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Using the method of completing the square, find (i) the coordinates of the vertex of the graph, and (ii) the maximum or minimum value of each of the following quadratic functions.

1. $y = x^2 + 13x - 13$

Solution:

$$\begin{aligned} y &= x^2 + 13x - 13 \\ &= x^2 + 13x + \left(\frac{13}{2}\right)^2 - \left(\frac{13}{2}\right)^2 - 13 \\ &= \left(x + \frac{13}{2}\right)^2 - \frac{221}{4} \end{aligned}$$

$\therefore$ Vertex $= \left(-\frac{13}{2}, -\frac{221}{4}\right)$, Minimum value $= -\frac{221}{4}$

2. $y = x^2 + 7x + 4$

Solution:

$$\begin{aligned} y &= x^2 + 7x + 4 \\ &= x^2 + 7x + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 4 \\ &= \left(x + \frac{7}{2}\right)^2 - \frac{33}{4} \end{aligned}$$

$\therefore$ Vertex $= \left(-\frac{7}{2}, -\frac{33}{4}\right)$, Minimum value $= -\frac{33}{4}$

3. $y = x^2 - 2x - 6$

Solution:

$$\begin{aligned} y &= x^2 - 2x - 6 \\ &= x^2 - 2x + \left(\frac{-2}{2}\right)^2 - \left(\frac{-2}{2}\right)^2 - 6 \\ &= (x - 1)^2 - 7 \end{aligned}$$

$\therefore$ Vertex $= (1, -7)$, Minimum value $= -7$

4. $y = x^2 - 4x + 7$

Solution:

$$\begin{aligned} y &= x^2 - 4x + 7 \\ &= x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + 7 \\ &= (x-2)^2 + 3 \end{aligned}$$

$\therefore$ Vertex $= (2, 3)$, Minimum value $= 3$

5. $y = x^2 - 10x - 5$

Solution:

$$\begin{aligned} y &= x^2 - 10x - 5 \\ &= x^2 - 10x + \left(\frac{-10}{2}\right)^2 - \left(\frac{-10}{2}\right)^2 - 5 \\ &= (x-5)^2 - 30 \end{aligned}$$

$\therefore$ Vertex $= (5, -30)$, Minimum value $= -30$

6. $y = x^2 - 9x - 13$

Solution:

$$\begin{aligned} y &= x^2 - 9x - 13 \\ &= x^2 - 9x + \left(\frac{-9}{2}\right)^2 - \left(\frac{-9}{2}\right)^2 - 13 \\ &= \left(x - \frac{9}{2}\right)^2 - \frac{133}{4} \end{aligned}$$

$\therefore$ Vertex $= \left(\frac{9}{2}, -\frac{133}{4}\right)$, Minimum value $= -\frac{133}{4}$

7. $y = x^2 - 3x - 15$

Solution:

$$\begin{aligned} y &= x^2 - 3x - 15 \\ &= x^2 - 3x + \left(\frac{-3}{2}\right)^2 - \left(\frac{-3}{2}\right)^2 - 15 \\ &= \left(x - \frac{3}{2}\right)^2 - \frac{69}{4} \end{aligned}$$

$\therefore$ Vertex $= \left(\frac{3}{2}, -\frac{69}{4}\right)$, Minimum value $= -\frac{69}{4}$

8. $y = x^2 - 6x + 3$

Solution:

$$\begin{aligned} y &= x^2 - 6x + 3 \\ &= x^2 - 6x + \left(\frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 + 3 \\ &= (x-3)^2 - 6 \end{aligned}$$

$\therefore$ Vertex $= (3, -6)$, Minimum value $= -6$

9. $y = x^2 + 14x + 3$

Solution:

$$\begin{aligned} y &= x^2 + 14x + 3 \\ &= x^2 + 14x + \left(\frac{14}{2}\right)^2 - \left(\frac{14}{2}\right)^2 + 3 \\ &= (x+7)^2 - 46 \end{aligned}$$

$\therefore$ Vertex $= (-7, -46)$, Minimum value $= -46$

10. $y = x^2 - 9x - 13$

Solution:

$$\begin{aligned} y &= x^2 - 9x - 13 \\ &= x^2 - 9x + \left(\frac{-9}{2}\right)^2 - \left(\frac{-9}{2}\right)^2 - 13 \\ &= \left(x - \frac{9}{2}\right)^2 - \frac{133}{4} \end{aligned}$$

$\therefore$ Vertex $= \left(\frac{9}{2}, -\frac{133}{4}\right)$, Minimum value $= -\frac{133}{4}$

11. $y = -2x^2 - 5x + 7$

Solution:

$$\begin{aligned} y &= -2x^2 - 5x + 7 \\ &= -2\left(x^2 + \frac{5}{2}x\right) + 7 \\ &= -2\left[x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 7 \\ &= -2\left[\left(x + \frac{5}{4}\right)^2 - \frac{25}{16}\right] + 7 \\ &= -2\left(x + \frac{5}{4}\right)^2 - 2\left(-\frac{25}{16}\right) + 7 \\ &= -2\left(x + \frac{5}{4}\right)^2 + \frac{81}{8} \end{aligned}$$

$\therefore$ Vertex $= \left(-\frac{5}{4}, \frac{81}{8}\right)$, Maximum value $= \frac{81}{8}$

12. $y = -2x^2 - 10x + 13$

Solution:

$$\begin{aligned} y &= -2x^2 - 10x + 13 \\ &= -2\left(x^2 + 5x\right) + 13 \\ &= -2\left[x^2 + 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right] + 13 \\ &= -2\left[\left(x + \frac{5}{2}\right)^2 - \frac{25}{4}\right] + 13 \\ &= -2\left(x + \frac{5}{2}\right)^2 - 2\left(-\frac{25}{4}\right) + 13 \\ &= -2\left(x + \frac{5}{2}\right)^2 + \frac{51}{2} \end{aligned}$$

$\therefore$ Vertex $= \left(-\frac{5}{2}, \frac{51}{2}\right)$, Maximum value $= \frac{51}{2}$

13. $y = 3x^2 + 7x - 11$

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Solution:

$$
\begin{aligned}
y &= 3x^2 + 7x - 11 \\
&= 3\left(x^2 + \frac{7}{3}x\right) - 11 \\
&= 3\left[x^2 + \frac{7}{3}x + \left(\frac{7}{6}\right)^2 - \left(\frac{7}{6}\right)^2\right] - 11 \\
&= 3\left[\left(x + \frac{7}{6}\right)^2 - \frac{49}{36}\right] - 11 \\
&= 3\left(x + \frac{7}{6}\right)^2 + 3\left(-\frac{49}{36}\right) - 11 \\
&= 3\left(x + \frac{7}{6}\right)^2 - \frac{181}{12}
\end{aligned}
$$

$\therefore$ Vertex $= \left(-\frac{7}{6}, -\frac{181}{12}\right)$, Minimum value $= -\frac{181}{12}$

14. $y = -2x^2 - 7x - 1$

Solution:

$$
\begin{aligned}
y &= -2x^2 - 7x - 1 \\
&= -2\left(x^2 + \frac{7}{2}x\right) - 1 \\
&= -2\left[x^2 + \frac{7}{2}x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] - 1 \\
&= -2\left[\left(x + \frac{7}{4}\right)^2 - \frac{49}{16}\right] - 1 \\
&= -2\left(x + \frac{7}{4}\right)^2 - 2\left(-\frac{49}{16}\right) - 1 \\
&= -2\left(x + \frac{7}{4}\right)^2 + \frac{41}{8}
\end{aligned}
$$

$\therefore$ Vertex $= \left(-\frac{7}{4}, \frac{41}{8}\right)$, Maximum value $= \frac{41}{8}$

15. $y = 4x^2 - 10x + 5$

Solution:

$$
\begin{aligned}
y &= 4x^2 - 10x + 5 \\
&= 4\left(x^2 - \frac{5}{2}x\right) + 5 \\
&= 4\left[x^2 - \frac{5}{2}x + \left(-\frac{5}{4}\right)^2 - \left(-\frac{5}{4}\right)^2\right] + 5 \\
&= 4\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] + 5 \\
&= 4\left(x - \frac{5}{4}\right)^2 + 4\left(-\frac{25}{16}\right) + 5 \\
&= 4\left(x - \frac{5}{4}\right)^2 - \frac{5}{4}
\end{aligned}
$$

$\therefore$ Vertex $= \left(\frac{5}{4}, -\frac{5}{4}\right)$, Minimum value $= -\frac{5}{4}$

16. $y = -4x^2 - 12x + 3$

Solution:

$$
\begin{aligned}
y &= -4x^2 - 12x + 3 \\
&= -4\left(x^2 + 3x\right) + 3 \\
&= -4\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] + 3 \\
&= -4\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 3 \\
&= -4\left(x + \frac{3}{2}\right)^2 - 4\left(-\frac{9}{4}\right) + 3 \\
&= -4\left(x + \frac{3}{2}\right)^2 + 12
\end{aligned}
$$

$\therefore$ Vertex $= \left(-\frac{3}{2}, 12\right)$, Maximum value $= 12$

17. $y = -x^2 - 2x - 10$

Solution:

$$\begin{aligned}
y &= -x^2 - 2x - 10 \\
&= -\left(x^2 + 2x\right) - 10 \\
&= -\left[x^2 + 2x + (1)^2 - (1)^2\right] - 10 \\
&= -\left[(x+1)^2 - 1\right] - 10 \\
&= -(x+1)^2 - (-1) - 10 \\
&= -(x+1)^2 - 9
\end{aligned}$$

$\therefore$ Vertex $= (-1, -9)$, Maximum value $= -9$

18. $y = -2x^2 - 4x - 18$

Solution:

$$\begin{aligned}
y &= -2x^2 - 4x - 18 \\
&= -2\left(x^2 + 2x\right) - 18 \\
&= -2\left[x^2 + 2x + (1)^2 - (1)^2\right] - 18 \\
&= -2\left[(x+1)^2 - 1\right] - 18 \\
&= -2(x+1)^2 - 2(-1) - 18 \\
&= -2(x+1)^2 - 16
\end{aligned}$$

$\therefore$ Vertex $= (-1, -16)$, Maximum value $= -16$

19. $y = 3x^2 + 9x - 1$

Solution:

$$\begin{aligned}
y &= 3x^2 + 9x - 1 \\
&= 3\left(x^2 + 3x\right) - 1 \\
&= 3\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] - 1 \\
&= 3\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}\right] - 1 \\
&= 3\left(x + \frac{3}{2}\right)^2 + 3\left(-\frac{9}{4}\right) - 1 \\
&= 3\left(x + \frac{3}{2}\right)^2 - \frac{31}{4}
\end{aligned}$$

$\therefore$ Vertex $= \left(-\frac{3}{2}, -\frac{31}{4}\right)$, Minimum value $= -\frac{31}{4}$

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20. $y = 3x^2 - 10x + 13$

Solution:

$$
\begin{aligned}
y &= 3x^2 - 10x + 13 \\
&= 3\left(x^2 - \frac{10}{3}x\right) + 13 \\
&= 3\left[x^2 - \frac{10}{3}x + \left(-\frac{5}{3}\right)^2 - \left(-\frac{5}{3}\right)^2\right] + 13 \\
&= 3\left[\left(x - \frac{5}{3}\right)^2 - \frac{25}{9}\right] + 13 \\
&= 3\left(x - \frac{5}{3}\right)^2 + 3\left(-\frac{25}{9}\right) + 13 \\
&= 3\left(x - \frac{5}{3}\right)^2 + \frac{14}{3}
\end{aligned}
$$

$\therefore$ Vertex $= \left(\frac{5}{3}, \frac{14}{3}\right)$, Minimum value $= \frac{14}{3}$