Part 1

Tuesday, 26 January 2021 12:44 pm

Activity 1: Conditional Probability

Only 4% of the population are color blind, but 7% of men are color blind. What percentage of color blind people are men?

Q: $P(M|C)$?

$P(C) = 0.04, \quad P(C|M) = 0.07$

$P(M|C) = \dfrac{P(C|M) \cdot P(M)}{P(C)}$

Assume $P(M) = 0.5$,

$= \dfrac{0.07 \times 0.5}{0.04} = 0.875$

$= 87.5\%$

Activity 2:

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

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a) • $P(\text{toothache} \wedge \neg\text{catch})$

$= 0.012$

b). $P(\text{catch}) = P(\text{catch} \wedge (\text{toothache} \cup \neg\text{toothache}))$

$= P(\text{catch} \wedge \text{toothache}) + P(\text{catch} \wedge \neg\text{toothache})$

∵ no overlap

$= (0.108 + 0.016) + (0.072 + 0.144) = 0.34$

c) $P(\text{cavity} \mid \text{catch})$

$= \dfrac{P(\text{cavity} \wedge \text{catch})}{P(\text{catch})}$ (Bayes' Rule)

$= \dfrac{(0.108 + 0.072)}{0.34} \doteq 0.5294$

d) $P(\text{cavity} \mid \text{toothache} \vee \text{catch})$

$= \dfrac{P(\text{cavity} \wedge (\text{toothache} \vee \text{catch}))}{P(\text{toothache} \vee \text{catch})}$

$= \dfrac{P[(\text{cavity} \wedge \text{toothache}) \vee (\text{cavity} \wedge \text{catch})]}{P(\text{toothache} \cup \text{catch})}$

$= \dfrac{P(\text{cavity} \wedge \text{toothache}) + P(\text{cavity} \wedge \text{catch}) - P(\text{cavity} \wedge \text{toothache} \wedge \text{catch})}{P(\text{toothache}) + P(\text{catch}) - P(\text{toothache} \wedge \text{catch})}$

$= \dfrac{(0.108 + 0.012) + (0.108 + 0.072) - 0.108}{(0.108 + 0.012 + 0.016 + 0.064) + (0.108 + 0.016 + 0.072 + 0.144) - (0.108 + 0.016)}$

$= \dfrac{0.192}{0.416} = 0.46$

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Verify the conditional independence claimed above, by showing that

$P(\text{catch} \mid \text{toothache} \wedge \text{cavity}) = P(\text{catch} \mid \text{cavity})$

$P(\text{catch} \mid \text{toothache} \wedge \text{cavity})$

$= \dfrac{P(\text{catch} \wedge \text{toothache} \wedge \text{cavity})}{P(\text{toothache} \wedge \text{cavity})}$

$= \dfrac{0.108}{0.108 + 0.012} = 0.9$

$P(\text{catch} \mid \text{cavity})$

$= \dfrac{P(\text{catch} \wedge \text{cavity})}{P(\text{cavity})}$

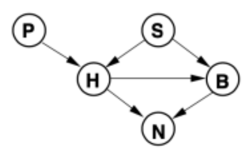
$= \dfrac{0.108 + 0.072}{0.108 + 0.012 + 0.072 + 0.008}$

$= \dfrac{0.18}{0.2} = 0.9$

Activity 3: Consider the following statements: Headaches and blurred vision may be the result of sitting to close to a monitor. Headaches may also be caused by bad posture. Headaches and blurred vision may cause nausea. Headaches may also lead to blurred vision.

(i) Represent the causal links in a Bayesian network. Let H stan for "headache", B for "blurred vision"' S for "sitting too close to a monitor", P for "bad posture" and N for "nausea". In terms of conditional probabilities, write a formula for the event that all five variables are true, i.e. P(H ∧ B ∧ S ∧ P ∧ N).

(i)



ex1.

$P(H \wedge B \wedge S \wedge P \wedge \neg N)$

$= P(H \mid S \wedge P) \cdot P(B \mid H \wedge S) \cdot P(S) \cdot P(P) \cdot P(\neg N \mid H \wedge B)$

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i \mid \text{parent}(X_i))$$

ex2. $P(H \wedge B \wedge \neg S \wedge P \wedge \neg N)$

$= P(H \mid \neg S \wedge P) \cdot P(B \mid H \wedge \neg S) \cdot P(\neg S) \cdot P(P) \cdot P(\neg N \mid H \wedge B)$

(iii) What is the probability that the patient suffers from bad posture given that they are suffering from headaches but not from nausea?

$P(P \mid H \wedge \neg N)$

$= \dfrac{P(H \wedge P \wedge \neg N)}{P(H \wedge \neg N)} = \dfrac{\sum_{b', s'} P(H \wedge b' \wedge s' \wedge P \wedge \neg N)}{\sum_{b', s', p'} P(H \wedge b' \wedge s' \wedge p' \wedge \neg N)}$

(By enumeration)