CS6202: Problem Set 2

Question 1

Give and derive the formula for the probability density function of the last iterate of Inverse Autoregressive Flow (IAF) $\log q(\mathbf{z}_T|\mathbf{x})$ in Variational Autoencoder. Assume that our base distribution $q(\mathbf{z}_0|\mathbf{x})$ is an isotropic Gaussian with parameter μ_0 and σ_0. This is similar to proof the Equation 11 in Inverse Autoregressive Flow paper.

Solution 1

Remember that in Normalizing Flows the probability density function of the final iterate is given as:

$$\log q(\mathbf{z}_T|\mathbf{x}) = \log q(\mathbf{z}_0|\mathbf{x}) - \sum_{t=1}^{T} \log \det \left| \frac{d\mathbf{z}_t}{d\mathbf{z}_{t-1}} \right| \tag{1}$$

Knowing that the base distribution $\log q(\mathbf{z}_0|\mathbf{x})$ is an isotropic Gaussian and log-determinant of the Jacobian in IAF is given as follow:

$$\log \det \left| \frac{d\mathbf{z}_t}{d\mathbf{z}_{t-1}} \right| = \sum_{i=1}^{D} - \log \sigma_i(\mathbf{z}_t) \tag{2}$$

$$= \sum_{i=1}^{D} - \log \sigma_{t,i} \tag{3}$$

We use $\sigma_{t,i} = \sigma_i(\mathbf{z}_t)$ to make the notation of dimension i of σ at iteration t shorter. We can compute the probability density function of the final iterate as:

$$\begin{aligned}
\log q(\mathbf{z}_T|\mathbf{x}) &= \log q(\mathbf{z}_0|\mathbf{x}) - \sum_{t=1}^{T} \log \det \left| \frac{d\mathbf{z}_t}{d\mathbf{z}_{t-1}} \right| \\
&= \sum_{i=1}^{D} \left(\log \left[\frac{1}{\sqrt{2\pi}\sigma_{0,i}} \right] - \frac{(\mathbf{z}_0 - \mu_{0,i})^2}{2\sigma_{0,i}^2} \right) - \sum_{t=1}^{T} \log \det \left| \frac{d\mathbf{z}_t}{d\mathbf{z}_{t-1}} \right| && \text{...log of isotropic Gaussian} \\
&= \sum_{i=1}^{D} \log \left[\frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_{0,i}} \right] - \frac{(\mathbf{z}_0 - \mu_{0,i})^2}{2\sigma_{0,i}^2} - \sum_{t=1}^{T} - \log \sigma_{t,i} && \text{...applying Equation 3} \\
&= \sum_{i=1}^{D} -\frac{1}{2} \log(2\pi) - \log \sigma_{0,i} - \frac{1}{2}\epsilon_i^2 - \sum_{t=1}^{T} - \log \sigma_{t,i} && \text{...by } \epsilon = \frac{\mathbf{z_0} - \mu(\mathbf{z_0})}{\sigma(\mathbf{z_0})} \\
&= -\sum_{i=1}^{D} \left(\frac{1}{2} \log(2\pi) + \frac{1}{2}\epsilon_i^2 + \sum_{t=0}^{T} \log \sigma_{t,i} \right) && \text{...merge the sum of } \sigma \text{ and take out negative.}
\end{aligned}$$

Question 2

asd

Solution 2