area of overlap as function of distance from optimum (d) and angle between optima (θ)

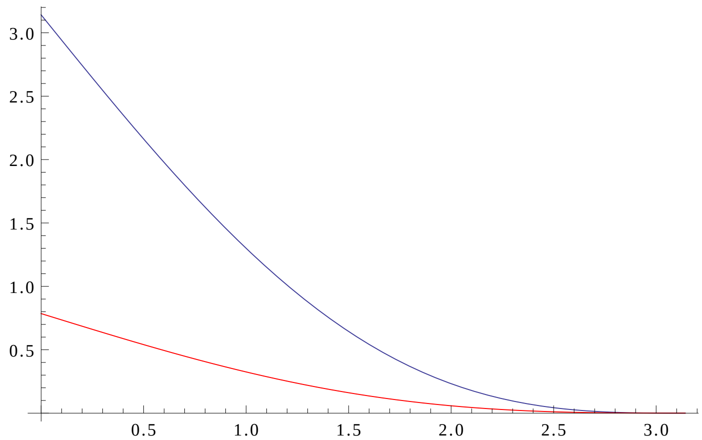
In[43]:=
```
A[d_, θ_] := 2 d^2 ArcCos[x/(2 d)] - x/2 Sqrt[4 d^2 - x^2] /. x -> d Sqrt[2 (1 - Cos[θ])]
```

$$A[d\_, \theta\_] := 2\, d^2\, \mathrm{ArcCos}\left[\frac{x}{2\, d}\right] - \frac{x}{2}\sqrt{4\, d^2 - x^2}\ /.\ x \to d\sqrt{2\,(1 - \mathrm{Cos}[\theta])}$$

overlap declines with angle (here from 0 to 180 degrees) for any distance

In[55]:=
```
Show[
 Plot[A[d, θ] /. d → 1, {θ, 0, π}],
 Plot[A[d, θ] /. d → 0.5, {θ, 0, π}, PlotStyle → Red],
 PlotRange → All
]
```

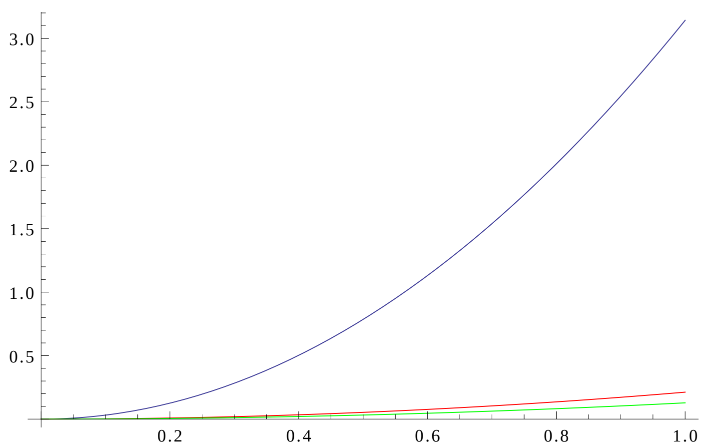
Out[55]=



overlap increases with distance for any angle

In[54]:=
```
Show[
 Plot[A[d, θ] /. θ → 0, {d, 0, 1}],
 Plot[A[d, θ] /. θ → 90, {d, 0, 1}, PlotStyle → Red],
 Plot[A[d, θ] /. θ → 180, {d, 0, 1}, PlotStyle → Green],
 PlotRange → All
]
```

Out[54]=



fraction overlap

note that the fraction overlap is independent of the distance, d:

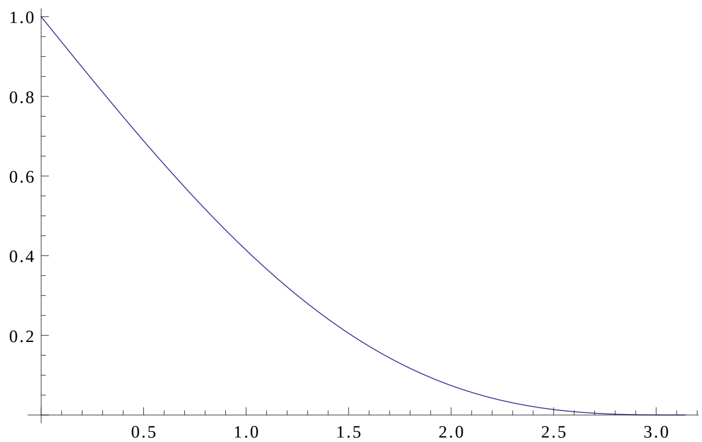
In[72]:=
```
Simplify[A[d, θ]/A[d, 0], d > 0]
```

Out[72]= $\dfrac{2\,\text{ArcCos}\left[\sqrt{\text{Sin}\left[\frac{\theta}{2}\right]^2}\right] - \sqrt{\text{Sin}[\theta]^2}}{\pi}$

and declines from 1 to 0 with angle

In[78]:=
```
Show[
 Plot[A[d, θ]/A[d, 0] /. d → 1, {θ, 0, π}],
 PlotRange → All
]
```

Out[78]=



the fraction overlap is 0 when the angle is 180 degrees

In[73]:=
```
A[d, θ]/A[d, 0] /. θ → π
```

Out[73]= 0

there is perfect overlap when the angle is 0

In[74]:=
```
A[d, θ]/A[d, 0] /. θ → 0
```

Out[74]= 1

when the angle is 90 degrees the fraction overlap is

In[77]:=
```
Simplify[A[d, θ]/A[d, 0] /. θ → π / 2, d > 0]
```

Out[77]= $\frac{1}{2} - \frac{1}{\pi}$