

# Big Data and Bayesian Nonparametrics

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## Big Data

The sample sizes are enormous. 200+ million obs

The data are super weird. density spikes, obese tails

'Big' and 'Strange' beg for nonparametrics.

In usual BNP you *model* a complex generative process with flexible priors, then apply that model directly in prediction and inference.

$$\text{e.g., } y = f(\mathbf{x}) + \epsilon, \text{ or even just } f(y|\mathbf{x})$$

However averaging over all of the nuisance parameters we introduce to be 'flexible' is a hard computational problem.

Can we do scalable BNP?

Frequentists are great at finding simple procedures (e.g.  $[\mathbf{X}'\mathbf{X}]^{-1}\mathbf{X}'\mathbf{y}$ ) and showing that they will 'work' regardless of the true DGP.

(DGP = Data Generating Process)

This is classical 'distribution free' nonparametrics.

- 1: Find some statistic that is useful regardless of DGP.
- 2: Derive the distribution for this stat under minimal assumptions.

Practitioners apply the simple stat and feel happy that it will work.

Can we Bayesians provide something like this?

## A flexible model for the DGP

$$g(\mathbf{z}) = \frac{1}{|\boldsymbol{\theta}|} \sum_{l=1}^L \theta_l \mathbb{1}[\mathbf{z} = \zeta_l], \quad \theta_l / |\boldsymbol{\theta}| \stackrel{iid}{\sim} \text{Dir}(\mathbf{a})$$

After observing  $\mathbf{Z} = \{\mathbf{z}_1 \dots \mathbf{z}_n\}$ , posterior has  $\theta_l \sim \text{Exp}(a + \mathbb{1}_{\zeta_l \in \mathbf{Z}})$ .  
(say every  $\mathbf{z}_i = [\mathbf{x}_i, y_i]$  is unique).

$a \rightarrow 0$  leads to  $p(\theta_l = 0) = 1$  for  $\zeta_l \notin \mathbf{Z}$ .

$$\Rightarrow g(\mathbf{z} \mid \mathbf{Z}) = \frac{1}{|\boldsymbol{\theta}|} \sum_{l=1}^L \theta_l \mathbb{1}[\mathbf{z} = \mathbf{z}_l], \quad \theta_i \sim \text{Exp}(1)$$

This is just the Bayesian bootstrap.

Ferguson 1973, Rubin 1981

## Example: Ordinary Least Squares

*Population* OLS is a posterior functional

$$\beta = (\mathbf{X}'\Theta\mathbf{X})^{-1}\mathbf{X}'\Theta\mathbf{y}$$

where  $\Theta = \text{diag}(\theta)$ . This is a random variable. (sample via BB)

Posterior moments for a first-order approx

$$\tilde{\beta} = [\mathbf{X}'\mathbf{X}]^{-1}\mathbf{X}'\mathbf{y} + \nabla\beta|_{\theta=\mathbf{1}}(\theta - \mathbf{1})$$

e.g.,  $\text{var}(\tilde{\beta}) \approx (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\text{diag}(\mathbf{e})^2\mathbf{X}'(\mathbf{X}'\mathbf{X})^{-1}$ , where  $e_i = y_i - \mathbf{x}_i'\hat{\beta}$ .

See Lancaster 2003 or Poirier 2011.

## User-Specific Behavior in Experiments

eBay runs lots of experiments: they make changes to the marketplace (website) for random samples of users.

Every experiment has response  $y$  (spend) and treatment  $d$  [0/1].  
In our illustrative example,  $d_i = 1$  for bigger pictures in my eBay.

$$i \in c : d_i = 0$$



$$i \in t : d_i = 1$$

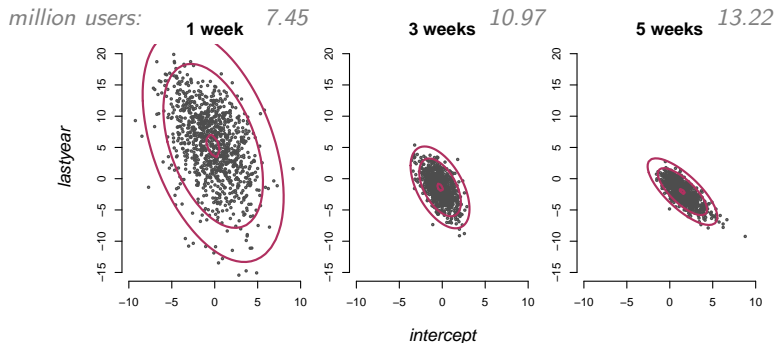


We know  $\mathbf{x}_i$  about user  $i$ : previous spend, last purchase date, items bought, sold, watched, bidden on, pages visited, ...  
And every user responds differently to the treatment.

We can *index* heterogeneity in treatment effect on  $\mathbf{x}_i$  via

$$\beta_t - \beta_c = (\mathbf{X}'_t \Theta_t \mathbf{X}_t)^{-1} \mathbf{X}'_t \Theta_t \mathbf{y}_t - (\mathbf{X}'_c \Theta_c \mathbf{X}_c)^{-1} \mathbf{X}'_c \Theta_c \mathbf{y}_c$$

e.g., coefficient on purchase within last-year vs an intercept:



Sample is from posterior, contours are normal approximation.  
This is a statistic we care about, even if the truth is nonlinear.

## Example: Decision Trees

Trees are great: nonlinearity, deep interactions, heteroskedasticity.



The 'optimal' decision tree is a statistic we care about (s.w.c.a).



## **CART:** greedy growing with optimal splits

Given node  $\{\mathbf{x}_i, y_i\}_{i=1}^n$  and DGP weights  $\theta$ , find  $x$  to minimize

$$\begin{aligned} |\theta| \sigma^2(x, \theta) = & \sum_{k \in \text{left}(x)} \theta_k (y_k - \mu_{\text{left}(x)})^2 \\ & + \sum_{k \in \text{right}(x)} \theta_k (y_k - \mu_{\text{right}(x)})^2 \end{aligned}$$

for a regression tree. Classification impurity can be Gini, etc.

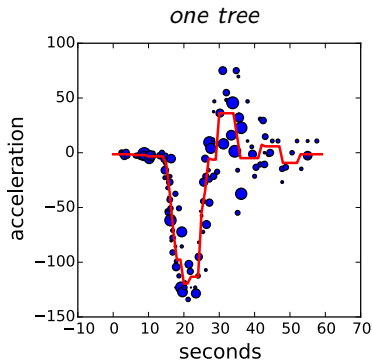
Population-CART might be a statistic we care about.

Or, in settings where greedy CART would do poorly (big  $p$ ), a randomized splitting algorithm might be a better s.w.c.a.

## Bayesian Forests: a posterior for CART trees

For  $b = 1 \dots B$ :

- draw  $\theta^b \stackrel{iid}{\sim} \text{Exp}(\mathbf{1})$
- run weighted-sample CART to get  $\mathcal{T}_b = \mathcal{T}(\theta^b)$



Random Forest  $\approx$  Bayesian forest  $\approx$  posterior over CART fits.

## Theoretical **trunk** stability

Given forests as a posterior, we can start talking about *variance*.

Consider the first-order approximation

$$\begin{aligned}\sigma^2(x, \boldsymbol{\theta}) &\approx \sigma^2(x, \mathbf{1}) + \nabla \sigma^2|_{\boldsymbol{\theta}=\mathbf{1}}(\boldsymbol{\theta} - \mathbf{1}) \\ &= \frac{1}{n} \sum_i \theta_i [y_i - \bar{y}_i(x)]^2\end{aligned}$$

with  $\bar{y}_i(x)$  the sample mean in  $i$ 's node when splitting on  $x$ .

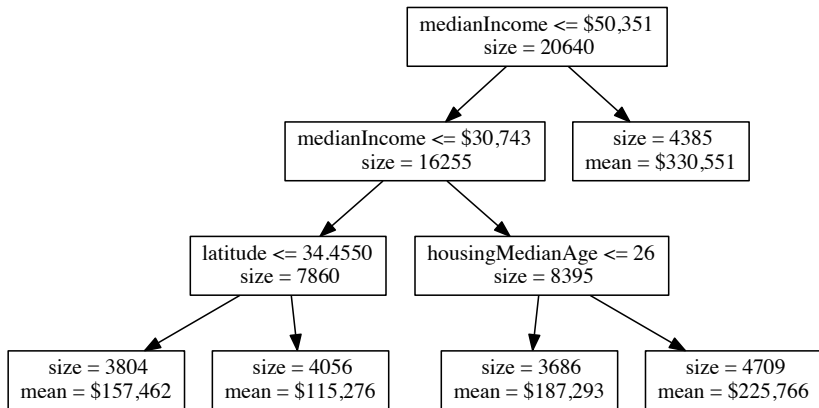
Based on this approx, we can say that for data at a given node,

$$p(\text{optimal split matches sample CART}) \gtrsim 1 - \frac{p}{\sqrt{n}} e^{-n},$$

with  $p$  split locations and  $n$  observations.

## California Housing Data

20k observations on median home prices in zip codes.



Above is the trunk you get setting min-leaf-size of 3500.



- ▶ sample tree occurs 62% of the time.
- ▶ 90% of trees split on income twice, and then latitude.
- ▶ 100% of trees have 1st 2 splits on median income.

Empirically and theoretically: trees are stable, at the trunk.

## Empirical Bayesian Forests (**EBF**)

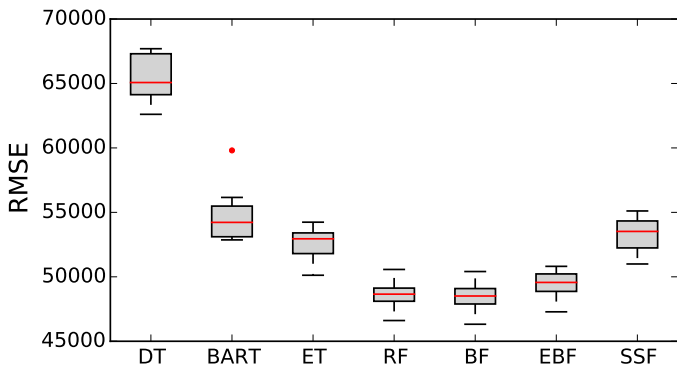
RFs are expensive. Sub-sampling hurts bad.

Instead:

- ▶ fit a single tree to a shallow **trunk**.
- ▶ Map data to each **branch**.
- ▶ Fit a full forest on the smaller branch datasets.

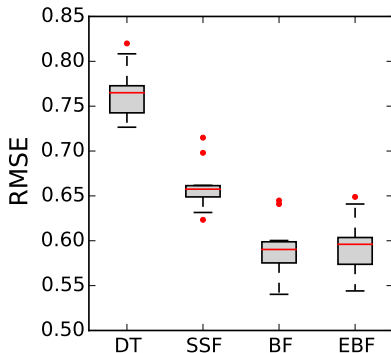
Empirical Bayes: fix plug-in estimates at high levels in a hierarchical model, focus effort at learning the hard bits.

Since the trunks are all the same for each tree in a full forest, our EBF looks nearly the same at a fraction of computational cost.



Here EBF and BF give nearly the same results. *SSF does not.*

## EBFs work all over the place

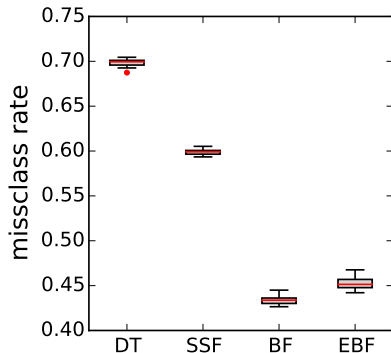


RMSE	% WTB	
0.5905	0.0	BF
0.5953	0.8	EBF
0.6607	11.9	SSF
0.7648	29.5	DT

Predicting wine rating from chemical profile



## EBFs work all over the place



MCR	% WTB	
0.4341	0.0	BF
0.4531	4.4	EBF
0.5989	38.0	SSF
0.6979	60.8	DT

or beer choice from demographics

## Choosing the trunk depth

Distributed computing perspective: **fix only as deep as you must!**

How big is each machine? Make that your branch size.

	CA housing			Wine			Beer		
<i>Min Leaf Size in <math>10^3</math></i>	6	3	1.5	2	1	0.5	20	10	5
<i>% Worse Than Best</i>	1.6	2.4	4.3	0.3	0.8	2.2	1.0	4.4	7.6

Still, open questions: e.g., more trees vs shallower trunk?

## Catching Bad Buyer Experiences at eBay

BBE: 'not as described', delays, etc.

$p(\text{BBE})$  is an input to search rankings.

Best way to improve prediction is more data.

EBFs via Spark: more data in less time.

On 12 million transactions, EBF with 32 branches yields a 1.3% drop in misclassification over the SSF alternatives.

Putting it into production requires some careful engineering, but this really is a very simple algorithm. **Big gain, little pain.**

## Heterogeneous Treatment Effects

Recall our earlier example: A/B experiment with user covariates  $\mathbf{x}_i$ .

Athey+Imbens propose indexing user HTE by fitting CART to

$$y_i^* = y_i \frac{d_i - q}{q(1 - q)} = \begin{cases} y_i / (1 - q) & \text{if } d_i = 0 \\ y_i / q & \text{if } d_i = 1 \end{cases}$$

where  $q$  is the probability of treatment (2/3 in our example).

This works because

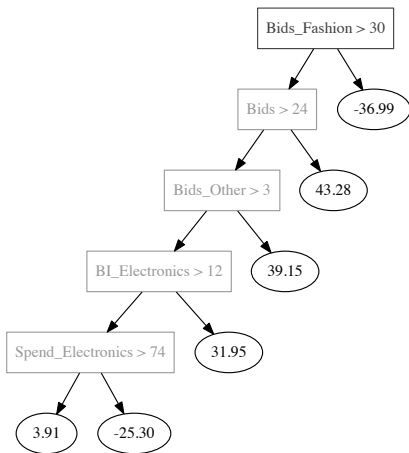
$$\mathbb{E}[y_i^* | \mathbf{v}_i] = v_i(1) - v_i(0)$$

where  $v_i(d)$  is the potential outcome for user  $i$  if  $d_i = d$ .

We only get to observe one of these:  $y_i = v_i(d_i)$ .

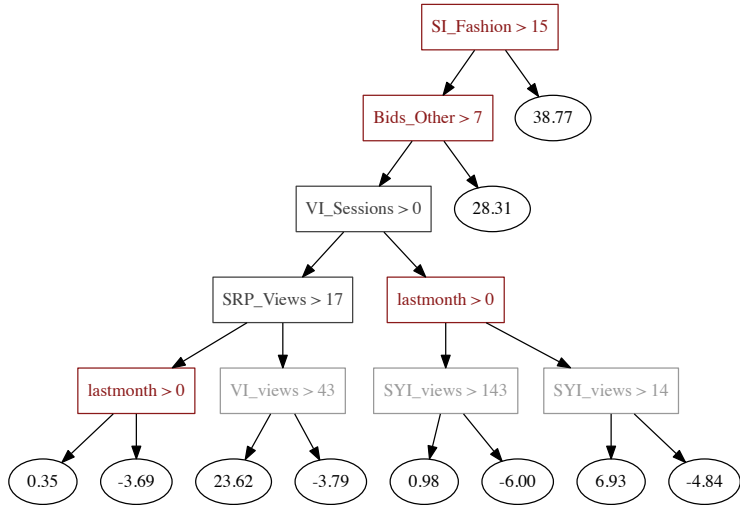
We can apply the Bayesian bootstrap (i.e., fit a BF) to assess *posterior uncertainty* about these treatment-effect trees.

e.g., sample depth-5 CART with min-leaf-size- $10^5$  after one week:



prob variable is node in tree:  $p < \frac{1}{3}$ ,  $p \in [\frac{1}{3}, \frac{1}{2})$ , and  $p \geq \frac{1}{2}$ .

After 5 weeks the tree is much more stable.



We can quantify uncertainty about all sorts of structure.

### Posterior probability of CART splitting on variable

Week 5	depth in tree				
	1	$\leq 2$	$\leq 3$	$\leq 4$	$\leq 5$
<i>SI Fashion</i>	.45	.50	.50	.50	.50
<i>Bids Other</i>	.30	.75	.75	.75	.75
<i>VI sessions</i>	.05	.05	.05	.10	.35
<i>lastmonth</i>	.05	.10	.15	.40	.65
<i>SRP views</i>	.00	.10	.15	.25	.40
<i>VI views</i>	.00	.00	.10	.20	.25
<i>SYI views</i>	.00	.00	.05	.15	.25

Simple scalable uncertainty quantification for complex algorithms.

## Big Data and distribution free BNP

I think about BNP as a way to analyze (and improve) algorithms.  
Decouple action/prediction from the full generative process model.

## Efficient Big Data analysis

To cut computation without hurting performance, we need to think about what portions of the 'model' are **hard** or **easy** to learn.

Once we figure this out, we can use a little bit of the data to learn the easy stuff and direct our full data at the hard stuff.

**thanks!**