Data-driven computer animation

Tutorial 1

Prof. Taku Komura
TA: Kemeng Huang(kmhuang@cs.hku.hk)

Tutorial 1.1: Basic Linear Algebra in Graphics

Computer Graphics

More dependent on Linear Algebra

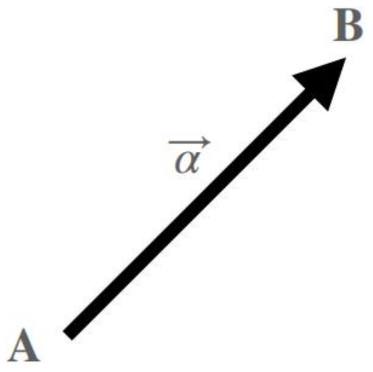
- Vectors
- Matrices

Examples

- direction is a vector
- operations like translating or rotating objects can be matrix-vector multiplication.

Vectors

- Usually written as $\overrightarrow{\alpha}$ or in bold α.
- Direction and length.
- No absolute starting position.

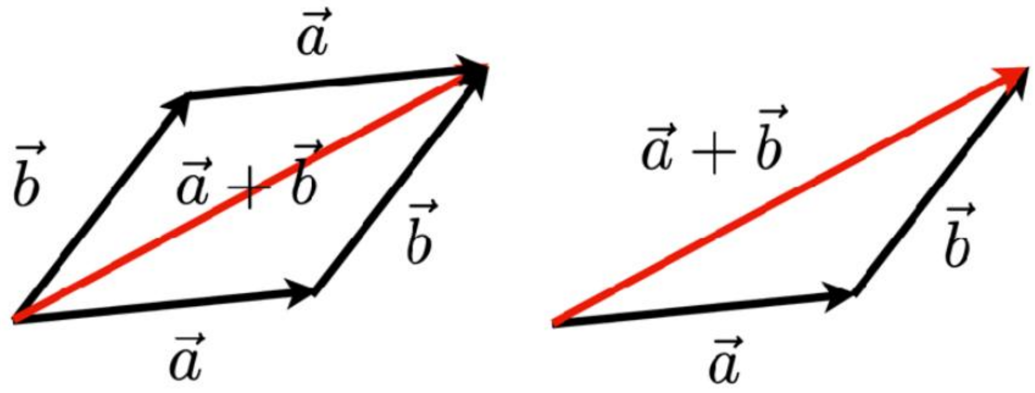


Vector Normalization

- Magnitude (length) of a vector written as $||\overrightarrow{\alpha}||$
- Unit Vector
  - A vector with a magnitude of 1
  - Finding the unit vector of a vector (normalization)
    - $\hat{a} = \dfrac{\vec{a}}{||\vec{a}||}$
  - Used to represent directions

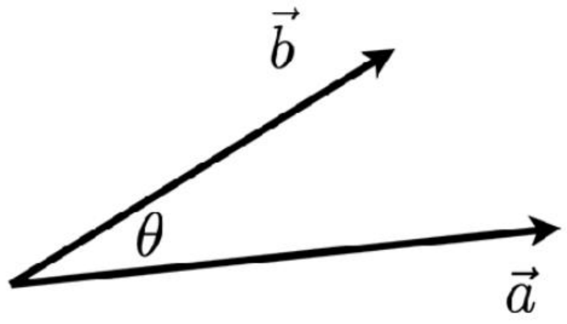
Vector Addition

- Algebraically: Simply add coordinates
- Geometrically: Parallelogram law & Triangle law



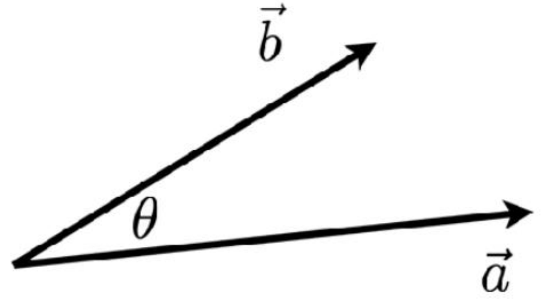
Vector Dot Product

- $\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$
- Component-wise multiplication, then adding up
  - $\vec{a} \cdot \vec{b} = \begin{pmatrix} x_a \\ y_a \end{pmatrix} \cdot \begin{pmatrix} x_b \\ y_b \end{pmatrix} = x_a x_b + y_a y_b$ in 2D.
  - $\vec{a} \cdot \vec{b} = x_a x_b + y_a y_b + z_a z_b$ in 3D.

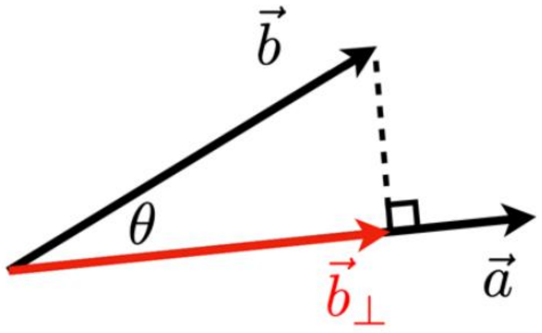


Dot Product in Graphics

- Find the angle between two vectors.
  - $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|}$

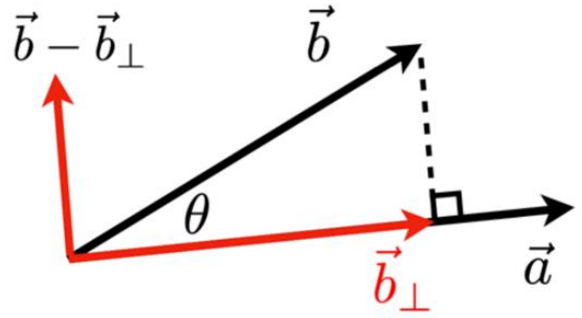


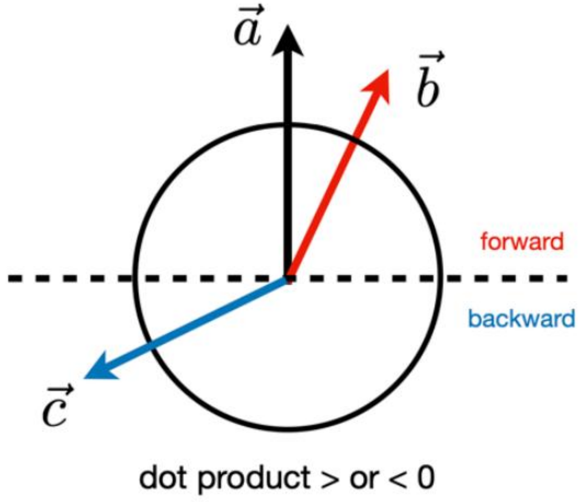
- Finding the projection of one vector on another
  - $\vec{b}_\perp$ is the projection of $\vec{b}$ onto $\vec{a}$.
  - $\|\vec{b}_\perp\| = \|\vec{b}\|\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|}$
  - $\vec{b}_\perp = \|\vec{b}_\perp\|\hat{a}$



Dot Product in Graphics

- Measure how close two directions are
- Decompose a vector
- Determine forward or backward

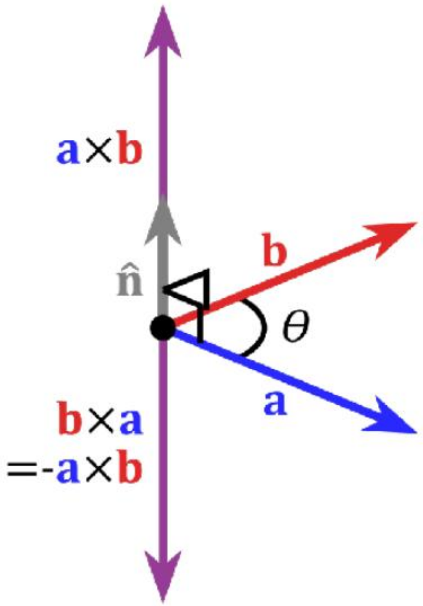




Cross Product

Properties

- Cross product is orthogonal to two initial vectors.
- Direction determined by the right-hand rule

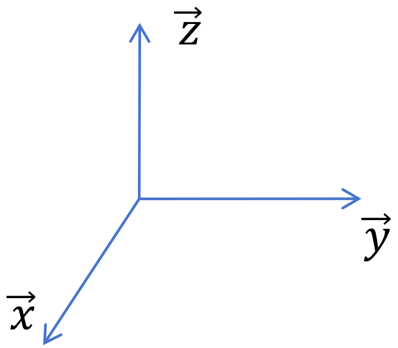


Cross Product

Formulation

$$\vec{a} \times \vec{b} = \begin{pmatrix} y_a z_b - y_b z_a \\ z_a x_b - x_a z_b \\ x_a y_b - y_a x_b \end{pmatrix}$$

Cross Product
Properties



$$\vec{x} \times \vec{y} = +\vec{z}$$

$$\vec{y} \times \vec{x} = -\vec{z}$$

$$\vec{y} \times \vec{z} = +\vec{x}$$

$$\vec{z} \times \vec{y} = -\vec{x}$$

$$\vec{z} \times \vec{x} = +\vec{y}$$

$$\vec{x} \times \vec{z} = -\vec{y}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \times \vec{a} = \vec{0}$$

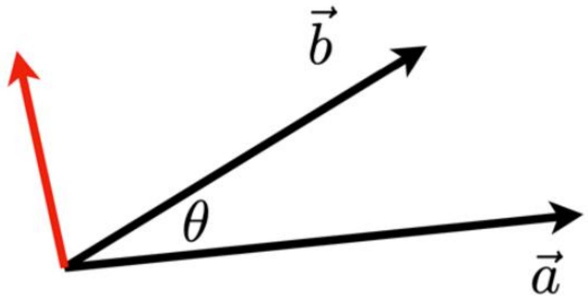
$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

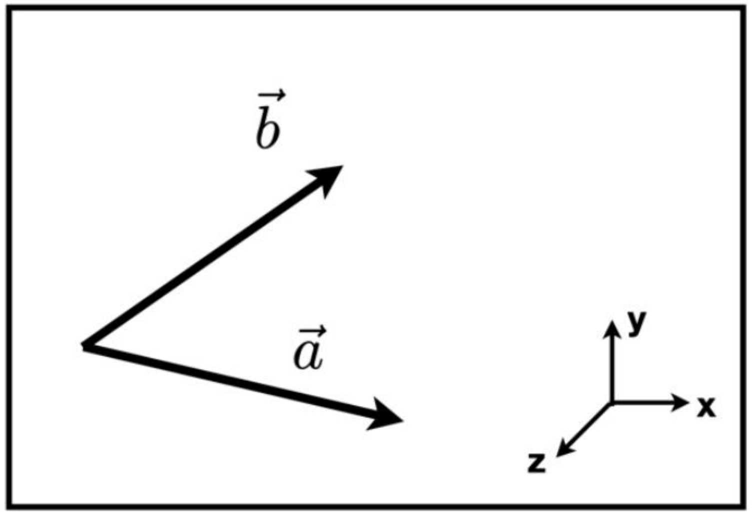
$$\vec{a} \times (k\vec{b}) = k(\vec{a} \times \vec{b})$$

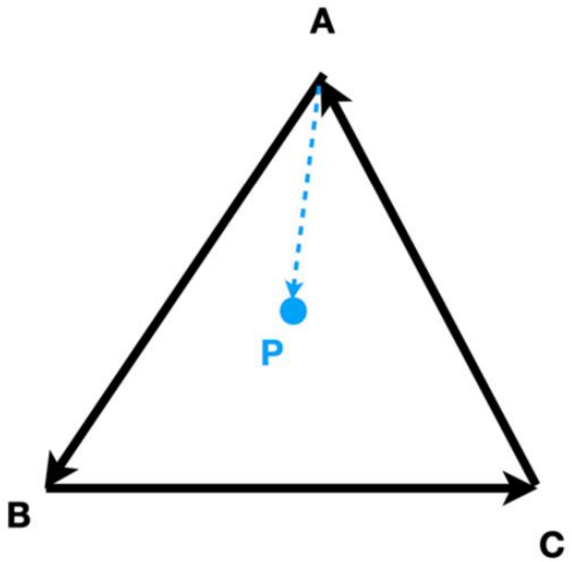
Cross Product

in Graphics

- Determine left / right
- Determine inside / outside







Matrices

- Magical 2D arrays that haunt every CS course
- In Graphics, pervasively used to represent transformations
  - Translation, rotation, shear, scale

Matrices
What is a matrix

- An m x n matrix is an array of numbers with m rows and n columns.

$$\begin{pmatrix} 1 & 3 \\ 5 & 2 \\ 0 & 4 \end{pmatrix}$$

- Addition and multiplication by a scalar are trivial:
  element by element.

Matrices

Matrix-Matrix Multiplication

- # (number of) columns in A must = # rows in B (M x N) (N x P) = (M x P)

$$\mathbf{AB} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix} = \begin{pmatrix} 11 & 14 & 17 & 20 \\ 23 & 30 & 37 & 44 \\ 35 & 46 & 57 & 68 \end{pmatrix} = \mathbf{C}$$

- $\mathbf{C}_{ij}$ = the dot product of the i-th row from matrix A and the j-th column from matrix B.

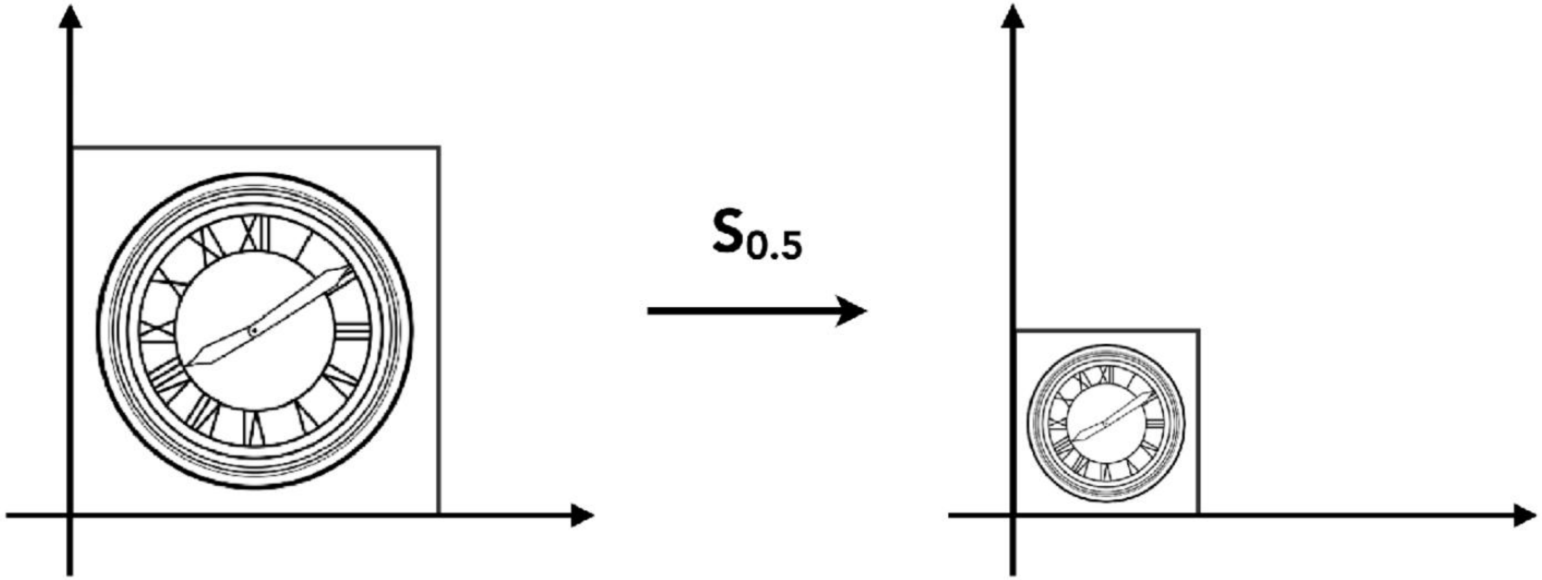
Matrices

Matrix-Vector Multiplication

- Treat vector as a column matrix ($m \times 1$)
- Key for transforming points

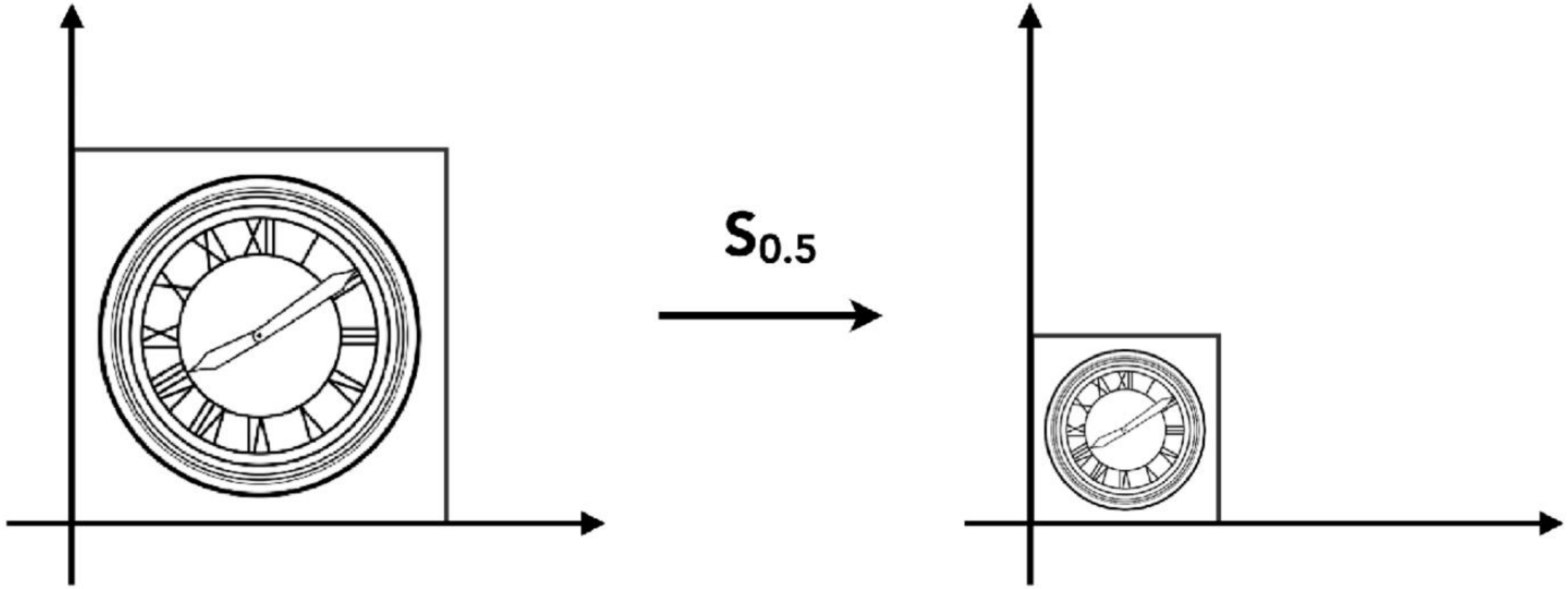
2D transformation

Scale



2D transformation

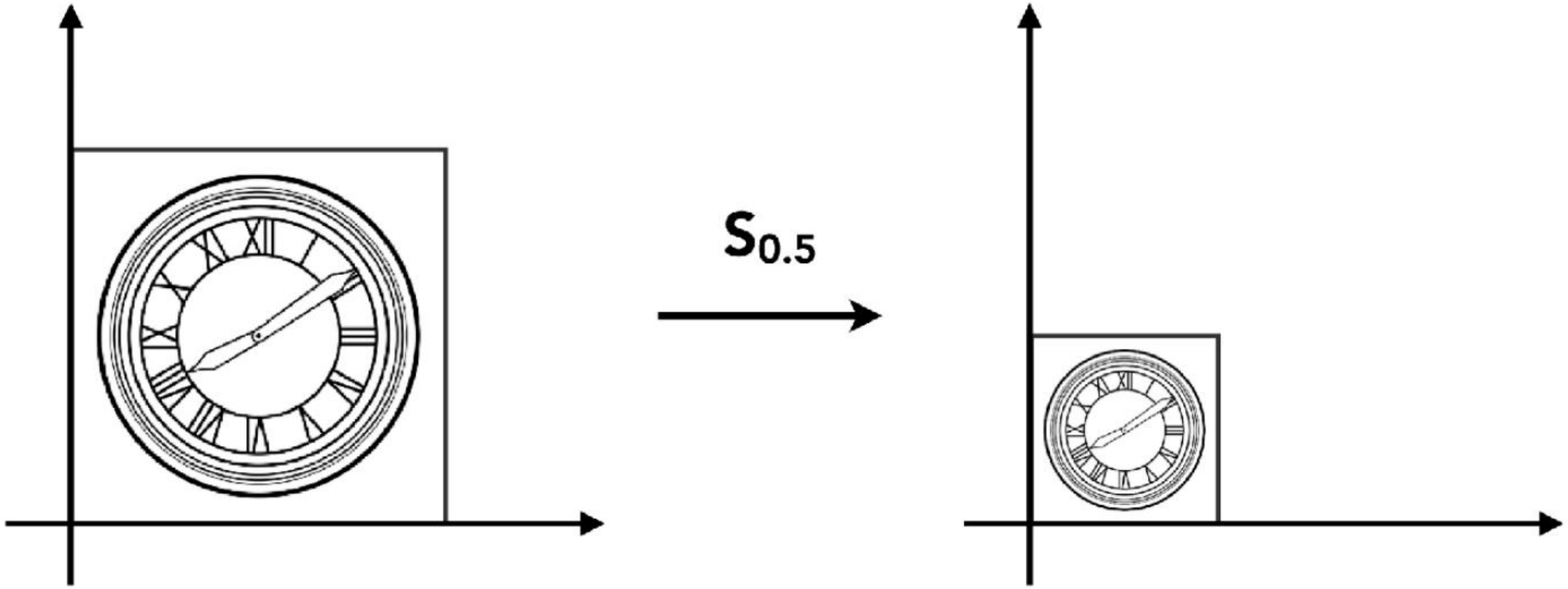
Scale



$$x' = sx$$
$$y' = sy$$

2D transformation

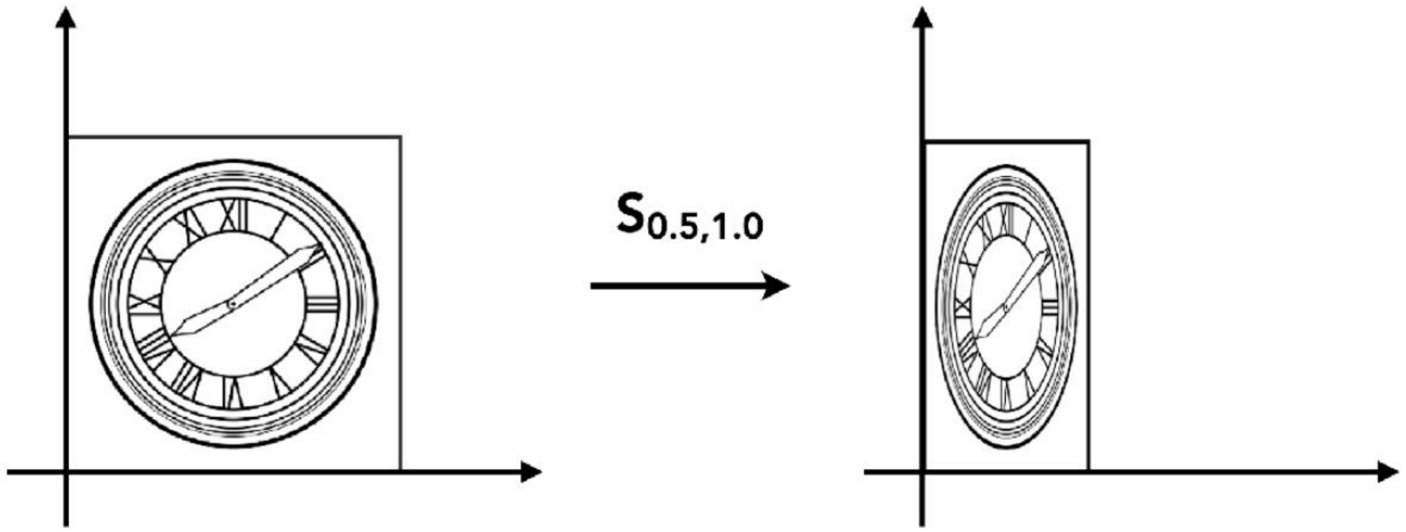
Scale



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D transformation

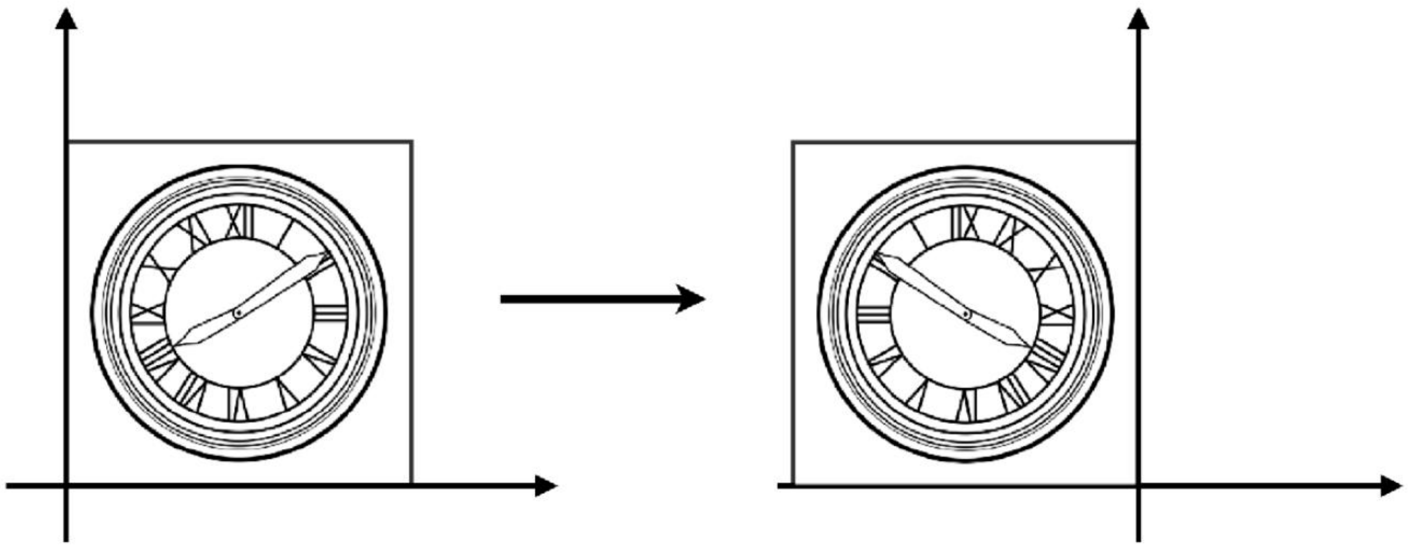
Scale (Non-Uniform)



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D transformation

Scale (Reflection)

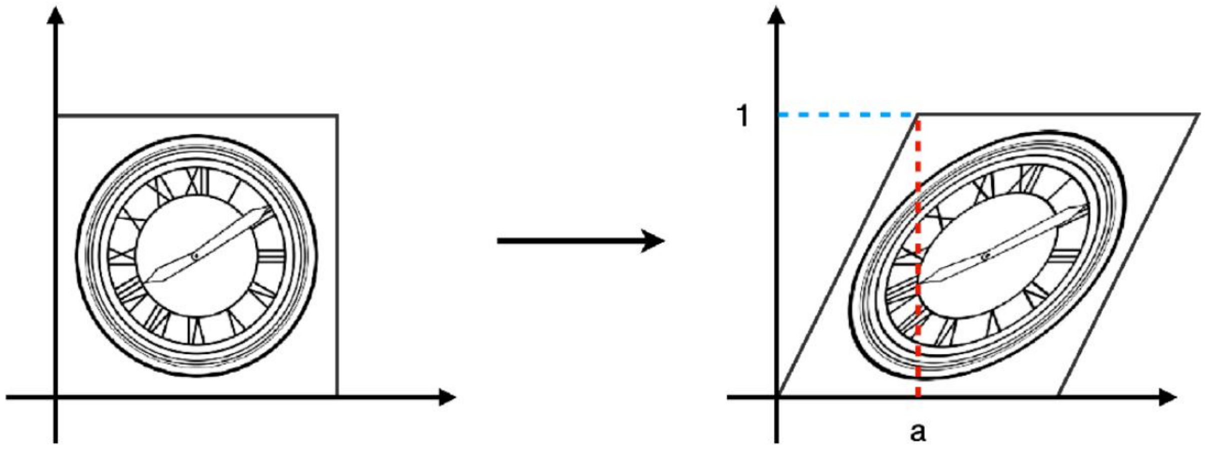


$$x' = -x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D transformation

Shear



Hints:

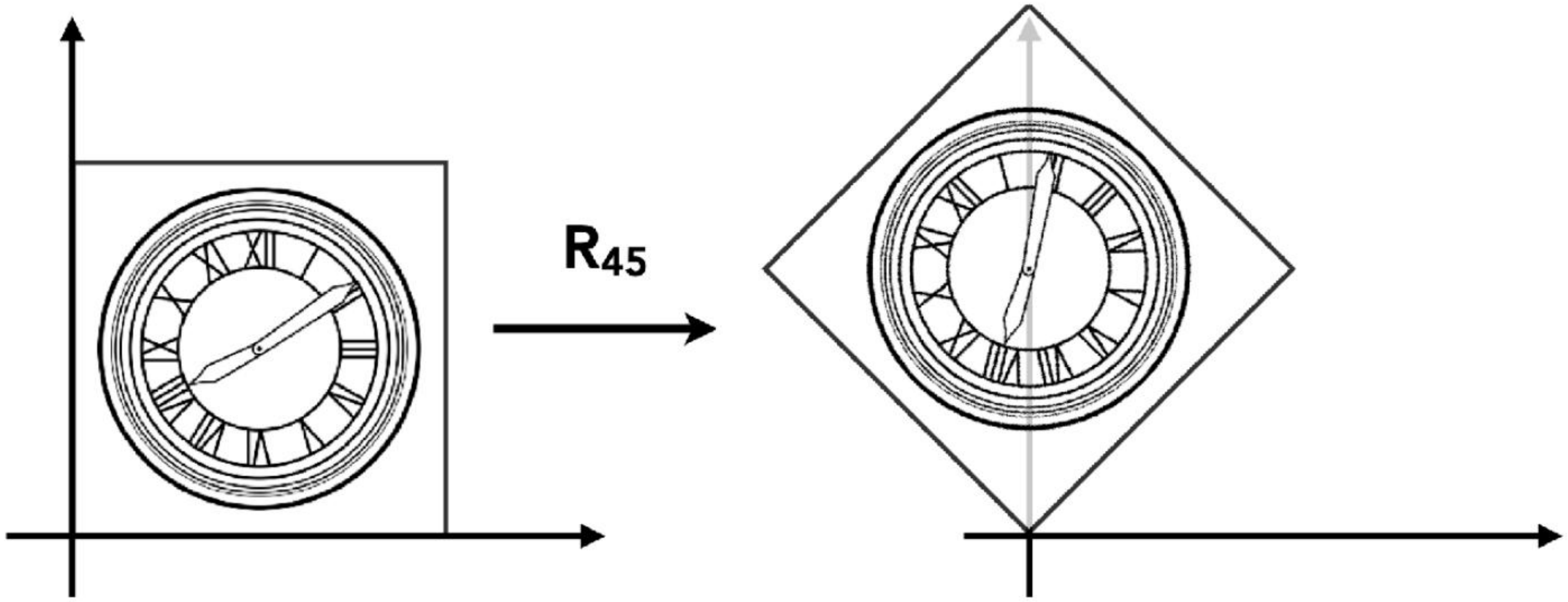
- Horizontal shift is 0 at y=0
- Horizontal shift is a at y=1
- Vertical shift is always 0

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x' = x + ay$$
$$y' = y$$

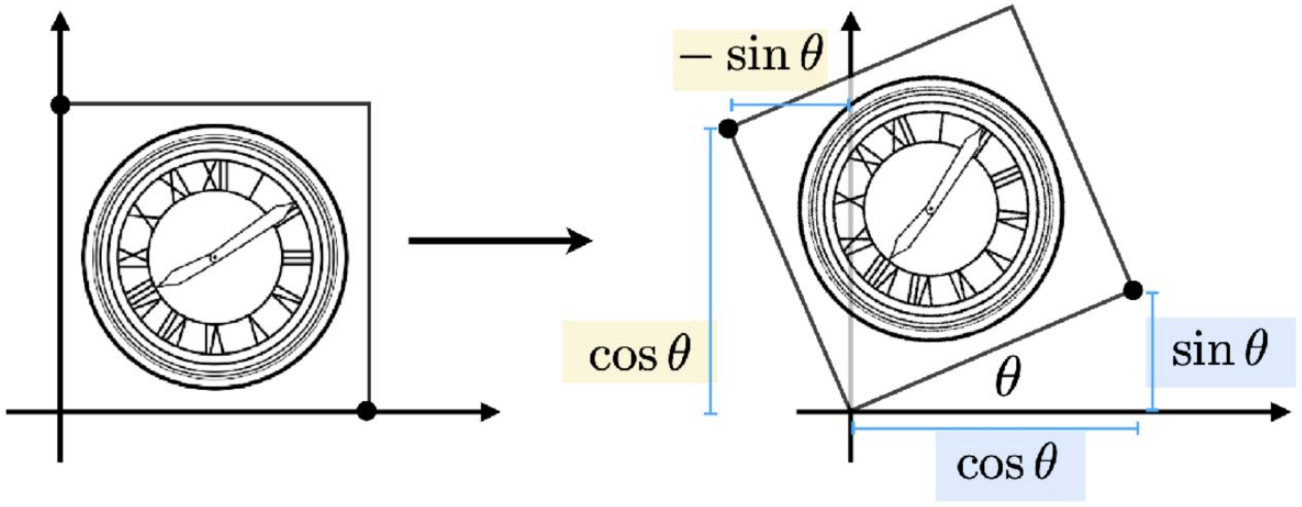
2D transformation

Rotate



2D transformation

Rotate



$$\mathbf{R}_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

2D transformation

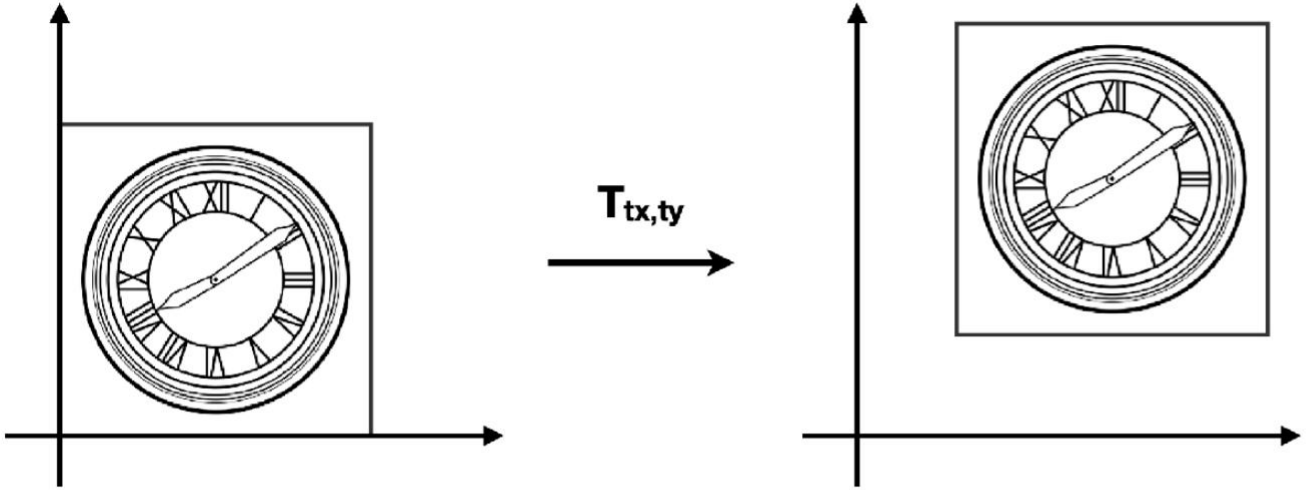
Linear Transformation = Matrices

$$x' = a\,x + b\,y$$
$$y' = c\,x + d\,y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

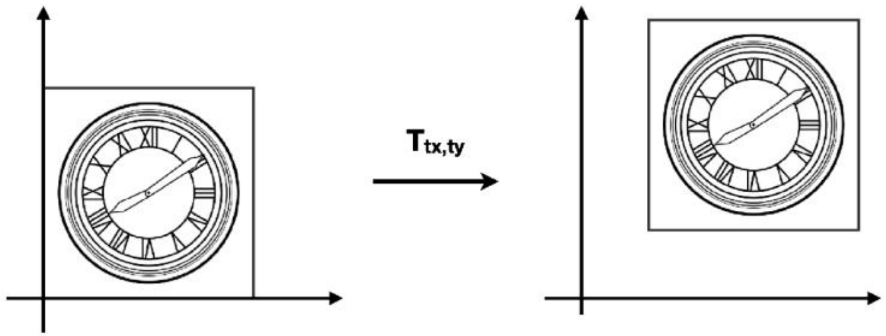
$$\mathbf{x}' = \mathbf{M}\,\mathbf{x}$$

Translation



Homogeneous coordinates

Translation



$$x' = x + t_x$$
$$y' = y + t_y$$

- Translation cannot be represented in matrix form.
- So, translation is NOT a linear transform.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Homogeneous coordinates

Add a new coordinate (w-coordinate.)

- 2d point (x, y, 1)^T, 3d point (x, y, z, 1)^T.
- 2d vector (x, y, 0)^T, 3d vector (x, y, z, 0)^T.

- Matrix representation of translations

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix}$$

Homogeneous coordinates

Valid operation

- Valid if the w-coordinate of the result is either 0 or 1.
  - Vector(0) + Vector(0) = Vector(0) VALID
  - Point(1) + Vector(0) = Point(1) VALID
  - Point(1) - Point(1) = Vector(0) VALID
  - Point(1) + Point(1) = ??(2) NOT VALID

Affine Transformation

- Affine map = linear map + translation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

- Using homogenous coordinates

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Affine Transformation

More 2D examples

Scale

$$\mathbf{S}(s_x, s_y) \;=\; \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rotation

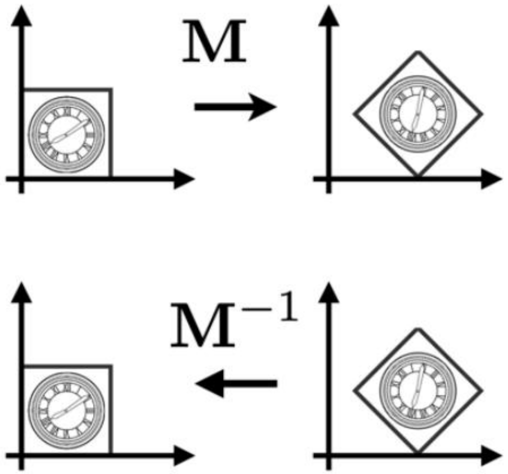
$$\mathbf{R}(\alpha) \;=\; \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Translation

$$\mathbf{T}(t_x, t_y) \;=\; \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

Inverse Transformation

- $\mathbf{M}$ is a transformation matrix. What is $\mathbf{M}^{-1}$?
- $\mathbf{M}^{-1}$ is the inverse of transform $\mathbf{M}$ in both a matrix and geometric sense.



Composite Transform

Transform Ordering Matters

- Matrix multiplication is not commutative.

$$R_{45} \cdot T_{(1,0)} \neq T_{(1,0)} \cdot R_{45}$$

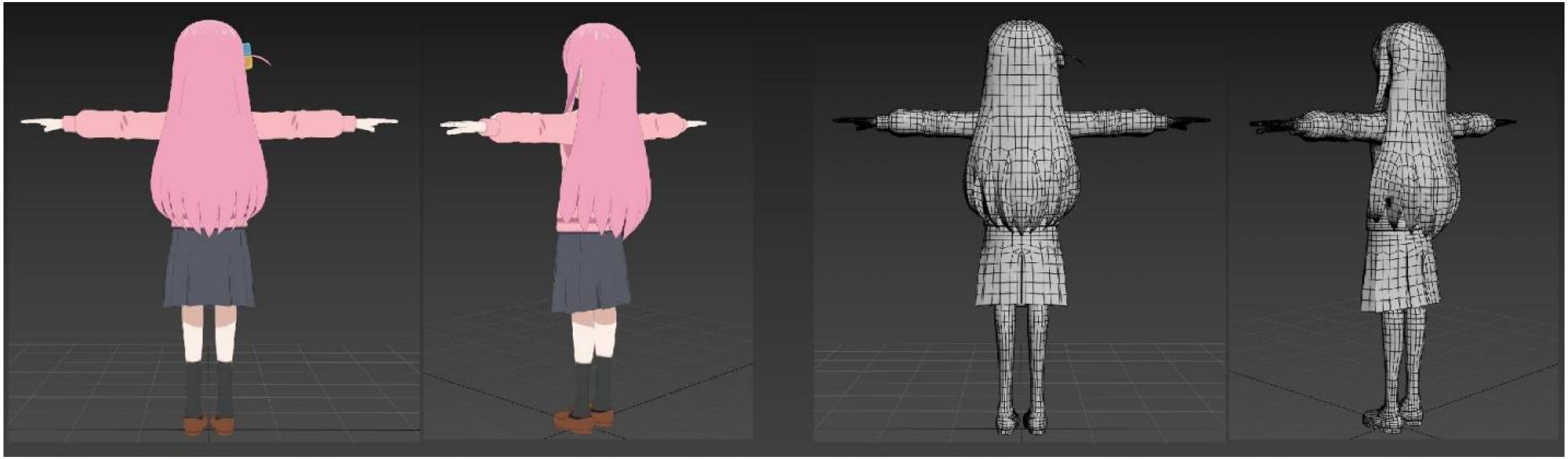
- Note that matrices are applied from right to left:

$$T_{(1,0)} \cdot R_{45} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Tutorial 1.2: Animation in Blender

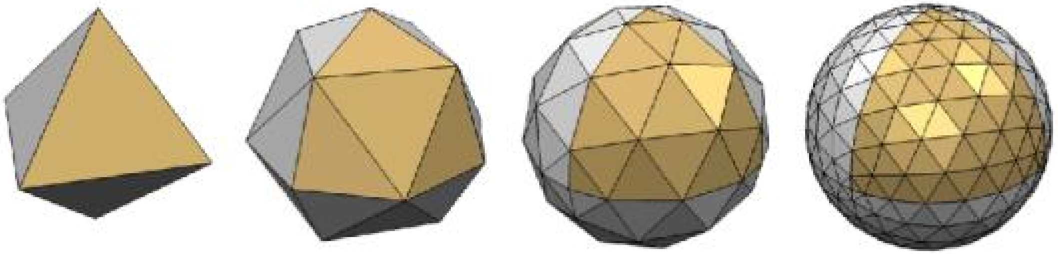
https://www.blender.org/download/

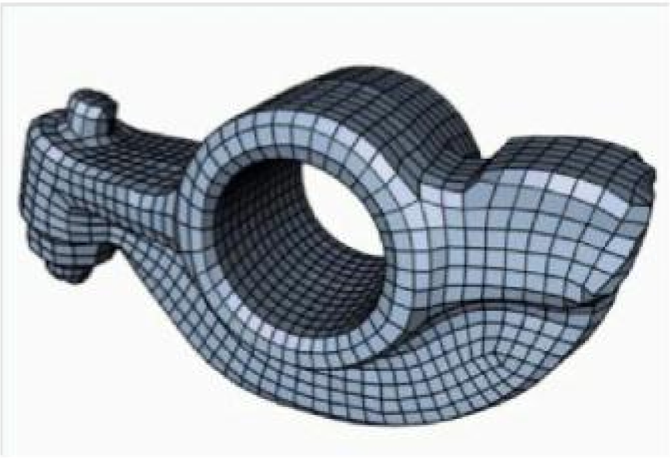
Mesh



Mesh

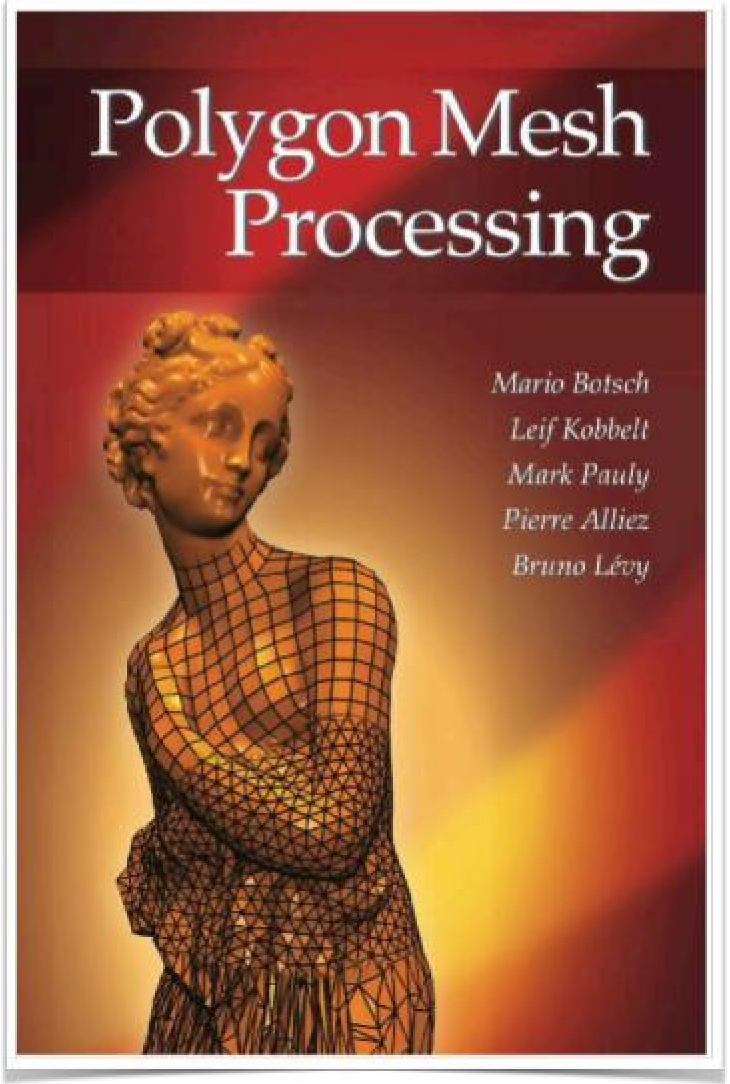
- Boundary representations of objects





Reference Book

- Polygon Mesh Processing Book, Chapter 2



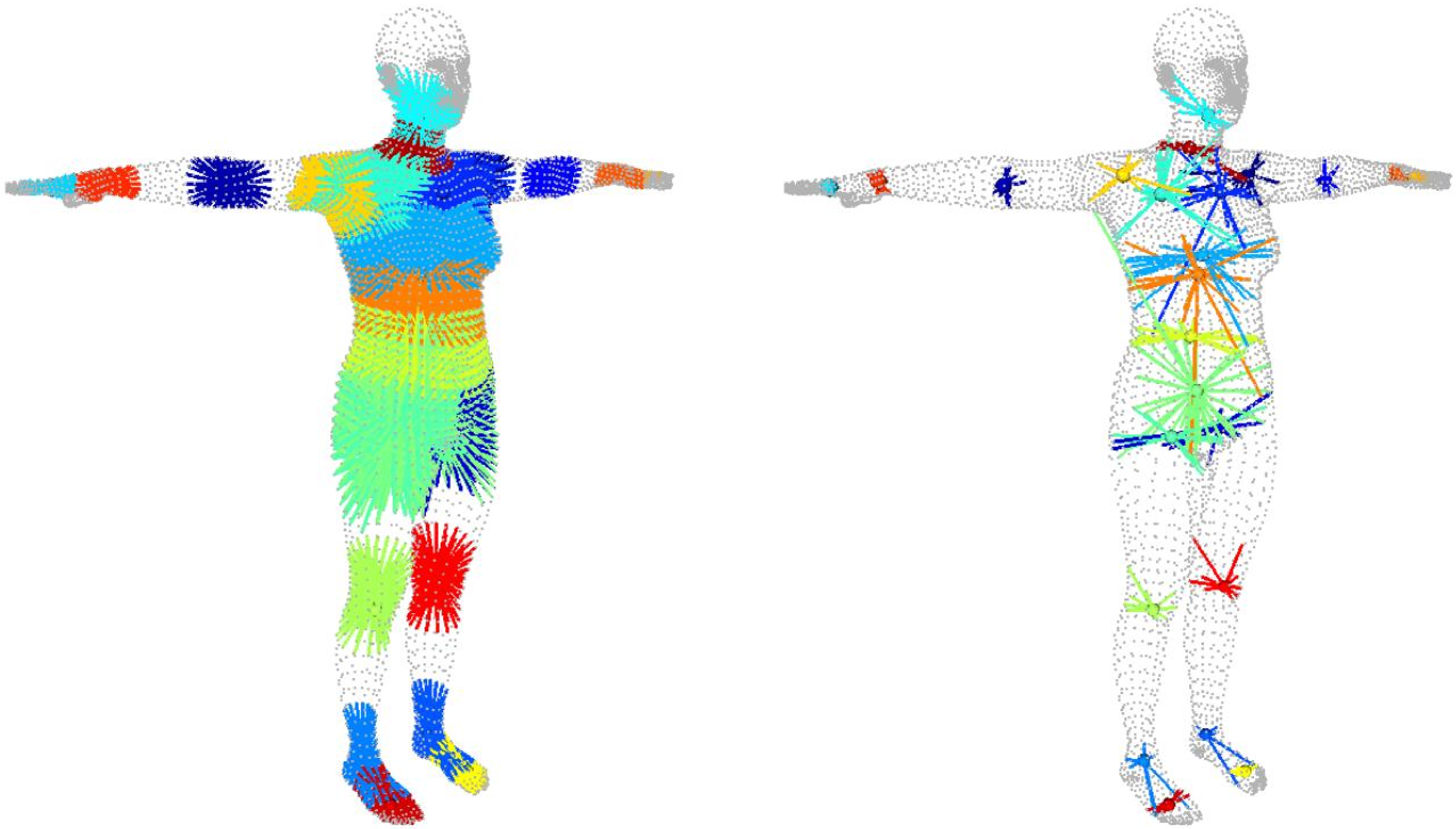
Control the key joints (skeleton)

Animation Example

Skeleton
(Bones)

Mesh

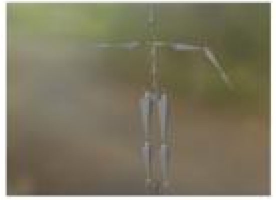
Example: skeleton_in_mesh.blend



Conclusion

- A polygon mesh is used to present digital humans.
- A skeleton will be in the mesh to rig the mesh.
- A weight-binding method is used to manipulate the mesh with a few key joints.

Import **Mesh** -> Import **Skeleton** -> **Bind(Rig)** Mesh and Skeleton



The link of the tutorial video

https://drive.google.com/file/d/1vdSngkQPU8G5gKtdVAe4EUlgB40mB7Oj/view?usp=sharing

Task1
40%

- Download Blender
- Import the provided mesh (feel free to use your mesh if you like)
- Define your key joints and skeleton
- Rigging/Skinning
- Design keyframes animation (feel free to make use of your creativity to add any objects you like)
- Render the video (make use of lights, camera)

Blender Tips
Keyframe Animation

- A new version will be welcomed
- It's free software, so please download the official version.

- Viewpoint Navigation - https://www.youtube.com/watch?v=ILqOWe3zAbk
- Add/delete Object - https://www.youtube.com/watch?v=JSAobQPRLwc
- Rigging - https://www.youtube.com/watch?v=f2pTkW-1JkE
- Keyframe Animation - https://www.youtube.com/watch?v=SZJswvw9wEs
- Google

Thank you