Amsterdam Macroeconomics Summer School 2010

Part II: Heterogeneous Agents

University of Amsterdam

Thursday Assignment

Badly behaved higher-order pertubation solutions &

why pruning is a bad idea

1 Goal

The goal of this assignment is to show that higher-order perturbation solutions are not always well-behaved. Simulations based on higher-order perturbation solutions can be explosive, especially when non-linearities are important. You will also show that pruning, the fix to this problem proposed by Kim, Kim, Schaumburg & Sims (2008), is a bad idea in that it does make the problem stationary but at a big cost, namely giving up recursivity.[1]

2 Model

The solution to the model of Deaton (1991) with the borrowing constraint replaced by a penalty function is characterized as follows

$$\frac{c_t^{-\gamma}}{1+r} = \beta \mathrm{E}_t \left(c_{t+1}^{-\gamma} \right) + \eta_1 \exp\left(-\eta_0 a_t \right) + \eta_2$$

$$c_t + \frac{a_t}{1+r} = a_{t-1} + y_t \equiv x_t$$

$$y_t = \mu + \varepsilon_t$$

$$\varepsilon_t \sim N\left(0, \sigma^2\right)$$

[1] There may be problems, especially outside economics, where one would not care much about the solution being recursive. There this cost would be smaller.

where c_t is consumption, a_t is assets (chosen in period t), y_t is income, and x_t is cash on hand. Of course, you can choose your own model parameters, but to make our programs comparable consider

Parameter	μ	σ	r	β	γ
Value	1.5	0.15	0.03	0.9	3.

3 Exercise 1: policy function

You should run the Matlab file `main.m`. The Dynare file `model.mod` called by this "mother" program. You can change the parameter values in the main program.

1. Solve the model for $\eta_0 = 10$ (with $\eta_1 = 0.054$ and $\eta_2 = -0.0116$) using 2^{nd}-order perturbation and plot the policy function $a_t(x_t)$. In which region is $E_t(x_{t+1}) > x_t$?

2. Solve the model for $\eta_0 = 30$ (with $\eta_1 = 0.045$ and $\eta_2 = -0.0018$) with 2^{nd}-order perturbation and plot the policy function $a_t(x_t)$. In which region is $E_t(x_{t+1}) > x_t$? Compare with exercise 1.1.

3. If y_t becomes more volatile it is obviously more likely to get into the explosive region when the policy function is kept fixed. But with 2nd-order perturbation the policy function is affected by the value of σ. Solve the model for $\sigma = 0.3$ (case $\eta_0 = 30$) with 2^{nd}-order perturbation and plot the policy function $a_t(x_t)$. In which region is $E_t(x_{t+1}) > x_t$? Compare with exercise 1.2. Does the shift in the policy function make it more or less likely to get into the explosive region.

4 Exercise 2: simulation

1. Simulate the model for $\eta_0 = 10$ (again with $\eta_1 = 0.054$ and $\eta_2 = -0.0116$).

2. Simulate the model for $\eta_0 = 30$ (again with $\eta_1 = 0.045$ and $\eta_2 = -0.0018$). What happens?

3. Complete the pruning procedure. Simulate the model for $\eta_0 = 30$ using the pruning procedure.

4. Based on the simulation in exercise 2.3, create a scatter plot with a_t on the y-axis and x_t on the x-axis. This is the so-called pruning policy "function".

5. Also include the policy function for $a_t(x_t)$ in the same figure. Does the pruning solution for the policy function make sense for a recursive problem?

5 Exercise 3: alternative to pruning

The purpose of this exercise is to show that it is not that difficult to come up with a better alternative to pruning. The alternative approximation is not explosive (like pruning) and its derivatives are equal to the true ones at the steady state *and* in contrast to pruning it is a function and, thus, does not violate the recursive nature of the problem.

1. Denote the first-order perturbation solution by $p^{(1)}(x_t - x_{ss})$ and the nth-order perturbation solution by $p^{(n)}(x_t - x_{ss})$. An easy way to ensure that the solution does not explode is to use the following weighted average:

$$\begin{aligned} a_t &\approx p^{\text{alt\_I}}(x - x_{ss}) \\ &= \exp\left(-\alpha\left(x_t - x_{ss}\right)^2\right) p^{(n)}\left(x_t - x_{ss}\right) + \left(1 - \exp\left(-\alpha\left(x_t - x_{ss}\right)^2\right)\right) p^{(1)}\left(x_t\right). \end{aligned}$$

   The idea is that you move towards the stable first-order approximation if you move away from the steady state. The parameter α controls how fast you bend the higher-order solution, $p^{(n)}(\cdot)$ away towards the first-order solution. The problem with $p^{\text{alt\_I}}(x - x_{ss})$ is that its derivatives at the steady state are *not* equal to the derivatives of the true policy function. Check this.

2. Now consider

$$
\begin{aligned}
a_t &\approx p^{\mathrm{alt\_II}}(x - x_{ss}) \\
&= \exp\left(-\left(x_t - x_{ss}\right)^2\right) \tilde{p}^{(n)}\left(x_t - x_{ss}\right) + \left(1 - \exp\left(-\left(x_t - x_{ss}\right)^2\right)\right) p^{(1)}\left(x_t\right)
\end{aligned}
$$

where we have replaced $p^{(n)}(\cdot)$ by something else, namely $\tilde{p}^{(n)}\left(x_t - x_{ss}\right)$. $\tilde{p}^{(n)}\left(x_t - x_{ss}\right)$ is the n^{th}-order approximation to a variable y. The values of a_t are generated by this equation (not its approximation). The idea is to choose y such that $p^{\mathrm{alt\_II}}(x - x_{ss})$ has the correct derivatives at the steady state. What is the variable y? Hint: just think of an equation to add to the Dynare file `model.mod` that implicitly defines y. Modify the model.mod Dynare file to solve for $\tilde{p}^{n}\left(x_t - x_{ss}\right)$.

6 References

- Deaton, A. (1991): "Saving and Liquidity Constraints," *Econometrica*, 59, 1221-1248.
- Kim, J., S. Kim, E. Schaumburg, and C. A. Sims (2008): "Calculating and Using Second-Order Accurate Solutions of Discrete Time Dynamic Equilibrium Models," *Journal of Economic Dynamics and Control*, 32, 3397-3414.