

Simplify and express the following expressions with positive indices.

1. $\frac{(x^4y^{-3})^4}{xy}$

Solution:

$$\begin{aligned}\frac{(x^4y^{-3})^4}{xy} &= \frac{x^{16}y^{-12}}{xy} \\ &= x^{16-1}y^{-12-1} \\ &= x^{15}y^{-13} \\ &= \frac{x^{15}}{y^{13}}\end{aligned}$$

2. $\frac{(x^{-5}y^{-4})^{-2}}{x^3y^2}$

Solution:

$$\begin{aligned}\frac{(x^{-5}y^{-4})^{-2}}{x^3y^2} &= \frac{x^{10}y^8}{x^3y^2} \\ &= x^{10-3}y^{8-2} \\ &= x^7y^6\end{aligned}$$

3. $\frac{(x^5y^3)^{-1}}{xy^2}$

Solution:

$$\begin{aligned}\frac{(x^5y^3)^{-1}}{xy^2} &= \frac{x^{-5}y^{-3}}{xy^2} \\ &= x^{-5-1}y^{-3-2} \\ &= x^{-6}y^{-5} \\ &= \frac{1}{x^6y^5}\end{aligned}$$

4. $\frac{(x^{-3}y^{-1})^2}{x^4y}$

Solution:

$$\begin{aligned}\frac{(x^{-3}y^{-1})^2}{x^4y} &= \frac{x^{-6}y^{-2}}{x^4y} \\ &= x^{-6-4}y^{-2-1} \\ &= x^{-10}y^{-3} \\ &= \frac{1}{x^{10}y^3}\end{aligned}$$

5. $\frac{(x^4y^{-5})^{-1}}{x^2y^5}$

Solution:

$$\begin{aligned}\frac{(x^4y^{-5})^{-1}}{x^2y^5} &= \frac{x^{-4}y^5}{x^2y^5} \\ &= x^{-4-2}y^{5-5} \\ &= x^{-6}y^0 \\ &= \frac{1}{x^6}\end{aligned}$$

6. $\frac{x^{-3}y^{-5}}{(x^2y^{-1})^{-2}}$

Solution:

$$\begin{aligned}\frac{x^{-3}y^{-5}}{(x^2y^{-1})^{-2}} &= \frac{x^{-3}y^{-5}}{x^{-4}y^2} \\ &= x^{-3-(-4)}y^{-5-2} \\ &= x^1y^{-7} \\ &= \frac{x}{y^7}\end{aligned}$$

7. $\frac{x^5y^{-3}}{(x^{-3}y^2)^5}$

Solution:

$$\begin{aligned}\frac{x^5y^{-3}}{(x^{-3}y^2)^5} &= \frac{x^5y^{-3}}{x^{-15}y^{10}} \\ &= x^{5-(-15)}y^{-3-10} \\ &= x^{20}y^{-13} \\ &= \frac{x^{20}}{y^{13}}\end{aligned}$$

8. $\frac{x^{-4}y^3}{(x^{-3}y^4)^{-2}}$

Solution:

$$\begin{aligned}\frac{x^{-4}y^3}{(x^{-3}y^4)^{-2}} &= \frac{x^{-4}y^3}{x^6y^{-8}} \\ &= x^{-4-6}y^{3-(-8)} \\ &= x^{-10}y^{11} \\ &= \frac{y^{11}}{x^{10}}\end{aligned}$$

9. $\frac{x^{-5}y^3}{(xy^4)^4}$

Solution:

$$\begin{aligned}\frac{x^{-5}y^3}{(xy^4)^4} &= \frac{x^{-5}y^3}{x^4y^{16}} \\ &= x^{-5-4}y^{3-16} \\ &= x^{-9}y^{-13} \\ &= \frac{1}{x^9y^{13}}\end{aligned}$$

10. $\frac{x^3y^3}{(x^3y^{-1})^{-5}}$

Solution:

$$\begin{aligned}\frac{x^3y^3}{(x^3y^{-1})^{-5}} &= \frac{x^3y^3}{x^{-15}y^5} \\ &= x^{3-(-15)}y^{3-5} \\ &= x^{18}y^{-2} \\ &= \frac{x^{18}}{y^2}\end{aligned}$$

11. $\frac{x^{-2}}{y} \left(\frac{y^3}{x^5} \right)^{-2}$

Solution:

$$\begin{aligned}\frac{x^{-2}}{y} \left(\frac{y^3}{x^5} \right)^{-2} &= \frac{x^{-2}}{y} \cdot \frac{y^{-6}}{x^{-10}} \\ &= x^{-2-(-10)} y^{-6-1} \\ &= x^8 y^{-7} \\ &= \frac{x^8}{y^7}\end{aligned}$$

12. $\frac{x^{-2}}{y^2} \left(\frac{y^4}{x} \right)^{-4}$

Solution:

$$\begin{aligned}\frac{x^{-2}}{y^2} \left(\frac{y^4}{x} \right)^{-4} &= \frac{x^{-2}}{y^2} \cdot \frac{y^{-16}}{x^{-4}} \\ &= x^{-2-(-4)} y^{-16-2} \\ &= x^2 y^{-18} \\ &= \frac{x^2}{y^{18}}\end{aligned}$$

13. $\frac{x^{-4}}{y^{-1}} \left(\frac{y^2}{x^{-3}} \right)^2$

Solution:

$$\begin{aligned}\frac{x^{-4}}{y^{-1}} \left(\frac{y^2}{x^{-3}} \right)^2 &= \frac{x^{-4}}{y^{-1}} \cdot \frac{y^4}{x^{-6}} \\ &= x^{-4-(-6)} y^{4-(-1)} \\ &= x^2 y^5\end{aligned}$$

14. $\frac{x^{-2}}{y^{-4}} \left(\frac{y^2}{x^{-5}} \right)^{-4}$

Solution:

$$\begin{aligned}\frac{x^{-2}}{y^{-4}} \left(\frac{y^2}{x^{-5}} \right)^{-4} &= \frac{x^{-2}}{y^{-4}} \cdot \frac{y^{-8}}{x^{20}} \\ &= x^{-2-20} y^{-8-(-4)} \\ &= x^{-22} y^{-4} \\ &= \frac{1}{x^{22} y^4}\end{aligned}$$

15. $\frac{x^{-2}}{y^{-1}} \left(\frac{y^5}{x^{-3}} \right)^{-4}$

Solution:

$$\begin{aligned} \frac{x^{-2}}{y^{-1}} \left(\frac{y^5}{x^{-3}} \right)^{-4} &= \frac{x^{-2}}{y^{-1}} \cdot \frac{y^{-20}}{x^{12}} \\ &= x^{-2-12} y^{-20-(-1)} \\ &= x^{-14} y^{-19} \\ &= \frac{1}{x^{14} y^{19}} \end{aligned}$$