GRA65515 - Quantitative Risk And Asset Management
Assignment - Modelling and Portfolio Management

April 18, 2024

Study of the market portfolio and Kelly criterion

There is clear evidence of a strong negative correlation between the Standard deviation and the Sharpe ratio of each industry as seen by the downward trend in the **Figure 1**, after computation this correlation is: -90.0% (Highly Significant). This means that higher standard deviation generally leads to lower Sharpe ratio, i.e. as the risk of the industry increases (measured by its volatility) their risk-adjusted return (measured by the Sharpe ratio) decreases too.

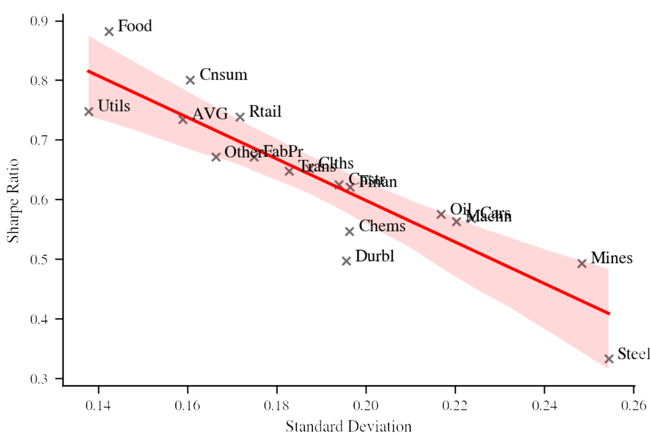

Figure 1. Sharpe Ratio vs. Standard Deviation for each industry

Assuming that the risk-free rate is 5% and that we can only invest in cash or in the market portfolio, the optimal leverage that we should take according to the half-Kelly Criterion is 1.345, which means that we should borrow and invest 34.5% more of our initial investment position. The optimal leverage being obtained by $\frac{f}{2} = \frac{\mu}{2\sigma^2}$ where μ is the excess return and σ is the market's volatility.

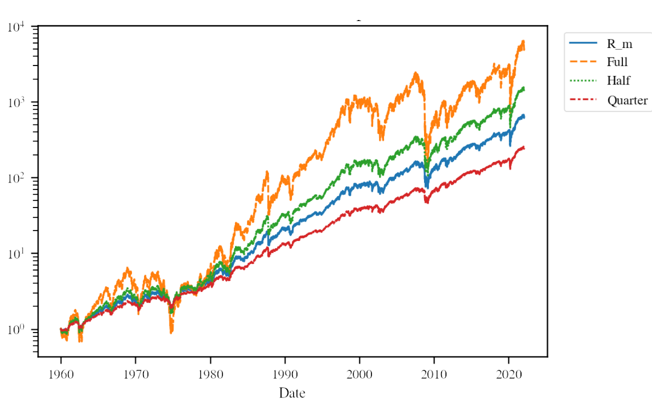

Figure 2. Cumulative returns comparison

As we can observe, a full Kelly Criterion would lead to more cumulative returns, but with monstrous drawdowns, up to 90%. The half-Kelly allows a significant reduction of those drawdowns, even though it could be still quite difficult to keep on the long run for investors. An alternative method would be to take a quarter of the Kelly Criterion (invest 67.3% in the market and 32.7% in cash). This implies a significant reduction of the cumulative returns, but it also allows for much lower drawdowns that are more bearable for risk-averse investors.

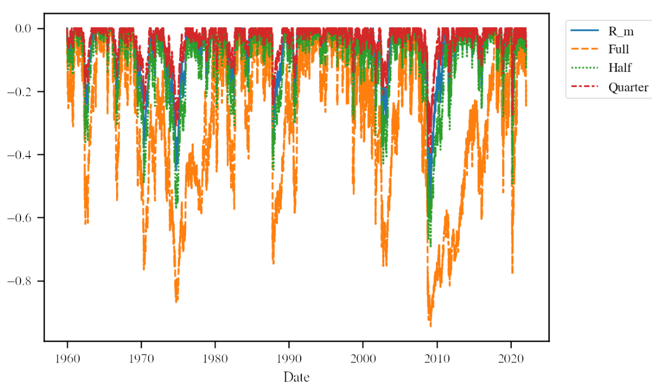

Figure 3. Drawdowns comparison

The spectrum of asset allocations between Quarter-Kelly and Half-Kelly being viable strategies that generate strong returns with decent volatilities, we would select one according to the investor risk-adversity. The more risk-averse the less leverage the investor should take, a good in-between position could be a full investment in the market portfolio.

To futher analysis the potential of allocation of the Kelly Criterion, we decided to dive into the construction of other portfolios, in the case of several risky assets, and we discovered very good performing portfolios which would attract investors interested in more returns in exchange for more risk.

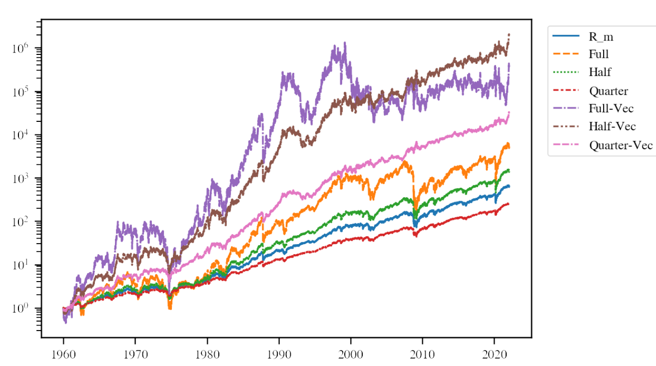

Figure 4. Cumulative returns comparison

While a full Kelly-Criterion would be disastrous for a manager, the half and quarter allocations out-perform our precedent portfolios with very large returns, however it doubles their volatility. We would conclude by proposing the quarter allocation with multi-risky assets, which is close in terms of Sharpe ratio to our previous quarter allocation.

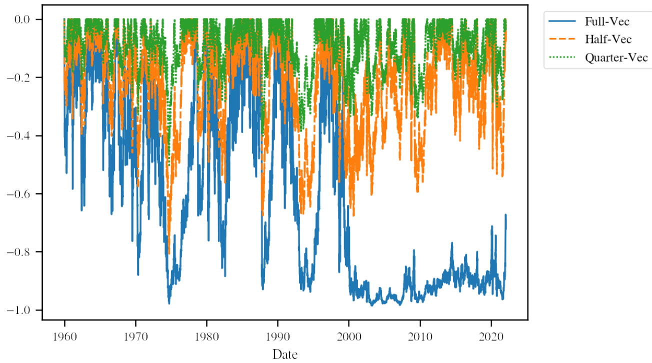

Figure 5. Drawdowns comparison

Table 1. Quarter Kelly (multi-asset) weights

	Food	Mines	Oil	Clths	Durbl
w (in %)	112.7	42.0	33.0	-29.4	-92.2
	Chems	Cnsum	Cnstr	Steel	FabPr
w (in %)	-43.9	56.9	9.0	-50.3	54.1
	Machn	Cars	Trans	Utils	
w (in %)	7.6	34.8	45.5	-55.0	
	Rtail	Finan	Other	RF	
w (in %)	14.8	-38.9	7.7	-8.4	

Table 2. Portfolios comparison

	R_m	Full	Half	Quarter
Average	11.7%	23.2%	14.0%	9.5%
Volatility	15.9%	42.8%	21.37%	10.7%
Sharpe ratio	0.734	0.542	0.656	0.884
		Full-V	Half-V	Quarter-V
Average		62.7%	33.8%	19.3%
Volatility		90.9%	45.4%	22.7%
Sharpe ratio		0.690	0.744	0.851

Evaluation of different strategies of Portfolio construction.

In this part, we compare and study three new allocations which are Inverse Volatility Weighted Portfolio (IVW), Sharpe ratio Momentum Portfolio (SRM), and Sharpe ratio Reversal Portfolio (SRR).

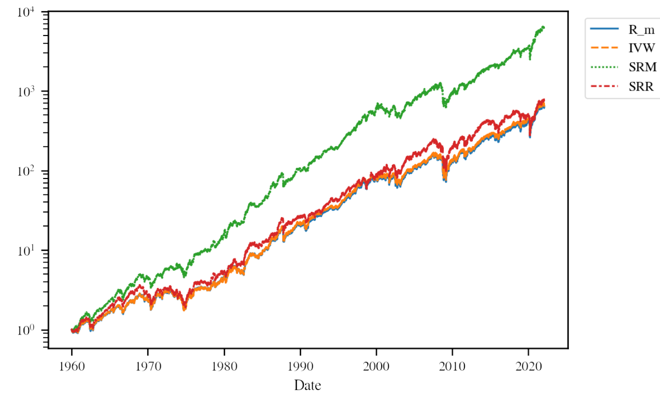

Figure 6. Portfolios comparison

Table 3. Portfolios comparison

	R_m	IVW	SRM	SRR
Average	11.7%	11.7%	15.4%	12.3%
Volatility	15.9%	15.5%	16.0%	17.6%
Sharpe ratio	0.734	0.756	0.959	0.699

Inverse Volatility Weighted Portfolio

The first portfolio tries to minimize risk by allocating more capital to less volatile industries and less capital to more volatile industries. By doing so, it reduces the impact of volatile assets and therefore reduces the volatility of the portfolio.

As we can observe from **Table 3**, it has the lowest volatility of the three strategies. This approach yields moderate returns, 11.7% per year, but with relatively low volatility, 15.5% per year. The Sharpe ratio for this strategy, 0.756 is also a good reflection of the good balance between the returns and the risk taken that this strategy aims to achieve. Moreover, we observe that this strategy is quite close to be fully invested in the market portfolio as we can see in **Figure 7**, since the attributed weight are really close to the one of an equal weight portfolio (**Figure 8**).

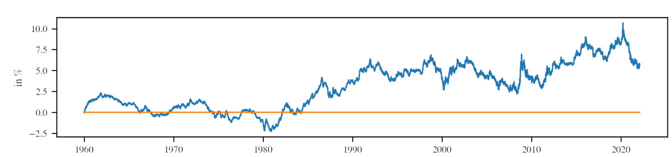

Figure 7. Relative difference between the evenly distributed and inverse-volatility-weighted market portfolios

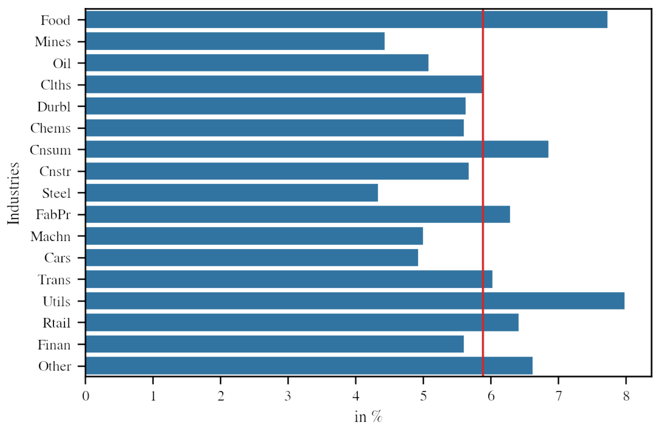

Figure 8. 1/std weights compared to evenly shared

Sharpe Ratio Momentum Portfolio

The second portfolio tries to capture a momentum effect in the Sharpe ratio of each industry, past good risk-adjusted returns could have a higher probability of having good-risk adjusted returns in the future. This strategy maximizes the risk-adjusted returns by investing in the industries that demonstrated the best past performance per unit of risk taken. It leads obviously to a high Sharpe ratio, 0.959 annualized, with significantly high returns, 15.4% annualized. This strategy has a higher volatility, 16.0% annualized, compared to the first portfolio, which is logical since there is less diversification as we invest in only five industries each time.

Sharpe Ratio Reversal Portfolio

The third portfolio tries to capitalize on potential reversals of the industries that perform the least, it assumes that there might be a market cycle and that, after a long time (12 months) of under performance, those industries will improve their performance. This strategy is riskier than the others because even though there is indeed a market cycle, it is very difficult to predict when the reversal will occur. That explains the annualized volatility of 17.6%, and the relatively low returns of 12.3% annualized. This leads to a poorer Sharpe ratio of 0.699.

Modelling the Market Portfolio Returns

The daily volatility computed as a rolling window of 63 trading days oscillates most of the time around 0.5% with some spikes that happen in clusters (stylized fact of financial returns) that can reach up to 4.5% due to market events or economic news.

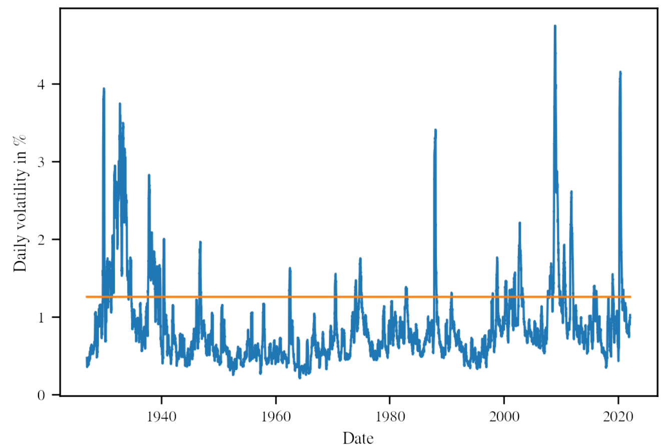

Figure 9. Daily volatility compared to a 20% annualized volatility

We study two periods 1927-2022 and 1990-2022 in fitting a mixture of 2 student-t distributions to the one of the returns using Maximum Likelihood Estimates. We identify the mixture distribution's parameter to this expression:

$$\pi \cdot t(\mu_1, \sigma_1, v_1) + (1 - \pi) \cdot t(\mu_2, \sigma_2, v_2)$$

Period 1927-2022

As we can observe on the **Figure 10**, the mixture fits well the returns if we look at the overall distributions. Yet, the model does not replicate very well the observations' skew and kurtosis indicating that the distribution of returns as fatter tails and is more left-skewed than the model actually predict this is further confirmed by the QQ-plot in the **Figure 11** that shows some divergence in the largest theoretical quantiles. However the model monthly VaR at 99% is quite well modelled by the mixture, which is a good thing if we want to assess the market risk of the portfolio, the monthly Expected Shortfall on the other hand are a underestimated by 1.22% which could lead to under estimation of the potential loss in case of bad events.

Table 4. Estimated parameters (1927-2022): $\pi = 50\%$

	First component	Second component
μ	0.90	-0.17
σ	0.80	1.28
ν	100.	6.21

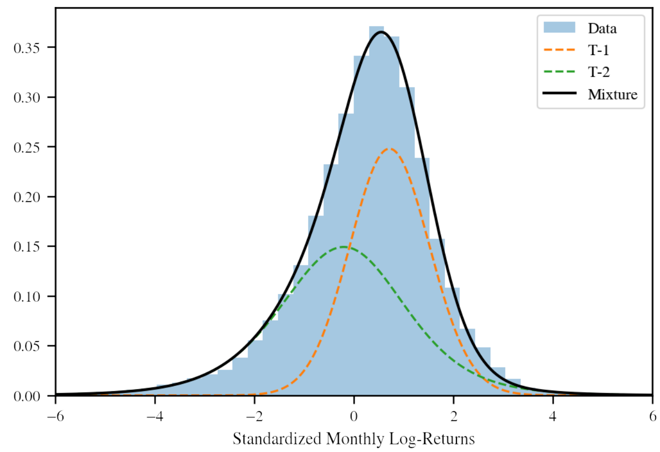

Figure 10. Distribution computed from 1927

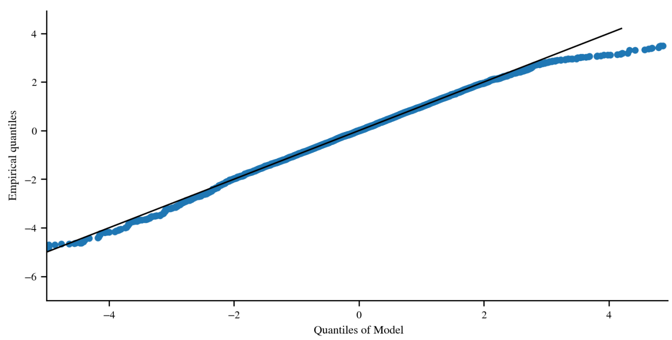

Figure 11. QQ plot

Table 5. Data & Models comparison since 1927

	Data	Model	$\sigma_{year} = 20\%$
Skew	-1.62	-0.66	-0.84
Kurtosis	13.28	6.17	12.06
VaR(99%)	-3.99	-3.67	-2.91
ES(99%)	-5.98	-4.76	-4.18

Table 7. Data & Models comparison since 1990

	Data	Model	$\sigma_{year} = 20\%$
Skew	-2.65	-0.96	-1.65
Kurtosis	28.98	15.58	13.99
VaR(99%)	-3.52	-3.23	-2.65
ES(99%)	-5.50	-4.56	-4.17

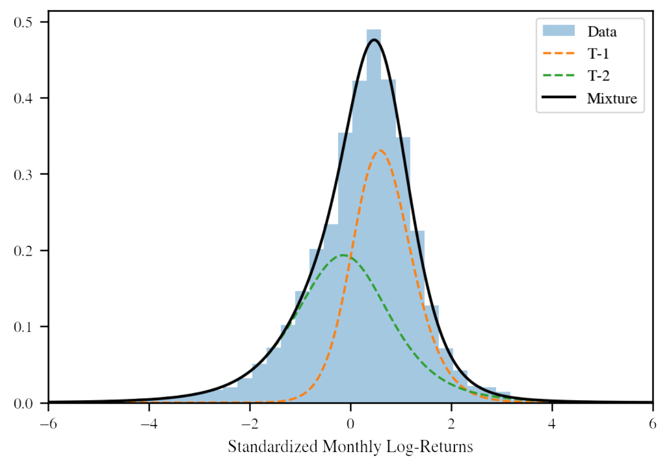

Figure 12. Distribution computed from 1990

Period 1990-2022

Here as well (**Figure 12**), the mixture fits well the overall distribution the returns, but the replication of the skew and kurtosis of the data is even worse as can be seen from the QQ plot **Figure 13**. Still the VaR is quite close to the data, indeed there is a probability of 99% that the worst loss will not exceed -3.52% per month for the data compared to -3.23% for the model, finally we also observe this difference in Expected Shortfall, demonstrating that the model is more conservative than the data.

Table 6. Estimated parameters (1990-2022): $\pi = 50\%$

	First component	Second component
μ	1.08	-0.17
σ	0.58	0.97
ν	11.64	4.00

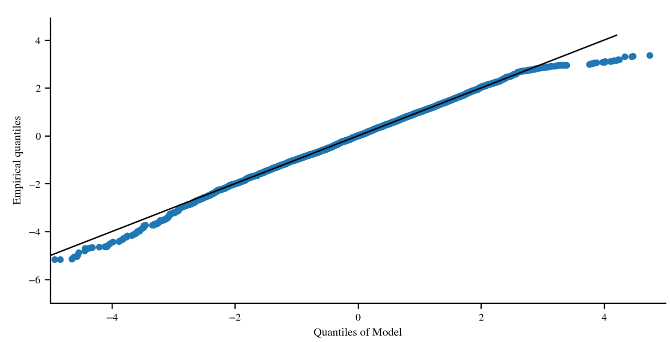

Figure 13. QQ plot