

**THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION
EXAMINATION**

142/1

ADVANCED MATHEMATICS 1
(For Both School and Private Candidates)

Time: 3 Hours

Monday, 05th May 2014 a.m.

Instructions

1. This paper consists of **ten (10)** questions, each carrying **ten (10)** marks.
2. Answer **all** questions.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).



1. Using a non-programmable calculator:

(a) (i) Evaluate $\frac{6.2 \ln \sqrt{7} \div \ln \sqrt{3}}{1782 \log 1783}$ and write your answer to six significant figures.

(ii) Compute $\frac{3^{1.75} \times \log_2 14}{\tan^{-1}(3.42) \times \log_e 13.27}$ to seven significant figures.

(b) The volume of a tetrahedron is given by $v = \frac{1}{6} a^3 (1 - \cos \theta)(1 + 2 \cos \theta)^{\frac{1}{2}}$ where a is the length of the edges and θ an angle made by the edges. By completing the table below, find the volume of the tetrahedron for the given values of a and θ and write your answers correct to three decimal places.

a	θ	$\cos \theta$	$2\sqrt{\frac{1 + \cos 2\theta}{2}}$	Volume (v) in cubic unit
1 unit	$\frac{\pi}{12}$			
2 unit	$\frac{\pi}{9}$			
3 unit	$\frac{\pi}{6}$			

2. (a) (i) Sketch the graphs of the functions $y = \cosh x$ and $y = \sinh x$ on the same $x - y$ plane.

(ii) Using part (a) (i), state the range of $\cosh x$ and $\sinh x$.

(b) (i) Prove that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$.

(ii) Solve the equation $3 \operatorname{sech}^2 x + 4 \tanh x + 1 = 0$ and write your answer correct to 4 decimal places.

(iii) Verify that $\sinh 3x = 3 \sinh x + 4 \sinh^3 x$.

(c) Show that $\frac{1}{e^{-3 \ln x}} \left(\frac{\cosh \ln x - \sinh \ln x}{\cosh \ln x + \sinh \ln x} \right) = x^2$.

3. (a) A farm is to be planted with cabbages and potatoes. The cost and the number of people needed for the work is indicated in the table below:

	Cabbages	Potatoes	Total available
Labour per hectare (Number of people)	2	4	10
Labour costs per hectare (Tshs)	28,000/=	24,000/=	168,000/=
Costs of fertilizer per hectare (Tshs)	60,000/=	80,000/=	480,000/=

- (i) Find the greatest number of hectares that can be planted.
- (ii) If the profit for a hectare of cabbages is 80,000/= and for potatoes is 60,000/=, how many hectares of each crop should be planted to maximize the profit?
- (b) One of the Tanzanian wine drink manufacturing firms has m plants located in different towns. The total production is absorbed by n retail shops in different towns.
- (i) Formulate the general transporting schedule that minimizes the total cost of transporting wine drinks from various plants to various retail shops.
- (ii) Construct the transportation table with 2 origins and 2 destinations using the following parameters: The Supply is a_i , demand b_j and the cost c_{ij} .
- (iii) From (b) (i) and (ii), deduce the conditions which satisfy the existence of a feasible solution to the transportation problem.

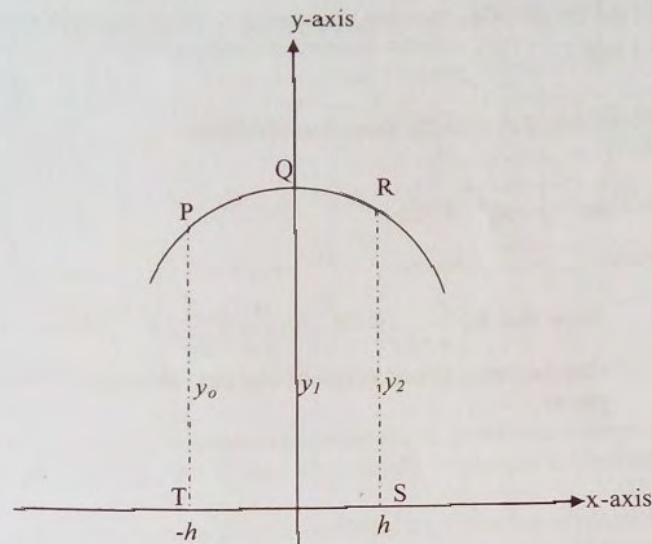
4. (a) The monthly wages of employees working in a certain factory are given in the table below:

Wages in shs ($\times 1000$)	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100	100 - 110	110 - 120
Number of employees	8	10	16	15	10	8	3

- (i) By using an appropriate formula, find the median and mode for the wages given above, giving your answer to the nearest thousand shillings.
- (ii) Find the semi-interquartile range of the given data.
- (b) The number of errors made by the typist on each page of a document with 100 pages were recorded in the table below.

Number of errors	Frequency
0	15
1	30
2	28
3	18
4	9

- (i) Find the variance and standard deviation of the number of errors per page, writing your answer correct to 4 decimal places.
- (ii) Find the 20th percentile of the data.
5. (a) Using Venn diagrams show that:
 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
- (b) By using set properties, prove that for any non-empty sets X and Y :
- (i) $X \cup (X \cap Y) = X$,
- (ii) $(X \cap Y') \cup (Y \cap X') = (X \cup Y) - (X \cap Y)$.
- (c) There are twenty five men at a meeting of which eleven are doctors, sixteen are teachers, and eight are both doctors and teachers. How many are neither doctors nor teachers?
6. (a) (i) A function is defined by $g : x \rightarrow x^2 - 10$ for all $x \in \mathbb{R}$. Find all the values of x for which $g \circ g(x) = 26$.
- (ii) If $f(x) = 3x - 2$, $g(x) = x + 7$ and $h(x) = \frac{1}{1+x}$, determine the intercepts and the asymptotes of $f \circ g \circ h$.
- (b) Given that $f(x) = x^4 - 2x^3 - x^2 + 2x$,
- (i) Find the value of x where the curve $f(x)$ cuts the x - axis.
- (ii) Sketch the graph of $f(x)$.
7. (a) (i) Apply the Newton - Raphson formula with three iterations to compute the value of $\sqrt[3]{7}$ correct to five significant figures. Use $x_0 = 2$.
- (ii) The figure below has points P, Q and R on the quadratic curve $f(x) = ax^2 + bx + c$. Derive the Simpson's rule with n -ordinates to approximate the area PQIRST.



- (b) (i) Evaluate $\int_0^{\pi/4} \cos^2 x dx$ by using the Simpson's rule with five ordinates and write your answer to 4 decimal places.
- (ii) Find the actual value of $\int_0^{\pi/4} \cos^2 x dx$ and compare your answers with part (b) (i).
8. (a) Sketch the diagram for the locus of points which move such that it covers a distance a units from the curve $x^2 + y^2 + 2x - 4y = 20$ where $|a| < 5$.
- (b) Find the length of the tangent from the point $(5, 7)$ to the circle $x^2 + y^2 - 4x + 6y + 9 = 0$.
- (c) If p and q are the lengths of the perpendiculars from the origin to the lines $x \cos \theta - y \sin \theta = k \cos 2\theta$ and $x \sec \theta + y \csc \theta = k$ respectively, prove that $p^2 + 4q^2 = k^2$.
9. (a) If the gradient of a certain function is $\frac{1}{7(e^x + 1)}$, find the function.
- (b) Evaluate the following integrals:
- (i) $\int_1^5 \frac{2t}{\sqrt{2t+1}} dt$ (leave your answer in surd form).
- (ii) $\left| \int_0^{\pi/2} \cos 2x \sin x dx \right|$.

- (c) Find the length of the arc of the curve $6xy = 3 + x^4$ between the points whose abscissa are 1 and 3.
10. (a) Differentiate $3x^2 + \cos 2x$ from first principles.
- (b) If $y = \sin^{n+1} x \cos^{m-1} x$, find $\frac{dy}{dx}$.
- (c) (i) Show that $\ln\left(\frac{x-1}{x+1}\right) = -2\left(\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \dots\right)$ for $|x| > 1$.
- (ii) Use the series in part (c) (i) to find the value of $\ln 0.5$ correct to three decimal places.