Amsterdam Macroeconomics Summer School 2010
Part I: The Essentials
University of Amsterdam

Wednesday Assignment
1. The difference between linear and loglinear perturbation
2. The difference between first-order and seconder-order perturbation
3. Calculate IRFs for non-linear models

4. TFP versus investment-specific technology shocks

1 Objective

The objectives of this assignment are the following:

1. Understand that there are several first-order perturbation solutions. Here we focus on two of them, namely linear in levels and linear in logs.
2. We will investigate whether a second-order perturbation solution is different from a first-order perturbation solution.
3. Learn how to calculate IRFs for non-linear policy functions.
4. We will investigate the behavior of a standard RBC model when there are TFP shocks and when there are investment specific shocks. Several articles point at the importance of investment-specific shocks (e.g. Jonas Fisher's 2006 JPE paper). This paper shows that in a *standard* RBC model investment-specific shocks have some odd implications.

2 Comparing perturbation solutions

Consider the following RBC model:

$$\max_{\{c_t,k_t\}_{t=1}^{\infty}} \quad \mathrm{E}\sum_{t=1}^{\infty}\beta^t \frac{c_t^{1-\nu}}{1-\nu},$$
$$\text{s.t.}$$

$$c_t + k_t = z_t k_{t-1}^{\alpha} + (1-\delta)k_{t-1} \tag{1}$$
$$z_t = (1-\rho) + \rho z_{t-1} + e_t \tag{2}$$

The first-order condition for this problem is:

$$c_t^{-\nu} = \mathrm{E}_t\left[\beta c_{t+1}^{-\nu}\left(z_{t+1}\alpha k_t^{\alpha-1} + 1 - \delta\right)\right] \tag{3}$$

You will solve this model using perturbation methods in four ways:

- 1st order approximation in levels
- 2nd order approximation in levels
- 1st order approximation in logs
- 2nd order approximation in logs

The Matlab file `Perturbation.m` is the master file. In particular, it sets the values of ν and σ (and the Dynare files will use these). These two parameters are the key parameters for this problem because the problem becomes more nonlinear if the values of these parameters increase. The files `RBClevels1st.mod` and `RBClevels2nd.mod` are the Dynare programs to generate a polynomial solution in levels for first and second-order, respectively.

You are asked to do the following:

1. Using the provided *.mod files as an example write the Dynare files that will generate a solution in *log levels*. !!! Do not change the law of motion for z_t. That is, you should only consider transformations of c_t and k_t.

2. Open the main file, `Perturbation.m`. It already generates artificial data series for `RBClevels1st.mod` and `RBClogs1st.mod` (i.e., the file you have just created). What you are asked to do is to write the part to generate artificial data from decision rules obtained from the two second-order approximations.

3. In the program provided the values of σ and ν are set equal to 0.007 and 3, two very reasonable values. Investigate what happens if you consider different values.
   For example, consider $\sigma = 0.07$. Are solutions very different when uncertainty is high? In which case does the approximation in logs make more sense than approximation in levels (hint: have a look at the consumption series for the models in levels)?

3 IRFs for non-linear models

For linear models the IRF is unique. For non-linear models the responses to a shock depend on (i) the values the state variables take on, (ii) the values *future* shocks take on (iii) they depend on the sign and the size of the shock even when the responses are scaled by the shock.

The file `IRFRBClevels2nd.m` computes the impulse responses for the three variables in the model.

1. Run the file `IRFRBClevels2nd.m` with the given parameters. It generates three impulse responses. Do not close the figure window.

2. Now change the starting values for the computation of the impulse response. For example, multiply the starting value for capital (k0) by 1.5 (so that the computation of the impulse response will start with capital 50% above the steady state). Compare the results with those you obtained in part #1. Are they the same?

3. Now undo the change made in part #2 and run the file again (that is do part #1 again). Again keep the figure window open. Now change the seed used to generate the random numbers. You'll see that there are some differences, although small (an easy way to see them is to click on the icon 'Data Cursor' on the figure an then click on one of the lines in the figure). Think why these differences occur. What would you have to do to if you wanted to obtain a 'typical' impulse-response function for the second order approximation?

4 A simple investment technology shock model

Consider the following model with investment-specific technology shocks.

$$\max_{\{c_t, i_t, k_t\}} \quad \mathrm{E} \sum_{t=1}^{\infty} \beta^t \frac{c_t^{1-\nu}}{1-\nu},$$

s.t.

$$c_t + i_t = k_{t-1}^{\alpha}$$

$$k_t = (1-\delta) k_{t-1} + exp(z_t) i_t$$

$$z_t = \rho z_{t-1} + e_t$$

First-order condition:

$$\frac{c_t^{-\nu}}{exp(z_t)} = \mathrm{E}_t \left[\beta c_{t+1}^{-\nu} \left(\alpha k_t^{\alpha-1} + \frac{1-\delta}{exp(z_{t+1})} \right) \right]$$

The purpose of this exercise is to investigate the differences between investment technology shocks and productivity shocks, and especially of the consequences they have for the comovement between variables. The file to solve this model with Dynare can be found in `RBCInvestment.mod`. The corresponding program for the model with productivity shocks is given in `RBCTechnology.mod`).[1] The file `Correlations.m` runs both models and computes business cycle statistics.

You are asked to do the following:

[1] Note that the model with productivity shocks is exactly the same as the model `RBClevels1st.mod`, with the only difference that investment series has been made explicit.

1. In `Correlations.m` the part that computes artificial time series for technology shocks has alredy been written. To investigate investment technology shocks, write the second part that (1) runs `RBCInvestment.mod` using Dynare, (2) loads the coefficient matrix (decision rules) and (3) generates artificial time series for output, consumption, and investment. Hint: you can draw heavily on what has already been written in the first part of `Correlations.m`.

2. When you complete part #1, the program will display the Dynare impulse responses and display correlations between output, consumption, and investment for the two models. Using US data we observe that corr(c,y)=0.78 and corr(i,y)=0.88. Does the model with investment technology shocks come close to these values? Think how you can explain the negative correlation between consumption and investment by looking at the impulse responses of the model with investment technology shocks.