Nome: Kemily Teixeira Cruz   CT11-317

Tarefa Básica - Sistemas Lineares - Regra de Cramer

01. a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$ $D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6-1 = 5$

$Dx = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6-3 = 3$ $x = \dfrac{Dx}{D} = \dfrac{3}{5}$ $V = \left\{ \left(\dfrac{3}{5}, \dfrac{-4}{5} \right) \right\}$

$Dy = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6-(-2) = -4$ $y = \dfrac{Dy}{D} = \dfrac{-4}{5}$

b) $\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$ $D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} \begin{matrix} 3 & -1 \\ 2 & 0 \\ 4 & 1 \end{matrix} = -10 - 13 = -23$

$Dx = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix} \begin{matrix} 1 & -1 \\ -1 & 0 \\ 7 & 1 \end{matrix} = -22 - 1 = -23$ $x = \dfrac{Dx}{D} = \dfrac{-23}{-23} = 1$

$Dy = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & -2 \end{vmatrix} \begin{matrix} 3 & 1 \\ 2 & -1 \\ 4 & 7 \end{matrix} = 32 - 55 = -23$ $y = \dfrac{Dy}{D} = \dfrac{-23}{-23} = 1$

$Dz = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{vmatrix} \begin{matrix} 3 & -1 \\ 2 & 0 \\ 4 & 1 \end{matrix} = 6 - (-17) = 23$ $z = \dfrac{Dz}{D} = \dfrac{23}{-23} = -1$

$$V = \{ (1, 1, -1) \}$$

$-5 - 12 + 48$

02.

$$\begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$$

$D = \begin{vmatrix} 3 & 4 & -1 & 3 & 4 \\ 4 & 5 & 2 & 4 & 5 \\ 1 & -2 & 3 & 1 & -2 \end{vmatrix} = 61 - 31 = 30$

$45 \quad 8 \quad 8$

$-12 + 48 + 12$

$D_y = \begin{vmatrix} 3 & 1 & -1 & 3 & 1 \\ 4 & 12 & 2 & 4 & 12 \\ 1 & 8 & 3 & 1 & 8 \end{vmatrix} = 78 - 48 = 30$

$108 + 2 - 32$

$y = \dfrac{D_y}{D} = \dfrac{30}{30} = 1$ Letra A

03.

$$\begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases}$$

$V = \{(a, b, c)\}$
$a + b + c = ?$

$-2 \quad -33 \quad -6$

$D = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -11 & 3 & 1 \\ 2 & 3 & -1 & 2 & 3 \end{vmatrix} = -36 - (-37) = 1$

$-1 \quad -44 \quad 9$

$1 \quad -33 \quad 4$

$D_x = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ -2 & 1 & -11 & -2 & 1 \\ 1 & 3 & -1 & 1 & 3 \end{vmatrix} = -29 - (-28) = -1$

$-1 \quad -22 \quad -6$

$x = \dfrac{-1}{-1} = -1$

$-4 \quad -11 \quad -3$

$D_y = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & -2 & -11 & 3 & -2 \\ 2 & 1 & -1 & 2 & 1 \end{vmatrix} = -17 - (-18) = 1$

$2 \quad -22 + 3$

$y = \dfrac{1}{1} = 1$

$2 \quad -6 + 6$

$D_z = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -2 & 3 & 1 \\ 2 & 3 & 1 & 2 & 3 \end{vmatrix} = 2 - 2 = 0$

$1 \quad -8 + 9$

$z = \dfrac{0}{1} = 0$

$V = \{(-1, 1, 0)\}$
$-1 + 1 + 0 = 0$
Letra C

04. $\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases}$

$-9 \quad -2 \quad -4$

$D = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} \begin{matrix} 1 & 2 \end{matrix} \quad 3 = 1 - (-15) = 16$

$-6 + 4 + 3$

$-72 \quad -58 \quad -16$

$Dx = \begin{vmatrix} 29 & 2 & 3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix} \begin{matrix} 29 & 2 \end{matrix} \quad 3 = -130 - (-146) = 16$

$-174 + 32 + 12$

$-12 \quad +16 \quad -58$

$Dy = \begin{vmatrix} 1 & 29 & 3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} \begin{matrix} 1 & 29 \end{matrix} \quad 4 = 26 - (-54) = 80$

$-8 + 58 - 24$

$87 \quad -4 \quad +16$

$Dz = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} \begin{matrix} 1 & 2 \end{matrix} \quad 3 = 3 - 99 = -96$

$24 + 8 - 29$

$x = \dfrac{16}{16} = 1 \qquad y = \dfrac{80}{16} = 5 \qquad z = \dfrac{-96}{16} = -6$

$x + y + z = 1 + 5 - 6 = 0$ Letra A

05.

$$\begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$$

$$D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} \begin{matrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{matrix} = 7 - 4 = 3$$

$0 + 4 + 0$

$4 + 3 + 0$

$$Dx = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} \begin{matrix} 5 & 1 \\ 3 & 2 \\ 7 & 2 \end{matrix} = 17 - 13 = 4$$

$0 + 10 + 3$

$10 + 7 + 0$

$x = \dfrac{4}{3}$

$$Dy = \begin{vmatrix} 2 & 5 & 0 \\ 0 & 3 & 1 \\ 3 & 7 & 1 \end{vmatrix} \begin{matrix} 2 & 5 \\ 0 & 3 \\ 3 & 7 \end{matrix} = 21 - 14 = 7$$

$0 + 14 + 0$

$6 + 15 + 0$

$y = \dfrac{7}{3}$

$$Dz = \begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 7 \end{vmatrix} \begin{matrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{matrix} = 37 - 42 = -5$$

$30 + 12 + 0$

$28 + 9 + 0$

$z = \dfrac{-5}{3}$

x, y, z

$\dfrac{4}{3} , \dfrac{7}{3} , \dfrac{-5}{3}$) Letra D

06.

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

$$\begin{cases} x = 3 \\ 2x + y = 7 \\ -x + 2y + 2z = -1 \end{cases}$$

$$\begin{bmatrix} x \\ 2x + y \\ -x + 2y + 2z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

$2 \cdot 3 + y = 7$
$y = 7 - 6$
$y = 1$

$-3 + 2 \cdot 1 + 2z = -1$
$2z = -1 + 1$
$z = 0/2$
$z = 0$

Letra E

Tarefa Básica - Escalonamento

01.
$$S = \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x \qquad - 5z = 3 \end{cases}$$

$$\begin{array}{ccc} -0.5 & -0.5 \\ +L_0 \\ \end{array} \left(\begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 1 & 3 & -1 & 11 \\ 1 & 0 & -5 & 3 \end{array}\right) \overset{\sim}{\underset{+3.5}{\underset{0.5}{}}} \left(\begin{array}{ccc|c} 0 & 3.5 & 0.5 & 13.5 \\ 0 & 0.5 & -3.5 & 5.5 \end{array}\right) \sim \left(\begin{array}{ccc|c} & & & \\ 0 & 12.5 & 0 & 50 \end{array}\right)$$

$x - 5z = 3$
$x - 5 \cdot (-1) = 3$
$\underline{x = -2}$

$0.5 \cdot y - 3.5z = 5.5$
$0.5 \cdot 4 - 3.5z = 5.5$
$2 - 5.5 = 3.5z$
$\dfrac{-3.5}{3.5} = z$
$\boxed{z = -1}$

$12.7 \cdot y = 50$
$\boxed{y = 4}$

02.
$$\begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases}$$

$x = 2y \rightarrow \boxed{y = \dfrac{x}{2}}$

$3z = 2y$
$3z = 2 \cdot \dfrac{x}{2}$
$3z = x$
$\boxed{z = \dfrac{x}{3}}$

$x + y + z = 11$
$x + \dfrac{x}{2} + \dfrac{x}{3} = 11$
$\dfrac{6x + 3x + 2x}{6} = 11$
$11x = 11 \cdot 6$
$x = \dfrac{66}{11}$
$x = 6$

$y = \dfrac{x}{2}$
$y = \dfrac{6}{2} = 3$

$z = \dfrac{x}{3}$
$z = \dfrac{6}{3} = 2$

$x + 2y + 3z = 6 + 2 \cdot 3 + 3 \cdot 2 = 18$
Letra B

03.
$$\begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases}$$

$$\begin{array}{c} -2 \\ +6 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \underset{L_b^+}{\sim}_2 \left(\begin{array}{ccc|c} 0 & -3 & -4 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \sim \left(\begin{array}{ccc|c} 0 & & & \\ 0 & 0 & -5 & -10 \end{array} \right)$$

$$-5z = -10$$
$$z = 2$$
Letra D ✓

04.
$$\begin{cases} A + B + C = 68 \\ B + C \cdot 0,2 = A \\ C + A \cdot 0,2 = 3B \end{cases}$$

$$A + B + C = 68$$
$$(B + C \cdot 0,2) + B + C = 68$$
$$2B + 1,2C = 68$$
$$2B = 68 - 1,2C$$
$$B = \frac{68 - 1,2C}{2}$$
$$B = 34 - 0,6C$$

$$C + A \cdot 0,2 = 3B$$
$$C + A \cdot 0,2 = 3(34 - 0,6C)$$
$$C + 0,2A = 102 - 1,8C$$
$$1,8C + C + 0,2A = 102$$
$$2,8C + 0,2A = 102$$
$$2,8C + 0,2(B + 0,2C) = 102$$
$$2,8C + 0,2B + 0,04C = 102$$
$$2,84C + 0,2B = 102$$
$$2,84C + 0,2(34 - 0,6C) = 102$$
$$2,84C + 6,8 - 0,12C = 102$$
$$2,72C = 102 - 6,8$$
$$C = \frac{95,2}{2,72}$$
$$C = 35$$

$$B = 34 - 0,6 \cdot 35$$
$$B = 13$$

$$A = B + 0,2C$$
$$A = 13 + 0,2 \cdot 35$$
$$A = 20$$

$$C - A = 35 - 20 = 15$$
Letra A

05. $A = 134$

$B = 115$

$C = 48$

$A \rightarrow \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix}$

$B \rightarrow A =$

$C \rightarrow$

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} 3y + 4z \\ x + 5z \\ 2x + y \end{bmatrix} = \begin{bmatrix} 134 \\ 115 \\ 48 \end{bmatrix}$$

$\begin{cases} 3y + 4z = 134 \\ x + 5z = 115 \\ 2x + y = 48 \end{cases}$

$x = 115 - 5z$
$x = 115 - 5 \cdot 20$
$x = 15$

$y = 48 - 2x$
$y = 48 - 2 \cdot 15$
$y = 18$

$3y + 4z = 134$
$3(48 - 2x) + 4z = 134$
$144 - 6x + 4z = 134$
$-6(115 - 5z) + 4z = -10$
$-690 + 30z + 4z = -10$
$34z = 680$
$z = \dfrac{680}{34}$
$z = 20$

$x + y + z = 15 + 18 + 20 = 53$ //

Letra A