pi n w mc y r g c nex yf;
```

## 4 Conclusion

In this notes use a simple New Keynesian model to illustrate what the economy will look like under interest rate pegs and demand shocks. We calibrated the model with China's macro-economy data. The simulation results show that under the interest rate peg periods, the GDP and domestic consumption actually increase a lot upon the demand shocks. Interest rate peg helps the households build up the expectation that real interest rate will fall upon demand shocks. Hence it helps to stimulate the domestic consumption.