Could Bid Cap Control Bankruptcy Rate?

Yu Zhou
econyz1216@gmail.com
Wuhan University

Outline

Topic

First-Price Auction with Shallow Pocket and Cap

First-price sealed-bid auction

A first-price sealed-bid auction is an auction where the highest bidder gets the object and pays the amount he bid.

Shallow pocket

Budget-constrained bidders can make their own financial decisions like borrowing, and default on their bids.

Cap

A cap is an upper bound on bids.

Examples of Topic

Examples of Topic

Examples of auction cap

- Salary caps
  - NBA, NFL
  - In this sports league, individual teams face annual caps on the sum of money they are allowed to spend on salaries.
- Technological caps
  - F1
  - Formula 1 racing cars must be constructed such that they cannot run faster than an absolute limit of 360 kilometers per hour.

Examples of Topic

Examples of auction with shallow pocket

- Spectrum auctions
- The FCC auctioned off the licenses for using the radio frequencies within a C-block spectrum.

Main Questions

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1. Does there exist a symmetric equilibrium with and without cap?
2. What are bidders' financial and default decisons?
3. Will setting a bid cap benefit an organizer who wishes to control bankruptcy rate?

The Model

Model and Notation

- n bidders compete for 1 indivisible risky prize by submitting bids x.
- Prize value v follows a binomial distribution, where $\overline{v}$ with probability $1-\theta$ and 0 with probability θ.
- Bidder i bears budget constraint e_i which is his private information.
- All bidders other than i perceive e_i as a random selection out of a support $[\underline{e}, \overline{e}] \in (0, \infty)$.
  - I.I.D
  - CDF: F, continuous differentiable
  - PDF: f, $f(e) > 0$ for all $e \in [\underline{e}, \overline{e}]$
- Exogenous borrowing rate is r while lending rate is 0.
- Organizer announces a bid cap d, where $d \in (0, +\infty)$.

Model and Notation

- Cost of bid is
$$c(x_i, e_i) = \begin{cases} x_i & \text{if } x_i \leq e_i \\ x_i + r(x_i - e_i) & \text{if } x_i > e_i \end{cases}$$
- Prize is given to only one bidder with the highest bid.
- Ties are broken randomly.
- After winning the object, the winning bidder gets to know its true value.
- Default is allowed whose penalty is the lose of his entire budget.
- $\widetilde{\beta}(e)$ is a symmetric bidding strategy without cap.
- $\beta(e, d)$ is a symmetric bidding strategy with cap.
- Bankruptcy rate is denoted by BR.

Timeline

Organizer announces bid cap d	Budget e_i is realized	Bidder *i* submits his bid x_i	Organizer announces the winner's index	The winner decides whether to default

Organizer Problem

Organizer Problem

The organizer selects the optimal bid cap to control/minimize the bankruptcy rate.

$$\min_{d \in (0,+\infty)} BR(d)$$

main result

Default decision

Proposition

Default only occurs at the winner who claims a debt (i.e.$x_i > e_i$) when 0 value happens.

Proof.

- If revelation of v is $\overline{v}$, then the winner will never default.
- If revelation of v is 0, then the winner will default only if he has claimed a debt.

$\square$

Equilibria without a Bid Cap

Proposition

(*Charles Z.Zheng, 2001*). In the case of $r \in [0, \frac{\theta}{1-\theta})$, there exists an unique continuous symmetric equilibrium of the auction game given by

$$\widetilde{\beta}(e) = \begin{cases} E_{e^L_{-i}}[\frac{\overline{v}+r' \min(e^L_{-i},(1-\theta)\overline{v})}{1+r} \mid e^L_{-i} > e] & \text{if } \overline{e} \leq e < (1-\theta)\overline{v} \\ (1-\theta)\overline{v} & \text{otherwise} \end{cases}$$

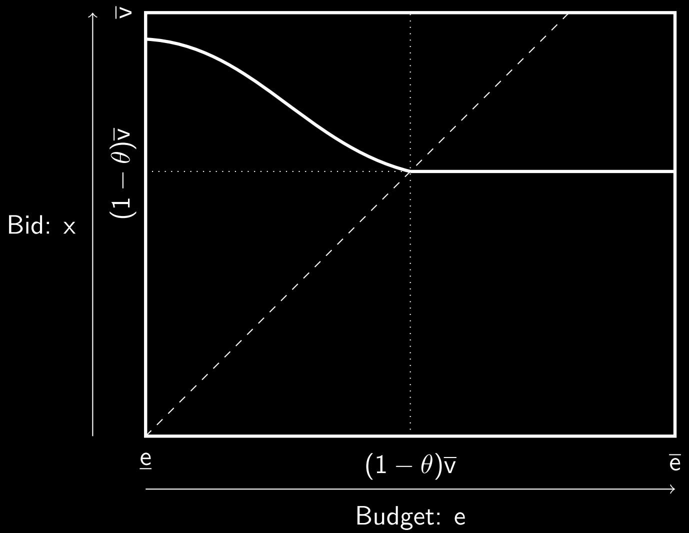
where $r' = r - \frac{\theta}{1-\theta}$ and e^L_{-i} denotes the lowest budget among a bidders' rivals.

Proposition

In the case of $r \in [0, \frac{\theta}{1-\theta})$,
bankruptcy rate is $\theta[1-(1-F((1-\theta)\overline{v}))^n]$.

Equilibria without a Bid Cap

Graph



Equilibria with an Inefficient Bid Cap

Proposition

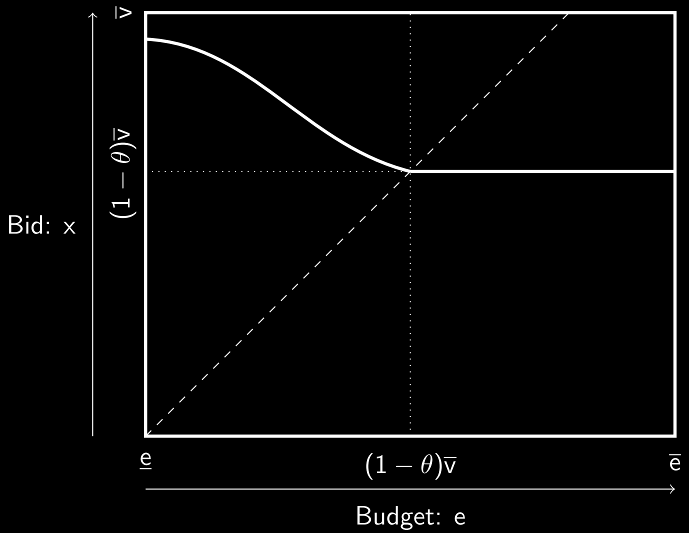
Let $r \in [0, \frac{\theta}{1-\theta})$ and $d > E_{e^L_{-i}}[\frac{\overline{v}+r' \min(e^L_{-i},(1-\theta)\overline{v})}{1+r}]$, there exists an unique continuous symmetric equilibrium of the auction game given by

$$\beta(e,d) = \widetilde{\beta}(e) = \begin{cases} E_{e^L_{-i}}[\frac{\overline{v}+r' \min(e^L_{-i},(1-\theta)\overline{v})}{1+r} \mid e^L_{-i} > e] & \text{if } \overline{e} \leq e < (1-\theta)\overline{v} \\ (1-\theta)\overline{v} & \text{otherwise} \end{cases}$$

where $r' = r - \frac{\theta}{1-\theta}$ and e^L_{-i} denotes the lowest budget among a bidders' rivals. And the bankruptcy rate is $\theta[1-(1-F((1-\theta)\overline{v}))^n]$.

Equilibria with an Inefficient Bid Cap

Graph



Equilibria with an Efficient Bid Cap

Proposition

Let $r \in [0, \frac{\theta}{1-\theta})$ and $(1-\theta)\overline{v} < d \leq E_{e^L_{-i}}[\frac{\overline{v}+r' \min(e^L_{-i},(1-\theta)\overline{v})}{1+r}]$. Then the bid cap is effective, and there exists a symmetric monotone equilibrium where bidding strategy is given by

$$\beta(e_i, d) = \begin{cases} d & \text{if } \underline{e} \leq e_i < \widetilde{e} \\ \widetilde{\beta}(e_i) & \text{if } \widetilde{e} \leq e_i < (1-\theta)\overline{v} \\ (1-\theta)\overline{v} & \text{if } (1-\theta)\overline{v} \leq e_i \leq \overline{e} \end{cases}$$

and the bankruptcy rate is given by

$$BR(d) = \theta[1 - (1 - F((1-\theta)\overline{v}))^n]$$

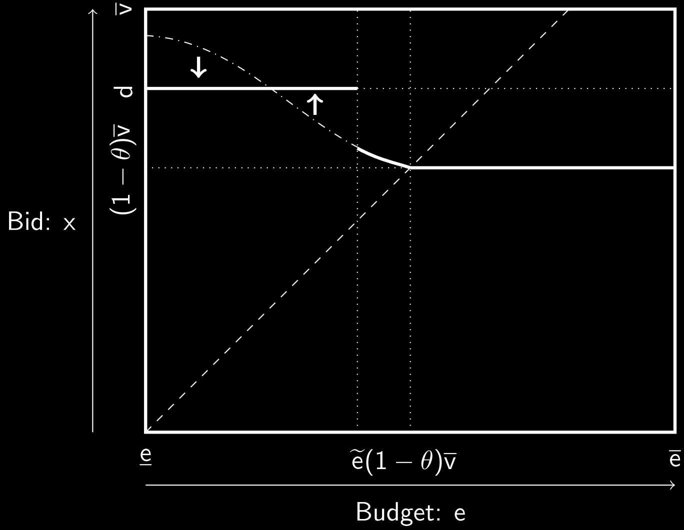
Equilibria with an Efficient Bid Cap

where the critical value $\widetilde{e} = \widetilde{e}(d)$ is strictly monotonic decreasing, and defined by

$$d = \frac{nF(\widetilde{e})(1-F(\widetilde{(e)}))^{n-1}}{1-(1-F(\widetilde{e}))^n}[\beta\widetilde{e} - \frac{\overline{v}+r'\widetilde{e}}{1+r}] + \frac{\overline{v}+r'\widetilde{e}}{1+r}$$

where $r' = r - \frac{\theta}{1-\theta}$

Equilibria with an Efficient Bid Cap



Equilibria with an Efficient Bid Cap

Proposition

Let $r \in [0, \frac{\theta}{1-\theta})$ and $0 < d \leq (1-\theta)\overline{v}$. Then the bid cap is effective, and there exists a symmetric monotone equilibrium where bidding strategy is given by

$$\beta(e_i, d) = d \ \ \forall e_i \in [\underline{e}, \overline{e}]$$

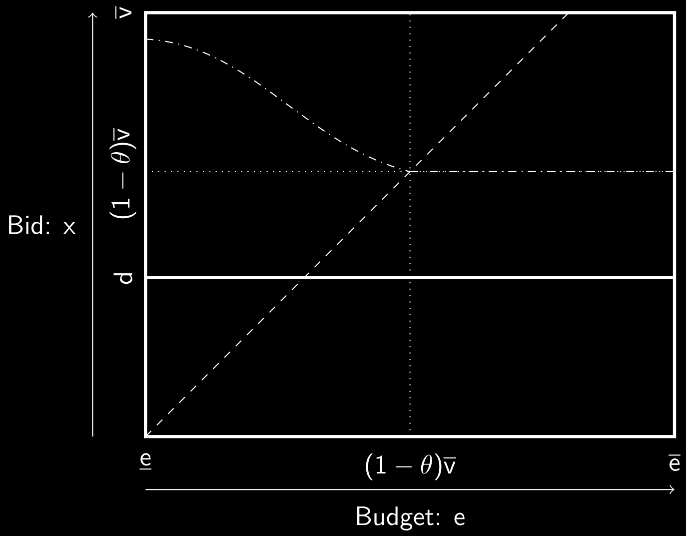
and the bankruptcy rate is given by

$$BR(d) = \theta F(d)$$

which is strictly increasing with respect to bid cap d.

Equilibria with an Efficient Bid Cap

Graph



Could Bid Cap Control Bankruptcy Rate?

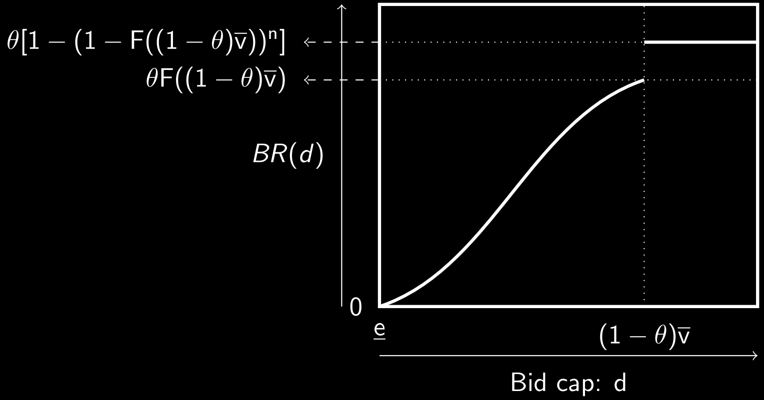
According to propositions, we could obtain

$$BR(d) = \begin{cases} \theta[1-(1-F((1-\theta)\overline{v}))^n] & \text{if } d > (1-\theta)\overline{v} \\ \theta F(d) & \text{if } 0 < d \leq (1-\theta)\overline{v} \end{cases}$$

- There is a jump at $d = (1-\theta)\overline{v}$.
- $BR(d)$ is a weakly increasing function w.r.t d.

Could Bid Cap Control Bankruptcy Rate?

Graph



Bibliography

Charles Z. Zheng.
High Bids and Broke Winners [J].
Journal of Economic Theory, 2001, 100: 129-171.

Chen B, Jiang X, Knyazev D.
On Disclosure Policies in All-pay Auctions with Stochastic Entry [J].
Journal of Mathematical Economics, 2017, 70: 66-73.

Gavious A, Moldovanu B, Sela A.
Bid Costs and Endogenous Bid Caps [J].
The RAND Journal of Economics, 2002, 33: 709-722.