Disclosure Policies in All-pay Auctions with Bid Caps and Stochastic Entry

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Abstract

This paper contributes to the literature that examines the effects of disclosing the actual number of bidders in contests with stochastic entry by considering resource constraint. We study an all-pay auction with complete information. The auction entails one prize and n potential bidders. Each potential bidder has an exogenous probability of participation and faces an exogenous bid cap. It is shown that the contest organizer prefers fully concealing the information about the number of participating bidders. We extend the result to a case with endogenous entry.

Keywords: contest, all-pay auction, stochastic entry, bid cap, disclosure.

JEL classification: C72, D44, D82.

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Introduction

Many real-world competitions, such as rent-seeking, political campaigns, R&D competitions, and job promotions, are commonly viewed as contests. In these contests, participants spend resources in order to win some prizes. In many competitions, an individual has no information about the actual number of contestants she has to compete. For instance, when an individual seeks a job promotion, she has to compete not only with colleagues whom she knows but also anonymous candidates from outside.

The existing literature models such competitions as standard contests and auctions but with stochastic entry and has discussed the effect disclosing the actual number of contestants on the expected total bid.[1] Lim and Matros (2009) are the first to study information disclosure policies in a Tullock contest in which each potential contestant has an exogenous probability of participation and has linear bid cost function. They find that the disclosure policies are irrelevant to the expected total effort. Fu et al. (2011) investigate the same problem in a similar setting with a more general contest success function. They find that the disclosure irrelevance principle does not hold in non-Tullock contests and the optimal disclosure policy depends on the curvature of the characteristic function they define.[2] Chen et al. (2017) go beyond these papers by introducing interdependent valuations of the prize with private and affiliated signals.[3] They link the optimal disclosure policy with the curvature of the cost function under both exogenous and endogenous probabilities of participation. McAfee and McMillan's (1987) relate the optimal disclosure policy to bidders' risk attitudes in a first price auction. They find that fully concealing the number of bidders is optimal if the bidders are risk-averse.[4]

On the other hand, what all these above studies have overlooked is that in many competitions, contestants face enforced constraints on the maximal bid or effort they could exert. U.S. Federal law limits both congressional election campaign contributions and spending. Job promotion candidates cannot work more than 24 hours per day even if they would like to do so. Recently, to control the housing price, the Chinese government enforced bid caps in land auctions. While the effects of bid caps have received wide attention and have been thoroughly examined by researchers (Che and Gale, 1998, 2006; Chen, 2019b; Einy et al., 2016; Gavious et al., 2002; Olszewski and

[1]There is another literature that studies the effects of contestants' abilities (e.g.,Chen, 2019a,c; Fu et al., 2014; Lu et al., 2018; Morath and Münster, 2008; Serena, 2017; Zhang and Zhou, 2016).

[2]Fu et al. (2016) study disclosure policies in a two-player Tullock contest with asymmetric valuations and asymmetric probabilities of participation.

[3]See Matthews (1987), Levin and Ozdenoren (2004), Levin and Smith (1994), and Ye (2004) for studies on auctions with a stochastic number of bidders.

[4]They also find that a (potential) bidder's interim expected utilities are the same across the two disclosure policies. Feng and Lu (2016) extend their setting to include more disclosure policies and risk-loving bidders.

Siegel, 2017; Szech, 2015), none of the former above mentioned studies addresses this constraint when considering optimal disclosure policies.

This paper contributes to the above literature by providing a comprehensive examination of the effect of disclosing the actual number of bidders on the expected total bid in all-pay auctions in which bidders face bid caps. The model is in the spirit of Che and Gale (1998) (an all-pay auction with complete information) but with exogenous stochastic entry. The key finding is that fully concealing the number of bidders dominates fully revealing the number in terms of expected revenue to the organizer. The key insight is that there is no high bids to mitigate low bids of a bidder under full concealment if bidders' bids are capped, which leads to an overall lower expected revenue than that under full concealment. I show that the result extends to a setting with endogenous entry in a two-potential-bidder case.

1 Model

Consider a contest with a set $N = \{1, 2, ..., n\}$ potential risk neutral bidders and one indivisible prize. The value of the prize is common to all potential bidders and is normalized to 1. Each potential bidder participates in the contest with an independent probability $p \in (0, 1]$. The number of participants is only observable to the contest organizer, and the organizer has to announce publicly and commit to his disclosure policy — either to **fully conceal (Policy C)** or **fully reveal (Policy D)** the information about the number of participating bidders.

Each participating bidder i faces an exogenously given bid cap h and submits a bid $b_i \leq h$. Bids are submitted simultaneously and independently of each other. The bidder with the highest bid wins the prize, but all participating bidders pay their bids. Ties are resolved by random allocation with equal probabilities. When there is a subset M of participating bidders, each bidder bids b_i and the payoffs are:

$$W_i = \begin{cases} 1 - b_i & \text{if } b_i > \max_{j \in M \setminus \{i\}} b_j \\ -b_i & \text{if } b_i < \max_{j \in M \setminus \{i\}} b_j \\ \frac{1}{\#\{k \in M : b_k = b_i\}} - b_i & \text{if } b_i = \max_{j \in M \setminus \{i\}} b_j. \end{cases}$$

In detail, the model has the following timing:

1. The contest organizer commits to reveal or conceal her private information before the contest starts.

2. Nature chooses the number of participating bidders, and participating bidders receive signals.

3. The organizer implements his commitment.[5]

4. Bidders submit their bids privately.

5. The one with the highest bid wins the prize, and ties are resolved by fair lotteries.

2 Results

We first consider the subgame in which the organizer commits to policy C. In this case, the organizer conceals the actual number of bidders before the participating bidders make their bids.

Proposition 2.1 (Full Concealment)**.** *Consider the subgame that follows policy C. There is a unique symmetric equilibrium, in which each bidder's equilibrium distribution of bids is given by*

$$F(x) = \begin{cases} \left[[x + (1-p)^{n-1}]^{1/(n-1)} - (1-p) \right] /p & \text{for } x \in [0, c] \\ \left[[c + (1-p)^{n-1}]^{1/(n-1)} - (1-p) \right] /p & \text{for } x \in (c, h) \\ 1 & \text{for } x = h, \end{cases}$$

where the critical value $c = c(h)$ is defined by

$$c = 0 \text{ if } h \leq \frac{1-(1-p)^n}{np} - (1-p)^{n-1};$$

$$h = \frac{1 - [c + (1-p)^{n-1}]^{n/(n-1)}}{n\left[1 - [c + (1-p)^{n-1}]^{1/(n-1)}\right]} - (1-p)^{n-1} \text{ if } h \in \left(\frac{1-(1-p)^n}{np} - (1-p)^{n-1}, 1-(1-p)^{n-1}\right].$$

The expected payment of a participating bidder is

$$EP^C = \begin{cases} h & \text{if } h \leq \frac{1-(1-p)^n}{np} - (1-p)^{n-1}; \\ \frac{1-(1-p)^n}{np} - (1-p)^{n-1} & \text{if } h \in \left(\frac{1-(1-p)^n}{np} - (1-p)^{n-1}, 1-(1-p)^{n-1}\right]. \end{cases}$$

The key observation in the above proposition is that the expected payment of a participating bidder (as well as the expected revenue of the organizer) is not affected by a bid cap unless the cap is below the threshold $\frac{1-(1-p)^n}{np} - (1-p)^{n-1}$, which makes the critical value $c(h)$ equal to 0.

We next consider the subgame in which the organizer commits to policy D. In this case, the organizer reveals the actual number of bidders before the participating bidders make their bids. The equilibrium behavior, in this case, is a degenerate case of Proposition 2.1.

[5] It is beyond the scope of our paper to provide a thorough analysis on the issue of commitment.

Corollary 2.1 (Full Revealing). *Consider the subgame that follows policy D. If there is $m = 1$ participating bidder, the only participating bidder will bid 0. Consider a contest among $m \geq 2$ bidders. There is a unique symmetric equilibrium, each bidder's equilibrium distribution of bids is given by*

$$F_m(x) = \begin{cases} x^{1/(m-1)} & \text{for } x \in [0, c_m] \\ c^{1/(m-1)} & \text{for } x \in (c_m, h) \\ 1 & \text{for } x = h, \end{cases}$$

where the critical value $c_m = c_m(h)$ is defined by

$$c_m = 0 \text{ if } h \leq 1/m;$$
$$h = \frac{1 - c_m^{m/(m-1)}}{m[1 - c_m^{1/(m-1)}]} \text{ if } h \in (1/m, 1].$$

The expected payment of a particiating bidder is

$$EP_m = \begin{cases} h & \text{if } h \leq 1/m; \\ 1/m & \text{if } h \in (1/m, 1]. \end{cases}$$

As a result, for $m = 1$, the expected revenue of the organizer is 0; for $m \geq 2$, the expected revenue is not affected by a bid cap and is equal to 1, unless the cap is below the threshold $1/m$, which makes the critical value $c_m(h)$ equal to 0. Hence, given a bid cap, the expected revenue under full concealment is lower than that in any case with more than 1 participating bidder under full revealing but strictly higher than that in the case with 1 participant under the same disclosure policy. In addition, the threshold of the bid cap for the corresponding critical value to be zero is decreasing in the number of participating bidders.

Now, we are ready to compare the expected revenue across the disclosure policies. Denote $B_{n-1}^{m-1}(p) := \binom{n-1}{m-1} p^{m-1}(1-p)^{n-m}$ as a participating bidder's probability of facing exactly $m - 1$ active competitors.

Theorem 2.1 (Revenue Ranking). *Suppose $h \geq 1/2$. The expected total bid is the same under the two disclosure policies. If $h < 1/2$, the expected total bid is higher under full concealment.*

Proof. If $h \geq 1/2$, then the expected payment of a participating bidder under full revealing is

$$EP^D = \sum_{m=2}^{n} B_{n-1}^{m-1}(p) \frac{1}{m} = \frac{1 - (1-p)^n}{np} - (1-p)^{n-1} = EP^C$$

Suppose $h \leq \frac{1-(1-p)^n}{np} - (1-p)^{n-1}$, Denote $\bar{m}(h) := \max\{m | h \leq 1/m, m = 2, ..., n\}$.[6] Then, the expected payment of a participating bidder is $EP^C = h$ under full concealment. Under full revealing, a participating bidder's expected payment is

$$\begin{cases} 0 & \text{if she meets no opponent} \\ h & \text{if she meets } \tilde{m} = 1, ..., \bar{m}(h) - 1 \text{ opponents} \\ \frac{1}{\tilde{m}+1} & \text{if she meets } \tilde{m} = \bar{m}(h), ..., n-1 \text{ opponents} \end{cases}$$

and her expected payment before knowing the number of opponents is then

$$\begin{aligned} EP^D &= \sum_{m=\bar{m}(h)}^{n} B_{n-1}^{m-1}(p)\frac{1}{m} + \left[1 - (1-p)^{n-1} - \sum_{m=\bar{m}(h)}^{n} B_{n-1}^{m-1}(p)\right] h \\ &= \left[1 - (1-p)^{n-1}\right] h + \sum_{m=\bar{m}(h)}^{n} B_{n-1}^{m-1}(p) \left[\frac{1}{m} - h\right] \\ &< \left[1 - (1-p)^{n-1}\right] h \\ &< EP^C. \end{aligned}$$

Suppose $h \in (\frac{1-(1-p)^n}{np} - (1-p)^{n-1}, 1/2)$. Then, there is an $l \geq 1$ such that under full revealing, each bidder bids h if she faces l or fewer (but a positive number of) opponents. Thus $EP_m^D = h < \frac{1}{m}$ for $m = 2, ..., l+1$. Hence,

$$\begin{aligned} EP^D &= \sum_{m=2}^{l+1} B_{n-1}^{m-1}(p)h + \sum_{m=l+2}^{n} B_{n-1}^{m-1}(p)\frac{1}{m} \\ &< \sum_{m=2}^{n} B_{n-1}^{m-1}(p)\frac{1}{m} = \frac{1-(1-p)^n}{np} - (1-p)^{n-1} = EP^C. \end{aligned}$$

□

The intuition for the result is as follows. First, consider the case when there is no bid cap. As in the standard case of Chen et al. (2017), the expected payment of a bidder would be same across the disclosure policies if there is no non-trivial bid cap. Under full concealment each participating bidder would implement the same strategy. Under full disclosure the bid strategy of a participating bidder varies. She bids more actively when she faces fewer opponents, except for the case when she faces no opponent, in which case she bids zero. High bids mitigate low bids. When her bids are caped, there is no high bid to mitigate low bids, and thus her average bid would be lower than that under full concealment.

[6] In fact, $\frac{1-(1-p)^n}{np} - (1-p)^{n-1} < 1/n$ holds for all $p \in (0,1)$ if $n = 2$.

A Discussion on Endogenous Participation

So far, we have assumed an exogenous probability of participation. As an extension, we now consider endogenous participation. Specifically, each potential participant faces a fixed entry cost c for participation. There is an entry stage in which all bidders simultaneously decide whether to participate. We derive a unique symmetric Nash equilibrium for the entry stage given that bidders play as described in Proposition 2.1 and Corollary 2.1. Rather than giving a thorough analysis of the general case, we focus on the two-potential-bidder case to show that full concealment still dominates full revealing.

Proposition 2.2. *Consider a two-potential-bidder contest with endogenous entry, in which the entry cost is c. Suppose $c < 1/2$.*[7]

i If $h \leq 1/2 - c$ or $h \geq 1/2$, then the organizer is indifferent between full concealment to full revealing.

ii If $h \in (1/2 - c, 1/2)$, then the organizer strictly prefers full concealment to full revealing.

Proof. Suppose bidder $-i$ participates with probability p in equilibrium. The symmetric equilibrium participation probability must be larger than 0. First, consider full concealment. I make several case distinctions.

Case C1: Suppose $h \leq \frac{1-(1-p)^2}{2p} - (1-p) = p/2$. Consider the second stage. Each active bidder will bid h and $EP^C = h$. Consider the first stage. Given that bidder $-i$ participates with probability p, bidder i's expected payoff will be

$$[(1-p) + p/2] - h - c$$

if she participates and 0 otherwise. If $[(1-p) + p/2] - h - c > 0$, she strictly prefers take participate in the contest. Hence,

$$\text{if } h \leq 1/2 - c, \text{ then } p = 1, \; pEP^C = h.$$

If $[(1-p) + p/2] - h - c = 0$, she is indifferent between participating and not. Hence,

$$\text{if } h \in (1/2 - c, 1/2 - c/2] \text{ then } p = 2(1-h-c), \; pEP^C = 2(1-h-c)h.$$

Case C2: Suppose $h > \frac{1-(1-p)^2}{2p} - (1-p) = p/2$. Consider the second stage. $EP^C = \frac{1-(1-p)^2}{2p} - (1-p) = p/2$. Consider the first stage. Given that bidder $-i$

[7] For the cases in which $c \geq 0$, just change the condition in case i to $h \geq 1/2$ and that in case ii to $h \in (0, 1/2)$.

participates with probability p, bidder i's expected payoff will be

$$[(1-p)+p/2]-\left[\frac{1-(1-p)^2}{2p}-(1-p)\right]-c$$

if she participates and 0 otherwise. She is indifferent between participating and not if $1-p-c=0$. Hence,

$$\text{if } h>1/2-c/2, \text{ then } p=1-c, \ pEP^C=(1-c)^2/2.$$

Next, consider full revealing. I make a few more case distinctions.

Case D1: Suppose $h<1/2$. Consider the second stage. A participating bidder will bid h if her opponent also participates and 0 otherwise. Hence, $EP^D=ph$. Consider the first stage. Given that bidder $-i$ participates with probability p, bidder i's expected payoff will be

$$[(1-p)+p/2]-ph-c$$

if she participates and 0 otherwise. If $[(1-p)+p/2]-ph-c>0$, she strictly prefers take participate in the contest. Hence,

$$\text{if } h\leq 1/2-c, \text{ then } p=1, \ EP^D=ph, \ pEP^D=h.$$

If $[(1-p)+p/2]-ph-c=0$, she is indifferent between participating and not. Hence,

$$\text{if } h\in(1/2-c,1/2) \text{ then } p=\frac{1-c}{1/2+h}, \ pEP^D=\left[\frac{1-c}{1/2+h}\right]^2 h.$$

Case D2: Suppose $h\geq 1/2$. The cap does not alter the expected payment of a bidder in the second stage. Hence, $EP^D=p/2$. Consider the first stage. Given that bidder $-i$ participates with probability p, bidder i's expected payoff will be

$$[(1-p)+p/2]-p/2-c$$

if she participates and 0 otherwise. She is indifferent between participating and not if $1-p-c=0$. Hence,

$$\text{if } h\geq 1/2, \text{ then } p=1-c, \ pEP^D=(1-c)^2/2.$$

Now, we can compare the expected revenue across the disclosure policies. For $h<1/2-c$ and $h\geq 1/2$, it is clear that the expected revenue is the same across the two policies.

Consider $h \in (1/2 - c, 1/2 - c/2]$. The difference between the expected revenue in this case is $2(1 - h - c)h - \left[\frac{1-c}{1/2+h}\right]^2 h$. The term is strictly increasing in h for $h \in (1/2 - c, 1/2 - c/2]$ because the first derivative of this term with respect to h is $-2 + 2(1-c)^2(1/2+h)^{-3} > 0$. The infimum of the term is 0 achieved at $h = 1/2 - c$. Hence in this case, full concealment renders a higher expected revenue.

Consider $h \in (1/2 - c/2, 1/2)$. The difference between the expected revenue in this case is $(1-c)^2/2 - \left[\frac{1-c}{1/2+h}\right]^2 h > 0$. Hence in this case, full concealment renders a higher expected revenue. □

3 Conclusion

In this paper we examined the effect of disclosing the actual number of bidders in an all-pay auction with stochastic entry when face a common exogenous bid cap. It is shown that the contest organizer prefers fully concealing the information about the number of participating bidders. The reason is that a bidder's bidding strategy varies under full disclosure and she would not be able to mitigate low bids by high bids if she faces a non-trivial bid cap. A similar analysis can be down on standard all-pay auctions with imcomplete information. It is technically more involving.

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Appendix

Lemma 3.1. *In a (symmetric) equilibrium bidding strategy, there is no mass point at any bid $b \in [0, h)$.*

Proof. See Che and Gale (1998). □

Lemma 3.2. *If $h \in (\frac{1-(1-p)^n}{np} - (1-p)^{n-1}, 1-(1-p)^{n-1})$, all m bidders have an infimum bid of zero. If $h < \frac{1-(1-p)^n}{np} - (1-p)^{n-1}$, all have an infimum of h.*

Proof. Let $b^* \equiv \inf\{z|D(z) > 0\}$ denote the infimum of a bidder's bids in a symmetric equilibrium. We first show that b^* could only be zero or h. Suppose the infimum bid is $b^* \in (0, h)$. By Lemma 3.1, take an $\epsilon > 0$ such that $D^{n-1}(b^* + \epsilon) - D^{n-1}(b^*) < b^*$. Then, any individual bidder i could profitably move density in $(b^*, b^* + \epsilon)$ arbitrarily close to zero. For $b \in (b^*, b^* + \epsilon)$, the payment would drop by b. The probability of winning would drop by $D^{n-1}(b) - D^{n-1}(b^*) < b^* \leq b$. Since such a profitable deviation exists, an infimum bid of $b^* \in (0, h)$ cannot occur in equilibrium. Thus, only zero and h are possible infimum bids in equilibrium. We next analyze these two possibilities one by one.

Suppose first that $h \in (\frac{1-(1-p)^n}{np} - (1-p)^{n-1}, 1-(1-p)^{n-1})$. We employ a proof by contradiction to show that the infimum bid in a symmetric equilibrium is zero. If a participating bidder bids 0, her expected payoff is at least $(1-p)^{n-1}$ (the probability that all others do not participate). Suppose that $b^* = h$. Then every participating bidder bids h, and this results into a tie, when there are more than two participating bidders. A participating bidder's expected payoff is thus

$$\sum_{k=0}^{n-1} B_{n-1}^k(p) \frac{1}{k+1} - h = \frac{1-(1-p)^n}{np} - h < (1-p)^{n-1}.$$

Thus, $b^* = h$ cannot occur in a symmetric equilibrium here.

Next, Suppose first that $h < \frac{1-(1-p)^n}{np} - (1-p)^{n-1}$. If a participating bidder bids h, her expected payoff is at least $\sum_{k=0}^{n-1} B_{n-1}^k(p) \frac{1}{k+1} - h > 0$ (which happens if all other participating bidder bids h). Suppose the infimum bid is zero, a bid near zero must be as good as a bid of h for a bidder. But if there is no mass at zero, then a bidder receives approximately $(1-p)^{n-1}$, which is less than $\sum_{k=0}^{n-1} B_{n-1}^k(p) \frac{1}{k+1} - h$, if she bids near zero. Therefore, there must have mass at zero, resulting in a contradiction to Lemma 3.1. Hence, the infimum bid in this case is $b^* = h$. □

Lemma 3.3. *Suppose that $h \in (\frac{1-(1-p)^n}{np} - (1-p)^{n-1}, 1-(1-p)^{n-1})$. There exists a constant c such that all bidders place nonzero density on every $b \in (0, c]$ and zero density on every $b \in (c, h)$. All bidders have mass points at h.*

Proof. We first show that there is a mass point at h in a symmetric equilibrium strategy. Lemmas 3.1 and 3.2 show that the infimum bid is zero and there is no mass at zero. If a participating bidder bids arbitrarily close to zero, her expected payoff is approximately $(1-p)^{n-1}$. If a participating bidder bids h, her expected payoff is approximately $1-h > (1-p)^{n-1}$. Hence there must be mass at h.

Let the mass at h be α. There must be a $b^* \in (0,h)$ such that the density is zero in (b^*, h). Because for any b close enough to h, a bidder could profitably move it to h: The payment would rise by less than a little bit but the probability of winning would rise by at lease $\sum_{k=0}^{n-1} B_{n-1}^k (p\alpha) \frac{1}{k+1}$.

Let b' denote the smallest $b^* \in (0,h)$ such that the density in (b^*, h) is zero. The density on almost every $b \in (0,b')$ must be positive. Suppose not. Let t' be the largest t such that there is an interval $(s_t, t) \subset (0, b')$ in which the density is zero. Then, a bidder could profitably move density from $(t, t+\epsilon')$ down to s_t, for some $\epsilon' > 0$. □

Proof of Proposition 2.1. Consider $h \in (\frac{1-(1-p)^n}{np} - (1-p)^{n-1}, 1-(1-p)^{n-1}]$. We first determine the distribution functions that make the bidders indifferent among all bids in $(0, c_{m,p}) \cup \{h\}$. We then find the equilibrium value of $c_{m,p}$.

Since a bidder must be indifferent among all bids in $(0,c) \cup \{h\}$, each bid in this set must yield the same expected payoff. That is, for all $b \in (0,c]$,

$$[pF(b) + (1-p)]^{n-1} - b = \sum_{k=0}^{n-1} B_{n-1}^k (p[1-F(c)]) \frac{1}{k+1} - h. \tag{1}$$

The left-hand side give the expected payoff from bidding $b \in (0,c]$, while the right-hand side corresponds to a bid of h. When a bidder bids h, there is a probability of $[pF(b) + (1-p)]^{n-1}$ that she faces k bidders who also bid h.

First, taking the limit of b to 0, we can pin down the expect payoff of a bidder with equation (1): for all $x \in (0,c]$

$$[pF(x) + (1-p)]^{n-1} - x = \lim_{b \to 0} [pF(b) + (1-p)]^{n-1} - b = (1-p)^{n-1}.$$

Hence, in equilibrium $F(x) = \left[[x + (1-p)^{n-1}]^{1/(n-1)} - (1-p) \right] / p$ for all $x \in [0,c]$.

Next, we can pin down the expect payoff of a bidder, again, with equation (1):

$$\begin{aligned}
&\sum_{k=0}^{n-1} B_{n-1}^k (p[1-F(c)]) \frac{1}{k+1} - h = (1-p)^{n-1} \\
\Rightarrow & h = \frac{1 - [(1-p) + pF(c)]^n}{n\,[1 - [(1-p) + pF(c)]]} \\
\Rightarrow & h = \frac{[1 - [c + (1-p)^{n-1}]^{n/(n-1)}}{n[1 - [c + (1-p)^{n-1}]^{1/(n-1)}]}.
\end{aligned}$$

Third, a potential bidder's ex ante probability of winning is $\frac{1-(1-p)^n}{n}$, and thus a participating bidder's probability of winning is $\frac{1-(1-p)^n}{np}$. The expected payment of a participating bidder equals to her probability of winning minus her expected payoff:

$$EP = \frac{1-(1-p)^n}{np} - (1-p)^{n-1}.$$

□

Proof of Corollary 2.1. The case of $m = 1$ is obvious. The proof for the case of $m \geq 2$ is degerate case of that of Proposition 2.1 with $n = m$ and $p = 1$. □