Caps on All-pay Auction with Stochastic Abilities

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Outline

Topic

All-Pay Auction and Cap

All-pay auction

An all-pay auction is an auction in which economic agents compete by making irreversible investments before the outcome of the competition.

Cap

A cap is an upper bound on bids.

Examples of contests with a cap

Examples of contests with a cap

- Salary caps
  - NBA, NFL
  - In this sports league, individual teams face annual caps on the sum of money they are allowed to spend on salaries.
- Technological caps
  - F1
  - Formula 1 racing cars must be constructed such that they cannot run faster than an absolute limit of 360 kilometers per hour.

Main Questions

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1. Does there exist a symmetric equilibrium?
2. Will setting a bid cap benefit an organizer who wishes to maximize the average bid?

The Model

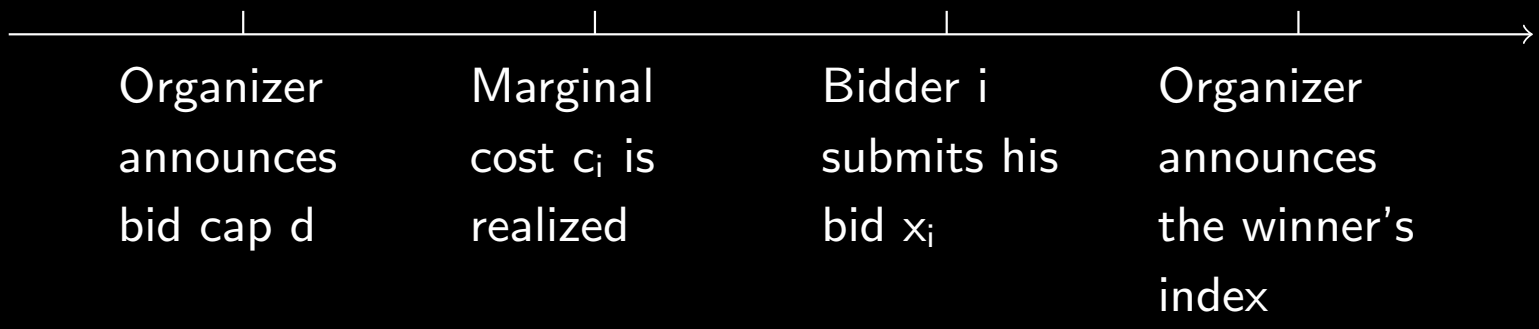
Model and Notation

- n bidders with linear cost function $c_i \cdot x_i$, where c_i is marginal cost of bidder i, and x_i is the bid/effort he excerts.
- There is single indivisible prize whose valuation is normalized to 1.
- c_i is a private information for bidder i.
- All bidders other than i perceive c_i as a random selection out of a support $[\underline{c}, \overline{c}] \in (0, \infty)$.
  - I.I.D
  - CDF: F, continuous differentiable
  - PDF: f, $f(c) > 0$ for all $c \in [\underline{c}, \overline{c}]$
- Organizer announces a bid cap d, where $d \in (0, +\infty)$.

Model and Notation

- Prize is given to only one bidder with the highest bid.
- Ties are broken randomly.
- $\widetilde{\beta}(c)$ is a symmetric bidding strategy without cap.
- $\beta(c, d)$ is a symmetric bidding strategy with cap.
- ER is the expected revenue for organizer.
- EV is the expected payoff for a bidder.

Timeline


Organizer announces bid cap d
Marginal cost c_i is realized
Bidder i submits his bid x_i
Organizer announces the winner's index

Organizer Problem

Organizer Problem

The organizer selects the optimal bid cap to maximize his expected revenue (i.e. the average bid).

$$\max_{d \in (0,+\infty)} \sum_{i=1}^{n} \int_{\underline{c}}^{\overline{c}} \beta(c_i, d)\, \mathrm{d}F(c_i)$$

main result

the Symmetric Assumption

- Each bidder chooses his bidding strategy $\beta_i()$ to maximize their expected payoff.
- The solutions set is too large to be specified.
- We focus on the symmetric solutions set.

Definition

A symmetric strategy is a strategy that make every bidder with a same marginal cost submit a same bid.

Equilibria without a Bid Cap

Lemma

Consider an incomplete-information all-pay auction without bid cap, there exists an unique symmetric equilibrium in which bidding strategy for bidder i is

$$\widetilde{\beta}(c_i) = \int_{c_i}^{\overline{c}} \frac{1}{y}(n-1)(1-F(y))^{n-2} f(y)\,\mathrm{d}y$$

and the expected revenue for organizer is

$$\widetilde{ER} = n\int_{\underline{c}}^{\overline{c}} \frac{1}{y}(n-1)(1-F(y))^{n-2} f(y)F(y)\,\mathrm{d}y$$

and the expected payoff for bidder i is

$$\widetilde{EV}(c_i) = (1-F(c_i))^{n-1} - c_i\int_{c_i}^{\overline{c}} \frac{1}{y}(n-1)(1-F(y))^{n-2} f(y)\,\mathrm{d}y$$

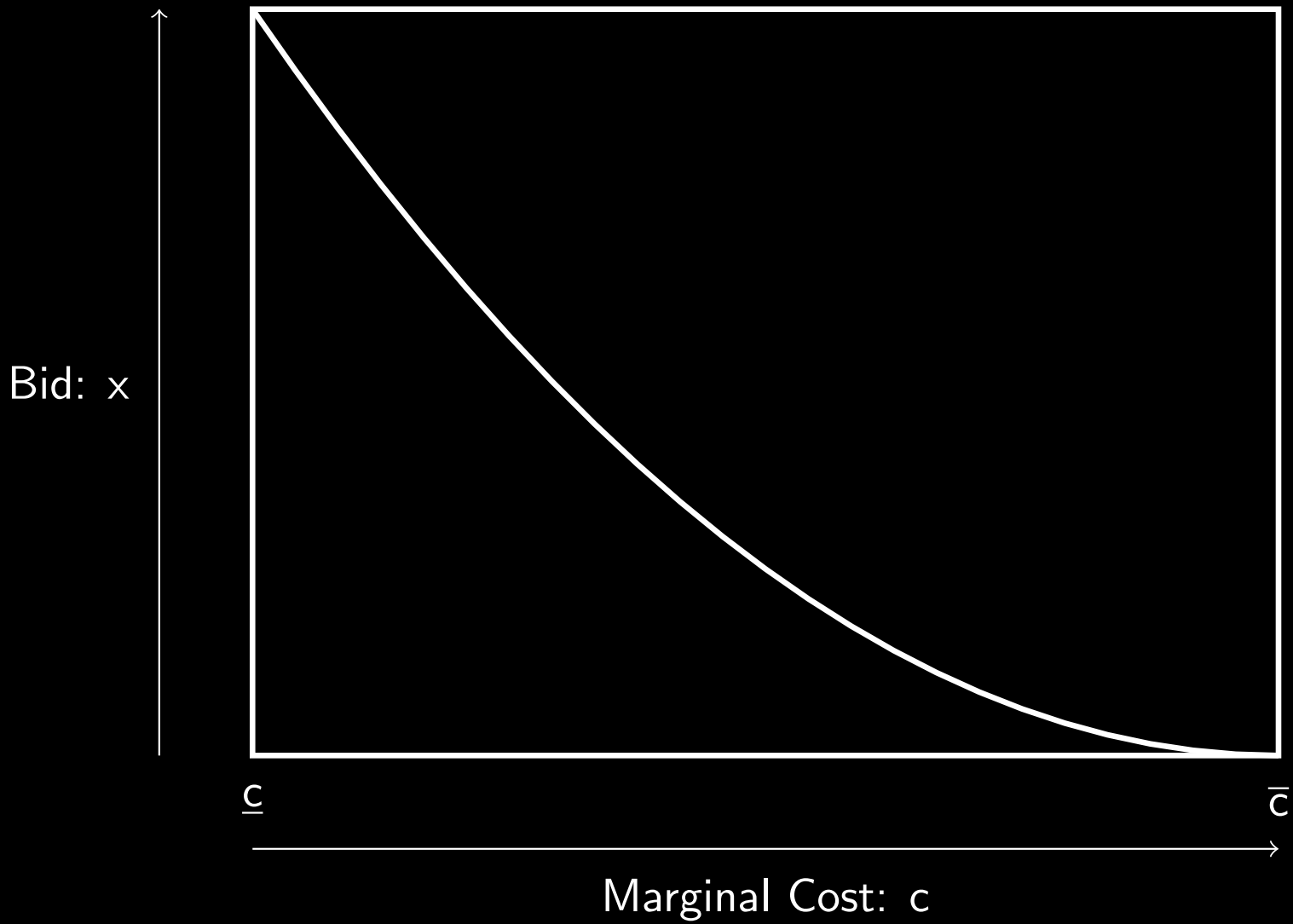
Equilibria without a Bid Cap

Properties of Bidding Strategy

1. *Weakly decreasing* $\widetilde{\beta}(\cdot)$ is weakly decreasing in $[\underline{c}, \overline{c}]$.
2. *Atomless bid* There is no subset $E \subseteq [\underline{c}, \overline{c}]$ having positive probability measure according to F, such that $\forall c, c' \in E$, $\widetilde{\beta}(c) = \widetilde{\beta}(c')$.
3. *Interval bid* $\widetilde{\beta}([\underline{c}, \overline{c}])$ is an interval.
4. *Strictly decreasing* $\widetilde{\beta}(\cdot)$ is strictly decreasing in $[\underline{c}, \overline{c}]$.
5. *Continuous* $\widetilde{\beta}(\cdot)$ is continuous in $[\underline{c}, \overline{c}]$.

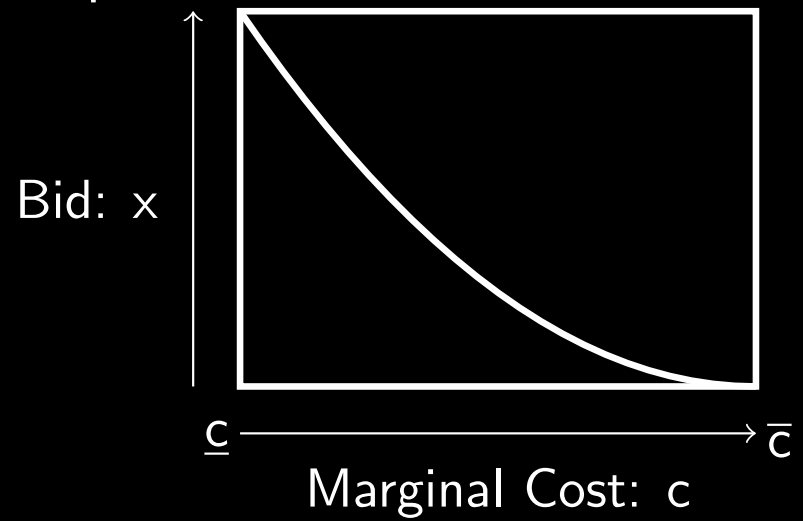
Equilibria without a Bid Cap

Graph



Equilibria with an Inefficient Bid Cap

- Consider an all-pay auction with a bid cap $d \geq \int_{\underline{c}}^{\overline{c}} \frac{1}{y}(n-1)(1-F(y))^{n-2} f(y)\,\mathrm{d}y$. Then the bid cap is redundant, and there exists an unique symmetric equilibrium where bidding strategy.
- Graph



Equilibria with an Efficient Bid Cap

Proposition

Consider an all-pay auction with a bid cap $0 < d < \int_{\underline{c}}^{\overline{c}} \frac{1}{y}(n-1)(1-F(y))^{n-2}f(y)\,\mathrm{d}y$. Then the bid cap is efficient, and there exists a symmetric monotone Nash equilibrium where bidding strategy is given by

$$\beta(c_i, d) = \begin{cases} d & \text{if } \underline{c} \leq c_i < \widetilde{c} \\ \widetilde{\beta}(c_i) & \text{if } \widetilde{c} \leq c_i \leq \overline{c} \end{cases}$$

and the ex ante expected total effort is given by

$$ER(d) = n[\int_{\widetilde{c}}^{\overline{c}} \frac{1}{y}(n-1)(1-F(y))^{n-2}f(y)F(y)\,\mathrm{d}y$$
$$+F(\widetilde{c})(\frac{1-(1-F(\widetilde{c}))^n}{nF(\widetilde{c})\widetilde{c}} - \frac{(1-F(\widetilde{c}))^{n-1}}{\widetilde{c}})]$$

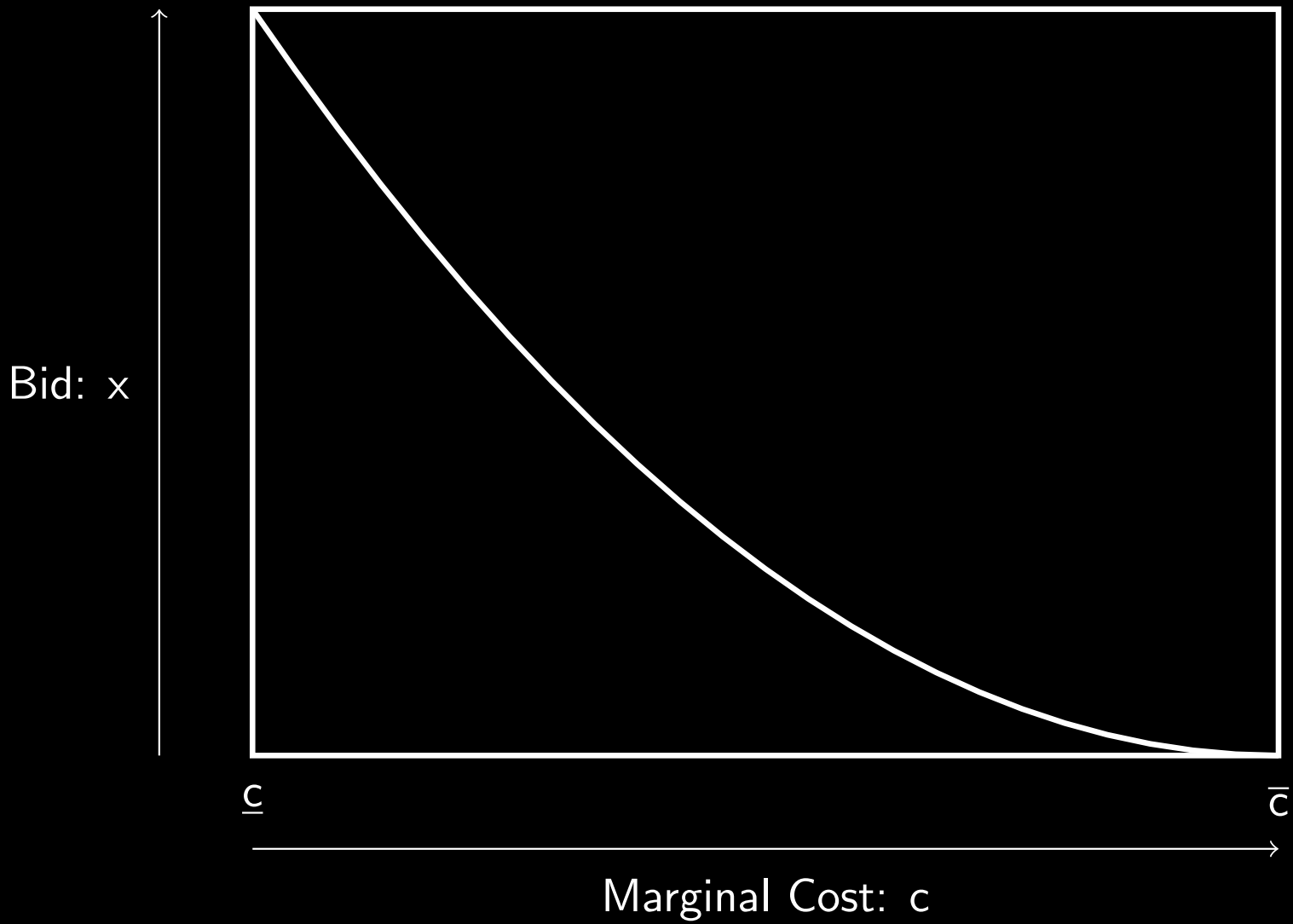
Equilibria with an Efficient Bid Cap

where the critical value $\widetilde{c} = \widetilde{c}(d)$ is strictly decreasing, and defined by

$$d = \int_{\widetilde{c}}^{\overline{c}} \frac{1}{y}(n-1)(1-F(y))^{n-2} f(y)\,\mathrm{d}y + \frac{1-(1-F(\widetilde{c}))^n - nF(\widetilde{c})(1-F(\widetilde{c}))^{n-1}}{nF(\widetilde{c})\widetilde{c}}$$

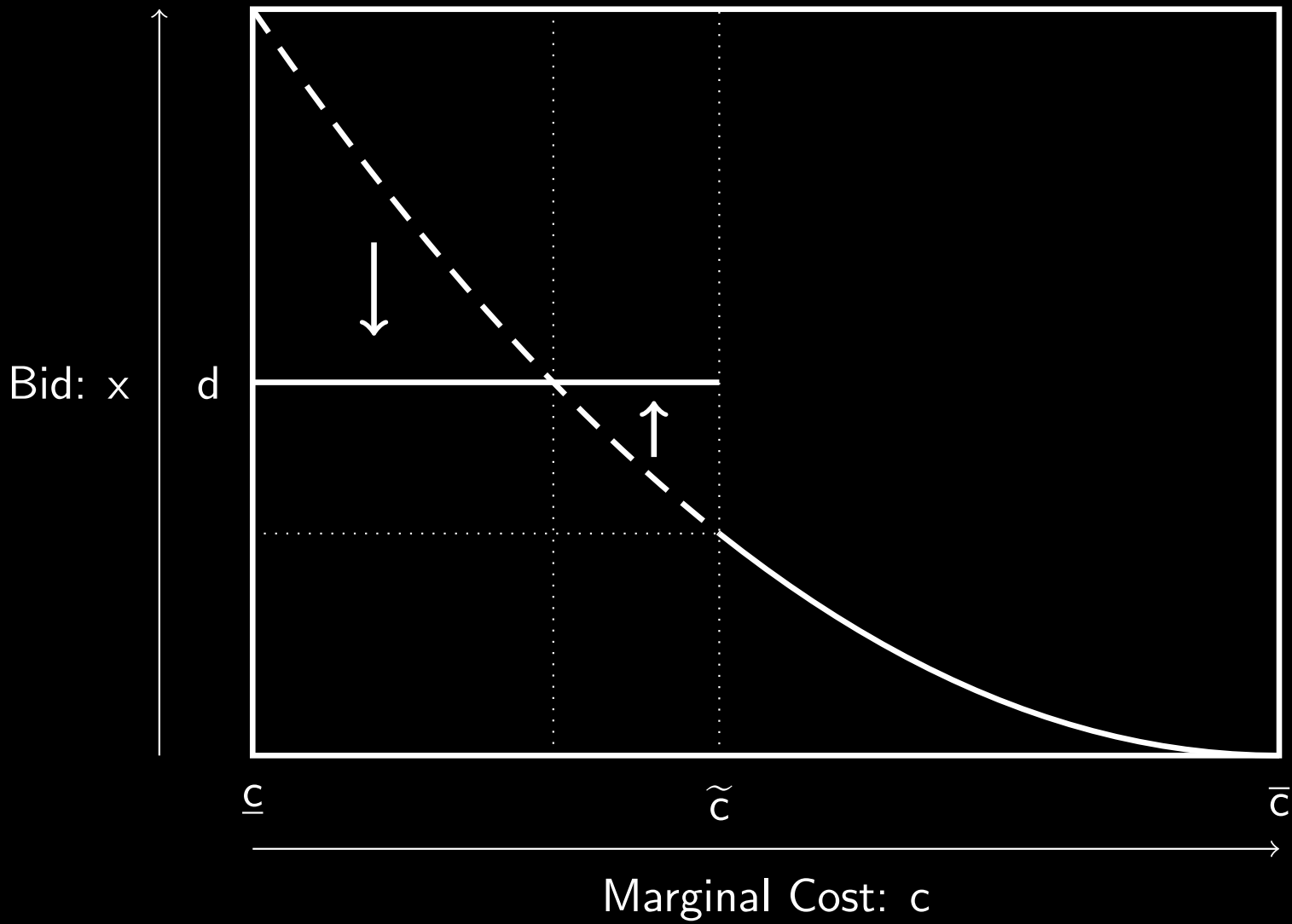
Equilibria with an Efficient Bid Cap

Graph



Equilibria with an Efficient Bid Cap

Graph



Bid Cap or Not?

Proposition

The expected revenue (average bid) of organizer is a strictly increasing function of the bid cap d, which means organizer will never use a cap.

Explaination

This proposition states that the organizer perfers no-cap policy, regardless of the marginal cost distribution and the number of bidders. With a bid cap, some middle-ability-level bidders will perfer a higher bid since there is a upper bound to limit bids submitted by higher-ability bidders. However, this gain is relatively small for organizer to offset lose from decrease of bid submitted by higher-ability bidders.

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