Big Data and Bayesian Nonparametrics
Example: Empirical Bayesian Forests

Matt Taddy, Chicago Booth

faculty.chicagobooth.edu/matt.taddy/research

Big Data

The sample sizes are enormous.

- we'll see 12, 21 and 200 million today.
- Data can't fit in memory, or even storage, on a single machine.
- Our familiar MCMC algorithms take too long.

The data are super weird.

- Internet transaction data distributions have a big spike at zero and spikes at other discrete values (e.g., 1 or $99).
- Big tails that matter.
- We cannot write down believable models.

Both ‘Big’ and ‘Strange’ beg for nonparametrics.

In usual BNP you *model* a complex generative process with flexible priors, then apply that model directly in prediction and inference.

$$\text{e.g.,} \quad y = f(\mathbf{x}) + \epsilon, \quad \text{or even just} \quad f(y|\mathbf{x})$$

However averaging over all of the nuisance parameters we introduce to be ‘flexible’ is a hard computational problem.

Can we do scalable BNP?

Frequentists are great at finding simple procedures (e.g. $[\mathbf{X}'\mathbf{X}]^{-1}\mathbf{X}'y$) and showing that they will 'work' regardless of the true DGP.

(DGP = Data Generating Process)

This is classical 'distribution free' nonparametrics.

1: Find some statistic that is useful regardless of DGP.

2: Derive the distribution for this stat under minimal assumptions.

Practitioners apply the simple stat and feel happy that it will work.

Can we Bayesians provide something like this?

Distribution free Bayesian nonparametrics

Find some *statistic of the DGP* that you care about.

- Derive it from first principles, e.g. moment conditions.
- Or a statistic could be an algorithm that we know works.

Call this statistic $\mathcal{S}(g)$ where $g(\mathbf{z})$ is the DGP (e.g., for $\mathbf{z} = [\mathbf{x}, y]$).

Then you write down a flexible model for the DGP g, and study properties of the posterior on $\mathcal{S}(g)$ induced by the posterior over g.

A flexible model for the DGP

Let's go back to Ferguson '73 and a multinomial sampling model:

$$g(\mathbf{z}) = \frac{1}{|\boldsymbol{\theta}|}\sum_{l=1}^{L}\theta_l \mathbb{1}[\mathbf{z} = \boldsymbol{\zeta}_l],$$

$\mathcal{Z} = \{\boldsymbol{\zeta}_1 \dots \boldsymbol{\zeta}_L\}$ is the *fixed* support of the DGP and $\theta_l \overset{iid}{\sim} \mathrm{Exp}(a)$.

After observing $\mathbf{Z} = \{\mathbf{z}_1 \dots \mathbf{z}_n\}$, posterior has $\theta_l \sim \mathrm{Exp}(a+\mathbb{1}_{\boldsymbol{\zeta}_l \in \mathbf{Z}})$.
(say every $\mathbf{z}_i$ is unique).

Taking $a \to 0$ leads to $\mathrm{p}(\theta_l = 0) = 1$ for $\boldsymbol{\zeta}_l \notin \mathbf{Z}$.
So the posterior has $g(\mathbf{z}) = \frac{1}{|\boldsymbol{\theta}|}\sum_{l=1}^{L}\theta_l \mathbb{1}[\mathbf{z} = \mathbf{z}_l]$ with $\theta_i \sim \mathrm{Exp}(1)$.

This is just the Bayesian bootstrap.

Example: Ordinary Least Squares

The *population* OLS projection is a posterior functional

$$\boldsymbol{\beta} = (\mathbf{X}'\boldsymbol{\Theta}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Theta}\mathbf{y}$$

where $\boldsymbol{\Theta} = \mathrm{diag}(\boldsymbol{\theta})$. This is a random variable (sample via BB).

We can derive posterior moments for a first-order approx

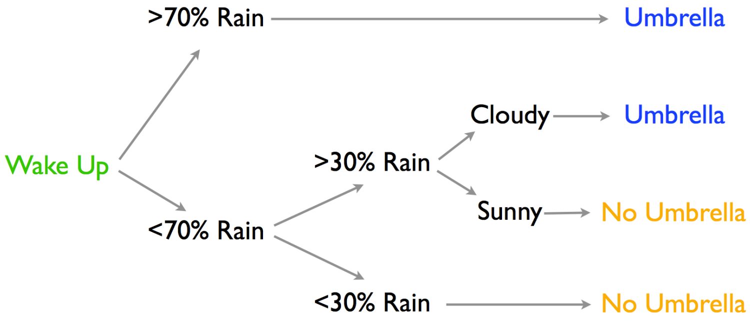
$$\tilde{\boldsymbol{\beta}} = [\mathbf{X}'\mathbf{X}]^{-1}\mathbf{X}'y + \nabla\beta\big|_{\boldsymbol{\theta}=\mathbf{1}}(\boldsymbol{\theta} - \mathbf{1})$$

e.g., $\mathrm{var}(\tilde{\boldsymbol{\beta}}) \approx (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathrm{diag}(\mathbf{e})^2\mathbf{X}'(\mathbf{X}'\mathbf{X})^{-1}$, where $e_i = y_i - \mathbf{x}_i'\hat{\boldsymbol{\beta}}$.

See Lancaster 2003 or Poirier 2011.

Example: Decision Trees

Trees are great: nonlinearity, deep interactions, heteroskedasticity.



The ‘optimal’ decision tree is a statistic we care about (s.w.c.a).

CART fits a tree by greedily growing with optimal splits.

Given parent node $\{\mathbf{x}_i, y_i\}_{i=1}^n$, the optimal split finds x_{ij} so that

$$\mathrm{left}(x_{ij}) : \{\mathbf{x}_k, y_k : \; x_{kj} \le x_{ij}\} \quad \text{and} \quad \mathrm{right}(x_{ij}) : \{\mathbf{x}_k, y_k : \; x_{kj} > x_{ij}\}$$

child sets are as homogeneous in response y as possible.
e.g., a regression tree recursively splits to minimize

$$|\boldsymbol{\theta}|\sigma^2(x, \boldsymbol{\theta}) = \sum_{k \in \mathrm{left}(x)} \theta_k (y_k - \mu_{\mathrm{left}(x)})^2 + \sum_{k \in \mathrm{right}(x)} \theta_k (y_k - \mu_{\mathrm{right}(x)})^2$$

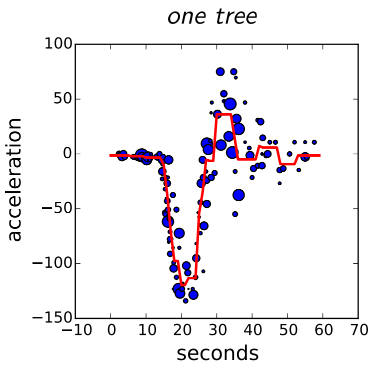
where θ_k are observation weights and $\mu_{\mathrm{c}(x)} = \frac{1}{|\boldsymbol{\theta}_{\mathrm{c}(x)}|} \sum_{k \in \mathrm{c}(x)} \theta_k y_k$.

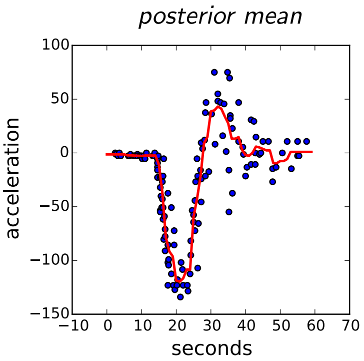
If population-CART would predict well, then it's a s.w.c.a.

Bayesian Forests: Sample the posterior for CART trees

For $b = 1 \dots B$:

- draw $\boldsymbol{\theta}^b \overset{iid}{\sim} \mathrm{Exp}(\mathbf{1})$
- run weighted-sample CART to get $\mathcal{T}_b = \mathcal{T}(\boldsymbol{\theta}^b)$





This is *very similar* to a random forest. So, RFs ≈ posterior over trees.

Theoretical **trunk** stability

Given forests as a posterior, we can start talking about *variance*.

As in simple OLS, look at the posterior for 1st order approx.

$$\sigma^2(x,\boldsymbol{\theta}) \approx \sigma^2(x,\mathbf{1}) + \nabla\sigma^2\big|_{\boldsymbol{\theta}=\mathbf{1}}(\boldsymbol{\theta}-\mathbf{1}) = \frac{1}{n}\sum_i \theta_i\left[y_i - \bar{y}_i(x)\right]^2$$

with $\bar{y}_i(x)$ the sample mean in i's node when splitting on x.

Based on this approx, we can say that for data at a given node,

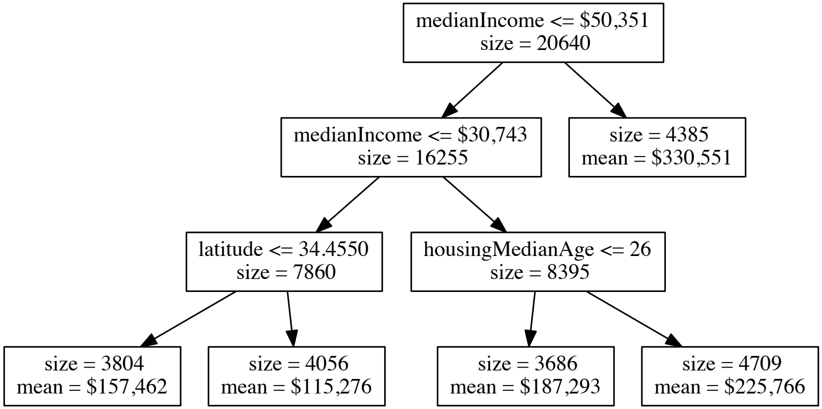
$$\mathrm{p}\left(\text{optimal split matches sample CART}\right) \gtrsim 1 - \frac{p}{\sqrt{n}}e^{-n},$$

with p split locations and n observations.

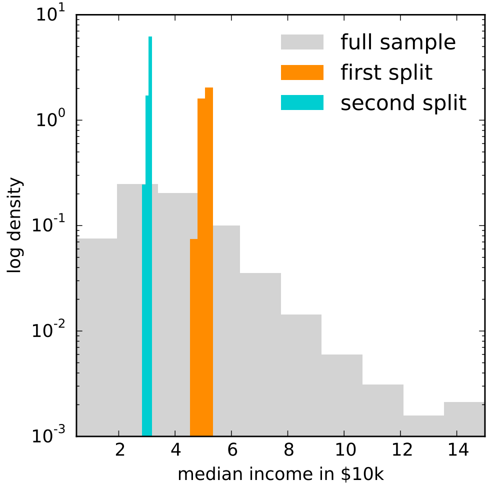
Things are pretty stable, until they aren't: as the tree grows, node sizes get smaller and chance of a non-optimal split multiplies.

California Housing Data

20k observations on median home prices in zip codes.



Above is the trunk you get setting min-leaf-size of 3500.



- sample tree occurs 62% of the time.
- 90% of trees split on income twice, and then latitude.
- 100% of trees have 1st 2 splits on median income.

Empirically and theoretically: trees are stable, at the trunk.

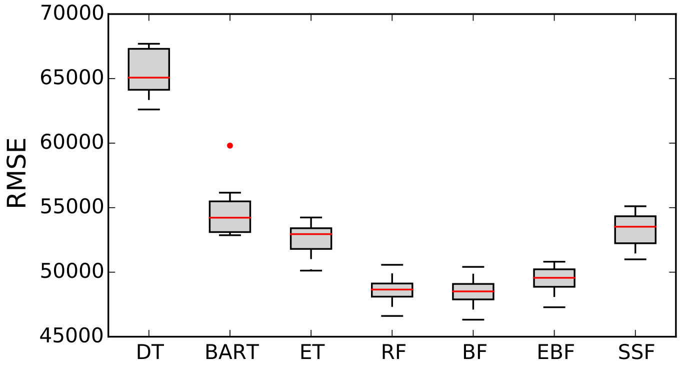
Forests are expensive when data is too big to fit in memory.
Subsampling forests lead to a big drop in performance.

Empirical Bayesian Forests (EBF):

- fit a single tree to a shallow trunk.
- Use this as a mapper to direct full data to each branch.
- Fit a full forest on the smaller branch datasets.

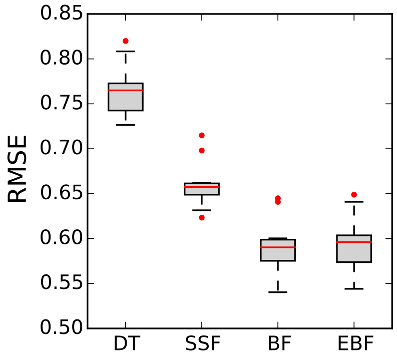
This is classic Empirical Bayes: fix higher levels in a *hierarchical model*, and direct your machinery+data at learning the hard bits.

Since the trunks are all the same for each tree in a full forest, our EBF looks nearly the same at a fraction of computational cost.



Here EBF and BF give nearly the same results. *SSF does not.*

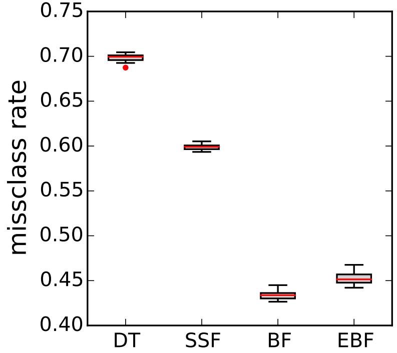
EBFs work all over the place



$\overline{\text{RMSE}}$	% WTB	
0.5905	0.0	BF
0.5953	0.8	EBF
0.6607	11.9	SSF
0.7648	29.5	DT

Predicting wine rating from chemical profile

EBFs work all over the place



$\overline{\text{MCR}}$	% WTB	
0.4341	0.0	BF
0.4531	4.4	EBF
0.5989	38.0	SSF
0.6979	60.8	DT

or beer choice from demographics

Choosing the trunk depth

Distributed computing perspective: fix only as deep as you must!

How big is each machine? Make that your branch size.

	CA housing			Wine			Beer		
Min Leaf Size in 10^3	6	3	1.5	2	1	0.5	20	10	5
% Worse Than Best	1.6	2.4	4.3	0.3	0.8	2.2	1.0	4.4	7.6

Still, open questions: e.g., more trees vs shallower trunk?

EBFs at EBay: predicting Bad Buyer Experiences

A BBE could be receiving something that is ‘significantly not as described’, or shipping delays, or any of many other problems.

The models are updated frequently, and information about p(BBE) is an input to search rankings and more.

The best thing to improve predictions is more data.
With millions of daily transactions, there’s little limit on data.

EBFs at EBay

Random forest runs take too long on full data.

Subsampling led to a noticeable and big drop in performance.

So: EBFs!

- trunk can be fit in distribution using Spark `MLLib`.
- this trunk acts as a sorting function to map observations to separate locations corresponding to each branch.
- Forests are then fit on a machine for each branch.

EBFs at EBay

On 12 million transactions, EBF with 32 branches yields a 1.3% drop in misclassification over the SSF alternatives.

Putting it into production requires some careful engineering, but this really is a very simple algorithm. Big gain, little pain.

A key point: EBFs are not inherently ‘better’ than forests fit to all of the data. But EBFs can be fit to more data in less time.

Efficient Big Data analysis

To cut computation without hurting performance, we need to think about what portions of the 'model' are hard or easy to learn.

Once we figure this out, we can use a little bit of the data to learn the easy stuff and direct our full data at the hard stuff.

Big Data and distribution free BNP

I think about BNP as a way to analyze (and improve) algorithms. Decouple action/prediction from the full generative process model.

thanks!