Empirical Asset Pricing
Problem Set 2

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Q1: Return decompostition

- Log-linear approximation of return:

$$r_{t+1} = \rho pd_{t+1} - pd_t + \Delta d_{t+1}$$
$$E_t[r_{t+1}] = \rho E_t[pd_{t+1}] - pd_t + E_t[\Delta d_{t+1}]$$

- Taking difference between realized return and expected return, we can obtain unexpected return:

$$\begin{aligned}
&r_{t+1} - E_t[r_{t+1}] \\
=&(E_{t+1} - E_t)[\rho pd_{t+1}] + (E_{t+1} - E_t)[\Delta d_{t+1}] \\
=&(E_{t+1} - E_t)\left[\sum_{j=1}^{\infty} \rho^j (\Delta d_{t+1+j} - r_{t+1+j})\right] + (E_{t+1} - E_t)[\Delta d_{t+1}] \\
=&\underbrace{(E_{t+1} - E_t)\left[\sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j}\right]}_{\text{cash-flow shocks/news}} - \underbrace{(E_{t+1} - E_t)\left[\sum_{j=1}^{\infty} \rho^j r_{t+1+j}\right]}_{\text{expected-return shocks/news}}
\end{aligned}$$

- Cash-flow news include news of both expected and contemporaneous cash flow.

Q1: Return decompostition (cont'd)

- Suppose $X_t = (r_t, pd_t)^T \sim VAR(1)$ where $X_{t+1} = AX_t + v_{t+1}$. Then, unexpected return turns to be:

$$r_{t+1} - E_t[r_{t+1}] = e_1^T(AX_t + v_{t+1}) - e_1^T AX_t = e_1^T v_{t+1}$$

- The expected-return shocks/news can be expressed as

$$\begin{aligned} Nr &= (E_{t+1} - E_t)\left[\sum_{j=1}^{\infty} \rho^j r_{t+1+j}\right] \\ &= \sum_{j=1}^{\infty} \rho^j e_1^T (A^j X_{t+1} - A^{j+1} X_t) \\ &= e_1^T \sum_{j=1}^{\infty} \rho^j A^j v_{t+1} \\ &= e_1^T \sum_{j=0}^{\infty} \rho^j A^j \rho A v_{t+1} \\ &= \rho e_1^T G v_{t+1}, \text{ where } G = (I - \rho A)^{-1} A \end{aligned}$$

Q1: Return decompostition (cont'd)

- The cash-flow shocks/news can be backed out:

$$Nc = e_1^T (I + \rho G) v_{t+1}$$

Q2: Backing out approach

- Suppose the state variables $X_t = (r_t, pdt)^T$ follows a $VAR(1)$ process where $X_{t+1} = A_0 + AX_t + \varepsilon$.
  - The long-run expected returns is given by $pd_t^r = e1^T GX_t$.
- The long-run expected dividend growth can be backed out as

$$pd^d_{t,\text{indirect}} = pd_t + pd_t^r$$

- This is equivalent to $e_d^T GX_t$.

Q2: Direct forecast approach

- Dividend growth rates follow a process as

$$\Delta d_{t+1} = b_1 + b^T X_t + u_{t+1}$$

  which can be estimated by running an OLS regression.
- Then, the long-run expected dividend growth rate can be expressed to be

$$\begin{aligned} pd^d_{t,\text{direct}} &= E_t \left[\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} \right] \\ &= b^T \sum_{j=1}^{\infty} \rho^{j-1} A^{j-1} X_t \\ &= b^T (I - \rho A)^{-1} X_t \end{aligned}$$

Q2: Indirect v.s. direct approach

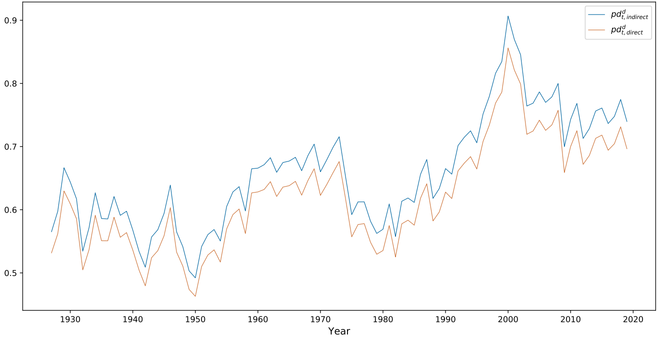

Figure: This figure plots $pd^d_{t,\text{direct}}$ and $pd^d_{t,\text{indirect}}$ under state variables $X_t = (r_t, pd_t)^T$. As you can see, the backing out approach works well. The reason is that by assuming r_t and pd_t follow $VAR(1)$, log-return identity indicates a linear relation between Δd_{t+1} and X_t. So backing out approach coincide with direct linear regression of Δd_{t+1} on X_t.

Q2: Different state variables

- Now, re-assuming the state variables used by investors to predict stocks' future performance to be: $X_t = (r_t, \text{term}_t, \text{def}_t)^T$, where
  - term_t is the difference in long-term and short-term Treasury yield;
  - def_t is the difference in yield between corporate bonds and long-term Treasury bonds.

Q2: Indirect v.s. direct approach

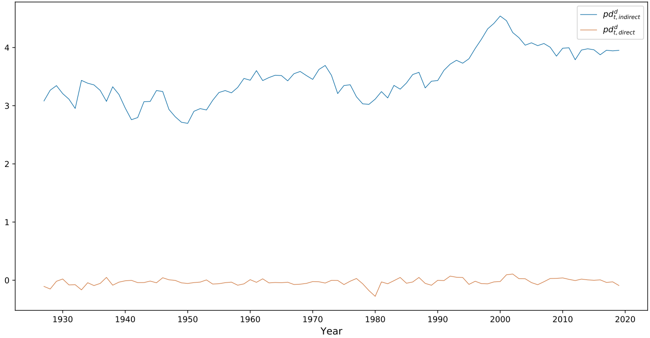

Figure: This figure plots $pd^d_{t,\text{direct}}$ and $pd^d_{t,\text{indirect}}$ under state variables $X_t = (r_t, \text{term}_t, \text{def}_t)^T$. These approaches work so differently. The reason is that by assuming r_t and pd_t follow $VAR(1)$, log-return identity indicates a linear relation between Δd_{t+1} and $\{X_t, pd_t\}$. So backing out approach departs from direct linear regression of Δd_{t+1} on X_t.

Q3: Cochrane (2007)

- Suppose a single-state-variable case where $X_t = dp_t$. Variables of interest in this world are follows a $VAR(1)$ process:

$$r_{t+1} = a_r + b_r dp_t + \varepsilon^r_{t+1}$$
$$\Delta d_{t+1} = a_d + b_d dp_t + \varepsilon^d_{t+1}$$
$$dp_{t+1} = a_{dp} + \phi dp_t + \varepsilon^{dp}_{t+1}$$

- The log-linear identity $r_{t+1} = -dp_{t+1} + dp_t + \Delta d_{t+1}$ implies the following restrictions (i.e., restricted $VAR(1)$)[1]

$$b_r = -\rho\phi + 1 + b_d$$
$$\varepsilon^r_{t+1} = -\rho\varepsilon^{dp}_{t+1} + \varepsilon^d_{t+1}$$

[1]Remark: dp_t is orthogonal to the residual terms

Q3: Cochrane (2007) (cont'd)

	slope	covariance (scaled by 100)		
r_{t+1}	0.08	3.7	1.87	-1.9
Δd_{t+1}	-0.01	1.87	2.06	0.19
dp_{t+1}	0.94	-1.9	0.19	2.17

Table: This table reports the estimates for slope coefficients and covariance matrix of residuals.

Q3: Cochrane (2007) (cont'd)

- Long-run expected return:

$$
\begin{aligned}
E_t \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} &= \sum_{j=1}^{\infty} \rho^{j-1} b_r \phi^{j-1} dp_t + \text{const.} \\
&= \frac{b_r}{1-\rho\phi} dp_t + \text{const.} \quad (\text{since } \rho \in (0,1), |\phi| < 1)
\end{aligned}
$$

- Long-run dividend growth rate:

$$
\begin{aligned}
E_t \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} &= \sum_{j=1}^{\infty} \rho^{j-1} b_d \phi^{j-1} dp_t + \text{const.} \\
&= \frac{b_d}{1-\rho\phi} dp_t + \text{const.} \quad (\text{since } \rho \in (0,1), |\phi| < 1)
\end{aligned}
$$

Q3: Cochrane (2007) (cont'd)

- the sample estimate of the long-run forecasting coefficients:

$$\hat{b}_r^{lr} = \frac{\hat{b}_r}{1 - \rho\hat{\phi}} = 0.93$$

$$\hat{b}_d^{lr} = \frac{\hat{b}_d}{1 - \rho\hat{\phi}} = -0.06$$

Q3: Simulation

- H_0: Stock returns are not predictable ($b_r = 0$). So $b_d = \rho\phi - 1 = -0.087$.
- Simulation results:

	b_r	b_d	ϕ	b_r^{lr}	b_d^{lr}
Point estimates	0.08	-0.01	0.94	0.93	-0.06
H_0	0	-0.087	0.94	0	1
Simulation estimates	0.04	-0.09	0.9	0.26	-0.74
p-value	0.24	0.03	0.18	0.04	0.04

Table: This table reports the point estimates, and simulation estimates under the null hypothesis.

Q3: Simulation (cont'd)

- Observations:
  - Start with H_0 that $b_r = 0$, the simulation estimates bias upward. And this bias is one-half of the point estimate which implies that the bias is large.
  - The p-value of $b_r^{\text{simulated}}$ is 0.24 which means that under the null hypothesis, our point estimate can realized with probability $0.24 > 0.05$. Hence, we can not reject the null hypothesis that return is not predictable.
  - The p-value of $b_d^{\text{simulated}}$ is $0.03 < 0.05$. Hence, we reject the null hypothesis that dividend growth is predictable. However, it is impossible that both return and dividend growth are unpredictable.
  - The p-value of $b_r^{lr,\text{ simulated}}$ is $0.04 < 0.05$. Hence, we reject the null hypothesis that return is not predictable.

Q3: Simulation (cont'd)

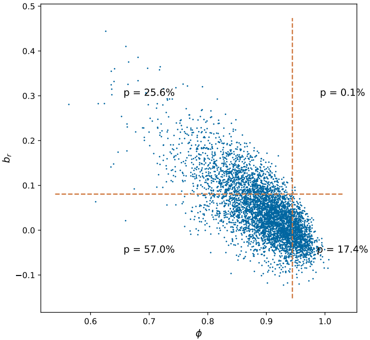

Figure: This figure presents a mechanically negative relation between $\hat{b}_r^{(n)}$ and $\hat{\phi}^{(n)}$. $\hat{b}_r^{(n)}$ is less stable since it decreases corresponding to $\hat{\phi}^{(n)}$. In constract, $\hat{b}_r^{lr,(n)} = \frac{\hat{b}_r^{(n)}}{1-\rho\hat{\phi}^{(n)}}$ increasing with $\hat{\phi}^{(n)}$ if keeping $\hat{b}_r^{(n)}$ unchanged.

Q4: Out-of-sample regressions

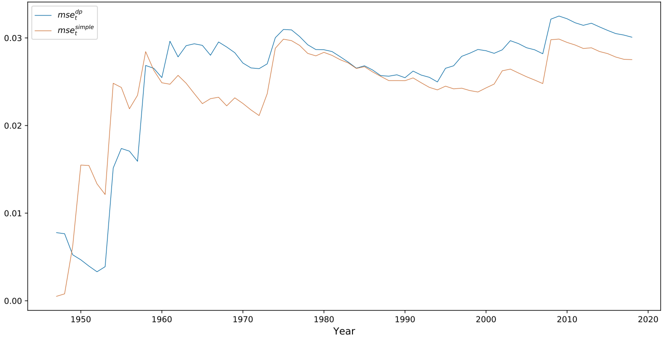

Figure: In the 1950s, the mean-squared errors of these two model increase sharply due to the fact that only few observations are used in in-sample regression. After 1960, the dp model perform poorly relatively to the simple model.

Q4: Out-of-sample regressions (cont'd)

- $OOSR^2 = 1 - \frac{\text{MSE}_T^{dp}}{\text{MSE}_T^{\text{simple}}} = -0.093 < 0$. The negative out-of-sample R^2 for the dp-model implies that sample trading strategy beats the trading strategy based on dp.

Q4: Simulation

- H_0: Dividend growth rates are not predictable ($b_d = 0$). So $b_r = 1 - \rho\phi = 0.087$.

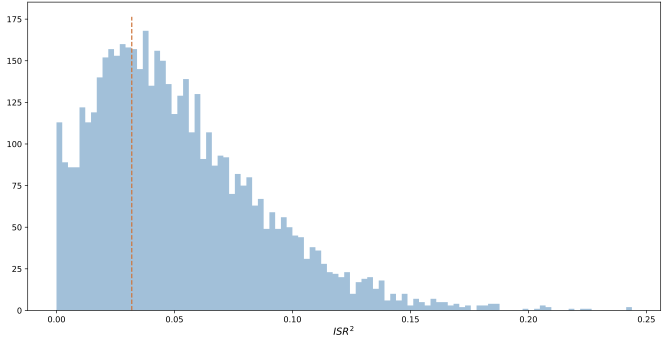

Figure: This figure plots the distribution of in-sample R^2 under the null hypothesis. Our point estimate 0.032 (dashed line) matches the distribution well. Hence we can not reject the null hypothesis.

Q4: Simulation

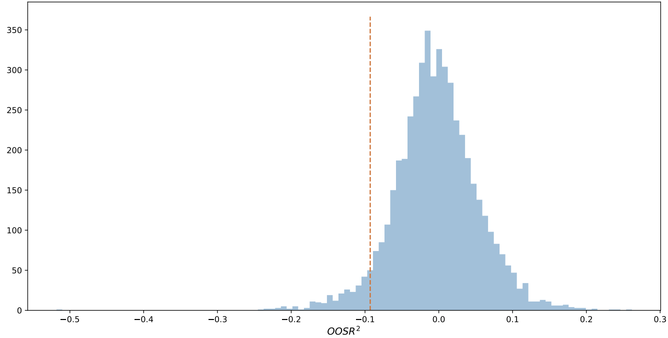

Figure: This figure plots the distribution of out-of-sample R^2 under the null hypothesis. Our point estimate -0.093 (dashed line) lies at the left tail, but there still exists many simulated $OOSR^2$ lower than our sample estimate. Hence we can not reject the null hypothesis.

Q4: Concluding remark

- The poor out-of-sample predictability implies that short-term (e.g., one-period) return is hard to predicted using fundamentals (e.g., dp). Short-term returns are quite volatile.
- However, this fact does not contradict against the long-term return predictability which says that time-varying risk premiums (or the long-term expected returns) dominate the variation in the stock price.