London School of Economics Summer School

Time-Varying VARs

1 Objective

The objective of this assignment is to program the procedure developed in Cogley and Sargent to estimate a time-series model with time-varying coefficients using Bayesian methods.

2 Model

The *dgp* is given by

$$y_t = c_t + \rho_{1,t} y_{t-1} + \rho_{2,t} y_{t-2} + \varepsilon_t, \tag{1}$$

where y_t is a scalar process, ε_t is an i.i.d. Normally distributed error term and c_t, $\rho_{1,t}$, and $\rho_{2,t}$ are the time-varying coefficients. Let θ_t be the vector with these three time-varying coefficients. The law of motion for θ_t is given by

$$\theta_t = \theta_{t-1} + \nu_t, \tag{2}$$

where ν_t is a 3×1 vector with Normally distributed error terms. We assume that $\mathrm{E}_t \left[v_t v_{t+\tau}' \right] = 0$ for $\tau \neq 0$. The variance of ε_t is denoted by R and the covariance matrix of ν_t is denoted by Q.

Note that these two equations form a standard state-space system in which y_t is the observable, and θ_t is the unobserved underlying state.

3 Included programs

The key programs are the following:

- `tvar.m`: this is the main program. It generates data for y_t and then estimates the time-varying coefficients.
- `sampleTH.m`: function which obtains a random draw for $\{\theta_t\}_{t=0}^{T}$. This is the first part of the Gibbs sampler.
- `createyhat.m`: (simple) function which calculates the prediction error for y_t.
- `iwishrnd.m`: function to draw from an inverted Wishart distribution. This is part of the Matlab Statistics toolbox, but it is included in case you don't have that toolbox.

There are some other programs added for "fun". Those are

- `progressbar.m`: program which gives a time estimate of the remaining time left in a for loop.
- `maximize.m`: program which blows up a graph to screen size.

The assignment, focuses on just one data set (randomly generated). The results will, of course, be specific for this one data set. `tvarmc.m` does a simple Monte Carlo to let you look at multiple eperiments so you can evaluate better how this estimation procedure performs.

4 Assignment

4.1 Generate data

- `p` is the lag length (start with 2, that is estimate both $\rho_{1,t}$ and $\rho_{2,t}$), `T` is the length of data available for estimation (equals total data points minus `p`), `k` is the number of time-varying parameters (equals 3 when `p=2`)
- `y` is the vector with LHS variables of length `T, X` is the `Tx(p+1)` matrix with RHS variables

To do:

- Do not look at this section in detail. That is, you should try to get estimates for c_t, $\rho_{1,t}$, and $\rho_{2,t}$ without knowing how they have been determined. In an actual empirical application, you wouldn't know how these coefficients are determined either.

4.2 Form priors

The parameter `np` is the number of observations in the training sample, `yp` is the vector with LHS variables of length `np, Xp` is the `np`×`(p+1)` matrix with RHS variables

To do:

1. The mean of the prior for θ is the OLS estimate for θ obtained in the training sample. Calculate this mean.
2. The variance of the prior for θ is 4 times the standard error of θ. Calculate this variance.
3. Take a look at how the prior for R and Q are formulated.

4.3 Gibbs Sampler

Section 3 of the program generates a long sequence for θ, R, and Q according to the posterior. This is done using the Gibbs sampler. When you have such a sequence, then you can calculate all kinds of statistics describing the posterior (like the mode or the mean or pecentiles).

To do:

1. Make sure you understand how the initialization is done.
2. Complete `sampleTH`, which does the first step of the Gibbs Sampler. Make sure you understand the timing and dimenstions of `TH(:,:,t)`, `Pe(:,:,t)`, and `Po(:,:,t).` Note that `TH(:,:,t)` is defined differently in the two for loops.
3. In the second step of the Gibbs sampler, you have to obtain draws for Q and R from the posterior (conditional on a time series for θ), that is from an inverted Wishart distribution. Calculate the correct scale matrix and degrees of freedom.

5 Additional exercises

Lag order: Check what happens if you set `p=1`. Note that $\rho_{2,t}$ is set equal to zero when the realizations of the time-varying coefficients are determined, so this simply imposes something that is true.

Prior of variability θ_t: With the prior for Q, we control how much time-variation we think there is in θ_t. In the program this is controlled by the coefficient `gam2`. Check the robustness of the results by using (much) higher and (much) lower values.

Other dgp: Play around with different *dgp*s to describe the time variation in the coefficients. In particular, explore *dgp*s such that the law of motions are random walks as assumed in the theory.

References

Cogley, T., and T. J. Sargent, 2001, Evolving Post-World War II U.S. Inflation Dynamics,

NBER Macroeconomics Annual, 16, pp. 331–373.

De Wind, J., 2014, Accounting for time-varying and nonlinear relationships in macroeconomic models, PhD dissertation, University of Amsterdam. Available at http://oatd.org/oatd/record?record=oai%5C:uvapub%5C:474570.