CMP418: Data Structures and Algorithms Analysis (3 units)

Lecture 1: Introduction

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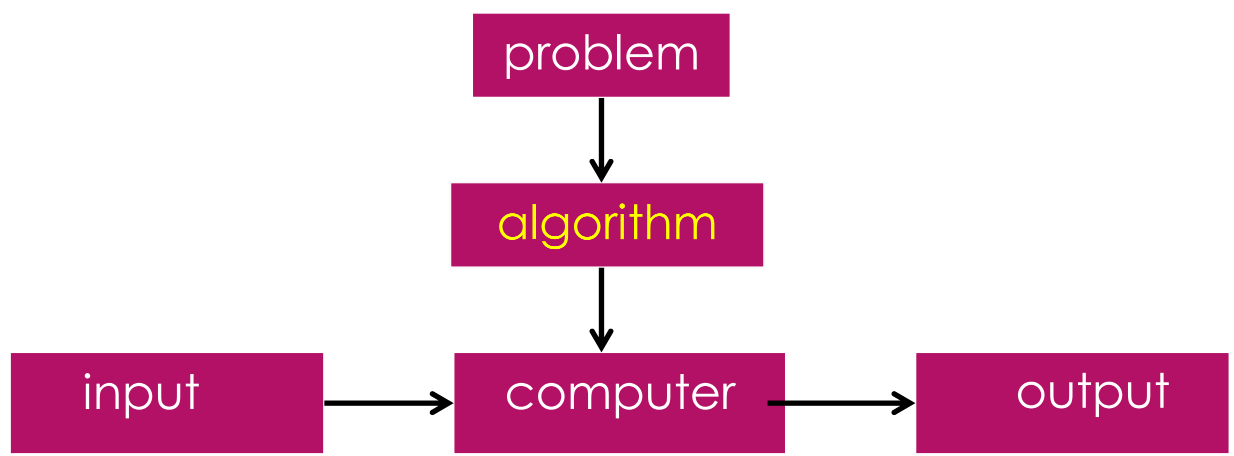
Outline

- What is an Algorithm?
  - Properties of Algorithms
- Algorithm design and analysis process.
  - How to Understand program development problem
  - Algorithm Design Techniques
  - …
- Important Problem Types
- Fundamental Data Structure

What is an Algorithm?

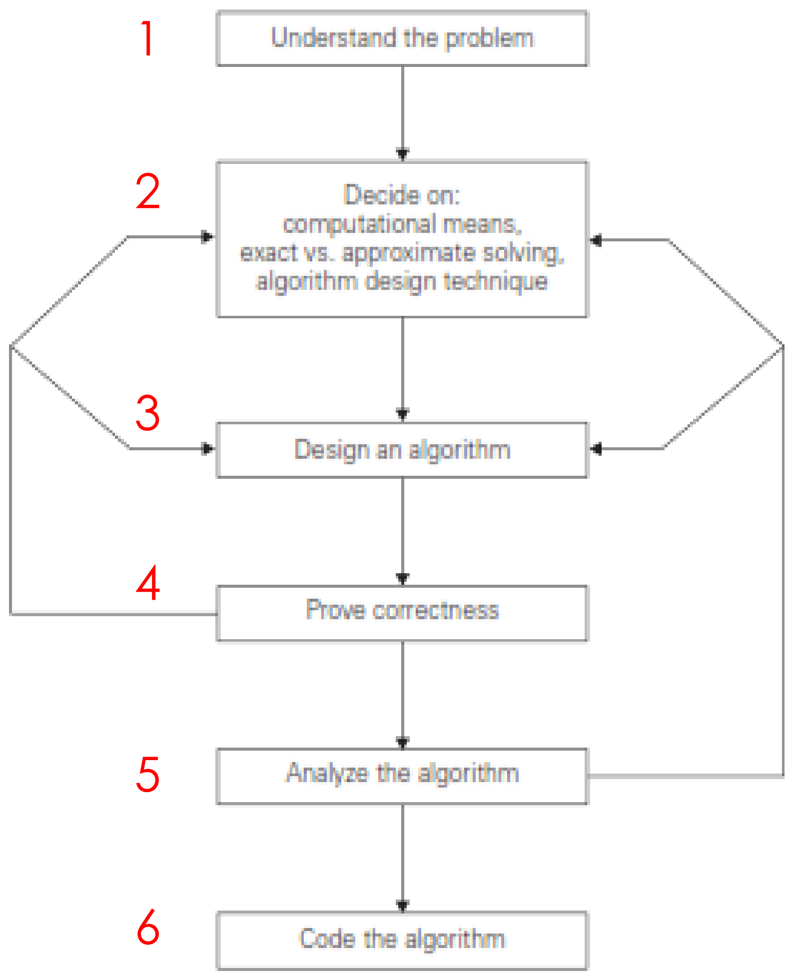
- A sequence of ***unambiguous*** instruction for solving a problem that is, for obtaining a required ***output*** for any legitimate ***input*** in a ***finite*** amount of time.



Properties of Algorithms

- **Finiteness:** terminates after a finite number of steps.
- **Definiteness:** each step must be rigorously and unambiguously specified, e.g., “stir until lumpy”
- **Input:** valid inputs must be clearly specified
- **Output:** data that result upon completion of the algorithm must be specified
- **Effectiveness:** steps must be simple and basic, e.g., check if m is prime

Algorithm Design and Analysis Process.



Algorithm Design and Analysis Process. Con't

1. Understanding the Problem

- Ask questions, do a few small examples by hand, think about special cases, etc.
- An input is an instance of the problem the algorithm solves
- Specify exactly the set of instances the algorithm needs to handle

2.1 Decide on Exact vs. approximate solution

- Approximate solution cannot solve exactly, e.g., extracting square roots, solving definite integrals, etc.
- Algorithms for exact solution can be unacceptably slow

Algorithm Design and Analysis Process. Con't

2.2 Algorithm Design Techniques

- is a general approach to solving problems algorithmically that is applicable to a variety of problems from different areas of computing such as.
- Brute force (Chapter 3)
- Decrease and Conquer (Chapter 4)
- Divide and Conquer (Chapter 5)
- Transform and Conquer (Chapter 6)
- Space and time tradeoffs (Chapter 7)
- Dynamic Programming (Chapter 8)

Algorithm Design and Analysis Process. Con't

3. Algorithm Design and Data Structure
   - Programs = Algorithms + Data Structures
   - Data structures: List, Array, Dictionary, Set, Queue, Stack, etc
4. Prove correctness of the algorithm
   - Algorithm yields required **output** for every **legitimate input** in **finite** time
   - Technique for proving correctness of;
     - **Exact algorithm** Mathematical induction can be used due to natural sequence of steps in an algorithm
     - **Approximate algorithm** show that the error produced by the algorithm does not exceed a predefined limit

Algorithm Design and Analysis Process. Con't

5. Analyzing an Algorithm

- **Time efficiency:** how fast it runs
- **Space efficiency:** How much extra memory it uses
- **Simplicity:** easier to understand, usually contains fewer bugs, sometimes simpler is more efficient, but not always!
- **Generality:** of the problem algorithm solves or the set of inputs it accepts

6. Coding Algorithm Write in a programming language for a real machine

- Standard tricks:
  - Compute loop invariant (which does not change value in the loop) outside loop
  - Replace expensive operation by cheap ones
    - E.g., check if n is even n%2 == 0 is more expensive than (n & 1) == 0

- Algorithm's ***optimality.*** *Actually, this question is not about the* efficiency of an algorithm but about the complexity of the problem it solves: min effort exert to solve a problem.

Important Problem Types

- Sorting
- Searching
- String Processing
- Graph Problem
- Combinatorial Problem
- Geometric Problem
- Numerical Problem

Sorting

- The *sorting problem is to rearrange the items of a given list in nondecreasing* order.
- Properties of sorting Algorithms
  - An algorithm is said to be ***stable*** *if it preserves the* ***relative order*** *of any* ***two equal elements*** *in its input*
  - An algorithm is said to be ***in-place*** if it does not *require* ***extra memory***, except, for a few memory units
- Statement of problem: rearrange the number is a given order
  - Input: a sequence of *N* numbers $<a_1, a_2, \ldots, a_N>$
  - Output: a reordering of the input sequence $<a'_1, a'_2, \ldots, a'_N>$ so that $a'_i \leq a'_j$ whenever $i < j$
  - Instance: the sequence <5, 3, 2, 8, 3> becomes <2, 3, 3, 5, 8> after sorting
- Algorithm: Bubble sort, Selection sort, Insertion sort, Merge sort, Quick sort

Searching

- The *searching problem deals with finding a given value, called a search key, in a* given set.
- Statement of problem: search a key number is a given set
  - Input: a sequence of *N* numbers $<a_1, a_2, \ldots, a_N>$ search key <k>
  - Output: the key <k> from the input sequence $<a'_1, a'_2, \ldots, a'_N>$
  - Instance: the sequence <5, 3, 2, 8, 3> search for 2 becomes return 2 or null if 2 doesn't exist
- Algorithm: sequential search, binary Search

String Processing

- A **string** is a sequence of characters from an alphabet.
- **String matching.** Is a particular problem of searching for a given word in a text.
- **Text strings:** letters, numbers, and special characters
- **Bit strings:** sequence of 0's and 1's
- **Gene sequence:** string of characters from four-character alphabet {A, C, G, T} for Adenine, Cytosine, Guanine, and Thymine, constituents of DNA
- **String matching:** search for a given word/pattern in a text or gene sequence

Graph Problem

- A *graph can be thought of as a collection of points called* ***vertices****, some of* which are connected by line segments called **edges**.
- The ***traveling salesman problem (TSP)*** *is the problem of finding the shortest tour* through n cities that visits every city exactly once. ***Shortest-path***
  - Circuit board and VLSI chip fabrication, X-ray crystallography,
- The ***graph-coloring problem*** *seeks to assign the smallest number of colors to* the vertices of a graph so that no two adjacent vertices are the same color.
  - event scheduling
- **Graph traversal** (how to reach all points in a network ?)
- **Topological sorting** (is a set of courses with prerequisites consistent?)

Combinatorial Problem

- These are problems that ask, explicitly or implicitly, to find a combinatorial object such as
  - a permutation,
  - a combination,
  - or a subset—

that satisfies certain constraints.

- Generally speaking, combinatorial problems are the most difficult problems in computing, from both a theoretical and practical standpoint.
  - the traveling salesman problem
  - graph-coloring problem

Geometric Problem

- ***GP** Deal with geometric objects such as points, lines, and polygons.*
- The ancient Greeks were very much interested in developing procedures for solving a variety of geometric problems,
  - Including problems of constructing simple geometric shapes triangles, circles, and so on with an unmarked ruler and a compass.
- The ***closest-pair*** problem is self-explanatory: given n points in the plane, find the closest pair among them.
- The ***convex-hull problem*** asks to find the smallest convex polygon that would include all the points of a given set.

Numerical Problem

- **NP** are problems that involve mathematical objects of continuous nature:
  - solving equations and systems of equations,
  - computing definite integrals,
  - evaluating functions, and so on.
- Principal difficulty stems from the fact that such problems typically **require manipulating real numbers**, which can be represented in a computer only **approximately**
- Since new applications require primarily algorithms for information storage, retrieval, transportation through networks, and presentation to users
  - **NP has lost its place in the computing industry**

Fundamental Data Structures

- Linear Data Structures
  - Array, Linked List, Stacks and Queues
- Graphs
- Trees

Linear Data Structures: Array

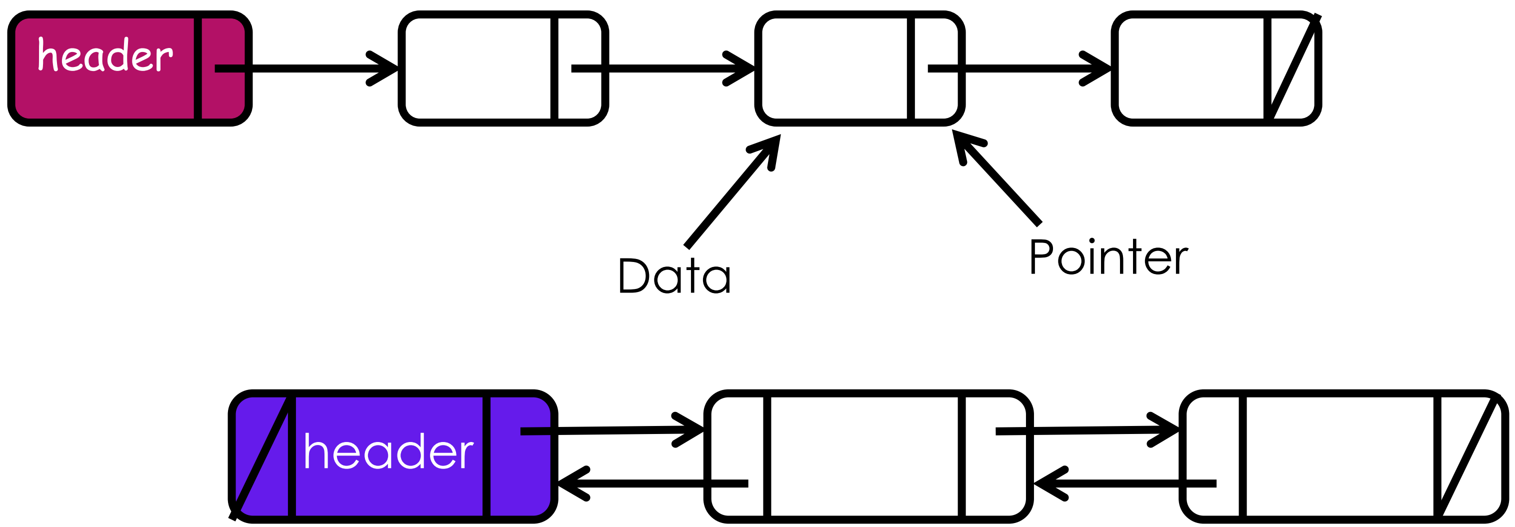
- **Array:** A sequence of n items of the same data type, stored contiguously in memory and made accessible through array indices.

indices →	0	1	2	3	4	5	6	7
A:	20	2	11	23	84	30	6	17

e.g.,
A[4] = 84,
A[2] = 1,
A[7] = 17

Linear Data Structures: Linked List

- Linked List
  - A sequence of nodes with links (called pointers) to neighbors
  - Singly or Doubly linked lists



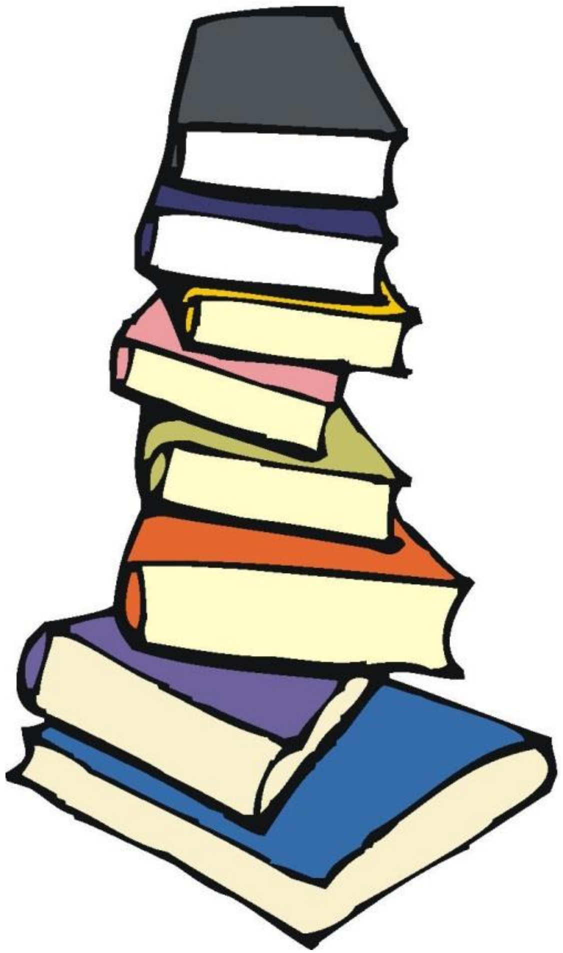
Linear Data Structures: Array vs Linked List

Differences between Array and Array List

Features	Array	Array List
Access time	Fixed	Variable
Update Cost	Higher	Lower
Storage Anticipation	Needed	Not Needed

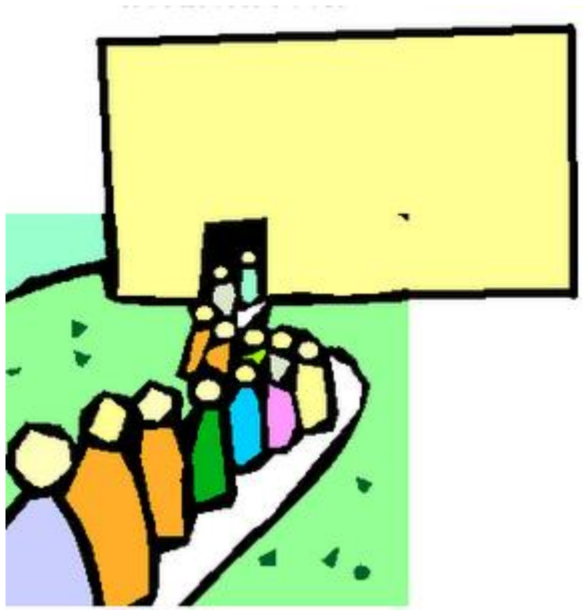
Linear Data Structures: Stack

- Stack
  - Insertion/deletion only at the top
  - Last In First Out (LIFO)
  - Two operations: push and pop
  - E.g., recursion needs stack
  - java.util.Stack



Linear Data Structures: Queue

- Queue
  - Insertion from rear, deletion from front
  - First In First Out (FIFO)
  - Two operations: enqueue, dequeue

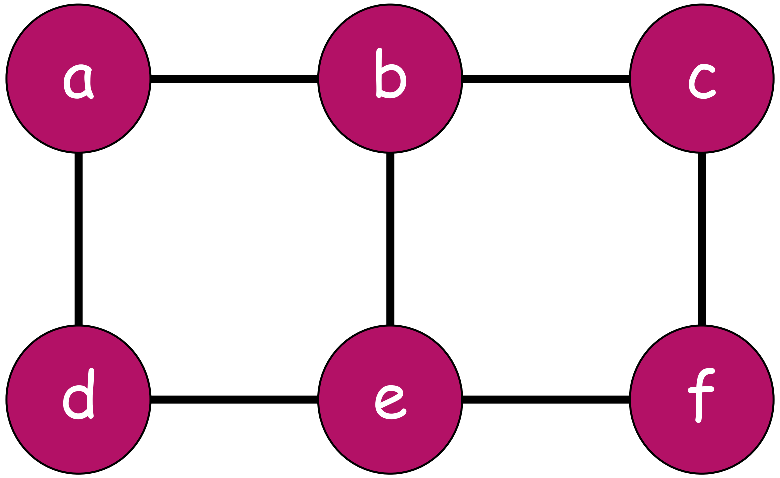


Linear Data Structures: Queue…

- Priority queues
  - Select item of highest priority among dynamically changing candidates. E.g., schedule job in a shared computer
  - Principal operations:
    - Find largest, delete largest, add new element
    - Add/delete must result in priority queue
  - Array or Sorted array based implementation is inefficient
  - "**Heap**" is better (We shall discuss in Transform and Conquer approach)

Graph

- A graph, G = < V, E > is defined by a pair of sets:
  - V is called vertices and E is called edges

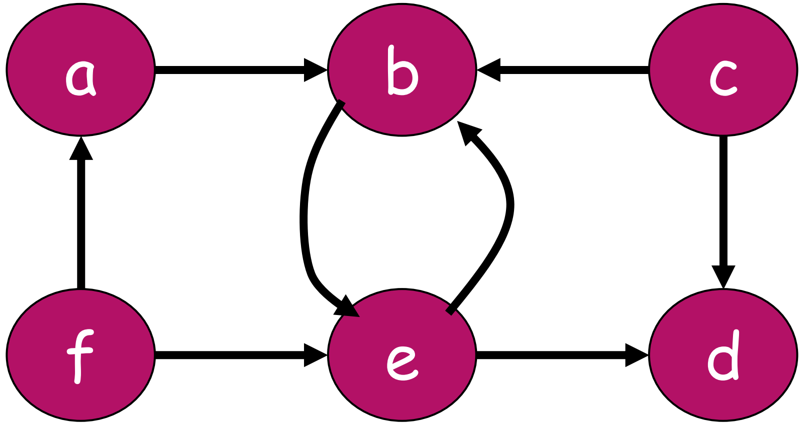


V = {a, b, c, d, e, f}

E = { (a, b), (b, c), (c, f), (f, c), (e, d), (d, a), (b, e) }

Graph

- Undirected graphs and Directed Graphs (Digraphs)
- We usually will not allow graphs to have loops
- What is the **maximum number of edges** in an undirected graph with |V| vertices?



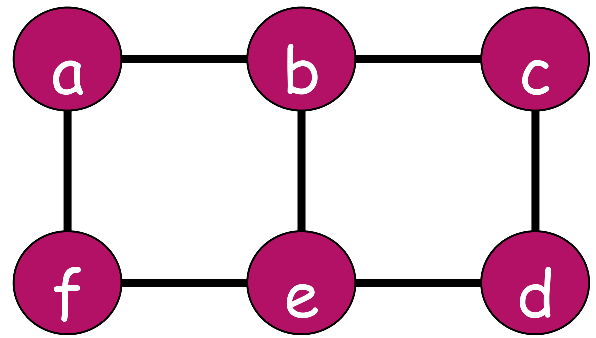
Directed graph

Representing Graphs

- Adjacency matrix
  - N × N boolean matrix
  - A[i, j] = 1 if there is an **edge** from i-th vertex to j-th vetex
  - Adjacency matrix of an undirected graph is symmetric.
- Adjacency list
  - A collection of linked lists, one for each vertex, that contain all neighbors of a vertex

Representing Graphs

$$
\begin{array}{c|cccccc}
 & a & b & c & d & e & f \\
\hline
a & 0 & 1 & 0 & 0 & 0 & 1 \\
b & 1 & 0 & 1 & 0 & 1 & 0 \\
c & 0 & 1 & 0 & 1 & 0 & 0 \\
d & 0 & 0 & 1 & 0 & 1 & 0 \\
e & 0 & 1 & 0 & 1 & 0 & 1 \\
f & 1 & 0 & 0 & 0 & 1 & 0
\end{array}
$$



a $\longrightarrow$ b $\longrightarrow$ f

b $\longrightarrow$ a $\longrightarrow$ c $\longrightarrow$ e

c $\longrightarrow$ b $\longrightarrow$ d

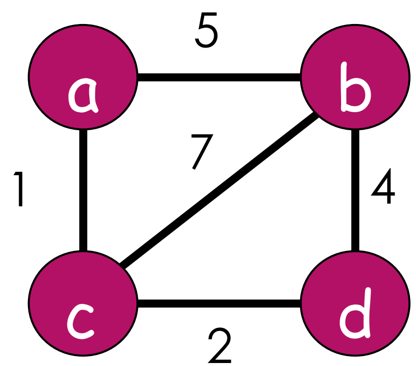
d $\longrightarrow$ c $\longrightarrow$ e

e $\longrightarrow$ b $\longrightarrow$ d $\longrightarrow$ f

f $\longrightarrow$ a $\longrightarrow$ e

Weighted Graphs

- Edges have numbers associated with them, e.g., distance between two cities



$$
\begin{array}{c}
\\
\begin{matrix} a \\ b \\ c \\ d \end{matrix}
\end{array}
\begin{array}{c}
\begin{matrix} a & b & c & d \end{matrix} \\
\begin{pmatrix}
\infty & 5 & 1 & \infty \\
5 & \infty & 7 & 4 \\
1 & 7 & \infty & 2 \\
\infty & 4 & 2 & \infty
\end{pmatrix}
\end{array}
$$

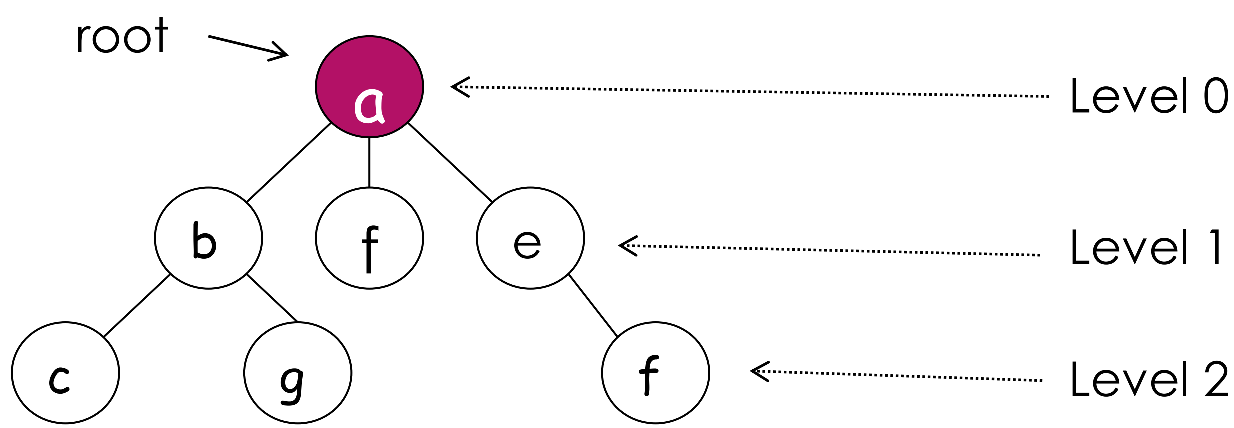
a	→ b, 5 → c, 1
b	→ a, 5 → c, 7 → d, 4
c	→ a, 1 → b, 7 → d, 2
d	→ b, 4 → c, 2

Tree

- A tree is a connected acyclic graph. $|E| = |V|-1$
- Forest: has no cycle but may be disconnected
- Between every two vertices there is exactly one simple path.
- Any vertex can be regarded as the root,
- and then we have a rooted tree.

Tree…

- Ancestors: For vertex v, all the vertices on the simple path from root to v are called v's ancestors.

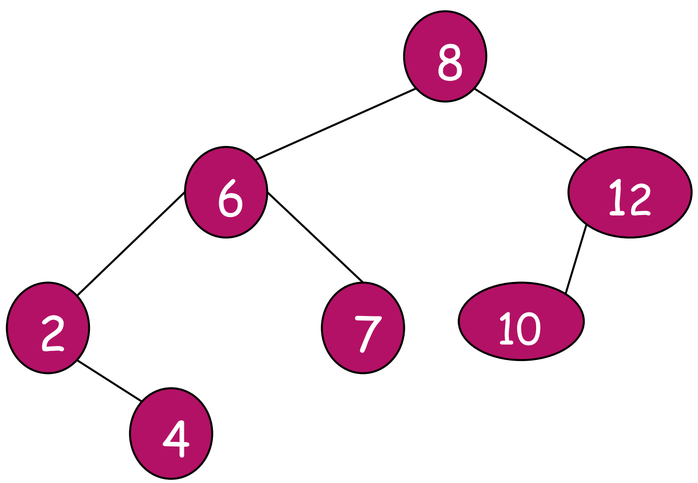


Tree…

- **Descendants:** All vertices for which v is an ancestor are descendants of v.
- **Parent, child, sibling:**
  - If (u, v) is the last edge of the simple path from root to v, then u is *parent* of v and v is a *child* of u.
  - Vertices having same parents are siblings.
- **Leaves:** A vertex without children is a leaf.
- **Subtree:** v with all its descendants is the subtree rooted at v.
- **Depth of vertex:** Length of the simple path from root to v is v’s depth.
- **Height of tree:** The length of the longest simple path from root to leaf.

Ordered Tree

- An ordered tree is a rooted tree in which all children of each vertex are ordered.
- **Binary tree:**
  - An ordered tree, each vertex has at most two children and each children is designated as either left child or right child.
  - $\lfloor \log_2 n \rfloor \leq h \leq n-1$, here n is the number of
  - nodes in the tree and h is tree's height
- **Binary Search Tree:**
  - Each vertex is assigned a number
  - A number assigned to a parental vertex is larger than all numbers in the left subtree and smaller than all numbers in the right subtree.



What we discussed about

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  - How to Understand program development problem
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Thank You