State Space Representation of Log-linearized Model

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The State Space Representation of a model is very useful to do some subsequent analysis after solution.

1 The State Space Representation

The log-linearized model can be written as the following form:

$$E_t \underset{(n+m)\times 1}{X_{t+1}} = \underset{(n+m)\times(n+m)}{M} \cdot \underset{(n+m)\times 1}{X_t}$$

where n, m stand for the number of jumps and states/predetermined variables respectively. If we define

$$X_t \equiv \begin{pmatrix} \underset{n\times 1}{X_{1t}} \\ \underset{m\times 1}{X_{2t}} \end{pmatrix}$$

and

$$M \equiv \begin{pmatrix} \underset{n\times n}{M_{11}} & \underset{n\times m}{M_{12}} \\ \underset{m\times n}{M_{21}} & \underset{m\times m}{M_{22}} \end{pmatrix}$$

After we have the policy function $X_{1t} = \underset{n\times m}{\phi} X_{2t}$,then we have

$$\begin{aligned} E_t X_{1t+1} &= M_{11} X_{1t} + M_{12} X_{2t} \\ &= \left(M_{11}\phi + M_{12}\right) X_{2t} \\ &\equiv \underset{n\times m}{C} X_{2t} \end{aligned} \tag{1}$$

and

$$\begin{aligned} E_t X_{2t+1} &= M_{21} X_{1t} + M_{22} X_{2t} \\ &= \left(M_{21}\phi + M_{22}\right) X_{2t} \\ &\equiv \underset{m\times m}{A} X_{2t} \end{aligned}$$

And by Rational Expectation,

$$X_{2t+1} = AX_{2t} + B\epsilon_{t+1}$$

or

$$X_{2t} = AX_{2t-1} + B\epsilon_t$$

Hence, by the policy function, we have

$$\begin{aligned} X_{1t} &= \phi X_{2t} \\ &= \phi AX_{2t-1} + \phi B\epsilon_t \\ &\equiv CX_{2t-1} + D\epsilon_t \end{aligned} \tag{2}$$

In the last equation, we use equivalent notation. You can numerically verify $C = \phi A$ using Matlab by looking at the two equations:(2,1) and a simple RBC model (see the Matlab code in Section B&K method). Hence the state space representation is

$$\boxed{\begin{aligned} X_{1t} &= CX_{2t-1} + D\epsilon_t \quad (3) \\ X_{2t} &= AX_{2t-1} + B\epsilon_t \quad (4) \end{aligned}}$$

2 The Code

see the Matlab code in Section B&K method.