2(c)

$Y \sim N(\mu, \sigma^2)$

$Y = \ln(X)$

$X = e^Y$

Let $f_Y(y)$ be the PDF of Normal ~~distr~~ distribution

$E(X) = \int_{-\infty}^{\infty} x f(x) dx$

$E(e^Y) = \int_{-\infty}^{\infty} e^Y f_Y(y)\, dy$

$= \int_{-\infty}^{\infty} e^Y \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy$

$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{y - \frac{y^2 - 2y\mu + \mu^2}{2\sigma^2}} dy$

$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y^2 - 2y\mu + \mu^2 + 2\sigma^2 y}{2\sigma^2}} dy$

$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(y-(\mu+\sigma^2))^2 + 2\mu\sigma^2 + \sigma^4}{2\sigma^2}}$

$= e^{\mu + \frac{\sigma^2}{2}} \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(y-(\mu+\sigma^2))}{2\sigma^2}} dy$

Since the integral is now integrating $N(\mu+\sigma^2, \sigma^2)$, so it is equal to

$E(X) = e^{\mu + \frac{\sigma^2}{2}}$

sub $\mu = 0.02$, $\sigma = 0.05$

$E(X) = e^{0.02 + \frac{0.05^2}{2}}$

$= 1.021477389$

$Var(X) = E(X^2) - E(X)$

$$E(X^2) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{2y - \frac{(y-\mu)^2}{2\sigma^2}} dy$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{y^2-4y\mu+\mu^2-4\sigma^2 y}{2\sigma^2}\right)} dy$$

$$= e^{2\mu+2\sigma^2} \times 1$$

$$= e^{2\mu+2\sigma^2}$$

sub $\mu = 0.02$, $\sigma = 0.05$

$$Var(X) = e^{2(0.02)+2(0.05)^2} - 1.021477389$$

$$= 0.02455047$$

Empircal data

sample	10	100	1000	10000	100000
μ	1.001957	1.013513	1.020112	1.021758	1.021626
σ^2	0.00244	0.002671259	0.002591146	0.00254836	0.002610844

Theoretical data

$E(X) = 1.021477389$

$Var(X) = 0.02455047$

From the compared result, when sample size ↑, E(X) will more accurate but Var(X) will less accurate