Empirical Asset Pricing
Problem Set 4

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Use GMM to estimate CCAMP

The model is

$$E_t[(C_{t+1}/C_t)^{-\gamma} R^e_{t+1}] = 0$$

Considering five excess returns:

- rmrf
- smb
- hml
- rmw: robust operating profitability portfolios minus weak operating profitability portfolios;
- cma: conservative investment portfolios minus aggresive investment portfolios.

Q1: Descriptive statistics

	C_{t+1}/C_t	rmrf	smb	hml	rmw	cma
mean	102.28%	6.78%	3.36%	4.66%	3.64%	3.70%
std	1.37%	17.53%	12.87%	16.13%	12.77%	9.06%
SR		0.39	0.26	0.29	0.29	0.41
corr						
C_{t+1}/C_t		0.20	0.10	0.11	-0.12	-0.11
rmrf			0.23	-0.14	-0.38	-0.28
smb				-0.10	-0.42	-0.02
hml					0.36	0.74
rmw						0.22
cma						

Table: The consumption growth is positively correlated with rmrf, smb and hml, but negatively correlated with rmw and cma. The market excess return has the highest correlation with the consumption growth.

Q2: The equity premium puzzle

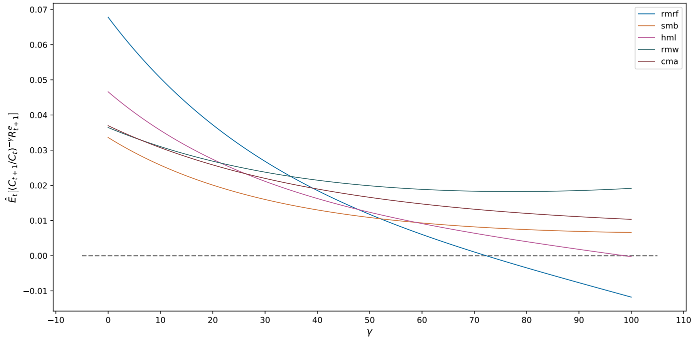

Figure: The price of excess return rmrf crosses zero around $\gamma = 72$, which features the risk premium puzzle. The line of hml crosses zero around $\gamma = 98$ which is higher than 72 because the correlation between C_{t+1}/C_t and hml is smaller. The other three never cross zero.

Q3: GMM

	rmrf	smb	hml
first-stage GMM			
γ	72.25	116.59	98.66
$\sigma(\gamma)$	48.79	1.08×10^{11}	63.53
χ^2	-	-	-
p-value	-	-	-
$E(MR^e)$	0	0.0064	0
$\sigma(E(MR^e))$	0	0	0
$E(R^e)$	6.78%	3.36%	4.66%
$a \times 1000$	-0.4	0	-0.2

Table: In the first-stage GMM, $W = I$. When using rmrf and hml alone, we obtain high $\hat{\gamma}$ which are insignificant. When using smb alone, there does not exist any γ making $g_T(\gamma) = 0$. So GMM sets $a_T = 0$. This result is meaningless.

Q3: GMM (cont'd)

	rmrf	smb	hml
second-stage GMM			
γ	72.25	116.59	98.66
$\sigma(\gamma)$	48.79	1.1×10^{11}	63.53
χ^2	-	-	-
p-value	-	-	-
$E(MR^e)$	0	0.0064	0
$\sigma(E(MR^e))$	0	0	0
$E(R^e)$	6.78%	3.36%	4.66%
$a \times 1000$	-16.6	0	-22

Table: In the second-stage GMM, $W = \hat{S}_{\text{First Stage}}$. Compared with the first-stage GMM, the absolute weight on sample moment increases.

Q3: GMM (cont'd)

	rmrf	smb	rmrf	hml
first-stage GMM				
γ	75.16		77.6	
$\sigma(\gamma)$	49.68		42.85	
χ^2	1.69	0.19	0.28	0.6
$E(MR^e)$	-0.13%	0.78%	-0.24%	0.45%
$\sigma(E(MR^e))$	0.001	0.006	0.007	0.013
$E(R^e)$	6.78%	3.36%	6.78%	4.66%
$a \times 1000$	-0.45	-0.077	-0.44	-0.23

Table: The pricing errors are not zero. J-test fails to reject the null.

Q3: GMM (cont'd)

	rmrf	smb	rmrf	hml
second-stage GMM				
γ	97		85.27	
$\sigma(\gamma)$	62.54		40.45	
χ^2	2.18	0.14	0	1
$E(MR^e)$	-1.06%	0.67%	-0.57%	0.28%
$\sigma(E(MR^e))$	0.007	0.004	0.017	0.009
$E(R^e)$	6.78%	3.36%	6.78%	4.66%
$a \times 1000$	-14.12	-22.37	-14.03	-28.41

Table: The pricing errors are not zero. J-test fails to reject the null.

Q4: Conceptual questions

(a)

- The beta pricing formula always holds when the tangency portfolio is taken to be the reference.

$$E[R_i] - R_f = \beta_{i,TAN}(E[R_{TAN}] - R_f)$$

- The beta pricing formula fails to hold when the value-weighted market portfolio is taken to be the reference, because investors have preference for those "lottery-like" stocks instead of purely mean-variance preference.

$$E[R_i] - R_f = \beta_{i,MKT}(E[R_{MKT}] - R_f)$$

Q4: Conceptual questions (cont'd)

(b)

- If the asset pricing model works well to explain expected return (that is to say the SDF lies in the factor span), then the intercept a_i should be 0.
- R^2 is not important because the objective of this model is to explain expected returns instead of realized returns.
- Low t-statistic for b_i doesn't mean the model is wrong. When one asset's return is close to risk-free, it should have low risk loading (i.e., small b_i that is insignificantly different from 0). What we really care about is whether a_i equal to 0.

Q4: Conceptual questions (cont'd)

(c)

- We can not evaluate this asset pricing model because a_i is no longer the pricing error. R^2 is also not important because the objective of this model is to explain expected returns instead of realized returns.

Q4: Conceptual questions (cont'd)

(d)

- For a good asset pricing model, a_i should be 0, b_i should be the risk premium of the factors, and R^2 should be large as this model tries to explain expected returns.

Q4: Conceptual questions (cont'd)

(e)

- For a good asset pricing model, c_i should be 0.
- It is acceptable to include X_i^2 and test the model.
- It is not acceptable to include β_i^2 because
  - The model predicts a linear relationship between the expected excess returns and factor loadings. So the coefficient of β_i^2 is not meaningful;
  - Including β_i^2 may make b_i estimates less accurate.