A Simple Example for Calculating Steady States

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The calculation of steady states differ from model to model. But the generally steps is alike. Some time it could be very difficult to calculate manually. You have to leverage some numerical program to help you out. Here is one simple example that you can calculate the steady states by hand.

1 The FOCs

$$\begin{aligned}
\frac{1}{C_t} &= \beta E_t\left(\frac{1}{C_{t+1}}\left(r_{t+1}+(1-\delta)\right)\right) && (1)\\
L_t^{\gamma} &= \frac{w_t}{C_t} && (2)\\
r_t &= \alpha A_t K_{t-1}^{\alpha-1}\left(L_t\right)^{1-\alpha} && (3)\\
w_t &= (1-\alpha) A_t K_{t-1}^{\alpha}\left(L_t\right)^{-\alpha} && (4)\\
K_t + C_t &= K_{t-1}(1-\delta) + \alpha A_t K_{t-1}^{\alpha}\left(L_t\right)^{1-\alpha} && (5)
\end{aligned}$$

2 How to calculate

First we need remove time subscripts,exogenous shocks and expectation operator from focs (we assume that the model is stationary).

$$\begin{aligned}
\frac{1}{C} &= \beta\left(\frac{1}{C}\left(r+(1-\delta)\right)\right) && (6)\\
L^{\gamma} &= \frac{w}{C} && (7)\\
r &= \alpha A K^{\alpha-1}\left(L\right)^{1-\alpha} && (8)\\
w &= (1-\alpha) A K^{\alpha}\left(L\right)^{-\alpha} && (9)\\
K + C &= K(1-\delta) + \alpha A K^{\alpha}\left(L\right)^{1-\alpha} && (10)
\end{aligned}$$

Second, technology shock $A = 1$. All parameters $\alpha, \beta, \gamma, \delta$ are all known at this points. We have C, r, K, L, w. From equation(6),

$$r = \frac{1}{\beta} - (1-\delta)$$

Then from Eq.(8), we have

$$\frac{K}{L} = \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\alpha}{\frac{1}{\beta} - (1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

Then Eq.(9) gives

$$w = (1-\alpha)\left(\frac{\alpha}{\frac{1}{\beta} - (1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}$$

Then Eq.(10) gives

$$\left(\left(\frac{\alpha}{\frac{1}{\beta} - (1-\delta)}\right)^{\frac{1}{1-\alpha}} - (1-\delta)\right) L + \frac{w}{L^{\gamma}} = \alpha L \left(\frac{\alpha}{\frac{1}{\beta} - (1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}$$

How to solve this equation depends on the value of γ. If $\gamma = 0, -1$, then this is a simple linear equation for L. If $\gamma = 1$, then this is a quadratic equation for L. If γ is not a integer or something greater that one, then you have to use some numerical program to find out the steady state for labor L.