Risk: Decisions with Asymmetric Costs

Lab Assignment

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(40) 1. A person doesn't feel well and goes to see the doctor. Let's assume there are only two possible states of nature here: this illness is due to a viral infection (ω_0), or this illness is due to a bacterial infection (ω_1). The doctor wants to decide between two possible treatment plans: rest (a_0), or a course of antibiotics (a_1). Suppose that the costs associated with all four (action,state) pairs are $L_{00} = L_{11} = 0$, $L_{01} = 10$, and $L_{10} = 1$.

(a) Why is it reasonable to model this situation with $L_{01} > L_{10}$?

In this case, taking rest with a bacterial infection is worse than taking antibiotics with a virus. The reason is because that is the most preventable situation. Since a viral infection is not affected by antibiotics, there would be no downside other than wasting antibiotics.

(b) Suppose that the doctor's prior guess about the patient, without doing any tests, is $P(\omega_1) = 0.1$. In this situation, compute the risks of both possible actions. In the absence of any testing, what does theory select as the correct action?

If the probability before tests is a 10% chance of a bacterial infection, the theory would select rest. P(omega1)*10 = 1 while P(omega0)*1 = .9. The expected cost for both are low, but choosing rest has a lower expected cost.

(c) Suppose the doctor gives the patient a test for bacterial infection, which has two possible results: x_0 (negative: no bacterial infection) and x_1 (positive: bacterial infection). Suppose this test has a false negative rate of 0.3 and a false positive rate of 0.2. Compute the four conditional risks that are relevant here: $R(a_0|x_0), R(a_1|x_0), R(a_0|x_1), R(a_1|x_1)$.

R(a0|x0) = Cost * P(state sick| negative test) = 0* .8 + 10*.3 = 3

R(a1|x0) = Cost * P(state sick| negative test) = 1* .7 + 1*.3 = 1

R(a0|x1) = Cost * P(state sick| negative test) = 10* .8 + 0*.2 = 8

R(a1|x1) = Cost * P(state sick| negative test) = 0* .8 + 1*.2 = .2

(d) Compute the total risk of the decision rule which says "if test is negative, prescribe rest; if test is positive, prescribe a course of antibiotics."

That would be adding two of the risks calculated in the last question which is

$$R(a0|x0) + R(a1|x1) = 3 + .2 = 3.2$$

(50) 2. This problem explores the idea of "binary classification with a REJECT option." More precisely, suppose there are two states of nature ω_1 and ω_2, but three possible actions available to us: a_1 (decide ω_1), or a_2 decide (ω_2), or a third option a_0 (refuse to state an opinion). Suppose that the costs of misclassifications are symmetric $L_{12} = L_{21} = 1$, that $L_{11} = L_{22} = 0$, and that refusal always costs $r = L_{01} = L_{02}$ for some real number $0 \leq r \leq 1$.

(a) Given a feature vector x, compute the conditional risks $R(a_1|x), R(a_2|x)$, and $R(a_0|x)$.

R(a1|x) = L12 * P(omega2|x) + L11* P(omega1|x)

R(a2|x) = L21 * P(omega1|x) + L22* P(omega2|x)

R(a0|x) = L01 * P(omega1|x) + L02* P(omega2|x)

(b) Determine the optimal decision rule (the decision rule which minimizes risk).
Express your answer in terms of the quantities $P(\omega_1|x), P(\omega_2|x)$, and r.

Based on what P(omega1|x) and P(omega2|x) and r, there are cases when rejection is the best way to reduce expected cost. This is only possible because r is between 0 and 1. This means that one could make a decision rule where min(P(omega1|x), P(omega2|x)) is at least 2 times greater than r than it is always more advantageous to reject to minimize risk.

min(P(omega1|x), P(omega2|x)) > 2 r

(c) Describe a real-world situation (perhaps one relevant to your major) which would be reasonably modeled by this setup.

A real world situation could possibly be investing in two company stocks where the state is which will do well. The cost would be not making a return on the investment and how much money is lost. If a profit is made than the cost is zero and the rejection option would be not investing at all which would lead to no profit which incurs a small cost.