CS1632: Static Analysis, Part 3

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Symbolic Model Checking

- Model checking can be categorized into:
  1. Enumerative model checking
     - What we learned in the last chapter
     - Hard to escape state explosion

  2. Symbolic model checking
     - What we will learn in this chapter
     - Model checking using *symbolic execution*
     - Can fundamentally solve the state explosion problem

Symbolic Execution

- **Symbolic execution**: Assigning *symbolic expressions* instead of actual values to variables during execution
  - Instead of `x = 1, y = true`, …
  - `x = A + 1`, `y = A * B`, …
- **Symbolic expression**: An expression using *symbolic values*
  - `A + 1`, `A * B`, …
- **Symbolic value**: Math symbol that stands for an *input value*
  - `A, B, …, X, Y, Z`
- Idea:
  - If `x == A+1`, `y == A+2` at source line `assert(x < y)`

  → Model checker can prove though math that it always passes, for every input value without having to try them one by one!

Notation We Will Use

- Program variables: lower case
  - `int x, y, z;`
- Symbolic values: UPPER CASE
  - `A, B, …, X, Y, Z`
  - Represent input values that are given to the code
  - Can be values from user input
  - Can be values from command line arguments
  - Can be values passed into method parameters

Example: Enumerative Model Checking

Code that swaps 2 integers

```
int x, y;  // parameters
if (x > y) {
  x = x + y;
  y = x – y;
  x = x – y;
  if (x > y)
    assert false;
}
```

Execution Path for x=1, y=0

x: 1, y: 0

↓

1 > 0 ? true

↓

x = 1 + 0 = 1

↓

y = 1 – 0 = 1

↓

x = 1 – 1 = 0

↓

0 > 1 ? false

- Must do this for all values of x and y.
- But is that how a human would do it?

Symbolic Model Checking

- Trace through a program like a human being would
- In a symbolic execution:
  - Inputs are *symbolic values* instead of concrete data values
  - Variables are *symbolic expressions* on the *symbolic values*
- Example:

```
int x, y; // parameters

x = x + y;

y = x – y;

x = x – y;
```

[Code]

x: 1, y: 0
↓
x = 1 + 0 = 1
↓
y = 1 – 0 = 1
↓
x = 1 – 1 = 0

[Concrete]

x: A, y: B
↓
x = A + B
↓
y = A + B – B = A
↓
x = A + B – A = B

[Symbolic]

- Symbolic execution proves that the swap works for all A and B!

Symbolic Model Checking

- What if there is path divergence?
  - if statement
  - for loop
  - while loop

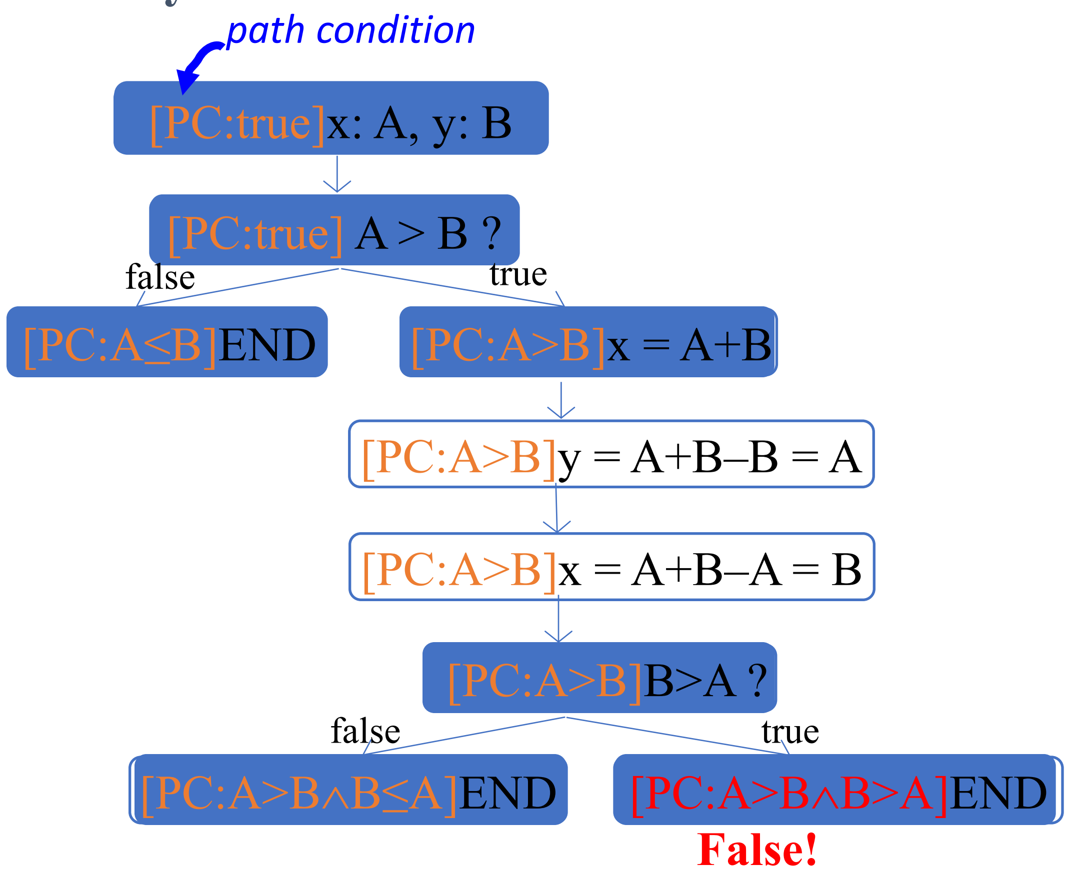
- For each path, build a Path Condition (PC)
  - Condition on symbolic values (the As and the Bs)

Example: Symbolic Execution

Code that swaps 2 integers:

```
int x, y;

if (x > y) {

  x = x + y;

  y = x – y;

  x = x – y;

  if (x > y)

    assert false;

}
```

Symbolic Execution Tree:



Is the Path Condition Feasible?

- Each path condition is checked using a constraint solver

$A > B \land B > A \Rightarrow$ $\boxed{\text{Constraint Solver}}$ $\Rightarrow$ {} Empty Set!

- If path is infeasible, does not continue down that path
  - Hence, assert false is never reached

Symbolic Model Checking Uses

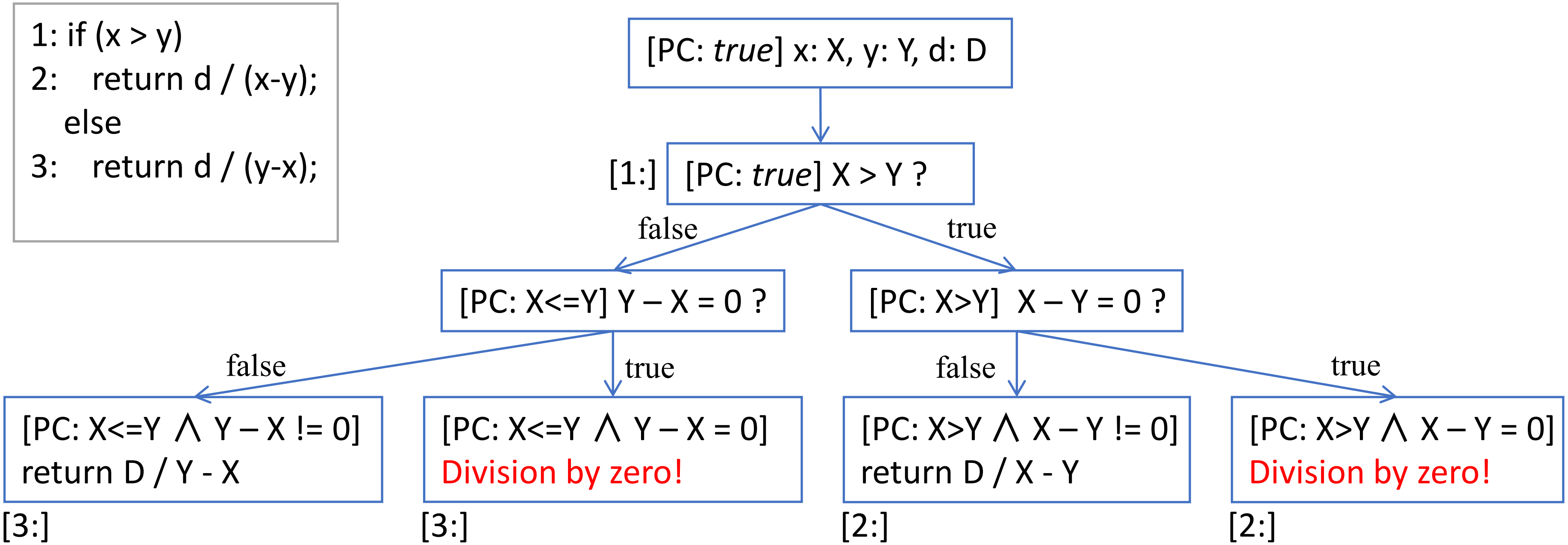
- Prove a program correct
  - Much less state explosion than enumerative checking
- Generate test vectors that exercise each path
  - For the purposes of increasing test coverage of code
  - The output sets of the constraint solver are the test vectors
- Generate program invariants
  - Invariants enhance programmer's understanding of code
  - The path conditions themselves *are* the invariants

Generating Test Cases out of Path Conditions

Method m (x, y, d):

```
1: if (x > y)
2:    return d / (x-y);
   else
3:    return d / (y-x);
```

Symbolic execution tree:



Solve path conditions → test inputs

Auto-generated JUnit Tests

- Constraint solver returns values satisfying each Path Condition

☛ Can achieve full code coverage by exercising each path!

```
@Test public void t1() {
   m(10, 20, 0);
}
```

Pass ✓ PC: $X<=Y \wedge Y - X \mathrel{!=} 0 \Leftrightarrow X < Y$

$(X=10, Y=20, D=0) \in PC$

```
@Test public void t2() {
   m(10, 10, 0);
}
```

Fail ✗ PC: $X<=Y \wedge Y - X = 0 \Leftrightarrow X = Y$

$(X=10, Y=10, D=0) \in PC$

```
@Test public void t3() {
   m(20, 10, 0);
}
```

Pass ✓ PC: $X>Y \wedge X - Y \mathrel{!=} 0 \Leftrightarrow X > Y$

$(X=20, Y=10, D=0) \in PC$

Skip ↷ PC: $X>Y \wedge X - Y = 0 \Leftrightarrow \{ \}$

Symbolic Model Checking Challenges

- Symbolic model checking does have challenges

  … Or everyone would be using symbolic model checking

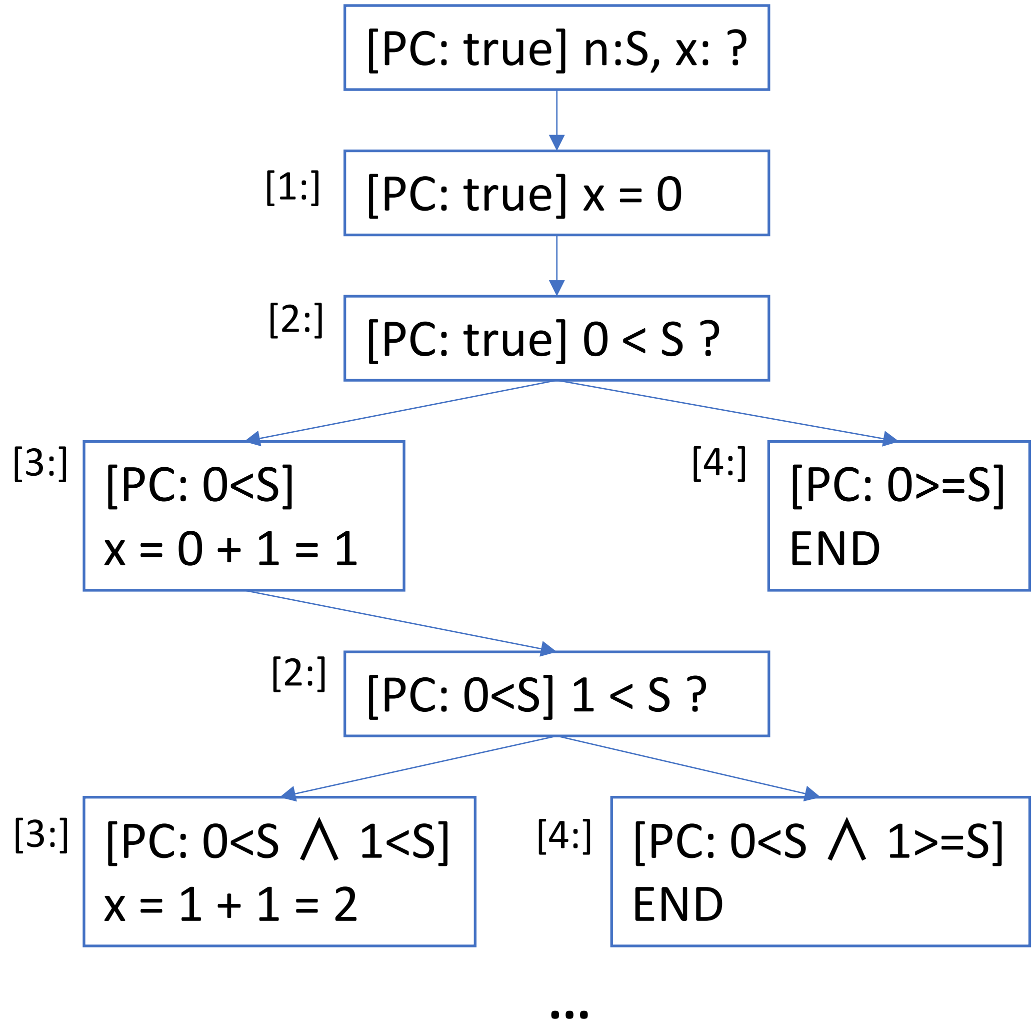
- Some examples are:
  - Loops
  - Complex math constraints
  - Complex data structures

Challenges: Loops

Example Code

```
void test(int n) {
1:  int x = 0;
2:  while(x < n) {
3:    x = x + 1;
4:  }
}
```

Infinite symbolic execution tree



Challenges: Complex Math Constraints

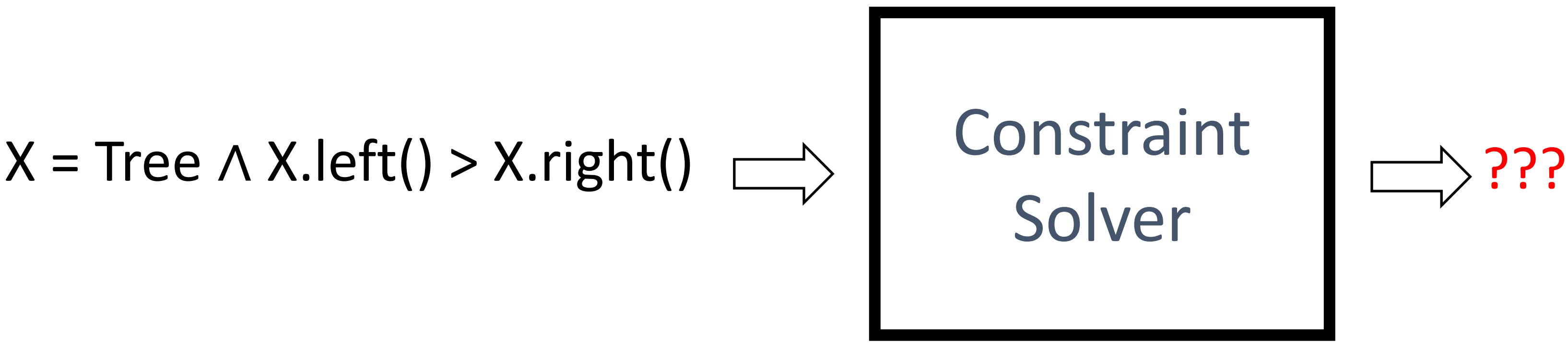
- Constraint solvers are not particularly good at math

$X < \log(Y)+1 \land \sin(X)^2 > \cos(Y) \Rightarrow$ **Constraint Solver** $\Rightarrow$???

- Constraint solving is NP-Complete in general
  - Research resulted in some subproblems being proved polynomial
  - State-of-the-art solvers have trouble when constraints are non-linear

Challenges: Complex Data Structures

- Complex data structures are confusing to solvers



- In order to solve above constraint, solver must know:
    - What a tree data structures looks like
    - What left() means and what right() means
- Solvers know some data structures, but not many

The Best of Both Worlds

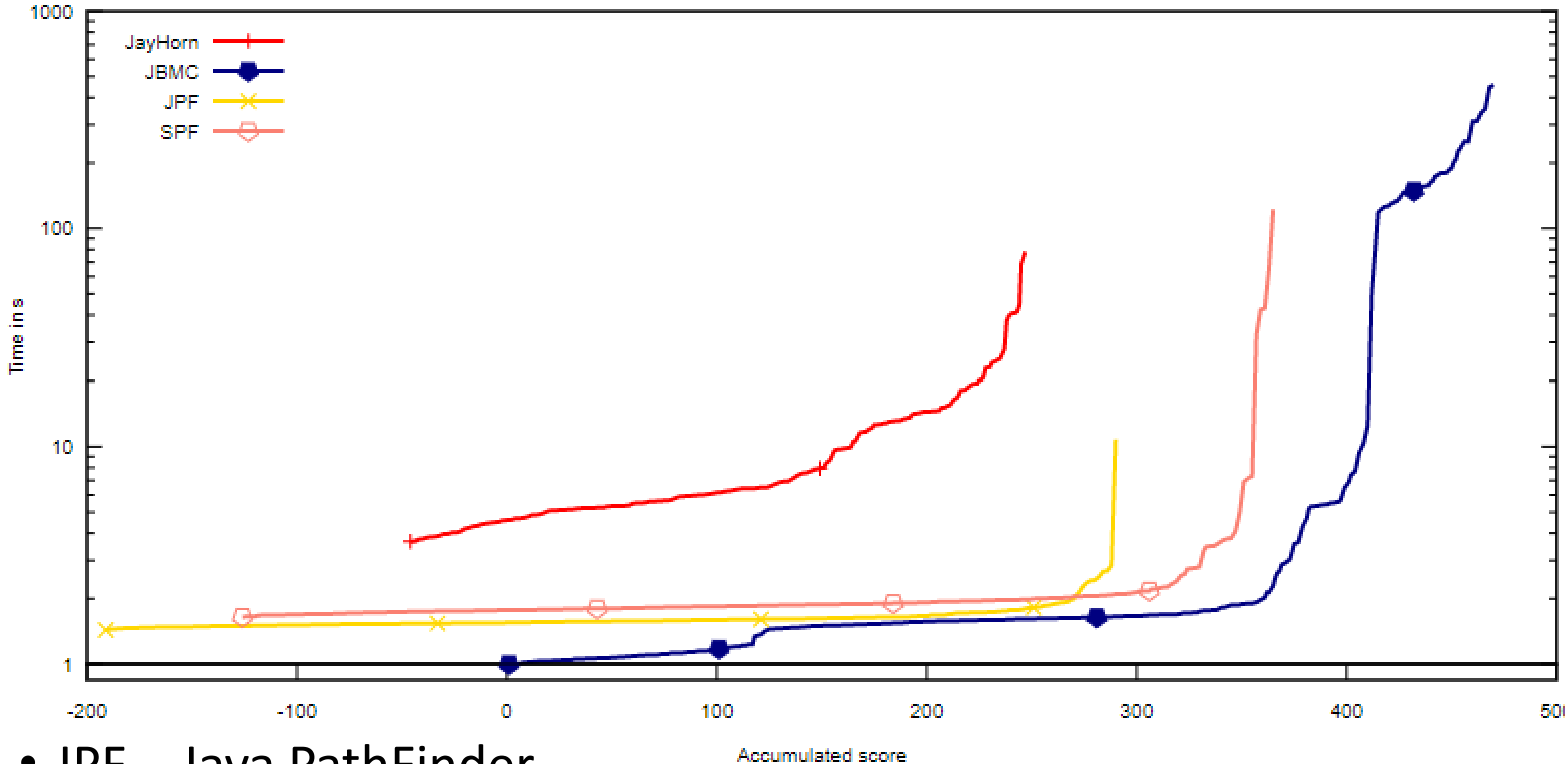
- Symbolic Model Checking (Symbolic Execution)
  + Much less state explosion
  - Hard time dealing with loops, math, data structures

- Enumerative Model Checking (Concrete Execution)
  - Serious state explosion
  + No problems with loops, math, data structures
    (just execute the loop, math, or data structure code)

- The best of both worlds: Concolic Execution
  - Concolic = Concrete + Symbolic
  - Symbolic Java Path Finder does exactly this!
  - Chooses between the two depending on the method

Model Checking is Getting Better Every Year

https://sv-comp.sosy-lab.org/2019/results/results-verified/



- JPF – Java PathFinder
- SPF – Symbolic Java PathFinder (JPF with symbolic execution)
- JBMC – Java Bounded Model Checker (2018 newcomer)

References

- Ranjit Jhala and Rupak Majumdar. 2009. “Software model checking”. ACM Computing Surveys: https://people.mpi-sws.org/~rupak/Papers/SoftwareModelChecking.pdf
- Cristian Cadar and Koushik Sen. 2013. “Symbolic execution for software testing: three decades later”. Communications of the ACM: https://people.eecs.berkeley.edu/~ksen/papers/cacm13.pdf
- 8th Competition on Software Verification (SV-COMP), 2019: https://sv-comp.sosy-lab.org/2019/results/results-verified/