Empirical Asset Pricing
Problem Set 5

Yu Zhou, HKUST

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Apply GMM to the CX of stock returns

- In this problem set, we are going to learn how to apply GMM to the cross-section of stock returns (in particular, 25 portfolios).
- Data from Kenneth French's website:
  - value-weighted monthly returns on the 25 portfolios sorted based on size and the book-to-market ratio;
  - factor returns.

Q1: Size effect and value effect

	Small	ME2	ME3	ME4	Large
Growth	0.59	0.65	0.73	0.75	0.67
	1.64	2.74	3.34	4.04	4.23
BM2	0.7	0.93	0.91	0.77	0.63
	2.41	4.18	4.72	4.25	4.03
BM3	0.98	0.97	0.9	0.85	0.7
	3.67	4.49	4.68	4.46	4.21
BM4	1.13	1.02	1.10	0.93	0.64
	4.59	4.63	4.91	4.58	3.26
Value	1.31	1.21	1.07	1.01	0.91
	4.77	4.67	4.24	3.91	3.6

Table: The value effect exist. However, for the low book-to-market portfolios (i.e., Growth and BM2), the size effect does not exist.

Q2: Time-series regression

- For each portfolio i, run OLS regressions

$$R^e_{it} = \alpha_i + \beta_i RMRF_t + \varepsilon_{it}$$

	Small	ME2	ME3	ME4	Large
Growth	-0.5	-0.21	-0.11	0.01	0.03
	-2.32	-1.76	-1.19	0.22	0.59
BM2	-0.24	0.11	0.15	0.04	-0.01
	-1.5	1.08	2.19	0.71	-0.14
BM3	0.06	0.16	0.15	0.1	0.05
	0.4	1.79	2.09	1.48	0.78
BM4	0.28	0.21	0.22	0.15	-0.1
	2.25	2.13	2.74	1.84	-1.18
Value	0.39	0.28	0.14	0.06	0.03
	2.69	2.19	1.24	0.47	0.24

Table: The estimated α_i and associated OLS t-statistics. The CAPM does not explain the value effect. α is larger for higher book-to-market stocks.

Q2: Time-series regression (cont'd)

	Small	ME2	ME3	ME4	Large
Growth	1.62	1.27	1.25	1.09	0.96
	11.67	24.69	36.23	49.63	66.62
BM2	1.40	1.23	1.13	1.08	0.95
	14.9	24.49	55.23	51.2	74.87
BM3	1.37	1.2	1.12	1.11	0.97
	19.77	26.09	35.96	31.21	32.87
BM4	1.27	1.21	1.17	1.16	1.10
	17.77	23.81	25.75	25.74	26.97
Value	1.37	1.38	1.38	1.41	1.31
	16.1	22.01	25.27	21.27	18.13

Table: The estimated β_i and associated OLS t-statistics. The CAPM explain the size effect within BM3, BM4, and Value portfolio.

Q2: Time-series regression (cont'd)

- GRS test statistic: 83.49 with p-value 0.
- Use GMM to conduct tests:
  - moment conditions:

$$E[(1\ RMRF)' \otimes \varepsilon] = 0$$

  - # moments = # params = 50 (exactly identified) $\Rightarrow A = I_{50}$.
  - ∂moment conditions$/\partial b'$:

$$\underbrace{D}_{50\times 50} = -\begin{bmatrix} 1 & E(RMRF) \\ E(RMRF) & E(RMRF^2) \end{bmatrix} \otimes I_{25}$$

  - F test statistic: $\hat{\alpha}' cov(\hat{\alpha})^{-1}\hat{\alpha} \sim \chi^2_{25}$
  - F test statistic: 83.49 with p-value 0. It is extremely similar to the GRS test statistic.
- These tests reject the CAPM.

Q2: Time-series regression (cont'd)

- For each book-to-market quintile, pick the portfolio with smallest stocks and largest stocks, to investigate whether the difference in betas are statistically significant or not.

	Growth	BM2	BM3	BM4	Value
$\gamma'\beta$	-0.66	-0.46	-0.4	-0.16	-0.07
t-stat	-13.35	-11.88	-12.56	-5.73	-1.94

Table: For every B/M quintile, small stocks have larger market betas than large stocks. These difference is significant except for the value stocks.

Q3: Cross-sectional regression

- Run a cross-sectional regression of average excess returns on the CAPM betas which have been estimated in time-series regressions.

$$E_T[R_{it}^e] = \lambda \beta_i + \alpha_i$$

- The OLS estimates for λ is $\hat{\lambda} = 0.72$, and the associated t-statistic is 20.56. This t-statistic is high. One reason is that the estimation errors on β_i have been ignored.

Q3: Cross-sectional regression (cont'd)

- Run the Fama-MacBeth regressions for every month to obtain the monthly λ_t.

$$R^e_{it} = \lambda_t \beta_i + \alpha_{it}$$

- the average factor risk premium is $\hat{\lambda}_{FMB} = \frac{1}{T}\sum \hat{\lambda}_t = 0.72$, and the associated t-statistic is 4.21 which is much lower than the OLS t-statistic. This may because FMB take considerations of the correlation among the error terms.
- To see if α is too large to not, we use $\hat{\alpha}' cov(\hat{\alpha})^{-1}\hat{\alpha} \sim \chi^2_{N-1}$.
  - the F statistic is 83.71 with p value of 0.
  - CAPM is rejected.

Q3: Cross-sectional regression (cont'd)

- Run the Fama-MacBeth regressions with intercepts.
  - $\hat{\lambda}_0 = 0.39$, $t(\hat{\lambda}_0) = 1.22$;
  - $\hat{\lambda}_1 = 0.40$, $t(\hat{\lambda}_1) = 1.14$.
- λ_0 is the excess return of zero-beta asset and λ_1 is the premium of market risk factor. If CAPM is the true asset pricing model, the λ_0 should be 0.

Q3: Cross-sectional regression (cont'd)

GMM

- We have 75 moment conditions and 51 parameters to be estimated. The GMM is overidentified.
- $\hat{\lambda} = 0.72$.
- D matrix:

$$D = -\begin{bmatrix} I_N & E(RMRF)I_N & 0 \\ E(RMRF)I_N & E(RMRF^2)I_N & 0 \\ 0 & \lambda I_N & \beta \end{bmatrix}$$

- A matrix:

$$A = \begin{bmatrix} I_{2N} & 0 \\ 0 & \beta' \end{bmatrix}$$

- The t-statistic is 4.43 which is not equal but quite similar to the FMB t-statistic.

Q3: Cross-sectional regression (cont'd)

- Pricing errors under the cross-sectional regressions:

	Small	ME2	ME3	ME4	Large
Growth	−0.57	−0.26	−0.16	−0.03	−0.01
	−1.57	−1.09	−0.72	−0.17	−0.08
BM2	−0.3	0.05	0.1	−0.01	−0.05
	−1.04	0.24	0.53	−0.03	−0.29
BM3	0	0.11	0.1	0.05	0.01
	−0.01	0.5	0.52	0.27	0.05
BM4	0.23	0.16	0.18	0.1	−0.15
	0.92	0.7	0.85	0.5	−0.77
Value	0.33	0.22	0.09	0	−0.02
	1.2	0.85	0.34	−0.01	−0.08

Table: Like the time-series regressions, the CAPM still does not fully explain the value effect under the cross-sectional regressions. α is larger for higher book-to-market stocks.

- Overidentification test statistic 78.96 with p-value of 0. So CAMP is rejected.

Q3: Cross-sectional regression (cont'd)

- As we have seen, the differences of t-statistic and χ^2-statistic between GMM and FMB are small.
- These differences reflect the fact that GMM has taken the estimation errors of $\hat{\beta}$ into consideration; while FMB treats the $\hat{\beta}$ estimated in the first-stage TS regressions as the true beta.
- In this example, the differences are small because portfolios' betas are estimated with less errors.

Q3: Cross-sectional regression (cont'd)

	CX	FMB	GMM	TS
$\hat{\lambda}$	0.72	0.72	0.72	0.67
$t(\hat{\lambda})$	20.56	4.21	4.43	4.23

Table: The point estimate of the market price of risk is smaller for TS regressions which is the average excess return of the market portfolio. The associated t-statistics are similar for FMB, GMM and TS but not for CX which implies that the cross-sectional correlation in monthly returns across assets matters a lot.

Q4: Allow for serial correlation in returns

- Serial correlation in returns is reasonable when the assets have some "stale" pricing.
- Allowing for serial correlation in monthly returns in the form of 12-month lags with Newey-West weights.
- $\hat{\lambda} = 0.72$, $t_{NW}(\hat{\lambda}) = 4.18$ which is slightly lower than t-statistic in no serial correlation case.