Project II: Optimizing US-global Health Commodity Shipment Plan

Ken Chen, Yinuo Sun, Yiting Wang, Suyu Zhou
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Abstract: This project focuses on optimizing the United States' global health supply chain to ship essential health commodities to countries in need. We designed cost-effective and environmentally sustainable shipment plans while minimizing CO2 emissions by reducing total route distances. We formulated the problem as Traveling Salesman Problems (TSP). We considered two scenarios: first, with a single aircraft departing from JFK and returning to LAX, and second, with shipments distributed between two warehouses near JFK and LAX. We provided optimized travel routes for both cases and reported the routes with visualization.

Keywords: TSP, Shipment, Linear Programming

1. Introduction

The U.S. government's Global Health Commodity Support Plan delivers essential health supplies, including antiretroviral (ARV) medications and HIV lab kits, to countries in need. This initiative seeks to balance cost-effectiveness and environmental sustainability by minimizing CO2 emissions associated with air transport. The project examines how to optimize shipment plans across three phases, focusing on route efficiency to reduce the total shipping distances.

Two scenarios are explored: a single-aircraft model, where deliveries originate from JFK and conclude at LAX, and a dual-warehouse model, where shipments are distributed from facilities near JFK and LAX using separate planes. Both scenarios are solved as Traveling Salesman Problems (TSP) to identify efficient delivery routes using integer programming techniques, ensuring timely and sustainable logistics for global health support. The rest of this paper is organized as follows. We find the optimized solution for the first scenario and discuss two possible cases for the second scenario. We briefly describe the data in Section 2. Section 3 provides the mathematical formulation of the problem and the optimization method. Section 4 discusses the results and performs robustness checks. Section 5 concludes.

2. Data Description

The raw data includes the airport code, longitude, and latitude of airports to be visited by the planes. We impute the point-wise distance between any two airports based on the haversine formula. The resulting data is the airport-level data, including the airport code, longitude, latitude, and distance to the other 45 airports.

3. Methods

This problem of sending medical supplies to a network of airports by visiting each airport once and once only fits with the design and purpose of the traveling salesman problem (TSP). We set each scenario up within the TSP framework and discuss the detailed objective functions, constraints, and parameters for each scenario.

3.1. The Problem of Optimizing Shipment

This is a special case of the traveling salesman problem (TSP). It differs from the classical TSP in that the starting point is fixed at JFK, and the ending point is fixed at LAX, and LAX is not JFK.

Parameters We define the parameters as follows:

- The set of airports is denoted by N and indexed by i and j;
- Let d_{ij} be the distance from airport i to j, $i \neq j$;

Decision Variables Let $x_{ij} = \begin{cases} 1 \text{ if route from airport } i \text{ to airport } j \text{ is traversed, } i \neq j; \\ 0 \text{ otherwise} \end{cases}$

Objective Function Minimizing the CO2 emission is equivalent to minimizing the distance traveled within the network:

$$\min \sum_{i \in N} \sum_{j \in N: j \neq i} d_{ij} x_{ij}. \tag{1}$$

Constraints Subject to:

- Exactly one trip entering each airport: $\sum_{i \in N: i \neq j} x_{ij} = 1, \forall j \in N$. (note: except JFK)
- Exactly one trip leaving each airport:: $\sum_{j \in N: j \neq i} x_{ij} = 1, \forall i \in N$. (note: except LAX)
- Binary decision: $x_{ij} \in \{0, 1\}, \forall i, j, \in N$
- We further formulate the problem using Miller-Tucker-Zemlin (MTZ). Define $|N| - 2$ variables $\mathbf{u} \equiv (u_2, u_3, \ldots, u_{n-1})$ having the interpretation that $u_j = k$ indicates that city j is located in the kth position after the first. We add the following constraints:

- Eliminate any possible subtour: $u_i + 1 \leq u_j + (n-2)(1 - x_{ij}), \forall i \in N, j \in N, i \neq j \neq 1$;
- There is no round trip to any city $x_{ii} = 0, \forall i \in N$
- There is a path from any city i to j: $x_{ij} \in \{0, 1\}$
- Fixed starting point at JFK: $u_1 = 1$
- Fixed ending point at LAX: $u_n = N$
- Number of cities $2 \leq u_i \leq n - 1$

3.2. The Problem of Optimizing Splitting Shipment

We formulate the problem as a multiple Traveling Salesman Problem (mTSP), as specified by [1]. Instead of the single starting point as in the classic TSP, we have two different starting points, with one plane at each starting point. The plane returns to the same starting point from which it started.

Parameters *same as problem 1* We define the parameters as follows:

- The set of airports is denoted by N and indexed by i and j;
- Let d_{ij} be the distance from airport i to j, $i \neq j$;

Decision Variables We add a third dimension w denoting the starting airport of the trip (0 for starting from JFK and 1 for LAX). Let

$$x_{wij} = \begin{cases} 1 \text{ if route from airport } i \text{ to airport } j \text{ is traversed, } i \neq j; \\ 0 \text{ otherwise} \end{cases}$$

Objective Function Let the set S contain the airports to be visited. Minimizing the CO2 emission is equivalent to minimizing the distance traveled within the network:

$$\min \left(\sum_{i \in S} \sum_{j \in S: j \neq i} d_{ij} x_{wij} \right) \tag{2}$$

We consider two cases for this problem:

1. There are two planes, one from JFK and one from LAX: $2 \leq S \leq N - 1$. Each plane returns to the same origin.
2. There are two planes, but the other one is not used for this trip. There is no additional constraint needed.

Constraints for Case 1 (one for JFK, one LAX) Subject to:

1. We modify the terminology to make the edge x_{ij} to include the starting airport of the trip as the first dimension: x_{0ij} denotes the trips starting from JFK, and x_{1ij} denotes the trips starting from LAX.
2. Exactly one trip entering each airport: $\sum_{i \in S: i \neq j} x_{0ij} = 1$ and $\sum_{i \in T: i \neq j} x_{1ij} = 1, \forall j \in S$

3. Exactly one trip leaving each airport:: $\sum_{j \in S: j \neq i} x_{0ij} = 1$ and $\sum_{j \in S: j \neq i} x_{1ij} = 1, \forall j \in S$.
4. Binary decision: $x_{0ij}, x_{1ij} \in \{0, 1\}, \forall i, j, \in S$.
5. Ensure that exactly one aircraft departs from the JFK airport: $\sum_{j \in S: j \neq 1} x_{01j} = 1$;
6. Ensure that exactly one aircraft returns to the JFK airport: $\sum_{j \in S: j \neq 1} x_{0j1} = 1$;
7. Ensure that exactly one aircraft departs from the LAX airport: $\sum_{j \in S: j \neq 1} x_{11j} = 1$;
8. Ensure that exactly one aircraft returns to LAX airport: $\sum_{j \in S: j \neq 1} x_{1j1} = 1$;
9. Both planes need to be deployed: $\sum_{j \in S: j \neq 1} x_{00j} + \sum_{j \in S: j \neq 1} x_{11j} = 2$
10. There's one place for each of LAX and JFK: $\sum_{j \in S: j \neq 1} x_{11j} = 1$ and, $\sum_{j \in S: j \neq 1} x_{00j} = 1$
11. Subtour elimination constraints MTZ is also added to each subset. In set S, define $|S| - 1$ variables $\mathbf{u} \equiv (u_2, u_3, \ldots, u_s)$ having the interpretation that $u_j = k$ indicates that city j is located in the kth position after the first. Add the following constraints:
    - Eliminate any possible subtour: $u_{wi} + 1 \leq u_{wj} + (s - 1)(1 - x_{wij}), \forall i \in S, j \in S, i \neq j \neq 1$ and $w \in \{0, 1\}$ denotes the starting airports ;
    - There is no round trip to any city $x_{wii} = 0, \forall i \in S$
    - There is a path from any city i to j: $x_{wij} \in \{0, 1\}$
    - Fixed starting point at JFK: $u_w 1 = 1$
    - Number of cities $2 \leq u_i \leq s$

Constraints for Case 2 (One plane may not get used) Subject to:

- Note that there's no need to separate the airports into two sets in this case. The setup of case 2 will be the same as case 1 above, except for the condition 9 and 10.
    - *The constraint 9th in case 1* will be changed to: "At least one plane needs to be deployed, and the other one may not be used": $\sum_{j \in S: j \neq 1} x_{01j} + \sum_{j \in S: j \neq 1} x_{11j} \geq 1$.
    - Moreover, *the 10th constraint from case 1* will also be deleted for case 2 to loose the requirement on plane from each warehouse.

4. Results and Discussion

We are able to find the optimized solution for both scenarios. We present our results below, accompanied with a visualized itinerary.

4.1. Optimizing Shipment

The result of optimizing shipment is shown in the figure above. The optimal objective value is **35,395.5 miles**. As shown from Figure 1, the aircraft departs from New York,

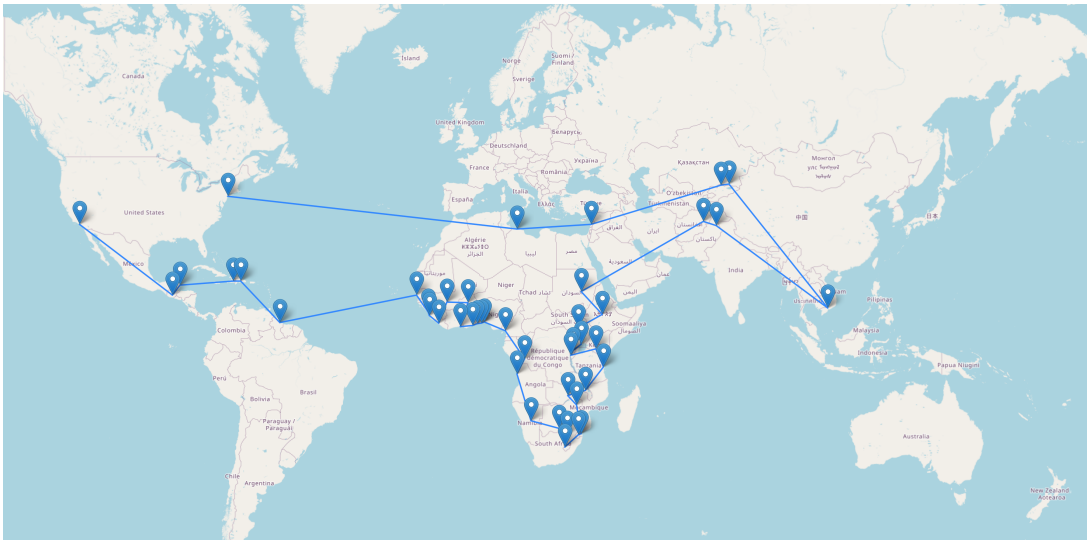

Figure 1: Visualization of the Optimized Itinerary, Question 1

travels across South America, and then travels to Africa, Europe and Asia, then back to Los Angeles.

4.2. Optimizing Splitting Shipment

4.2.1. Case 1

The result of optimizing the splitting shipment for Case 1 is shown in Figure 2 below. The optimal objective value is **37,960.8 miles**. In this solution, one aircraft departs from LAX and services destinations such as Belize and Guatemala, while the other aircraft departs from JFK and handles all other airports in the network.

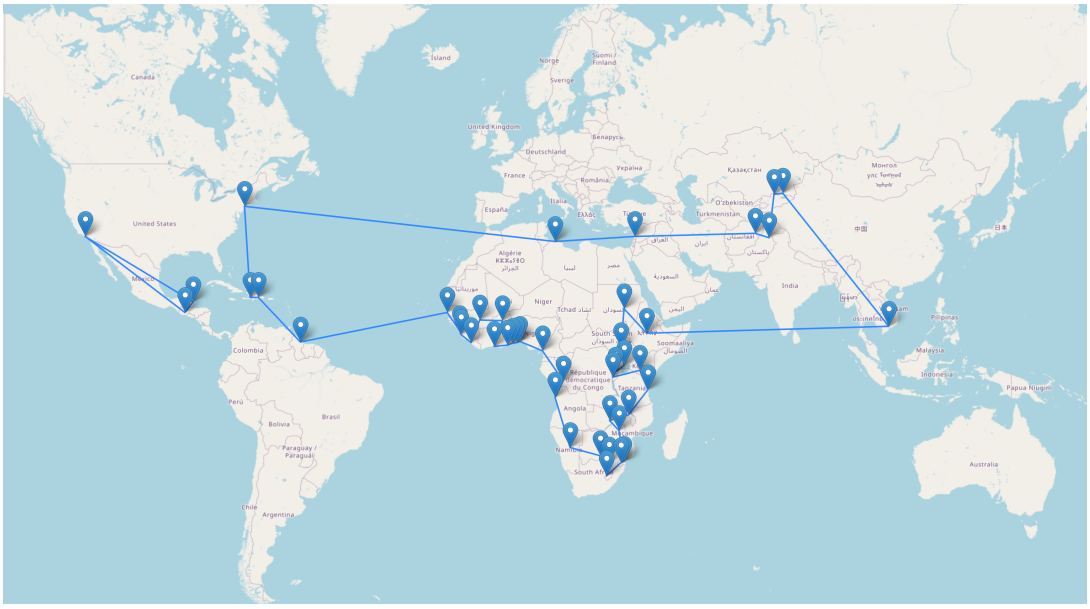

Figure 2: Splitting Shipment Question 2 - Case 1 Result Visual

4.2.2. Case 2

The result of optimizing splitting shipment (Q2) case 2 is shown in the Figure 3 below. The optimal objective value is **35,116.5 miles**. Although two aircrafts are allowed in this case, only one will be utilized from JFK. That is, there is an aircraft departing from JFK, following a large loop travel plan to cover all destinations efficiently, and one aircraft idle at LAX airport.

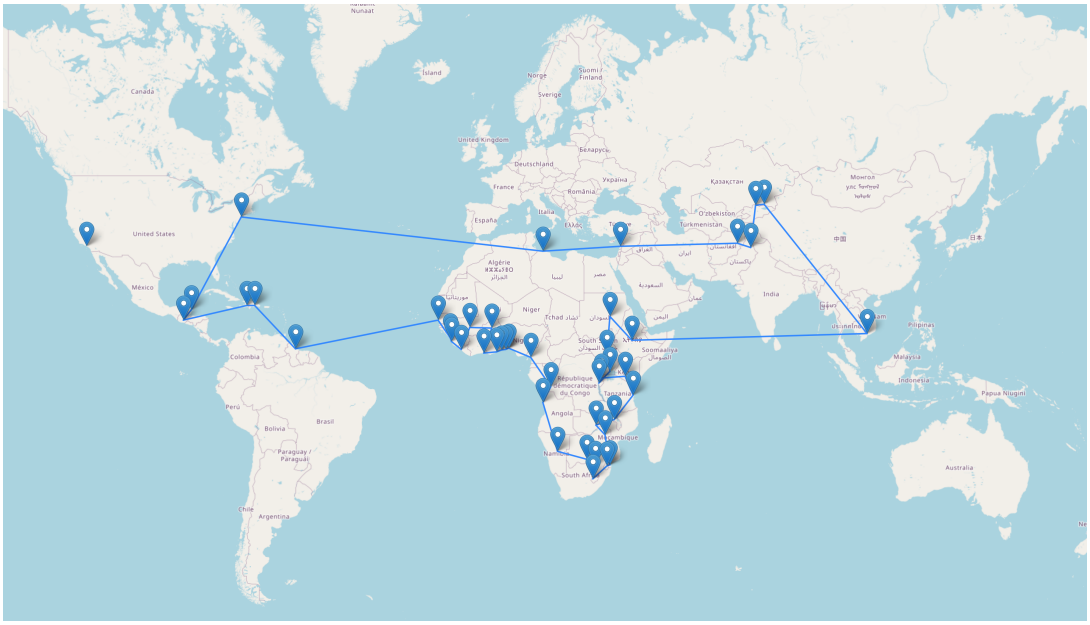

Figure 3: Splitting Shipment Question 2 - Case 2 Result Visual

4.2.3. Comparing 2 cases of Question 2

Comparing results from Case 1 and Case 2, the latter achieves a shorter total travel distance, having a smaller total distance of **2,844.3 miles**. Case 2 is more efficient in terms of travel distance and cost. Although case 1 offers more flexibility as two aircraft can operate independently, potentially reducing overall delivery times for destinations, in the specific setting of this project, Case 2 offers a more efficient solution, it is chosen as the solution for Question 2, and the subsequent sensitivity test will be based on Case 2.

4.3. Sensitivity Test

A sensitivity test is performed to test the 2 models under potential **measurement error of airport locations**. Modifying the longitude of Tan Son Nhat International Airport in Vietnam, changing it from (10.818797, 106.651856) to (10.818797, 180.000000), the adjustment places the airport near the International Date Line, significantly changing its geographical position. Re-run the models to test for impact the optimized travel routes and total distance under measurement error of airport locations.

The optimized results with modified data for optimizing shipment (Q1) is shown in

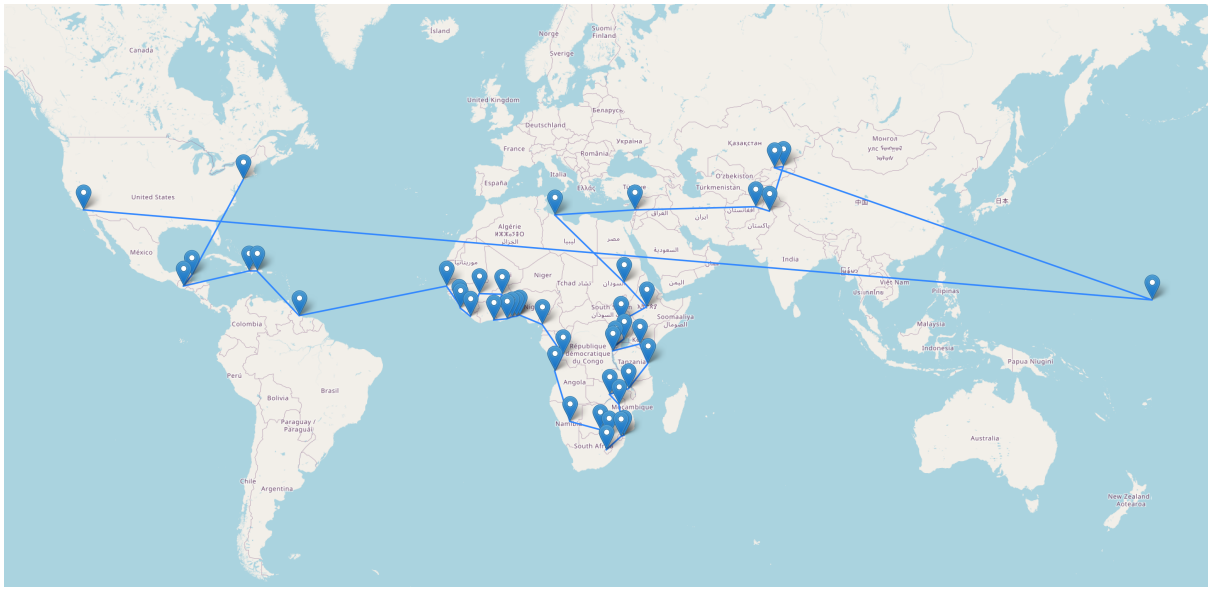

Figure 4: Sensitivity Test in Optimizing Shipment Scenario 1

the figure above. The optimized distance under misplaced airport is **35211.6 miles**, while if actually following this itinerary **(flying with Tan Son Nhat International Airport's location in Vietnam in real Earth)** the actual travel distance is **35665.4 miles**, indicating a moderate **269.9 miles** increase in the optimized travel distance compared to the original Q1 objective. The adjusted itinerary will include a route from the modified Tan Son Nhat International Airport to Los Angeles that spans the other side of the Earth, crossing the Pacific Ocean, while the other itineraries through the rest of the locations are unchanged. This change demonstrates that **minor singular geographical location measurement error will have some but moderate effect on the model's result of an optimized travel route and total distance traveled.**

The results with the modified data for splitting shipment Case 1 and Case 2 are shown in Figure 5 and Figure 6, respectively. In Case 1, two aircraft depart separately from LAX and JFK. The itinerary of the aircraft departing from LAX is altered to travel across the Pacific Ocean to reach Vietnam. With this modified itinerary, the optimized distance based on the erroneous longitude data is 38,762.26 miles, while the actual distance is 46,720.16 miles (Q2 - case 1 has 37,960.8 miles). In Case 2, the aircraft departs from LAX instead of JFK as in the original configuration. The optimized distance based on the erroneous longitude data is 35,229.07 miles, while the actual distance is 35,727.94 miles (Q2 - case 2 has 35,116.5 miles). These results show that the erroneous geographic data and measurement error could change optimized itinerary and worsen the resulting optimized travel distances.

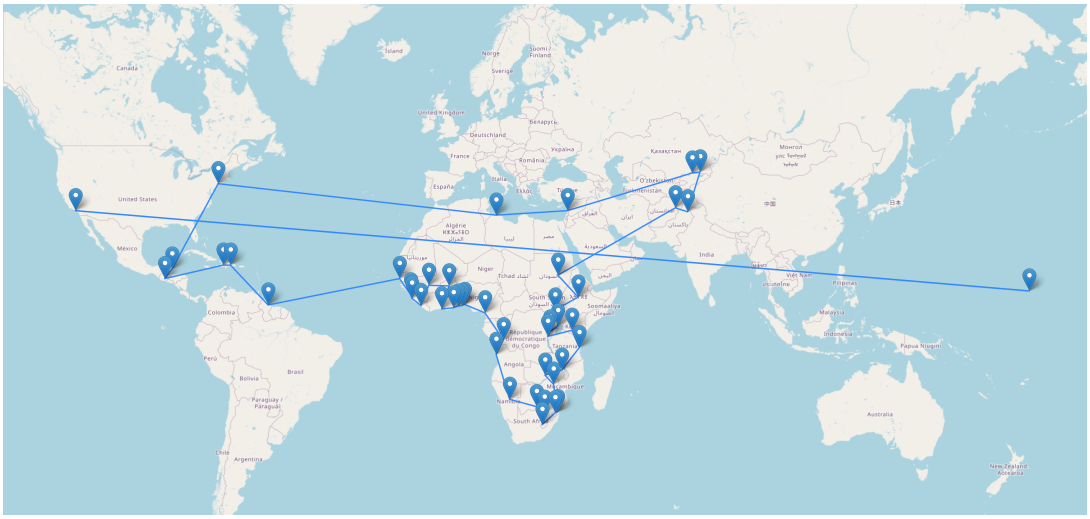

Figure 5: Sensitivity Test in Splitting Shipment Scenario 2, Case 1

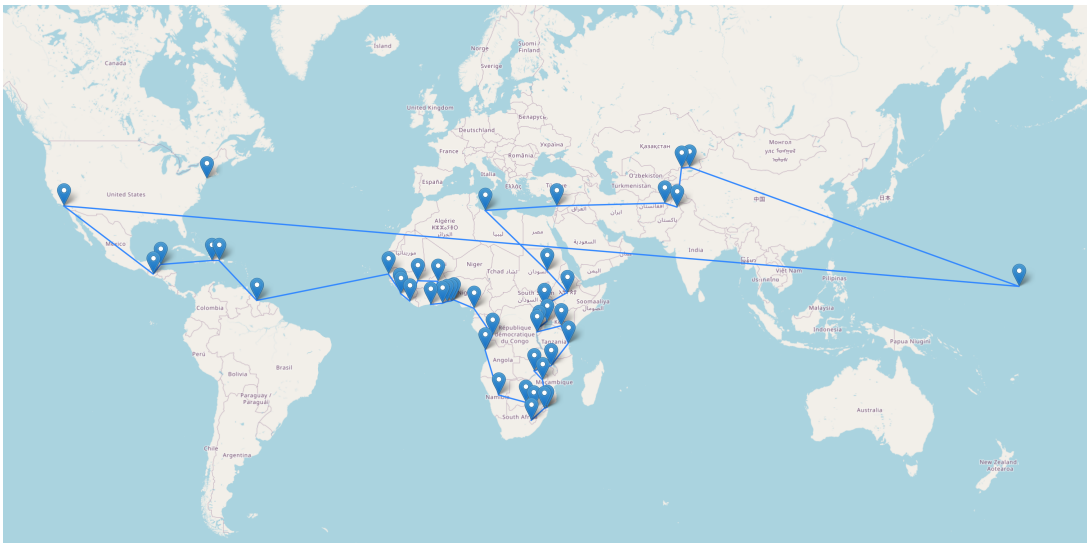

Figure 6: Sensitivity Test in Splitting Shipment Scenario 2, Case 2

5. Conclusion

This study explored optimizing global health commodity shipments by addressing the problem as variations of the Traveling Salesman Problem (TSP). Two scenarios were evaluated: a single-aircraft model departing from JFK and concluding at LAX, and a splitting shipment model with two aircraft departing from LAX and JFK. The single-aircraft optimizing shipment model provided a cost-effective and environment-friendly solution by minimizing total travel distance to 35,395.5 miles, reducing fuel consumption and associated CO2 emissions. Case 2 of the splitting shipment model demonstrated superior efficiency compared to Case 1, achieving a shorter total travel distance of 35,116.5 miles (a 2,844.3-mile advantage over Case 1). Sensitivity tests revealed that erroneous geographic data (measurement error in locations) can negatively impact the optimization process.

References

[1] Tolga Bektas. The multiple traveling salesman problem: an overview of formulations and solution procedures. *Omega*, 34(3):209–219, 2006.