LSE Macroeconomics Summer Program
Part II: Heterogeneous Agents
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Friday Assignment

1 Objective

This assignment has the following objective. You will see that the tools you learned this week can also be used to solve models with heterogeneous beliefs. In particular, we will look at models in which there are agents that are not rational and agents that are. "Our" rational agents are truly rational and take into account the presence of both the other rational and the irrational agents in the economy. In contrast, there are numerous models in the literature in which so called rational agent just now a bit more than other agents, but are still not rational, that is, ignore information. Typically these agents have some key information like the fundamental value of an asset, but also ignore aspects of the model that the model maker does have, that is, they are not rational. You'll see that solving for the behavior of the rational agents is not necessarily that difficult.

2 Model

Production function. The economy consists of a unit mass of risk neutral firms. In this version of the model, firms are either "rational" firms or "type A" firms *and* their type never changes. Moreover, half of them is rational and half of them is not. Technology is the same for all firms. The idea is that firms have to allocate labor to two different production lines and both face their own random productivity levels. The tricky part is that firms have to choose period $t+1$ employment levels in period t. Thus expectations play a key role. The production function is given by

$$y_i = z_i \left(z_1 n_{1,-1}^{\alpha} + z_2 n_{2,-1}^{\alpha} \right),$$

where z_i is an idiosyncratic productivity shock, z_1 and z_2 are aggregate productivity shocks, and $n_{1,-1}$ and $n_{2,-1}$ are the levels of employment used in production processes 1 and 2, respectively. These are set in period $t-1$.

Exogenous random variables. The laws of motion for the three productivity levels are given by

$$\begin{aligned}
z_i &= 1-\rho_i+\rho_i z_{i,-1}+e_i \\
z_1 &= 1-\rho_1+\rho_1 z_{1,-1}+e_1 \\
z_2 &= 1-\rho_2+\rho_2 z_{2,-1}+e_2
\end{aligned}$$

Firm problems Firms face a quadratic adjustment cost in adjusting the firm employment level. That is, firms can costlessly switch labor from one process to another, but there is a cost in changing the total employment level. Rational firms solve the following problem

$$v(n_{1,-1}, n_{2,-1}, z) = \max_{n_1, n_2} \begin{array}{c} z_i \left(z_1 n_{1,-1}^{\alpha} + z_2 n_{2,-1}^{\alpha}\right) - w(n_{-1}) - 0.5\eta(n - n_{-1})^2 \\ +\beta \mathrm{E}_t \left[v(n_1, n_2, z_{+1})\right] \end{array}$$

where

$$n = n_1 + n_2.$$

The set of first-order conditions is given by

$$\begin{aligned} \alpha\beta \mathrm{E}\left[z_{i,+1}\right] \mathrm{E}\left[z_{1,+1}\right] n_1^{\alpha-1} - \beta \mathrm{E}\left[w_{+1}\right] - \eta\left(n - n_{-1}\right) + \eta\beta \mathrm{E}\left[n_{+1} - n\right] &= 0 \\ \alpha\beta \mathrm{E}\left[z_{i,+1}\right] \mathrm{E}\left[z_{2,+1}\right] n_2^{\alpha-1} - \beta \mathrm{E}\left[w_{+1}\right] - \eta\left(n - n_{-1}\right) + \eta\beta \mathrm{E}\left[n_{+1} - n\right] &= 0 \end{aligned}$$

The first-order conditions for type A firms are assumed to be identical, except that the forecasts are not made using the true conditional expectation. Thus,

$$\begin{aligned} \alpha\beta \widehat{\mathrm{E}}\left[\hat{z}_{i,+1}\right] \widehat{\mathrm{E}}\left[z_{1,+1}\right] \hat{n}_1^{\alpha-1} - \beta \widehat{\mathrm{E}}\left[w_{+1}\right] - \eta\left(\hat{n} - \hat{n}_{-1}\right) + \eta\beta \widehat{\mathrm{E}}\left[\hat{n}_{+1} - \hat{n}\right] &= 0 \\ \alpha\beta \widehat{\mathrm{E}}\left[\hat{z}_{i,+1}\right] \widehat{\mathrm{E}}\left[z_{2,+1}\right] \hat{n}_2^{\alpha-1} - \beta \widehat{\mathrm{E}}\left[w_{+1}\right] - \eta\left(\hat{n} - \hat{n}_{-1}\right) + \eta\beta \widehat{\mathrm{E}}\left[\hat{n}_{+1} - \hat{n}\right] &= 0 \end{aligned}$$

A circumflex above a variable indicates it is a variable of a type A firm. Similarly for the expectations operator. Below we will consider different choices to model the expectations of type A firms.

Labor supply We assume that labor supply is determined by the following labor supply equation

$$w = \omega_0 + \omega_1 \left(N_{-1} - N_{ss}\right)$$

where N is the per capita employment level. Thus

$$N = \frac{\int n_i di + \int \hat{n}_i di}{2}$$

and N_{ss} equal to the steady value of N. We set

$$\omega_0 = \alpha 0.5^{\alpha-1}$$

This ensures that in the steady state

$$\begin{aligned} n_1 &= n_2 = \hat{n}_1 = \hat{n}_2 = 0.5 \\ w &= \omega_0 \\ N &= 1 \end{aligned}$$

3 Equilibrium

The equilibrium consists of

1. a set of policy rules for both types of agents
2. a law of motion for aggregate employment
3. a wage rule

such that

1. the policy rules for each type of agent solve their optimization problem given the wage rate and the law of motion for aggregate employment
2. the aggregated individual policy rules add up to the assumed law of motion for aggregate employment.
3. the labor market is in equilibrium

4 Law of motion for aggregate employment

The law of motion depends in principle on the whole cross-sectional distribution. Here we use as an approximation:

$$\widetilde{N} = c_{z_1}(z_1 - 1) + c_{z_2}(z_2 - 1) + c_N \widetilde{N}_{-1}. \tag{1}$$

Rational agents will use the best possible choices for c_{z_1}, c_{z_2}, and c_N, whereas type A agents may use something else.

5 Main programs

The two main programs are `heterobeliefs1.mod` and `ratexhetero1.m`. With these programs you can make the type A agents both rational and irrational. The structure is as follows:

1. In `ratexhetero1.m` you set both the true parameter values and the values perceived by type A firms.
2. Next you are going to iterate on the law of motion for aggregate employment until the law of motion used by the rational agents to determine their behavior is consistent with the aggregate behavior of both rational and irrational agents.

If you want to make the type A agents rational you have to do the following in `ratexhetero1.m`

- set their perceived values equal to the true values
- update their beliefs about the law of motion for aggregate employment together with the beliefs of the rational agents.

Two comments:

- The choices made are arbitrary and are more meant to show you what is possible.
- If your computer isn't that fast and your programs runs very slowly you can reduce the number of periods and/or persons used in the simulation.

6 Exercises

1. Run the representative agent model. The relevant Dynare file is `heterobeliefs1repagent.mod`. The only thing you have to do is to run the beginning of `ratexhetero1.m`.
2. Solve the model for the case when all agents are rational.
   - You have to do the following:
     - In `ratexhetero1.m` fill out the part where it says `"if A_rational == 1"`
     - In `heterobeliefs1.mod` write down (i) the Euler equations for type A firms and (ii) the laws of motions for N as they are perceived by the rational and the irrational agents.
   - You should find that the law of motion for aggregate employment (reported in the last column) is close to, but not exactly equal to the one found in part #1. Why are they not exactly equal? Which parameter do you have to change to get them closer and closer to each other?
3. Now suppose that the type A firms are not fully rational. In particular assume that they think that there is no autocorrelation in the values of z_1, z_2, and w. This means that type A firms do no longer response to aggregate productivity shocks. They are assumed to be rational about their forecasts of idiosyncratic productivity and their own future employment levels. The other firms are fully rational. You should do the following:
   - In `ratexhetero1.m` set `A_rational` equal to zero and fill out the part where it says `"if A_rational == 1"`
   - Compare the employment IRFs to a shock to z_1 in this case to the IRFs found in part 2. Are the responses half as big because only half of the firms respond? You can use Dynare to calculate the IRFs.
4. Now we consider the case where there is a fixed probability of switching types. You have to do the following:
   (a) Copy `heterobeliefs1.mod` and call it `heterobeliefs2.mod`.
   (b) Incorporate the following two changes into `heterobeliefs2.mod`:

    i. Firms of type A follow a fixed rule, namely

$$\hat{n}_1 = \hat{n}_2 = 0.5 \quad \forall t$$

    ii. Each period 2% of all firms switch type

(c) Run `ratexhetero2.m`. This file is like `ratexhetero1.m` except that it takes care of the switching of types in the simulation. Even though there are a finite number of agents it does the switching in such a way that each period the number of persons switching types is exactly equal to the expected number of switches. That is, we impose the law of large numbers when we can even though our number of agents is not infinite.

    i. You have to make the same changes as you did in `ratexhetero1.m`
    ii. note that we already had defined and written the switching probability even though we didn't need it yet.

5. How would you change `heterobeliefs2.mod` if the rule for type A fixed is not as simple as here, but depend on lagged employment levels. Hint: make sure that each type of firm always uses his own lagged values.

6. You don't have to program this, but what would you have to do to make the probability of switching endogenous?