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BHU/20/04/05/0010
COMPUTER SCIENCE

1. In clear and defined steps show the derivation of the Bresenham's line drawing algorithm and plot the pixels for the following points A (2,5) and B (7, 10)

i) Derivations

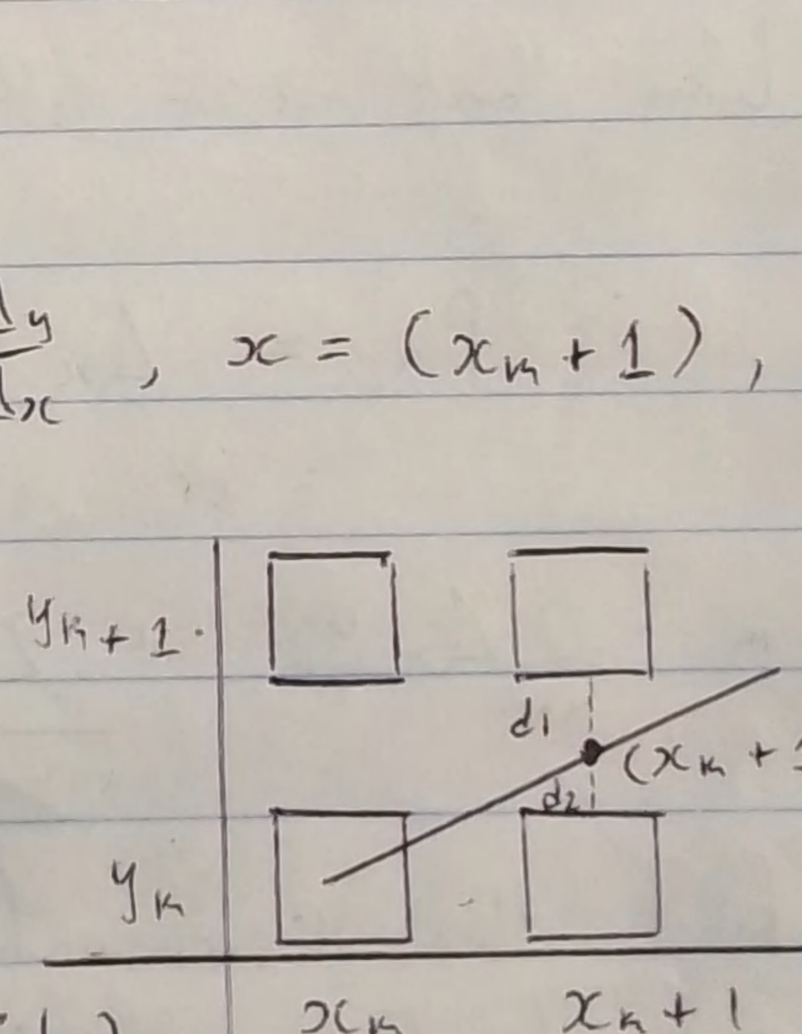
In a case where the tan θ of a line is less than 45°, Bresenham's Algorithm tries to decide which pixels to select, in other words, whether or not y_k changes or stays the same. My derivation will be based on the instance, for instance if a line passes through 2 pixels, bresenham's algorithm checks which pixel is closer in distance to the line and whichever pixel ends up being that, it selects that one over the other which results in smoother lines when displayed on a Raster Graphics display as supposed to DDA's crooked lines result we get when the line goes through multiple pixels at a time

Formula for a vector line

$y = mx + c$

where m (slope) $= \frac{\Delta y}{\Delta x}$, $x = (x_k + 1)$, $c =$ y-intercept

$y = m(x_k + 1) + c$



Calculate distance 1 (d_1) and distance 2 (d_2)

$d_1 = y - y_k$

$d_2 = y_k + 1 - y$

Replace y value

$d_1 = y - y_k = (m(x_k + 1) + c) - y_k$

$d_2 = y_k + 1 - y = y_k + 1 - (m(x_k + 1) + c)$

Open brackets

$d_1 = m(x_k + 1) + c - y_k$

$d_2 = y_k + 1 - m(x_k + 1) - c$

Finding which distance is closer

if $d_1 - d_2 < 0$ (d_1 is closer which means we select y_k)

if $d_1 - d_2 > 0$ (d_2 is closer which means we select $y_k + 1$)

$$d_1 - d_2 = [m(x_k+1)+c-y_k] - [y_k + 1 - m(x_k+1) - c]$$

Open brackets

$$= m(x_k+1)+c-y_k-y_k+1+m(x_k+1)+c$$

$$= 2m(x_k+1) - 2y_k + 2c$$

Substitute $m = \frac{\Delta y}{\Delta x}$ and multiply both sides by Δx because we don't want any fractions or decimals in bresenham's algorithm

$$\Delta x(d_1-d_2) = \cancel{\Delta x}\left[2\frac{\Delta y}{\cancel{\Delta x}}(x_k+1) - 2y_k + 2c - 1\right]$$

Open brackets

$$\Delta x(d_1-d_2) = 2\Delta y\, x_k + 2\Delta y - 2\Delta x\, y_k + 2\Delta x c - \Delta x$$

Organise based on constants (~~terms~~ terms without y_k or x_k)

$$\Delta x(d_1-d_2) = 2\Delta y\, x_k - 2\Delta x\, y_k + 2\Delta y + 2\Delta x c - \Delta x$$

And just like that we've gotten our initial decision variable formula (P_k)

$$P_k = \Delta x(d_1-d_2)$$

or

$$P_k = 2\Delta y\, x_k - 2\Delta x\, y_k + \underbrace{2\Delta y + 2\Delta x c - \Delta x}_{\text{constants}}$$

Next we want to find the incremental decision variable (P_{next} or P_{k+1}), to do this we climinate constants from our P_k formula leaving us with

$$P_k = 2\Delta y\, x_k - 2\Delta x\, y_k$$

With the above we can say P_{k+1} is

$$P_{next} = 2\Delta y\, x_{next} - 2\Delta x\, y_{next}$$

Now as our line ~~edge~~ on our screen moves from one pixel to another we want to subtract $P_{next} - P_m$

$$P_{next} - P_m = [2\Delta_y x_{next} - 2\Delta_x y_{next}] - [2\Delta_y x_m - 2\Delta_x y_m]$$

Open brackets

$$P_{next} - P_m = 2\Delta_y x_{next} - 2\Delta_x y_{next} - 2\Delta_y x_m + 2\Delta_x y_m$$

#

$$P_{next} - P_m = 2\Delta_y (x_{next} - x_m) - 2\Delta_x (y_{next} - y_m)$$

Now as we move from one screen pixel to another x_{next} is always incremented, while y_{next} given the tan θ of the line < 45° degrees, y_{next} doesn't always increment. So our focus is on y_{next}, to find out it's current value on each step

NOTE:

If $P_{next} - P_m < 0$ — x_{next} increases by 1, y_{next} stays the same

If $P_{next} - P_m > 0$ — x_{next} increases by 1, y_{next} increases by 1

Formula for P_{next}, when $P_{next} - P_m < 0$

$$P_{next} - P_m = 2\Delta_y (x_{next} - x_m) - 2\Delta_x (y_{next} - y_m)$$

move $-P_m$ across

$$P_{next} = P_m + 2\Delta_y (x_{next} - x_m) - 2\Delta_x (y_{next} - y_m)$$

substitute x_{next} and y_{next}

$$P_{next} = P_m + 2\Delta_y (\cancel{x_m} + 1 - \cancel{x_m}) - \cancel{2\Delta_x (y_m - y_m)}$$

$$\hookrightarrow 2\Delta_x \cancel{y_m} - 2\cancel{\Delta_x y_m}$$

$$P_{next} = P_m + 2\Delta_y$$

Formula for P_{next}, when $P_{next} - P_m > 0$

$$P_{next} - P_m = 2\Delta_y (x_{next} - x_m) - 2\Delta_x (y_{next} - y_m)$$

move $-P_m$ across

$$P_{next} = P_m + 2\Delta_y (x_{next} - x_m) - 2\Delta_x (y_{next} - y_m)$$

$$P_{next} = P_m + 2\Delta_y(\cancel{x_m} + 1 - \cancel{x_m}) - 2\Delta_x(\cancel{y_m} + 1 - \cancel{y_m})$$

$$P_{next} = P_m + 2\Delta_y - 2\Delta_x$$

FORMULA SUMMARY

If $P_m < 0$ $\quad P_{next} = P_m + 2\Delta_y$

if $P_m >= 0$ $\quad P_{next} = P_m + 2\Delta_y - 2\Delta_x$

Next we want to derive Initial value to our descision parameter (P_1), using our P_m formula from earlier on with it's constants

$$P_m = 2\Delta_y x_m - 2\Delta_x y_m + 2\Delta_y + 2\Delta_x C - \Delta_x$$

Remove c (y-intercept)

$$y_1 = mx_1 + c$$

$$y_1 = \frac{\Delta y}{\Delta x} x_1 + c$$

$$y_1 - \frac{\Delta y}{\Delta x} x_1 = c \qquad c = y_1 - \frac{\Delta y}{\Delta x} x_1$$

Substitute c in our P_1 equation

$$P_1 = 2\Delta_y x_1 - 2\Delta_x y_1 + 2\Delta_y + 2\Delta_x\left[y_1 - \frac{\Delta_y}{\Delta_x} x_1\right] - \Delta_x$$

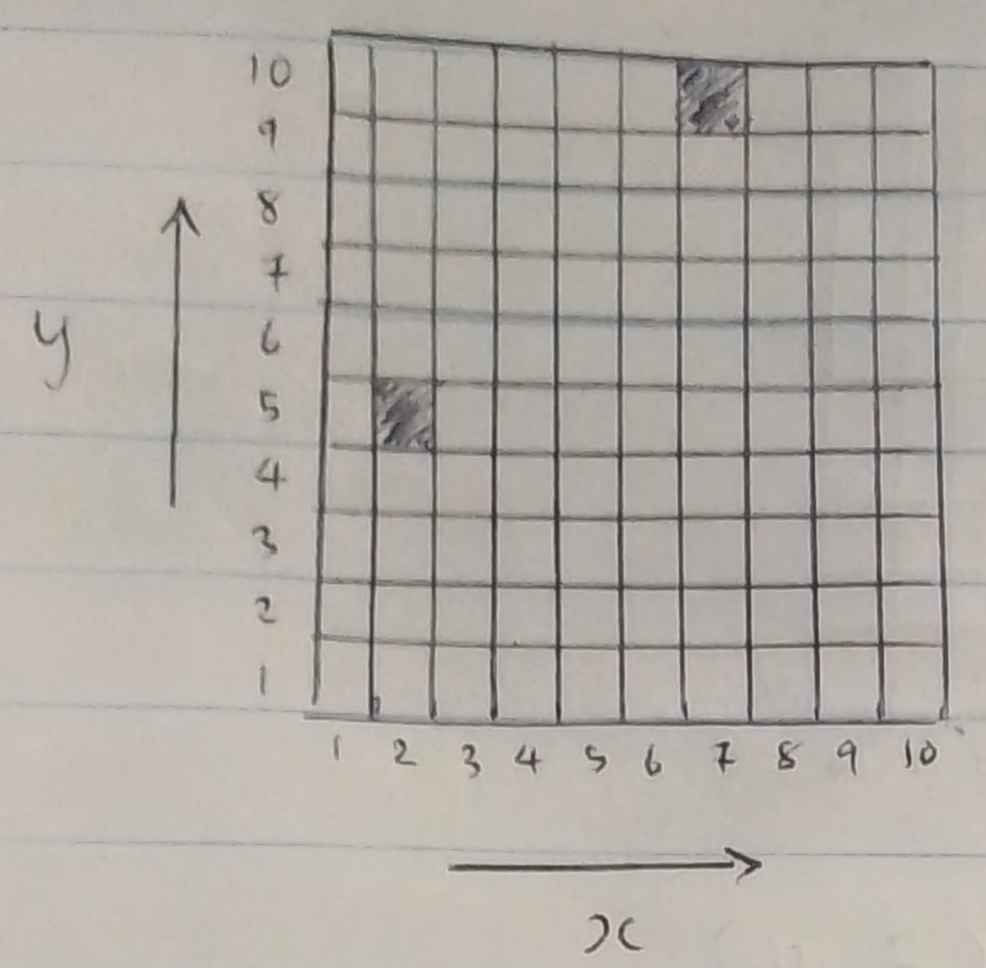
$$P_1 = \cancel{2\Delta_y x_1} - \cancel{2\Delta_x y_1} + 2\Delta_y + \cancel{2\Delta_x y_1} - \cancel{2\Delta_x y_1} - \Delta_x$$

$$P_1 = 2\Delta_y - \Delta_x$$

We've successfully derived our needed descision variables

- $P_1 = 2\Delta_y - \Delta_x$
- If $P_m < 0$ $\quad P_{m+1} = P_m + 2\Delta_y$
- If $P_m >= 0$ $\quad P_{m+1} = P_m + 2\Delta_y - 2\Delta_x$

ii Point A (x_1, y_1) (2, 5) and Point B (x_2, y_2) (7, 10)



$m = 1 \quad \tan\theta = 45°$

STEP 1

Formulas from derivation

- $P_1 = 2\Delta_y - \Delta_x$
- if $P_k < 0 \qquad P_{k+1} = P_k + 2\Delta_y$
- if $P_k >= 0 \qquad P_{k+1} = P_k + 2\Delta_y - 2\Delta_x$

where
$$\Delta_y = y_2 - y_1 = 10 - 5 = 5$$
$$\Delta_x = x_2 - x_1 = 7 - 2 = 5$$
$$P_1 = 2\Delta_y - \Delta_x$$
$$= 2(5) - 5$$
$$= 10 - 5$$
$$= 5$$

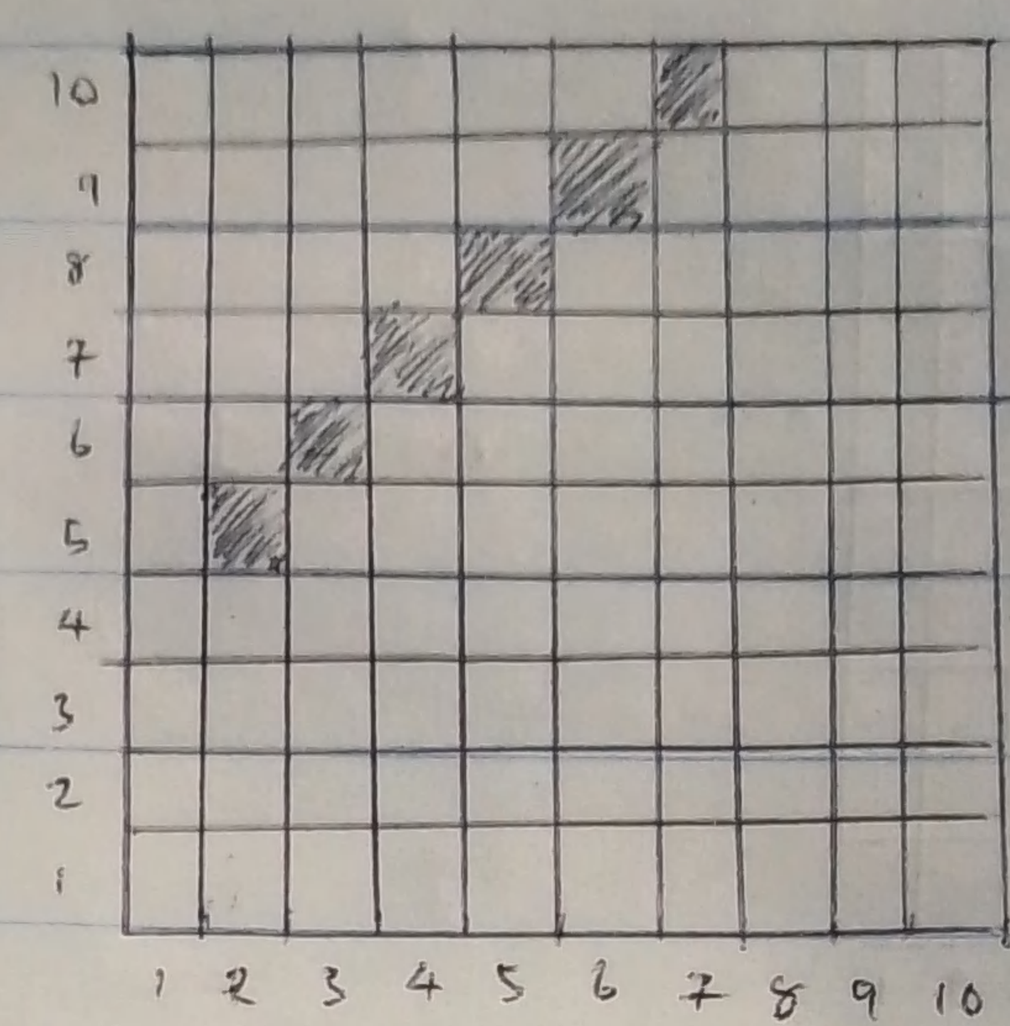
STEP 2: Take Points (Starts from Point A (x_1, y_1) (2, 5))

$x_{inc} = 1 \quad y_{inc} = 1 \quad \Delta_x = 5 \quad \Delta_y = 5$

x	y	p	
2	5	5	# since P >= 0 $\quad P_{next} = P_k + 2\Delta_y - 2\Delta_x = 5 + 2(5) - 2(5) = 5$
3	6	5	# same
4	7	5	# same
5	8	5	# same
6	9	5	# same
7	10	5	# same

Notice: because the $\tan\theta$ of our line is 45°, on each step we increment x and y, if m ~~was~~ $\leq$ 1 we would only increment x consistently and y occassionally on each step of the way

STEP 3: Plot Points on Screen



Points

(2, 5) (3, 6) (4, 7) (5, 8) (6, 9) (7, 10)