

## Discrete Task 4 Analysis

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### Affine Decryption Rule:

$$D = a^{-1} (c - b) \bmod m$$

c: position of ciphered letter in alphabet

a: 1<sup>st</sup> key

b: 2<sup>nd</sup> key

m: number of letters in alphabet

D: Deciphered letter

### Mod Inverse

Calculated with Extended Euclidean algorithm:

### Multiplicative Inverse

Ex: Find MI of 11 in  $\mathbb{Z}_{26}$

q	r <sub>1</sub>	r <sub>2</sub>	r	t <sub>1</sub>	t <sub>2</sub>	t
2	26	11	4	0	1	-2 ?
2	11	4	3	1	-2	5
1	4	3	1	-2	5	-7
3	3	1	0	5	-7	26
x	0	x	x	x	x	x

$\rightarrow \text{gcd}(11, 26) = 1$        $\rightarrow \text{M.I.} = -7$

GCD(161, 28) = ?

q	r <sub>1</sub>	r <sub>2</sub>	r	s <sub>1</sub>	s <sub>2</sub>	s	t <sub>1</sub>	t <sub>2</sub>	t
5	161	28	21	1	0	1	0	1	-5
1	28	21	7	0	1	-1	1	-5	6
3	21	7	0	1	-1	4	-5	6	-23
x	7	0	x	-1	4	x	6	-23	x

GCD(161, 28) = 7

$$a \cdot s_1 + b \cdot t_1 = d$$

$$s = s_1 - s_2 \cdot q$$

$$t = t_1 - t_2 \cdot q$$

$$s_1 = 1; s_2 = 0$$

$$t_1 = 0; t_2 = 1$$

$$s = 1 - 0 \times 5 = 1$$

$$t = 0 - 1 \times 5 = -5$$

$$s = 0 - 1 \times 1 = -1$$

$$t = 1 - (-5 \times 1) = 6$$

$$s = 1 - (-1 \times 3) = 4$$

$$t = -5 - (6 \times 3) = -23$$

Formulas

Initialization  
for first step

### Analytical Calculation for my case:

$a=7$   $b=13$   $m=27$

language: English

Modulus Inverse = 4

$\gcd(27, 7) = 1$

## Decipher

$$a = 7 \quad b = 13 \quad m = 27 \quad t = t_1 - t_2 * q$$

$$q = r_1 / r_2 \quad r = r_1 \% r_2 \quad S = S_1 - S_2 * q$$

q	r <sub>1</sub>	r <sub>2</sub>	r	t <sub>1</sub>	t <sub>2</sub>	t	S <sub>1</sub>	S <sub>2</sub>	S
3	27	7	6	0	1	-3	1	0	1
1	7	6	1	1	-3	4	0	1	-1
6	6	1	0	-3	4	-27	1	-1	7
X	(1)	0	X	(4)	-27	X	(-1)	7	X

$$\gcd(1)$$

$$m \cdot I = 4$$

### Proof:

my case	
$1 = S, m + t_1 a$	$1 = (-1)(27) + (4)(7)$
$1 \bmod m = t_1 a$	$7^{-1} \bmod 27 = 4$
$\frac{1}{a} \bmod m = t_1$	
$a^{-1} \bmod m = t_1$	

### Implementation Steps:

- here m and a will be copied to r1 and r2
- initialization of t1 and t2:  $t_1=0$  ,  $t_2=1$
- quotient  $q=r_1/r_2$  and remainder  $r=r_1\%r_2$  calculated
- calculate  $t = t_1 - t_2 * q$
- shift r2 to r1 , remainder to r2 and t2 to t1
- Keep Looping until r2 reaches 0

### At the end:

- $r_1$  will be gcd , it must be 1 for multiplicative inverse to be present , the 2 numbers must be coprime with each other
- ( gcd(a,m) must be 1 )
- $t_1$  will be the multiplicative inverse I am looking for but the number might be negative and it should be between 0 and m-1
- We loop and add m till it is inside range.
- Last block I check if gcd (  $r_1$  ) equals 1 if not then no multiplicative inverse so returns -1

```
int modInverse(int a,int m){
    int t1=0,t2=1,t,r1=m,r2=a;
    while(r2!=0){ // applying extended euclidean algorithm to get t1 wh
        int q,r;
        q=r1/r2;
        r=r1%r2;
        t=t1-t2*q;
        r1=r2;
        r2=r;
        t1=t2;
        t2=t;
    }
    if(r1==1){
        while(t1<0){ // mod inverse must be bewteen 0 & m-1
            t1=t1+m;
        }
        return t1%m; // returning mod inverse of a and m with gcd of 1
    }
    else return -1; // not coprime so no mod inverse
}
```

### Main function

**Input Prompts:** Message to be Deciphered , a and b

a=7 , b=13 , message = "ROVKMWUKKTHUMVKMRJQMKUGDUR"

**Language:** English Alphabet with space at position 0

**m:** 27

```
int main() {
    string cipherText,alphabet;
    int a, b;
    cout << "Enter the affine ciphered message: ";
    getline(cin, cipherText);
    cout << "Enter a: ";
    cin >> a;
    cout << "Enter b: ";
    cin >> b;
    alphabet=" ABCDEFGHIJKLMNOPQRSTUVWXYZ"; // SPACE is added
    int m = alphabet.size();

    // Decrypt the message
    string decryptedMessage = affineDecrypt(cipherText, a, b, m,alphabet);

    if (!decryptedMessage.empty()) {
        cout << "The decrypted message is: " << decryptedMessage << endl;
    }

    return 0;
}
```

## Decipher Function

- 1) Modulus inverse of a is calculated using function described above
- 2) Checks if valid modulus inverse or not
- 3) Loops over each letter of ciphered message
- 4) Calculates Deciphered letter offset in alphabet array using :  
$$a\_inv * (\text{position of letter in alphabet} - b) \% m$$
- 5) Position calculated can be negative so we have to make it between 0 and m-1 by adding m until value in range
- 6) Access position of deciphered letter in our alphabet array ( alphabet[] array of 27 letter )
- 7) Add letter to decrypted message

```
// Function to decrypt the Affine ciphered message
string affineDecrypt(string cipherText, int a, int b, int m, string alphabet) {
    string decryptedText = "";
    int a_inv = modInverse(a, m);

    if (a_inv == -1) {
        cout << "Modular inverse of 'a' does not exist!" << endl;
        return "";
    }

    for (char c : cipherText) {
        int L;
        L = (a_inv * (search(alphabet, c) - b)) % m;
        while (L < 0) {
            L = L + m;
        }
        decryptedText += alphabet[L];
    }
    return decryptedText;
}
```

## Example for 1<sup>st</sup> letter in message:

1<sup>st</sup> letter of ciphered message : R

$$4(18 - 13) \% 27 = 20$$

T=20

## Output:

```
Enter the affine ciphered message: ROVKMWUKKTHUMVKMRJQMKUGDUR
Enter a: 7
Enter b: 13
The decrypted message is: THIS MESSAGE IS TOP SECRET
```