Futures ,Forwards, and Swaps

Financial Engineering and Computations

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此書內容

- Financial Engineering & Computation

  教課書第12章 Forwards, Futures , Futures Options, Swaps

- Options, Futures, and Other Derivatives—John C. Hull

  第七章　Swaps

Outline

- Brief Introduction of Derivatives
- Forwards
- Relationships Between Forward Price and Spot Price
- Futures
- Relationships between Forward and Futures Prices
- Cost of Carry
- Hedging and Futures
- Minimum Variance Hedge Ratio
- Brief Introduction of Swaps

Derivatives

- Payoffs depend on other more fundamental assets:
  - Underlying assets
    - Commodity
    - Index
    - Interest rate
    - Other derivatives
- Four types of derivatives
  - Forwards(遠期契約)
  - Futures(期貨)
  - Swaps(交換合約)
  - Options(選擇權)

遠期契約 (Forwards)

- 定義：遠期契約雙方同意在未來某一天以某一價格(履約價格)交易某一特定資產或商品。

- Terms:

  Maturity date:到期日

  Strike price:履約價格

  Underlying asset: 標的物

Why we Need Forwards?
A Simple Story

- 考慮XYZ公司欲投資x元生產棉花,生產期為三個月
- 三個月後:
  - 棉花價格上升:公司大賺一筆
  - 棉花價格狂洩:公司虧本甚至倒閉
- 如何避免棉花的價格風險?

A Simple Story on Forwards

- 假定有另一紡布公司ABC,三個月後需要棉花紡布
  - 棉花不能今日進貨:倉儲問題
  - 三個月後進貨
    - 棉花價格上升:公司有虧本甚至破產之虞
    - 棉花價格狂洩:公司大賺一筆
  - 如何規避棉花的價格風險?

A Simple Story on Forwards

- XYZ和ABC之間可簽訂合約
  - 使用預定好的金額K於三個月後交易棉花
  - 價格風險消失,但也無法得額外利潤
- XYZ (Forward的報酬為 K-P)

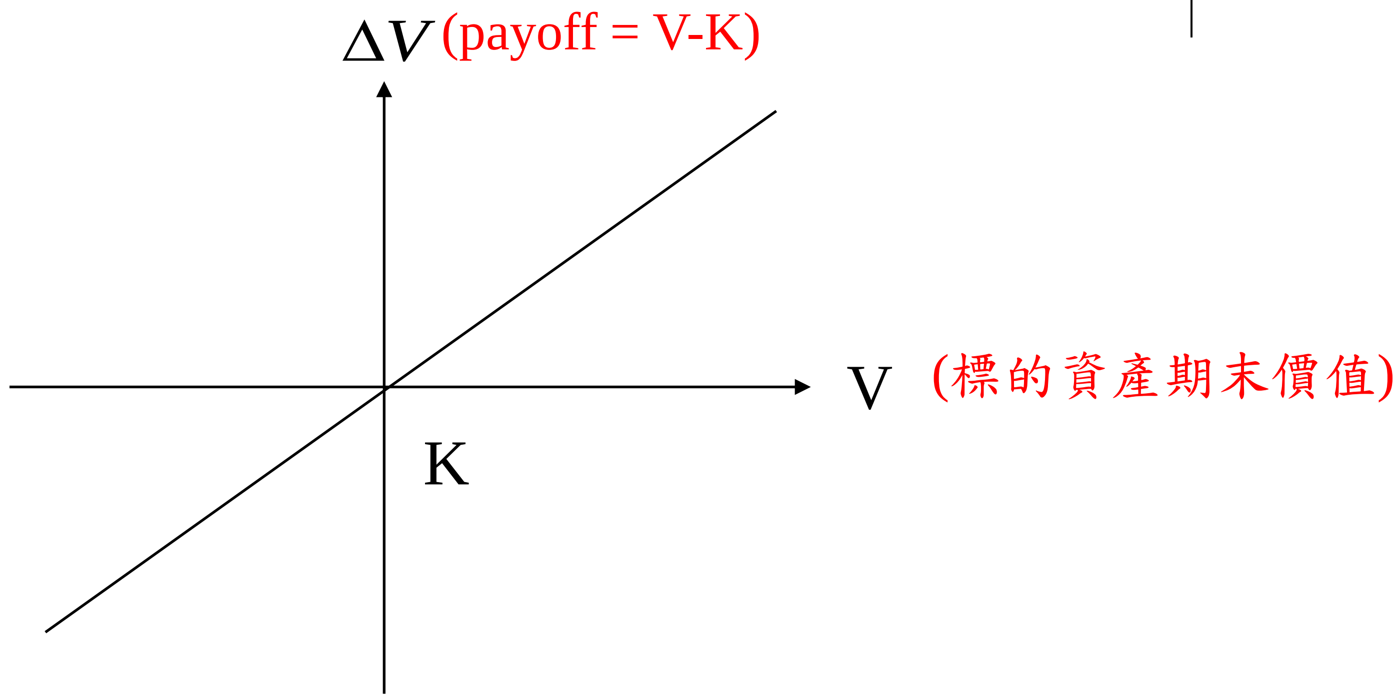
	不簽合約	簽合約	
		遠期合約	市場價格
價格上升至P1	P1(額外利潤)	K-P1	P1
價格下降至P2	P2(虧損)	K-P2	P2

總報酬鎖定為K

A Simple Story on Forwards

- 上述為一遠期合約
- 標的物:棉花
- 到期日:三個月後(假定棉花價格為P)
- 履約價格:K
- 買方:ABC (報酬為 P-K)
- 賣方:XYZ (報酬為 K-P)
- 交易方式:
  - 實物交易 (Negative Oil Future Price)
  - 現金清算

從 Buyer (ABC)來看 Forward的報酬



1. 報酬構面(payoff profile)是直線的
2. 遠期合約期初價值為零，期中無任何支付，僅有期末會有支付產生。

Forward的缺點

- 當棉花價格上升,XYZ失去大賺一筆的機會
- 當棉花價格下降,ABC失去減少成本的機會
  - Options
- Default risk(違約風險)
  - XYZ可能失火,導致無法履約
  - 當棉花價格上升,XYZ有違約的可能
  - 當棉花價格下降,ABC有違約的可能
    - 採用期貨合約每日清算的制度,減少違約風險

Forward的缺點

- 合約未標準化
  - 採用標準化的期貨合約
  - 以台指期貨為例
    - 交易標的物:加權指數
    - 規格:近三個月和兩個季月到期的合約
      - Ex: Now: Oct.,　　(Introduced later)
      - 交易合約到期日: Oct., Nov., Dec., Mar., Jun.
- 市場撮合和流通
  - 期貨合約可在交易所流通

Reasonable Strike Price

- 在理想的假定下(no tax, transaction cost, etc.)遠期的履約價格和標的物的現貨價格有一定的關係
- 當該關係不滿足時,存在套利機會
- 凱因斯(Keynes)使用 Interest parity指出該狀況
  - 考慮遠期匯率合約
  - 匯率和本國利率,外國利率有連動的關係

Interest Rate Parity

- Terms:
  - S: Spot exchange rate (Domestic/Foreign)
  - F: Forward exchange rate (Maturity: T year)
  - R_f: Foreign interest rate
  - R_l: Local interest rate
- Then we have $F = S \times e^{(R_l - R_f)T} = \dfrac{F}{S} = e^{(R_l - R_f)T}$
- Otherwise, there exists arbitrage opportunity.

Arbitrage Opportunity

- Assume $\frac{F}{S} > e^{(R_l - R_f)\mathrm{T}}$
- Now:
  - Borrow 1 domestic dollar, change to 1/S foreign dollar, save it at a interest rate of Rf
  - Sign a exchange rate forward with price F.
- t year later:
  - You get $\frac{1}{S}e^{R_f \mathrm{T}}$ foreign dollars.
  - Convert them at the rate F (Forward contract)
  - Since $\frac{F}{S}e^{R_f \mathrm{T}} > e^{R_l \mathrm{T}}$ → get free lunch

In Class Exercise

- Show that arbitrage opportunity exists as $\frac{F}{S} < e^{(R_l - R_f)\mathrm{T}}$

考慮存款借款利率不同下的套利分析

Time	Domestic dollar	Foreign dollar
0	-1	$\frac{1}{S}$
T	$-e^{R_l^l T}$	$\frac{1}{S}e^{R_f^b T}$

- $\frac{F}{S}e^{R_f^b T} - e^{R_l^l T} > 0$
- $\frac{F}{S} > e^{(R_l^l - R_f^b)T}$

Time	Domestic dollar	Foreign dollar
0	+S	-1
T	$Se^{R_l^b T}$	$-e^{R_f^l T}$

- $\frac{S}{F}e^{R_l^b T} - e^{R_f^l T} > 0$
- $\frac{S}{F} > e^{(R_f^l - R_l^b)T}$
- $\frac{F}{S} < e^{(R_l^b - R_f^l)T}$

R_f^b:在外國銀行存錢利率
R_f^l:向國外銀行借錢利率
R_l^b:在國內銀行存錢利率
R_l^l:向國內銀行借錢利率
右下index：domestic/ foreign
右上index：borrow / lend

考慮存款借款利率不同
以及交易費率下的套利分析

<table>
<tr><td></td><td colspan="2">情況一</td><td colspan="2">情況二</td></tr>
<tr><td>Time</td><td>Local dollar</td><td>Foreign dollar</td><td>Local dollar</td><td>Foreign dollar</td></tr>
<tr><td>0</td><td>-1</td><td>$\frac{1}{S}\times(1-\tau_{Exchange})$</td><td>$S\times(1-\tau_{Exchange})$</td><td>-1</td></tr>
<tr><td rowspan="3">T</td><td>$-e^{R_l^l T}$</td><td>$\frac{1}{S}\times(1-\tau_{Exchange})$
$\times e^{R_f^b T}$</td><td>$S\times(1-\tau_{Exchange})\times$
$e^{R_l^b T}$</td><td>$-e^{R_f^l T}$</td></tr>
<tr><td colspan="2">履行遠期合約</td><td colspan="2">履行遠期合約</td></tr>
<tr><td>$-e^{R_l^l T}+\frac{F}{S}$
$\times(1-\tau_{Exchange})\times e^{R_f^b T}$
$\times(1-\tau_{Forward})$</td><td>0</td><td>0</td><td>$-e^{R_f^l T}+\frac{S}{F}$
$\times(1-\tau_{Exchange})\times e^{R_l^b T}$
$\times(1-\tau_{Forward})$</td></tr>
</table>

R_l^b：在本地銀行存錢的利率；R_l^l：在本地銀行借錢的利率；R_f^b：在外國銀行存錢的利率；
R_f^l：在外國銀行借錢的利率；$\tau_{Exchange}$：兌換外幣的交易費率；$\tau_{Forward}$：遠期合約的交易費率。

情況一：

當 Local $dollar_T > 0$，存在套利空間。

整理Local $dollar_T$，得：$\frac{F}{S} > e^{(R_l^l - R_f^b)T} \times \left(1-\tau_{Exchange}\right)^{-1} \times (1-\tau_{Forward})^{-1}$，存在套利空間。

情況二：

當 Foreign dollar $_T > 0$，存在套利空間。

整理Foreign $dollar_T$，得：$\frac{F}{S} < e^{(R_l^b - R_f^l)T} \times \left(1-\tau_{Exchange}\right) \times (1-\tau_{Forward})$，存在套利空間。

Forward Price

- 遠期合約在期初時不會有現金流
  - 遠期合約初始價值為0 (買方不需付權利金給賣方)
- Forward price (F) 代表一合理的價格,使得當履約價格(K)等於F時,遠期合約的初始價值為0
- Forward price會隨時間而改變
  - 令$\mathbf{F_t}$為時間t的Forward price
  - 假定合約起始日為0,到期日為T
  - $\mathbf{S_T}$為標的物在時間T的價格
  - 可類推到期貨(future price)

Contract Value

- 遠期合約在期初的價值為0,
- 遠期合約在時間t的價值為 $f_t = S_t - Ke^{-r(T-t)}$
- Proof:
  - At time t
    - Long a forward contract
    - Save $Ke^{-r(T-t)}$
    - Short underlying asset
  - At time T

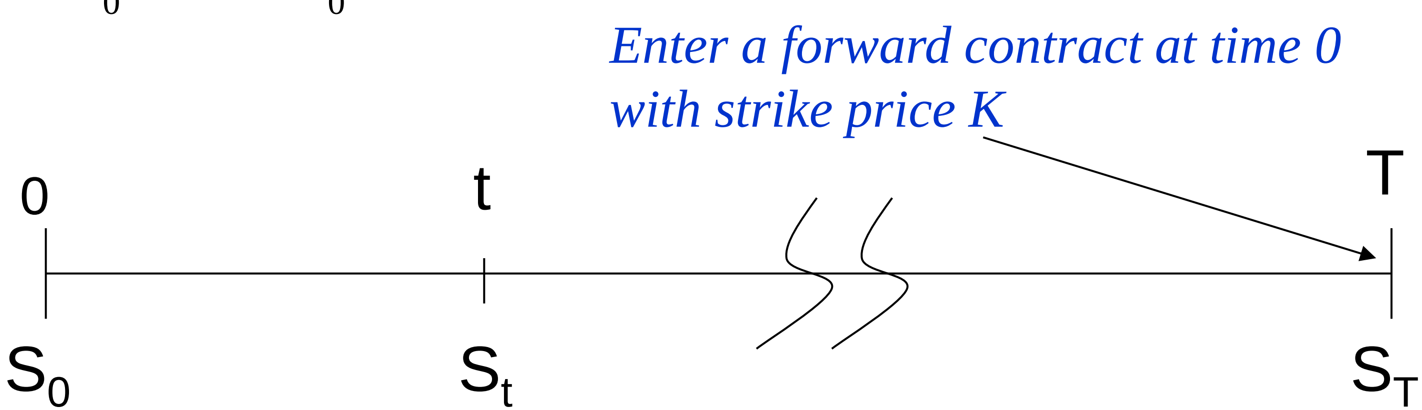
$$\left(S_T - K\right) + K - S_T = 0$$ Riskless, Benefit=0

Forward Price和Spot Price關係
(標的物於T前無支付現金)

- 在時間 0時,現貨價格為S_0
- 假定標的物在T前不會產生 cash flow (like dividend)
- 則可得 $F_0 = S_0 e^{rT}$
- 如上式不滿足,則存在套利空間

令 $f_0 = S_0 - Ke^{-r(T-0)} \equiv 0$

$F_0 = K = S_0 e^{rT}$



Arbitrage Opportunity

- 假定 $F_0 > S_0 e^{rT}$
- 時間0時簽合約,K= F_0

Cash flow

時間0	借S0	Buy the stocks	Short Forward	0
時間T	還 $S_0 e^{rT}$	得S_T	還S_T-K (K=F_0)	F_0-$S_0 e^{rT}$

$-S_0 e^{rT} + S_T - (S_T - F_0) > 0$ ，期初不用支付任何成本，期末可得正的報酬!

In Class Exercise

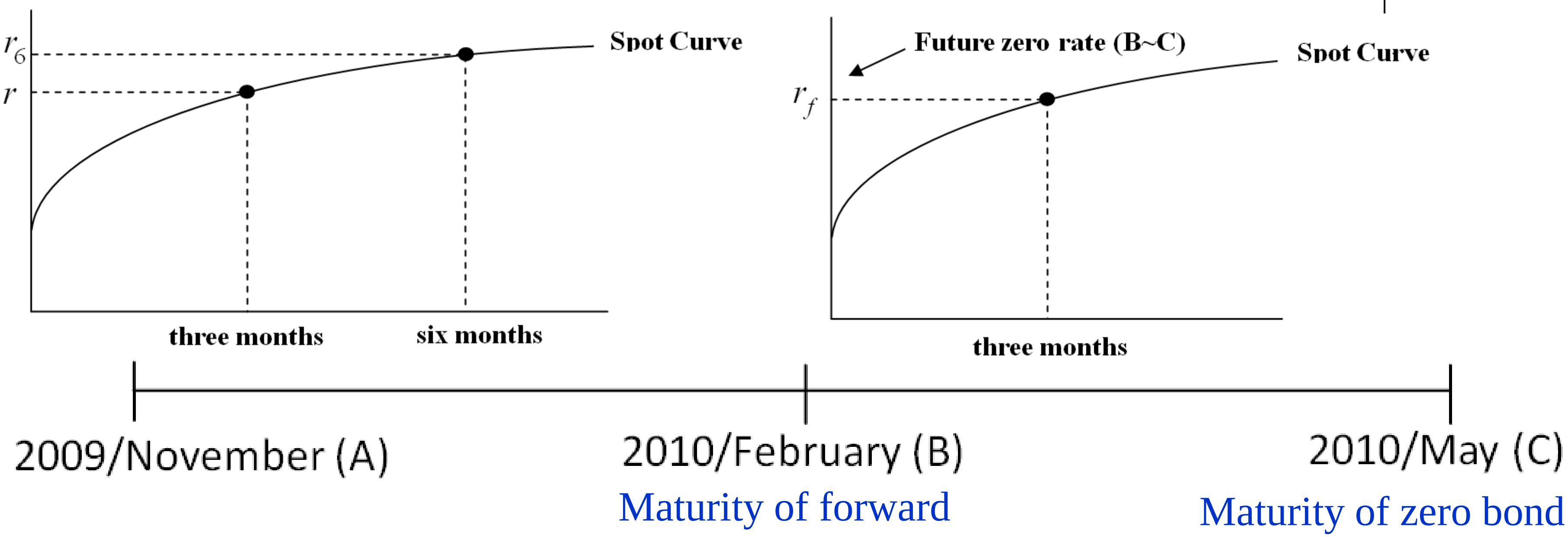
- 考慮 $F_0 < S_0 e^{rT}$ 的套利機制

A Real Example

- r is the annualized 3-month riskless interest rate.
- S is the spot price of the 6-month zero-coupon bond.
- A new 3-month forward contract on a 6-month zero-coupon bond should command a delivery price of $Se^{r/4}$.
- So if r = 6% and S = 970.87, then the delivery price is

  $970.87 \times e^{0.06/4} = 985.54.$

Forwards on Bond and Interest Rates



F *is face value of the 6-month zero-coupon bond*

S_t *is the spot price of the 6-month zero-coupon bond at time t*

At time A:

$$S_A = F \times e^{-r_6 \frac{1}{2}} = 970.87$$

$$F_A = S_A \times e^{r \frac{1}{4}} = 970.87 \times e^{\frac{0.06}{4}} = 985.54$$

At time B:

$$S_B = F \times e^{-r_f \frac{1}{4}}$$

$$reward\ of\ buyer = S_B - 985.54$$

Forward Price和Spot Price關係 (標的物於T前支付固定報酬)

- 假定該報酬(例如股息)的現值為 **I**
- 可得 $F_0 = (S_0 - I)e^{rT}$
- 假定 $F_0 > (S_0 - I)e^{rT}$
  - At time 0
    - short forward (K=F_0), long underlying asset, borrow S_0
    - Initial cash flow=0
  - At time T
    - $-S_T + F_0 + S_T + Ie^{rT} - S_0 e^{rT} = F_0 - (S_0 - I)e^{rT} > 0$

Example

- Consider a 10-month forward contract on a stock with a price of \$50. We assume that the risk-free rate interest is 8% per annum for all maturity, and that dividends of \$0.75 per share are expected after three months, six months and nine months. Please calculate the forward price.

$$\because I = 0.75e^{-0.08\times 3/12} + 0.75e^{-0.08\times 6/12} + 0.75e^{-0.08\times 9/12} = 2.162$$

$$\Rightarrow F = (50 - 2.162)e^{0.08\times 10/12} = \$51.14$$

$$where\ \ I\ is\ the\ PV\ of\ the\ dividends$$

Homework 7

- 1.Show that arbitrage opportunity exist if

  $$F_0 < (S_0 - I)e^{rT}$$

- 2.Assume that the underlying stock pays a continuous dividend yield at rate q. Show that the forward price is $F_0 = S_0 e^{(r-q)T}$

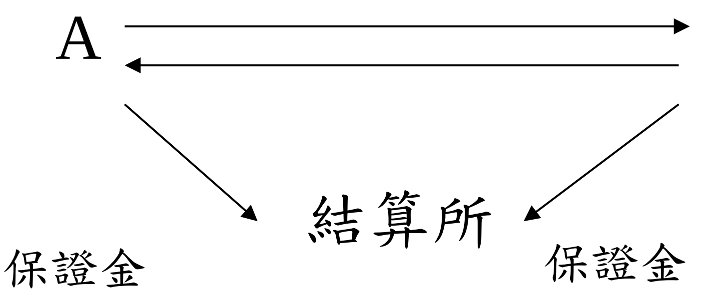
期貨合約(Futures)

- 期貨合約和遠期合約很類似，主要差別在於：

  1.每日現金結算損益(marked-to-market)

  2.買賣雙方都必須提供保證金(margin)

  — 交易所藉由marked-to-market和margin account來降低違約風險



雙方交易的共同保證人

期貨合約(Futures)

3. 合約標準化(期貨均在集中交易所交易)。

4. 透過集中市場競價來決定交易價格。

5. 可以反向契約來抵銷原有契約。

- Black: Futures可視為一系列的遠期合約。在每日將前一天的Forward清算掉，並重新訂約。

有關期貨相關資訊可至台灣期交所 http://www.taifex.com.tw/

月期貨

- 月期貨權存續期間為一個月

102年2月						
一	二	三	四	五	六	日
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28			

以102年三月小型台指期(MTX)為例

日期	到期契約	上市之新契約
2/20(三)	MTX 2月	MTX 3月
3/20(三)	MTX 3月	MTX 4月

102年3月						
一	二	三	四	五	六	日
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

週期貨

- 週期貨存續期間為一週，
- 交易當月每週三加掛MTX一週到期契約，惟第2個星期三不加掛
- 上市日與最後交易日如非為營業日，則順延至次一營業日
- 第2個禮拜三不加掛周期貨，第3個禮拜三月期貨到期
- 第1、4、5個星期三，將有2個MTX一週到期契約同 時交易

到期週別	新契約上市日	最後交易日即結算日
第1週	上個月最後1個星期三	當月第1個星期三
第2週	當月第1個星期三	當月第2個星期三
第4週	當月第3個星期三	當月第4個星期三
第5週	當月第4個星期三	當月第5個星期三

週期貨

- 以102年三月小型台指期(MTX)為例

102年3月						
一	二	三	四	五	六	日
		2/27		1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

日期	到期契約	上市之新契約
2/27(三)		MTX 3月 W1
3/06(三)	MTX 3月 W1	MTX 3月 W2
3/13(三)	MTX 3月 W2	
3/20(三)	MTX 3月	MTX 3月 W4
3/27(三)	MTX 3月 W4	MTX 4月 W1

週期貨

- 以102年二月小型台指期(MTX)為例

102年2月						
一	二	三	四	五	六	日
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28			

日期	到期契約	上市之新契約
2/06(三)	MTX 2月 W1	MTX 2月 W2
2/18(一)	MTX 2月 W2	
2/20(三)	MTX 2月	MTX 2月 W4
2/27(三)	MTX 2月 W4	MTX 3月 W1

- 2013/2/6封關，2013/2/18開紅盤
- 原2/13到期之MTX 2月W2契約，遇連續假日因此順延至次一 營業日(18日)到期

台灣的期貨市場

期指報價：台指期 小台指 電子期 金融期 台灣50期 元富預約開戶

台指期 (權值表) 資料日期：2004/10/13

名稱	時間	成交價	買進	賣出	漲跌	總量	基差	昨收	開盤	最高	最低	元富期貨下單
台指期10	10:07:51	6000	6000	6004	△9	10806	-33.72	5991	6020	6028	5985	買 賣 口數 送出
台指期11	10:07:49	5990	5990	5995	△10	605	-23.72	5980	5980	6022	5980	買 賣 口數 送出
台指期12	10:04:49	5990	5951	6025	△20	1	-23.72	5970	5990	5990	5990	買 賣 口數 送出
台指期03	08:50:22	5921	5921	5980	▽29	1	+66.23	5950	5921	5921	5921	買 賣 口數 送出
台指期06		—	5901	5950	—	—	—	5929	—	—	—	買 賣 口數 送出

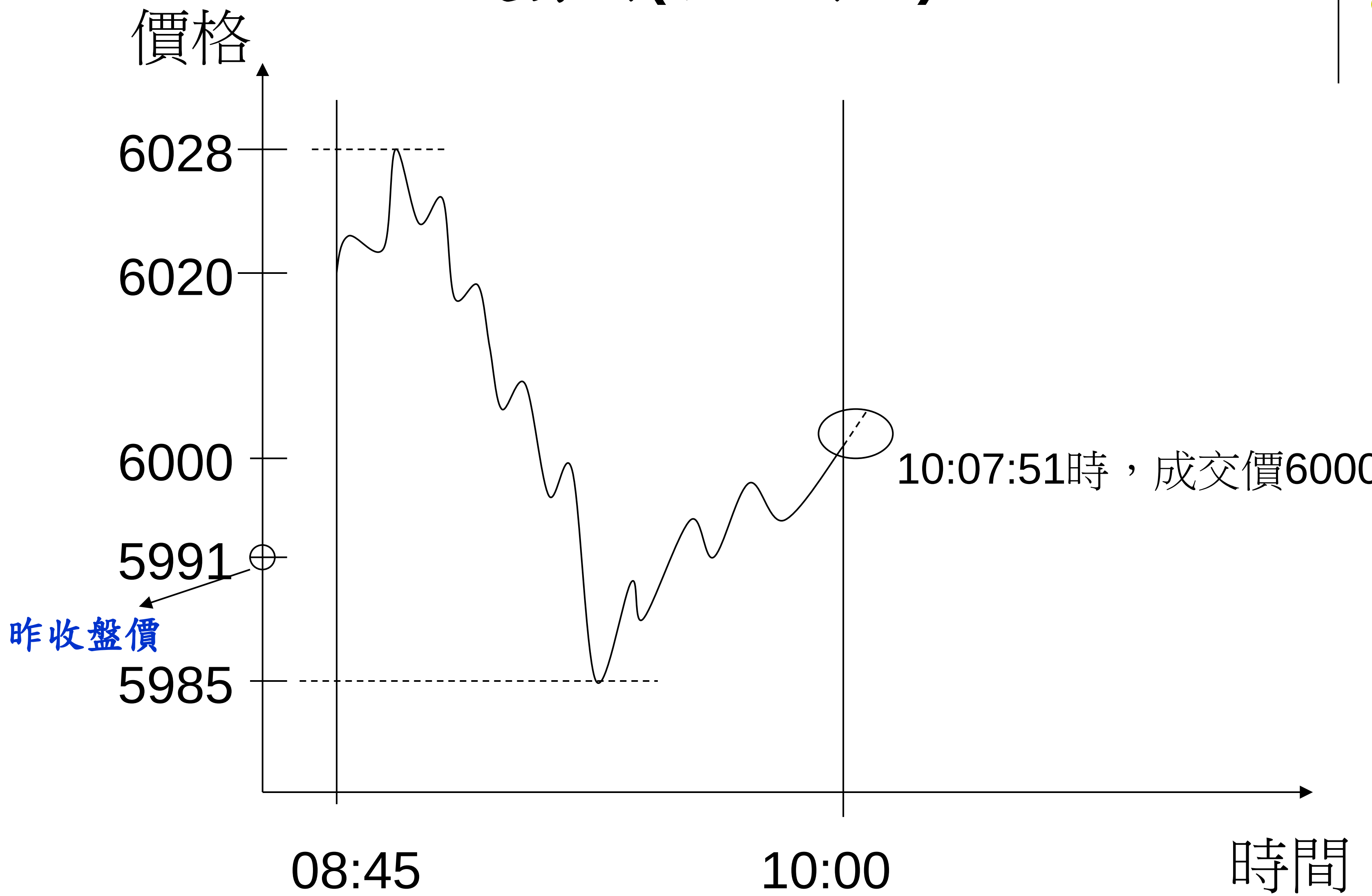
小台指 (權值表) 資料日期：2004/10/13

名稱	時間	成交價	買進	賣出	漲跌	總量	基差	昨收	開盤	最高	最低	元富期貨下單
小台指10	10:07:51	6000	5996	6005	△9	2021	-33.72	5991	6022	6025	5985	買 賣 口數 送出
小台指11	10:07:23	5988	5987	5998	△8	186	-21.72	5980	6000	6020	5982	買 賣 口數 送出
小台指12		—	5988	6014	—	—	—	5970	—	—	—	買 賣 口數 送出
小台指03	08:46:19	5947	5931	5989	▽3	3	+19.28	5950	5930	5947	5930	買 賣 口數 送出
小台指06	09:52:12	5938	5911	5947	△9	3	+41.86	5929	5922	5938	5922	買 賣 口數 送出

標的物: 股票指數

Check: http://tw.stock.yahoo.com/future/

走勢圖(台指期10)

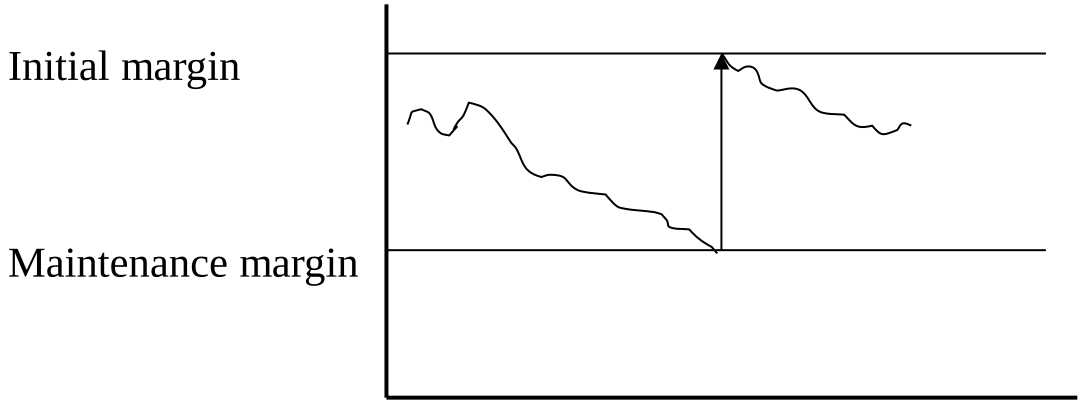

價格
6028
6020
6000
5991
5985
昨收盤價
10:07:51時，成交價6000
08:45
10:00
時間

報價表(名詞解釋)

- 名稱：台指期10，指交割月份為10月份的台指期貨。
  (期貨報價通常會揭露近三個連續近月與二個接續季月共五個資訊)
- 時間：10:07:51，指系統揭露成交的時間。
- 漲跌：△9，指成交價減去昨天收盤價的價差(6000-5991=9)。
- 總量：10806，指08:45~10:07期間市場成交的總數量。
- 基差：-33.72，指現貨價格減去期貨的價格，由此報價表可得知台股指數約為5966點。(基差反應的是持有現貨的成本、收益與市場對未來走勢的預期心理)
- 最高：6028，指09:00~10:07期間內最高價。
- 平倉：以等量但相反買賣方向沖銷原有的部位。
- 未平倉量(open interest)：買（賣）的期貨契約在未平倉以前稱為未平倉量，代表等量的多頭部位與空頭部位。(報價表中常見的名詞)

保證金清算

- 替代原則:結算公司介入每筆交易，成為買方的賣方，也成為賣方的買方，承擔雙方之信用風險



- 保證金低至觸及維持保證金時，投資人會收到保證金催繳通知，必須將保證金補足至原始保證金水準，否則期貨商可逕自予以平倉

Example:期貨保證金清算

- 假設某投資人於2006年11月23日買進一口台股指數期貨契約，成交價格是5600點，原始保證金為105,000元，維持保證金為81,000元，此契約的總價值為$1,120,000元(大台指契約乘數每點200元)。在11月24、25、26日時，台指期收盤價為分別為5500、5450、5700點，期貨保證金清算過程如下:

Example:期貨保證金清算

日期	台指期收盤價	當日損益	保證金餘額
11/23	5600	—	105,000
11/24	5500	-20,000	85,000
11/25	5450	-10,000	75,000
11/26	5700	50,000	155,000

保證金餘額低於維持保證金$81,000，該投資人會接到經紀商通知(margin call)，並將保證金補足至原始保證金，故26日當天一開始的餘額變為$105,000元。

Daily cash flows

- 假定第i天的future price為f_i
- The contract cash flow at day i should be $f_i - f_{i-1}$
- Net cash flow:

$$(f_1 - f_0) + (f_2 - f_1) + (f_3 - f_2) + \ldots + (f_T - f_{T-1})$$

$$\because S_T = F_T$$

$$= f_T - f_0 = S_T - f_0$$

- It may differ because of the reinvestment and the margin system.

A futures contract has the similar accumulated payoff to a forward contract.

Relationships between Forward and Futures Prices

- Forward price = futures prices, if the interest rate is not stochastic.
- Let Forward price is **F** Futures price is **f**
- Consider forward and futures contracts on the same underlying asset with *n* days to maturity.
- The interest rate for day *i* is r_i.
- One dollar at the beginning of day *i* grows to $R_i \equiv e^{r_i}$ by day's end.

Proof

- Let fi be the futures price at the end of day i.
- So \$1 invested in the n-day discount bond at the end of day n will be worth \$**R**

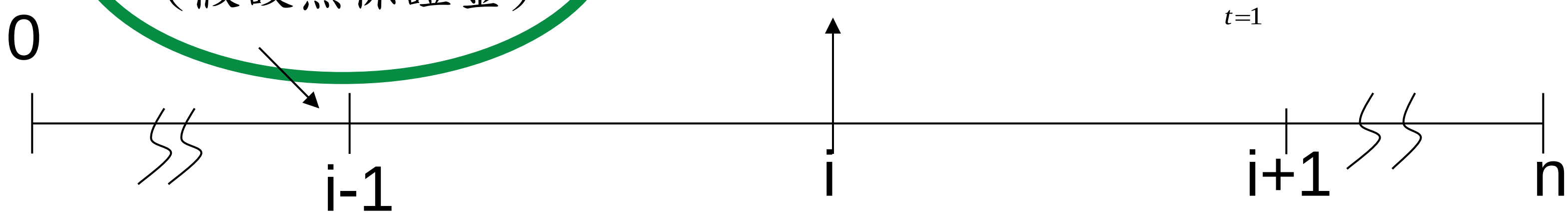
$$1 \times e^{r_1} e^{r_2} \ldots\ldots e^{r_n} = R_1 R_2 \ldots R_n = \prod_{t=1}^{n} R_t = R$$

- Consider the follow strategy:
  - Long $\prod_{t=1}^{i} R_t$ futures positions at the end of day $i-1$ and invest the cash flow at the end of day i in riskless bonds maturing on delivery day n.

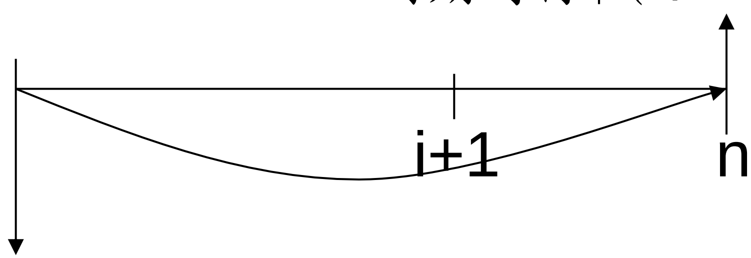
Proof

買入$\prod_{t=1}^{i} R_t$ 單位期貨
（假設無保證金）

在第i日的損益為$(f_i - f_{i-1})\prod_{t=1}^{i} R_t$

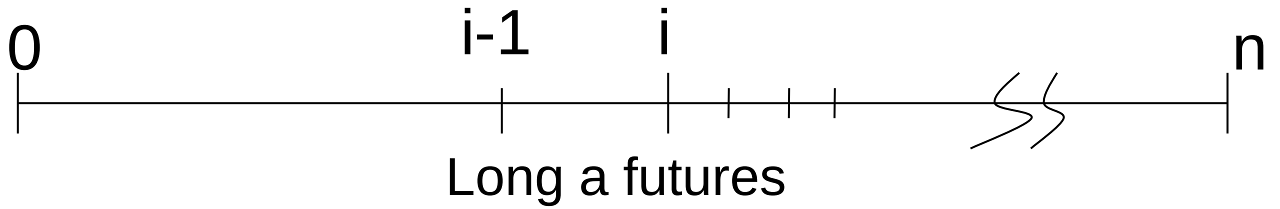


到期時得$\$\left(f_i - f_{i-1}\right)R$



拿$\$\left(f_i - f_{i-1}\right)\prod_{t=1}^{i} R_t$ 再投資無風險債券

$$\because \left(f_i - f_{i-1}\right)\prod_{t=1}^{i} R_t \prod_{t=i+1}^{n} R_t = \left(f_i - f_{i-1}\right)\prod_{t=1}^{n} R_t = \left(f_i - f_{i-1}\right)R$$



Forwards 可視為n個futures所組成！

- The value at the end of day n is

$$\sum_{i=1}^{n}\left(f_i - f_{i-1}\right)R = \left(f_n - f_0\right)R = \left(S_T - f_0\right)R$$

- Recall the value of R units of forwards is $\left(S_T - F_0\right)R$

- Arbitrage opportunity exists if $f_0 \neq F_0$.

Relationships between Forward and Future Prices

- 當利率隨機波動時, future price和forward price的理論價格不等

  一般而言,兩者差距不大
- Unless stated otherwise, assume forward and futures prices are identical.

Futures on Commodities

- For a commodity hold for investment purposes and with zero storage cost, the futures price is $F_0 = S_0 e^{rT}$
- In general, **U** stands for the PV of storage cost incurred during the life of a futures contract, and the futures price on commodity is $F_0 = (S_0 + U)e^{rT}$

$$\because F_0 = (S_0 - I)e^{rT} \quad I \text{ is the present value of income}$$

$$\because U \text{ is the present vaule of storage cost}, U = -I$$

$$\therefore F_o = (S_0 + U)e^{rT}$$

Futures on Commodities (convenience yield)

- Because user of the commodity may feel that ownership of the physical commodity provides benefit that not obtained by holders of futures contract.
- The benefit are the convenience yield (**y**) provided by the product. $F_0 e^{yT} = (S_0 + U)e^{rT}$
  - Ex: Oil refiner would like to take crude oil
- The convenience yield for investment asset=0 to prevent arbitrage.

Cost of Carry

- The relationship between futures prices and spot prices can be summarized in term of the cost of carry.
- This measures the dollar cost of carrying the asset over a period and consists of interest rate **r**, storage cost at the rate of **u**, minus cash flow at the rate of q generated by the asset.
  - The cost of carry is $C \equiv r + u - q$
- The futures price is $F_o = S_o e^{CT}$

The relationship cost of carry, convenience yield and basis

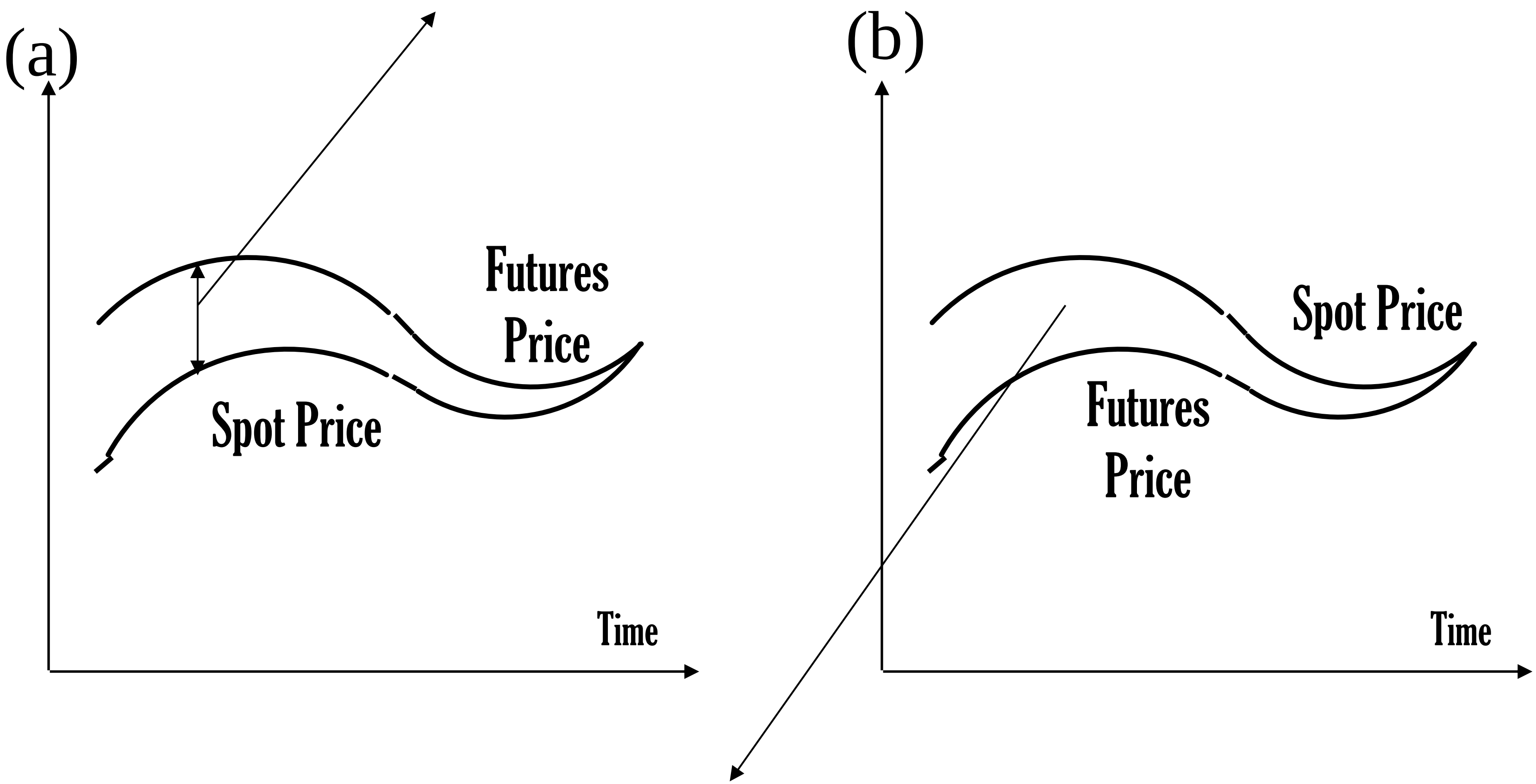
- **basis=$S_0 - F_0$**

$$F_o e^{yT} = S_o e^{CT} \rightarrow F_o = S_o e^{(C-y)T}$$

- In case of consumption assets, the futures price is greater than the spot price (basis$<$0) reflecting that the cost of carry.

Convergence of Futures Price to Spot Price

在正常市場下，基差(basis)為負，反應的就是標的
資產的持有成本，隨著到期日接近基差就會越來越小。



思考:為何現貨價格會大於期貨價格(逆向市場)?

Hedging with Futures

- A long hedge is that involve taking a long position in a futures contract, when the hedger will have to purchase a certain asset in the future and wants to lock in a price now.
- A short hedge is that involve taking a short position in a futures contract, when the hedger owns an asset and expects to sell it at some time in the future.

(see example)

Example－Short Hedge

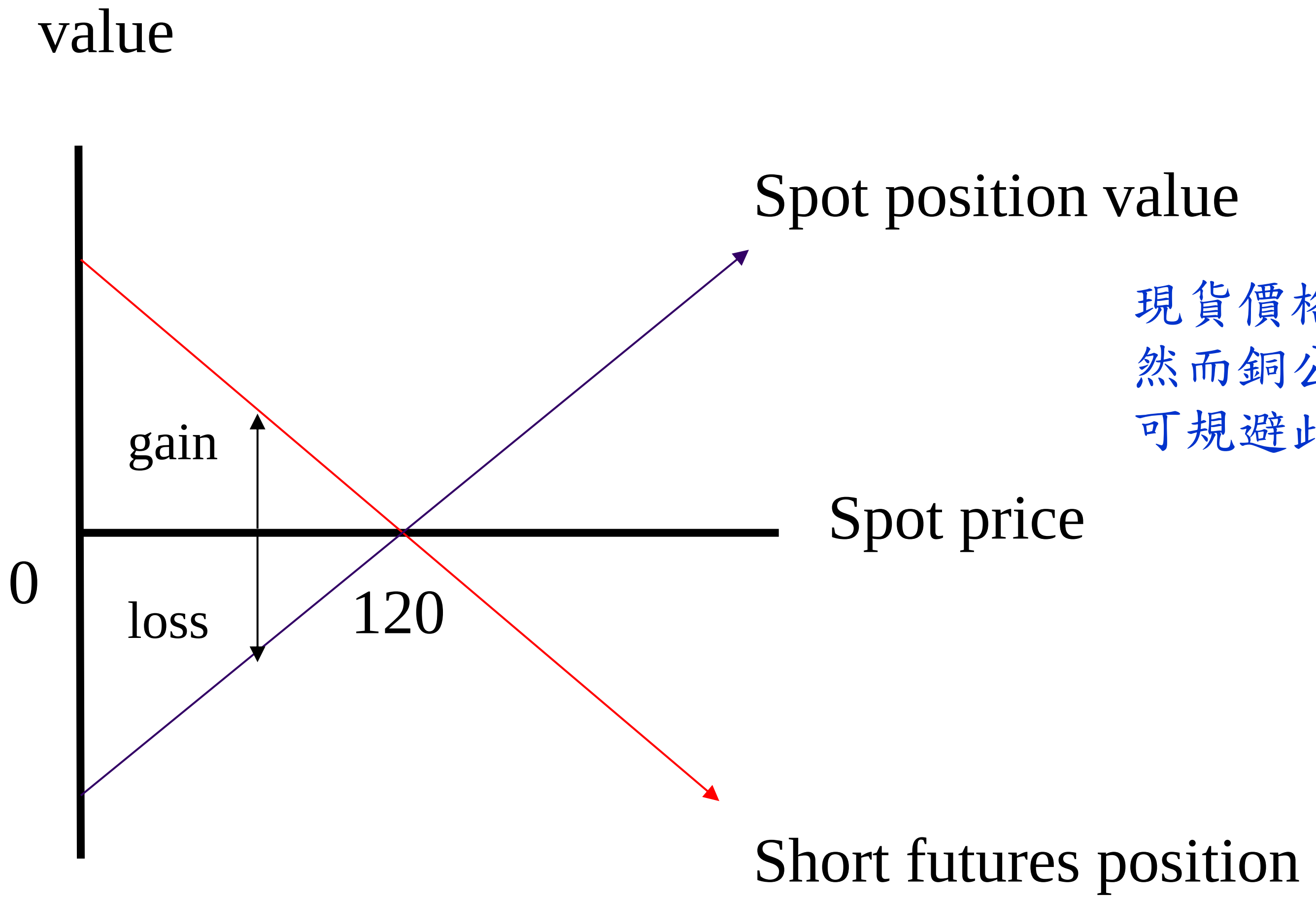
- 假設今天(1月25日)銅的現貨價格為每磅120美元，某一國外銅開採公司預計其將在5月15日賣出100,000磅銅，且在COMEX交易的五月份到期(到期日假設為5月15日)之銅期貨價格為每磅120美元(契約乘數)，其中每一口銅期貨合約大小為25,000磅。(假設不用繳保證金)
- 如果5月15日銅的現貨價格為每磅105美元，該國外銅開採公司如何利用COMEX(紐約商品期貨交易所)銅期貨合約來規避未來銅的價格風險？其淨收益為何?

該國外銅開採公司擔心未來銅價格下跌，導致生產收入減少，故它可以賣出四口COMEX銅期貨合約來避險。

Spot Market	Futures Market
1/25: 現貨100000磅價值為 $\$120 \times 100000 = 12,000,000$	1/25: 賣出四口5月份到期之銅期貨
5/15: 賣出現貨100000磅現貨價值減少為 $\$105 \times 100000 = 10,500,000$	5/15: 結算四口銅期貨結算的利得為 $\$(120-105) \times 4 \times 25000 = \$1,500,000$
Opportunity Loss $= \$12,000,000 - 10,500,000 = \$1,500,000$	Gain = $1,500,000

價格鎖定120美元

空頭避險損益圖



現貨價格雖然在5/15下跌，然而銅公司透過放空期貨可規避此價格的損失。

In class Exercise

- 如果到期時銅的現貨價格為每磅125美元, 請分析其現金流

- 思考：假如市場上沒有想要避險現貨的標的資產之期貨?或是現貨的交易日與期貨的到期日不相同?該怎麼辦?

完全避險的條件

- 從事避險之期貨標的物要和現貨商品一模一樣。
- 到期日要等於避險沖銷日。
- 對應於要避險之現貨部位，所買賣的期貨口數為某一整數。

期貨合約避險之合約選擇

- 交叉避險(Cross Hedge)：實務上要找到與現貨完全相同的標的資產之期貨合約非常困難，故乃以與現貨相關性高標的資產替代，此種策略稱為交叉避險。

- 避險者使用期貨合約避險，其效果好壞決定於期貨價格和現貨價格相關性高低。因此避險者選擇期貨合約需考慮的因素有二：

  (1)期貨合約標的資產的選擇

  (2)期貨合約交割月份的選擇

最適期貨避險數量

- 決定最適的期貨合約數量有二種方法：

  (1)單純避險法(Naive Hedge Method)

  (2)最小變異數避險比率法(Minimum Variance Hedge Ratio Method)

單純避險法

- 又稱完全避險法(Perfect Hedge Method)，指避險者買進或賣出和欲避險之現貨部位金額相同，但部位相反的期貨合約。
- 假設基差風險不存在，亦即現貨價格和期貨價格之變化是完全一致。
- **公式**：期貨合約口數=欲避險之現貨部位金額÷每口期貨合約價值。
- 實際上現貨價格的變化和期貨價格的變化未必會完全一致，因此單純避險法並不一定能將現貨價格風險完全規避。

Example

- 假設某一基金經理人持有價值新台幣20億的股票，而台股指數期貨目前價格為5000點，其每一點值新台幣200元，那麼該基金經理人為了防止其現股價格下跌的風險，他應該出售多少口台股指數期貨來避險？

每口台股指數期貨價值 = 5000 ×200 = 1,000,000

應出售的期貨合約口數 = 2,000,000,000 ÷1,000,000= 2,000（口）

最小變異數避險比率法

- **定義**：找出使避險投資組合風險(變異數)最小的避險比率的方法。又稱為迴歸分析法。
- **假設**：

  △S：避險期間內現貨價格的變動

  △F：避險期間內期貨價格的變動

  σ_S：現貨價格變動的標準差

  σ_F：期貨價格變動的標準差

  ρ：△S和△F間的相關係數
- **公式**：

$$h^*\text{（最小變異數避險比率）} = \rho\frac{\sigma_s}{\sigma_F}$$

最小變異數避險比率

- 如果$\rho=1$且$\sigma_F=\sigma_S$，則$h^*=1$。(此時最小變異數避險比率=單純避險法之避險比率，因為期貨合約價格的變動和現貨價格的變動完全一致。)

- 假設其他情況不變，若σ_S愈大或ρ愈大，則h愈大
- 假設其他情況不變，若σ_F愈大，則h愈小。

最小變異數避險比率的推導

- Suppose we expect to sell N_A units of an assets at time t_2 and to hedge at time t_1 by shorting futures contracts on N_F units of a similar asset.
  - The hedge ratio is $h = \dfrac{N_F}{N_A}$

- S_1 and S_2 are the asset prices at time t_1 and t_2, and F_1 and F_2 are the futures prices at time t_1 and t_2.

最小變異數避險比率的推導

- Total amount realized for the asset denoted by Y

$$Y = S_2 N_A - (F_2 - F_1) N_F$$

$$\Rightarrow Y = S_1 N_A + (S_2 - S_1) N_A - (F_2 - F_1) N_F = S_1 N_A + N_A (\Delta S - h \Delta F)$$

- When the variance of △S－h△F is minimized, the variance of Y is minimized.

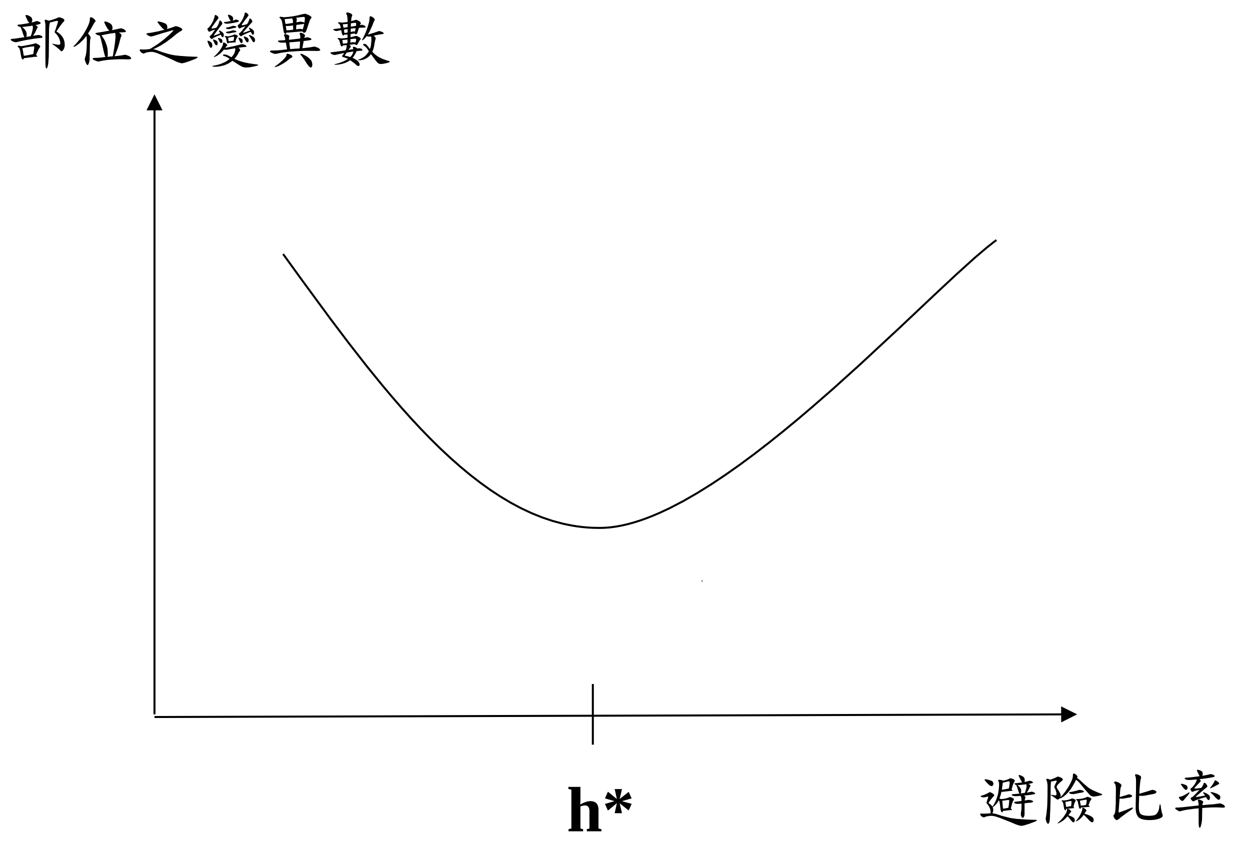
$$\because Var(\Delta S - h\Delta F) = \sigma_S^2 + h^2 \sigma_F^2 - 2h\rho\sigma_S\sigma_F = (h\sigma_F - \rho\sigma_S)^2 + \sigma_S^2 - \rho^2\sigma_S^2$$

$$\because MinVar(\Delta S - h\Delta F)$$

$$\Rightarrow (h\sigma_F - \rho\sigma_S)^2 = 0$$

$$\Rightarrow h = \rho \frac{\sigma_S}{\sigma_F}$$

避險者的部位之變異數和避險比率之關係



Example

- 假設台股指數價格變動百分比和大台指期貨價格變動百分比的相關係數為0.8，而台股指數價格變動百分比之標準差為0.6，而期貨合約價格變動百分比之標準差為0.4，某一基金經理人持有價值新台幣20億的股票，而台股指數期貨目前價格為5000點，其每一點值新台幣200元，那麼該基金經理人為了防止其現股價格下跌的風險，他應該出售多少口台股指數期貨來避險？

Ans: 最小避險變異數比率為0.8×(0.6÷0.4)=1.2

應出售的口數為2000×1.2=2400(口)

Swaps

- Swaps are agreements by two parties to exchange cash flows in the future according to a prearranged formula.
- There are two basic types of swaps:
- ➔interest rate swaps
- ➔Currency swaps
- Swaps on commodities are also available.

Interest rate Swaps

- An example for a “ Plain Vanilla ” Interest rate Swaps
- An agreement by Microsoft to receive 6-month LIBOR & pay a fixed rate of 5% per annum every 6 months for 3 years on a notional principal of $100 million

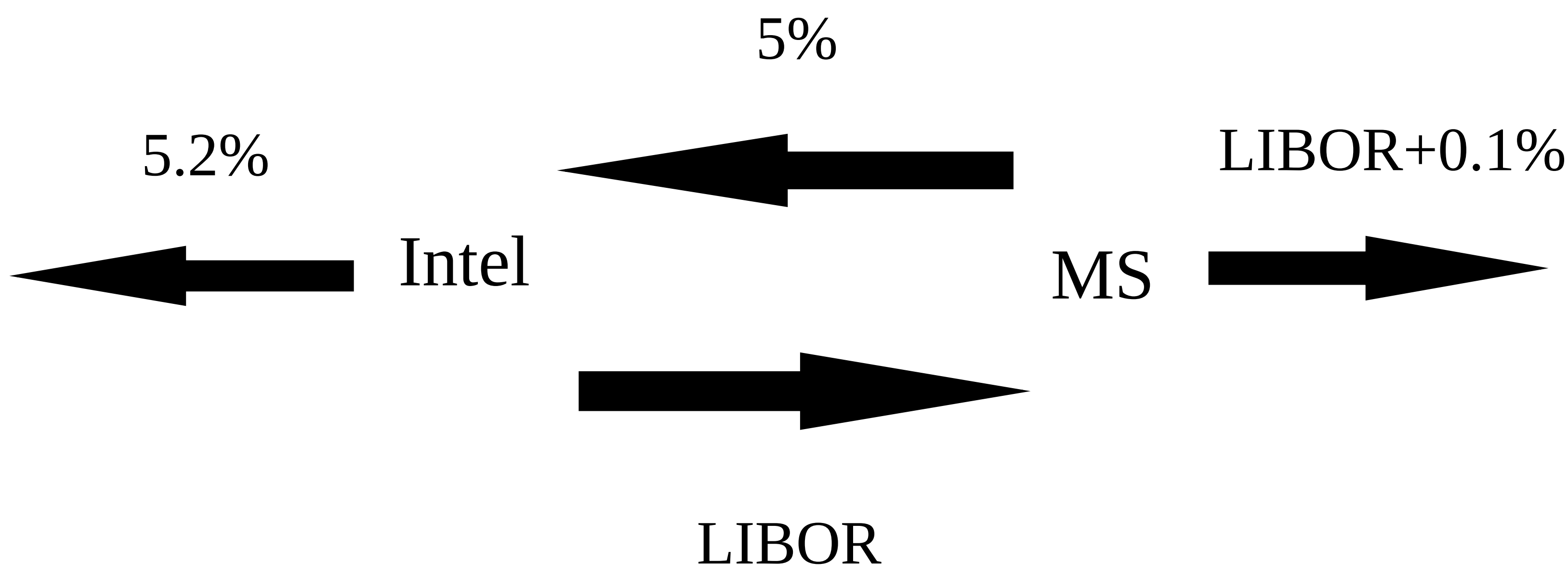
Cash Flows to Microsoft

		---------Millions of Dollars---------		
	LIBOR	*FLOATING*	*FIXED*	Net
Date	Rate	Cash Flow	Cash Flow	Cash Flow
Mar.5, 2004	4.2%			
Sept. 5, 2004	4.8%	+2.10	–2.50	–0.40
Mar.5, 2005	5.3%	+2.40	–2.50	–0.10
Sept. 5, 2005	5.5%	+2.65	–2.50	+0.15
Mar.5, 2006	5.6%	+2.75	–2.50	+0.25
Sept. 5, 2006	5.9%	+2.80	–2.50	+0.30
Mar.5, 2007	6.4%	+2.95	–2.50	+0.45

Typical Uses of an Interest Rate Swap

- Converting a liability or an investment from
  - fixed rate to floating rate
  - floating rate to fixed rate

Intel and Microsoft (MS) Transform a Liability



Suppose that Microsoft has arranged to borrow $100 million at LIBOR plus 10 basis points.

Suppose that Intel has a 3-year $100 million loan outstanding on which it pays 5.2% .

Using the Swap to Transform a Liability

- For Microsoft, the swap could be used to transform a floating-rate loan into a fixed –rate loan. After Microsoft has entered into the swap, it has the following three sets of cash flows:

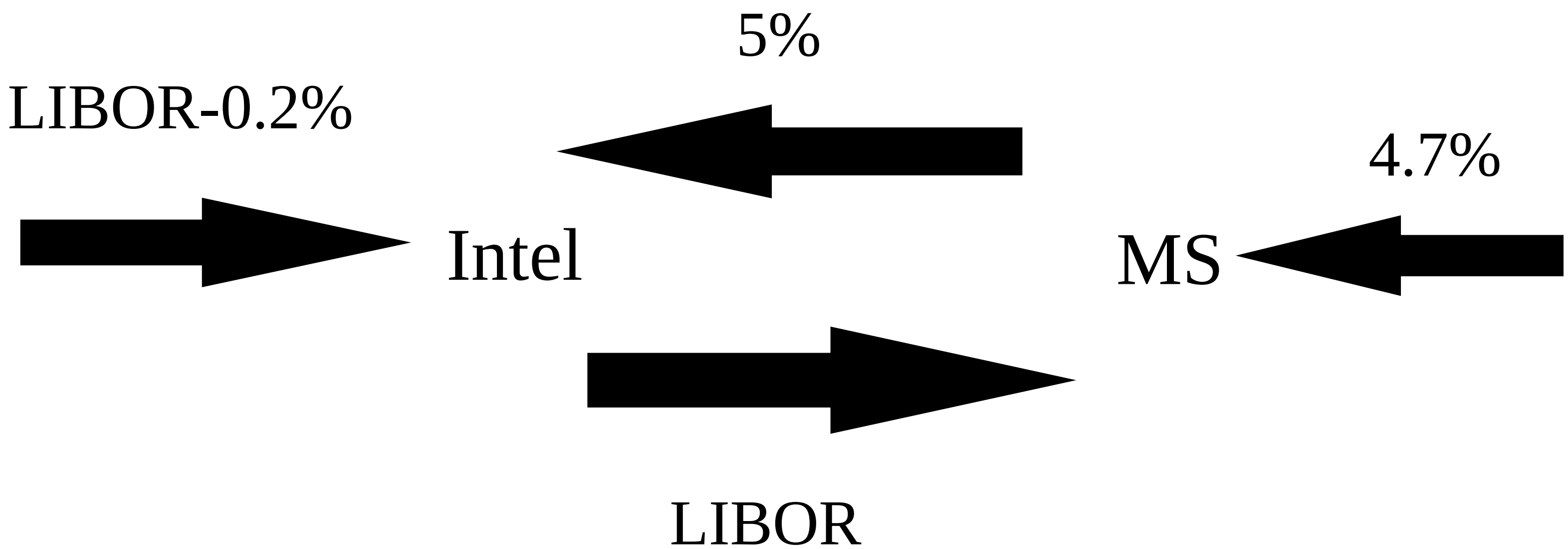
1. It pays LIBOR plus 0.1% to its outside lenders.
2. It receives LIBOR under the terms of the swap.
3. It pays 5% under the terms of the swap.

Using the Swap to Transform a Liability

- For Intel, the swap could have the effect of transforming a fixed-rate loan into a floating –rate loan. After Intel has entered into the swap, it has the following three sets of cash flows:

1. It pays 5.2% to its outside lenders.
2. It pays LIBOR under the terms of the swap.
3. It receives 5% under the terms of the swap.

Intel and Microsoft (MS) Transform an Asset



Suppose that Microsoft owns $100 million in bonds that will provide interest at 4.7% per annum over the next 3 years.

Suppose that Intel has an investment of $100 million that yields LIBOR minus 20 basis points.

Using the Swap to Transform an Asset

- For Microsoft, the swap could have the effect of transforming an asset earning a fixed rate of interest into an asset earning a floating rate of interest. After Microsoft has entered into the swap, it has the following tree sets of cash flows:

1. It receives 4.7% on the bonds.
2. It receives LIBOR under the terms of the swap.
3. It pays 5% under the terms of the swap.

Using the Swap to Transform an Asset

- For Intel, the swap could have the effect of transforming an asset earning a floating rate of interest into an asset earning a fixed rate of interest. After Microsoft has entered into the swap, it has the following tree sets of cash flows:

1. It receives LIBOR minus 20 basis points on its investment.

2. IT pays LIBOR under the terms of the swap.

3. It receives 5% under the terms of the swap.

Quotes By a Swap Market Maker

Maturity	Bid (%)	Offer (%)	Swap Rate (%)
2 years	6.03	6.06	6.045
3 years	6.21	6.24	6.225
4 years	6.35	6.39	6.370
5 years	6.47	6.51	6.490
7 years	6.65	6.68	6.665
10 years	6.83	6.87	6.850

Using Swap Rates to Bootstrap the LIBOR/Swap Zero Curve

- Consider a new swap where the fixed rate is the swap rate
- When principals are added to both sides on the final payment date the swap is the exchange of a fixed rate bond for a floating rate bond
- The floating-rate rate bond is worth par. The swap is worth zero. The fixed-rate bond must therefore also be worth par
- This shows that swap rates define par yield bonds that can be used to bootstrap the LIBOR (or LIBOR/swap) zero curve

假設F = swap rate,   P = par,

s_i = i-th period (year) spot rate,

N= # of swap periods,

$$P * F * e^{-s_1 * 1} + P * F * e^{-s_2 * 2} + \ldots + P(1 + F)e^{-s_N * N} = P$$

Valuation of an Interest Rate Swap that is not New

- Interest rate swaps can be valued as the difference between the value of a fixed-rate bond and the value of a floating-rate bond
- Alternatively, they can be valued as a portfolio of forward rate agreements (FRAs)

#Homework 8

- 請分析如何使用拔靴法（bootstrap method）和市場新台幣利率交換（Interest Rate Swap，IRS）報價資料來計算零息債券的利率（zero coupon rate），並撰寫計算零息利率的程式。
- 假定一年交換一次,第4,6年資料用內插估計

到期日 (年)	Bid (%)	Offer (%)	Swap rate (%)
1	2.25	2.27	2.26
2	2.26	2.29	2.275
3	2.27	2.30	2.285
5	2.33	2.38	2.355
7	2.41	2.47	2.44

資料來源(此為參考資料)

2008年9月15日

Swap rate為買賣價之平均值

KGI凱基證券網站

http://www.kgieworld.com.tw/bond/bond_01_02.htm

Currency Swaps

- A currency swap involves two parties to exchange cash flows in different currencies.
- Consider the following fixed rates available to party A and party B in U.S. dollars and Japanese yen:

	Dollars	Yen
A	D_A%	Y_A%
B	D_B%	Y_B%

- Suppose A wants to take out a fixed-rate loan in yen, and B wants to take out a fixed-rate loan in dollars.

Currency Swaps

- A straightforward scenario is for A to borrow yen at Y_A% and B to borrow dollars at D_B%.
- But suppose A is *relatively* more competitive in the dollar market than the yen market, and vice versa for B.
  - ➔ $Y_B + D_A < D_B + Y_A$.
- Alternative arrangement:
  - ➔A borrows dollars.
  - ➔B borrows yen.
  - ➔They enter into a currency swap with a bank as the intermediary.

	USD	JPY
BA(A)	3%	1%
JAL(B)	5%	2%

Compare spread

Currency Swaps (concluded)

- The counterparties exchange principal at the beginning and the end of the life of the swap.
- This act transforms A's dollar loan into a yen loan and B's yen loan into a dollar loan.
- The total gain is $((D_B - D_A) - (Y_B - Y_A))\%$:

  ➔ The total interest rate is originally $(Y_A + D_B)\%$.

  ➔The new arrangement has a smaller total rate of $(D_A + Y_B)\%$.
- Transactions will happen only if the gain is distributed so that the cost to each party is less than the original.

Swaps & Forwards

- A swap can be regarded as a convenient way of packaging forward contracts
- The value of the swap is the sum of the values of the forward contracts underlying the swap
- Swaps are normally “at the money” initially
  - This means that it costs nothing to enter into a swap
  - It does not mean that each forward contract underlying a swap is “at the money” initially